

Imposing an energy balance about the heat source, one can write:

$$\dot{m} = \frac{Q_{in}}{\int_{T_o}^{T_i} C_p(T) dT} \quad (1)$$

Consider an infinitesimal element in the primary radiator. Applying energy conservation:

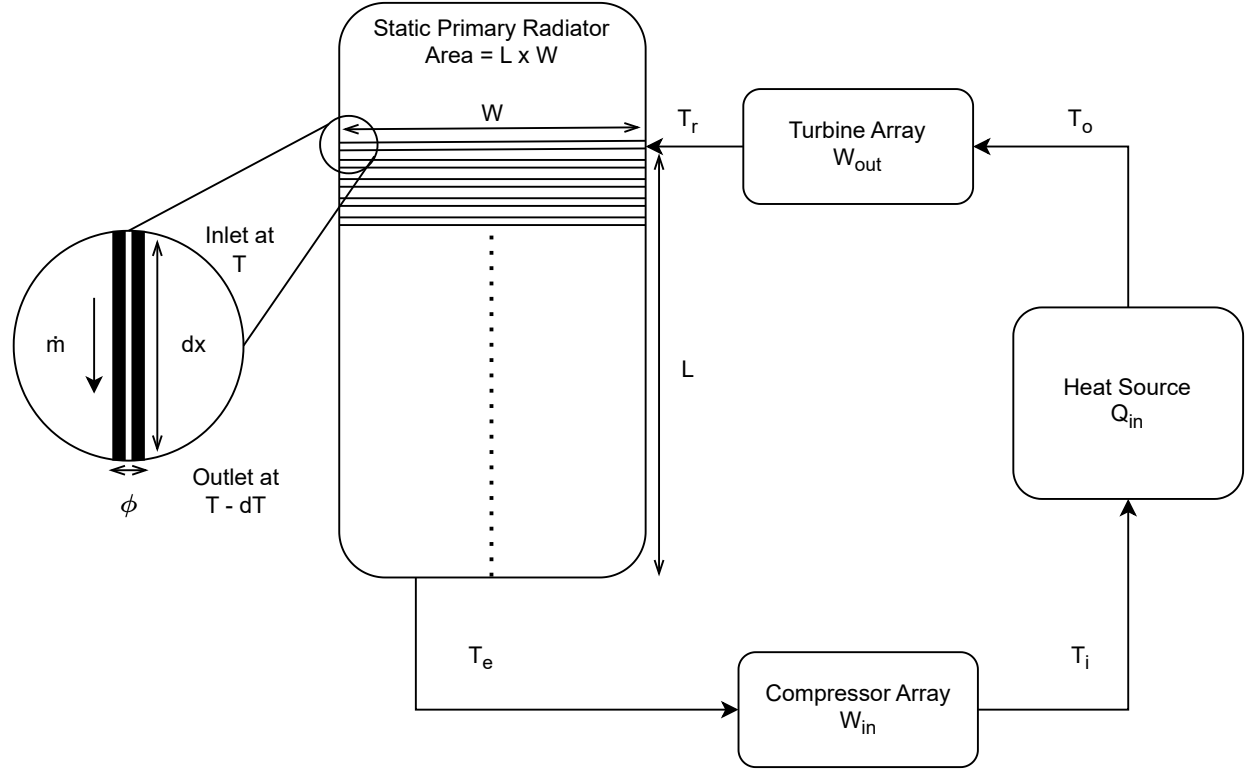
$$dQ = \dot{m} C_p(T) dT = \epsilon \sigma_b \phi (T^4 - T_a^4) dx \quad (2)$$

Where T_a is the ambient temperature and ϵ is the surface emissivity coefficient

The above equations in \dot{m} and T can be numerically solved if $C_p(T)$ is known. The maximum thermal to electrical conversion efficiency is then:

$$\eta_{th} = \eta_{gen} \frac{W_{out} - W_{in}}{Q_{in}} \quad (3)$$

Where η_{gen} is the generator's efficiency



For He-Xe cooled systems, $C_p = 2.5 \frac{R}{MM}$ and it is constant for $T > 400K$. The constant C_p value leads to simpler solutions for \dot{m} and T . Rearranging (2):

$$\int \frac{dT}{T^4 - T_a^4} = \frac{\epsilon \sigma_b \phi}{\dot{m} C_p} \int dx \quad (4)$$

However, $T_a \ll T$ in space, which simplifies the LHS. The limits for the LHS integration are from T_r to T_e . For the RHS, using an assumption that the radiator is a single long tube folded into N sections of W length and ϕ apparent diameter, one has $\phi \int dx = \phi N W = \phi \frac{L}{\phi} W = L W$. Putting that result and (1) in (4):

$$\left[\frac{1}{3T_e^3} - \frac{1}{3T_r^3} \right] \left[\frac{1}{T_o - T_i} \right] = \frac{\epsilon \sigma_b L W}{Q_{in}} \quad (5)$$

If the pressure ratio (PR) is isentropically defined as the ratio of the outlet to the inlet temperature raised to $\frac{\gamma}{\gamma-1}$, the pressure balance in the system is:

$$CPR = \frac{F_p}{TPR} \quad (6)$$

Where CPR is the compressor array pressure ratio, TPR is the turbine array pressure ratio and F_p is a pressure factor to account for pressure losses in the cycle from phenomena like friction and fluid expansion.

If one defines T_o , TPR and F_p , that results in:

$$T_r = T_o (TPR)^{\frac{\gamma-1}{\gamma}} \quad \& \quad T_i = T_e \left[\frac{F_p}{TPR} \right]^{\frac{\gamma-1}{\gamma}}$$

Substituting for T_r and T_i in (5) enables one to find T_e via numerical solution.

Finally, one can express the system's conversion efficiency using an energy balance: $\eta_{th} = \eta_{gen} \frac{T_o - T_r - T_i + T_e}{T_o - T_i}$. Substituting for T_r and T_i yields:

$$\eta_{th} = \eta_{gen} \frac{T_o \left[1 - (TPR)^{\frac{\gamma-1}{\gamma}} \right] - T_e \left[\left(\frac{F_p}{TPR} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{T_o - T_e \left(\frac{F_p}{TPR} \right)^{\frac{\gamma-1}{\gamma}}} \quad (7)$$