IB Mathematics Analysis and Approaches SL

Internal Assessment Mathematics Exploration

Mathematical Analysis of Modelled Sustainable Community

Pages: 19

1. Introduction

1.1 Exploration Purpose and Aim

As a child, I found it incredibly fascinating to see the unique architecture in the modern world. Whenever I stroll around downtown Toronto, I always look around to find uniquely shaped structures and not just the normal rectangular buildings that are so common. I also loved solving problems and creating solutions as a child. I always begged my parents to get more puzzles as I adore taking the time, separating the colours, putting the puzzle together, and most importantly persevering when the puzzle just never came together. These skills led me to discover computer science and architecture, both suiting my personality in many different ways. Through this I found Blender, a 3D modelling software used to create and model various buildings, architecture and animations. As an aspiring architect and computer scientist, I fell in love with the software, as it allowed me to be creative and whenever I ran into a problem, I always had to persevere to get through. With climate change becoming a prevailing issue, it is essential to implement sustainable communities to allow future generations to meet their needs and reduce the impact of global warming. The significance of climate change made me realize that my passion for architecture may also be relevant in developing sustainable communities amid environmental degradation. Thus, ***the aim of the exploration is to create a sustainable community using a 3D software called Blender and use mathematics to analyze various functions and their properties in the context of architecture.***

1.2 Terminology

Table 1. Definitions of Important Terminology

Term	Definition
Blender	A 3D platform that allows individuals to model and design various models, animations, 3D applications (Blender Foundation, 2019).
Mesh	A collection of vertices, edges, and faces that tell us the shape and the orientation of a 3D object (Walt, 2021).
Sustainable Development	The development that meets the demands of society now, without compromising the ability for future generations to meet their needs (International Institute for Sustainable Development, 2022).

2. Data Collection

2.1 Planning Process

Before I began modelling, I first decided to research how sustainability can be achieved in a community. This led me to find a few sustainability principles such as limiting levels of pollution, implementing sustainable energy to ensure resources are being sustained and allowing individuals to meet needs within walking distance (Low Carbon and Sustainable Communities | Stafford Borough Council, n.d.). Through these guiding rules, I attempted to create a plan for my community.

Furthermore, I also wanted to challenge myself to create new and innovative architecture through mathematical manipulation. Many uncommon buildings always use unique mathematical functions, deviating from the regular rectangular shape that we commonly see. For those reasons, I wanted to try to use as much of the mathematical knowledge I have when designing the community and specific structures. After careful consideration, I decided to choose a quadratic and a sinusoidal function since those are the two functions that were quite common in the architecture world. This led me to create the blueprint in *Figure 1*.

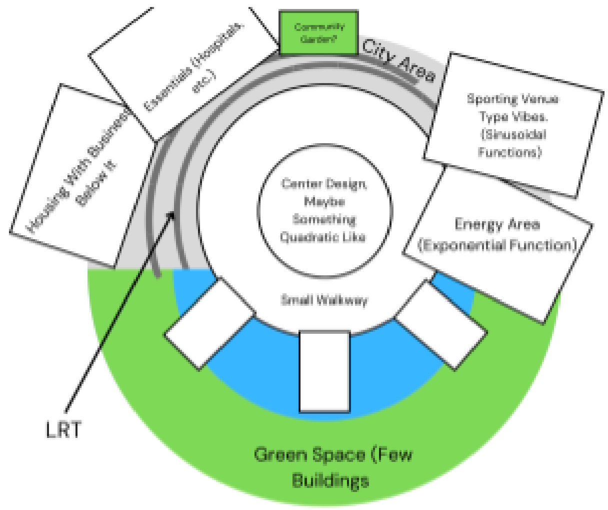

Figure 1. Blueprint Plan for Sustainable Community

2.2 Building Procedure

When I first started building the community, I was quite intimidated by Blender as I had never used it before. I had to watch numerous videos and tutorials in an attempt to learn the software. My first look into the software was difficult, there were so many tools and ways you could work with. However, as I progressed through building the structures, it started to become straightforward as I had the opportunity to apply the knowledge gained from the tutorials.

I started by building the main structures that I wanted to analyze and then moved on to building the residential areas with the commercial areas on the main floor. The progress at this point was shown in *Figure 2*. It is important to note that throughout the exploration, various pictures are generated through a feature in Blender, which allows you to render and create images of the model based on the perspective you choose. *Figures 2, 3 and 4* are examples of such rendered images. I then proceeded to create a green space and some background buildings to showcase the city's looks. I ended up with the following results as shown in *Figures 3 and 4*. I was quite proud of how it turned out given that it was challenging at first to figure out how to use the software. I also felt as though I had effectively met the sustainability criteria outlined previously and I created an accessible community that has all of society's necessities. However, I could have made it look more like a city by adding more buildings and having a variety of shapes in the larger structures. Furthermore, more thought could have gone into the engagement aspect of the community to ensure that communities have opportunities to play a role in the development process. Due to time constraints, both limitations were unattainable.

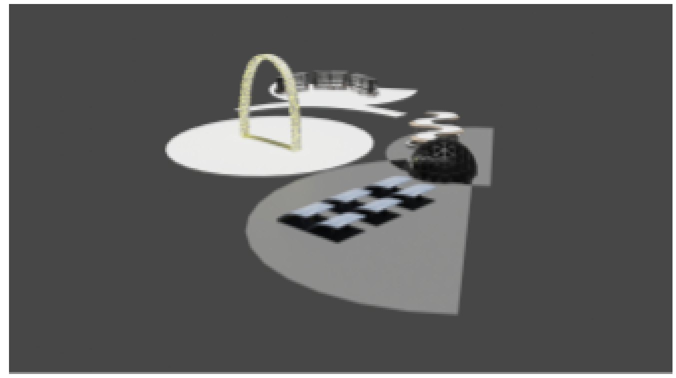

Figure 2. Architecture Progress Point

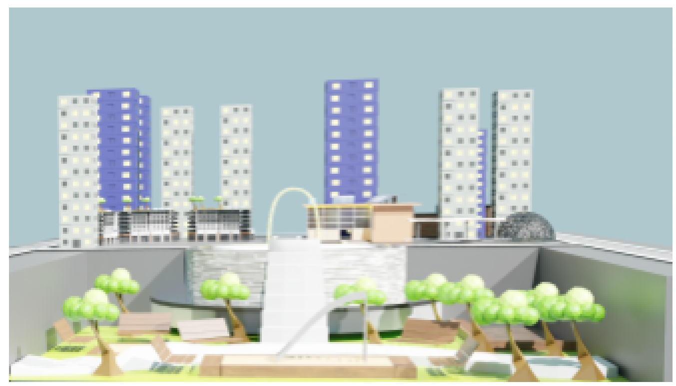

Figure 3. Final Build of the Community

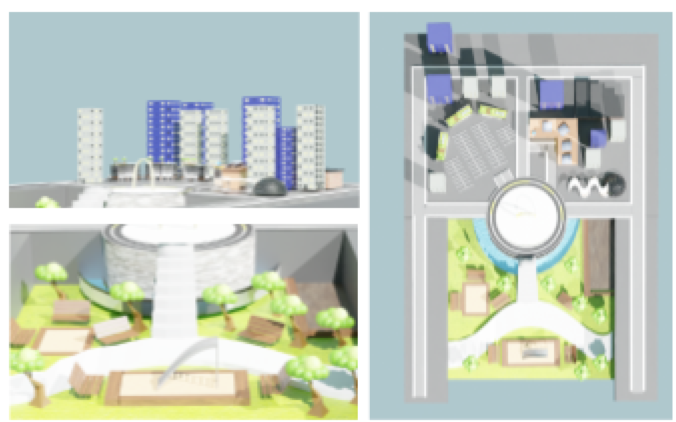

Figure 4. Different Views of the Final Build

3. Quadratic Arc Attraction

3.1 Description of The Architecture

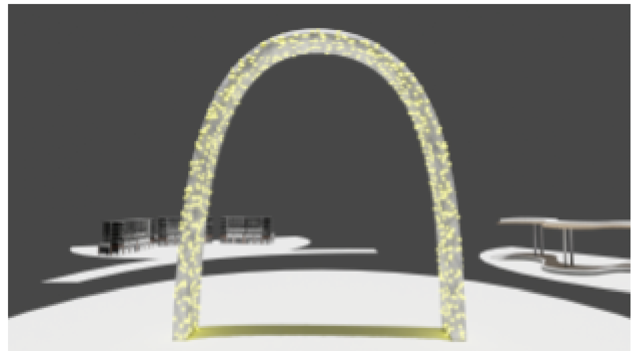
Figure 5. Close-Up Image of Quadratic Arc

When I first planned the community, I wanted to add a centrepiece/attraction that the community revolves around. Tourist attractions are what define communities; it is what tourists love to see. Furthermore, some attractions have a historical significance which adds to the identity of the community. This led me to create a quadratic arc, as shown in *Figure 5*. The dimensions of the arc were provided through Blender, as the software has a feature that gives you the dimensions of the world and the mesh. The arc was 1258 m in length and 1379 m in height. It is important to note that the scale of the architecture is quite unrealistic. If this were modelled in the real world, it would most likely be abnormally large and it would require significant amounts of material.

3.2 Determining An Equation For The Arc

Looking at the arc, it is evident that it seems to be a quadratic function, due to it being similar to a concave-down function with one local maximum point. It is important to note that when I was attempting to model the quadratic, I decided to do calculations by hand, using a few significant points. This was due to two reasons: the regression created by Desmos was not accurate to the model and in real-world perspectives, many architects use paper blueprints to model their designs and identify properties. Hence, for this exploration, I stuck to calculations without technology. To determine the equation for the arc, I first added the image to Desmos and scaled out to where 1 unit in Desmos was equivalent to 1 m in the mesh. I used the leftmost point at the base of the structure as the origin point and stretched it out until the rightmost point touched the x-axis at 1258. It is important to note that the leftmost and rightmost points have the same y-coordinate, and in the graph, they are the zeros. I then identified the y-coordinate of the vertex, which is the maximum point of the function. Since the height is the distance from the bottom of the structure to the top, the maximum y-coordinate is simply the height. Hence the y-coordinate of the vertex is y = 1379. Since the length of the function is 1258m, the arc starts at the origin, and both points are zeros, I determined the x-coordinate of the vertex by determining the equation of the axis, which is a symmetrical line that goes through the vertex, dividing the quadratic into two symmetrical parts.

$$equation\ of\ the\ axis\ =\ \frac{zero_{min} + zero_{max}}{2}$$

$$equation\ of\ the\ axis\ =\ \frac{0 + 1258}{2}$$

$$x\ =\ 629$$

Therefore the x-coordinate of the vertex is 629, hence the vertex is (629, 1379). From there, I decided to use the factored form of a quadratic equation, $y\ =\ a(x\ -\ r)(x\ -\ s)$, where *r* and *s* are the zeros. I substituted the zeros into r and s, creating the new equation y = ax(x - 1258). We are left with the unknown variable *a*, which is known as the vertical stretch, so I substituted the vertex into the equation to isolate for *a*. The calculation can be found below.

$$y\ =\ ax(x\ -\ 1258)$$

$$1379\ =\ a(629)(629\ -\ 1258)$$

$$1379\ =\ a(629)(629\ -\ 1258)$$

$$a\ \approx\ -\ 0.003485\ (4\ s.f)$$

Therefore, the equation that had been identified is y ≈ -0.003485x(x - 1258). I rounded the *a* value to 4 significant figures, as the stretch deals with larger numbers in the thousands, thus an additional decimal place can affect how the stretch affects the values. The graph is shown in *Figure 6.*

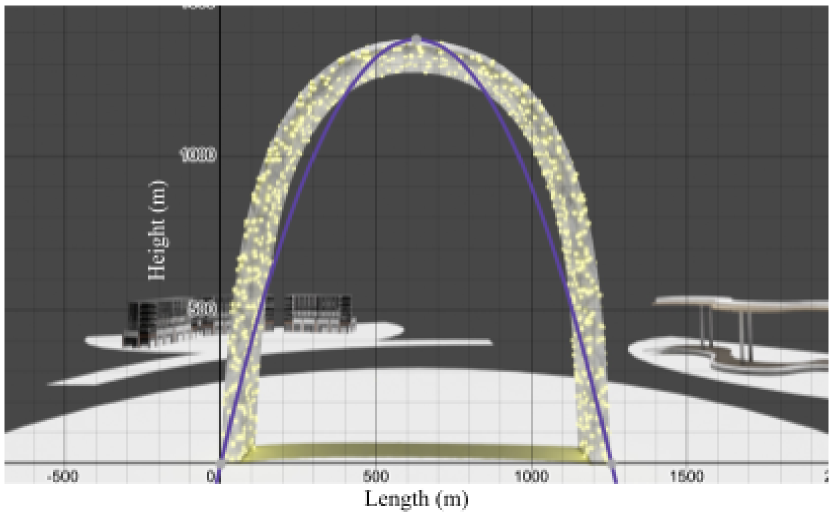

Figure 6. The Graph of the Function y ≈ -0.003485x(x - 1258)

When I looked at the graph, I noticed that it was cutting into various important portions of the arc and it was not an accurate model as I anticipated it would be. I started to play around with the function by changing the form of the quadratic from factored to vertex form, by completing the square. The vertex form of a quadratic is given by $y = a(x - h)^2 + k$, where *h* is the x-coordinate of the vertex and *k* is the y-coordinate of the vertex.

$$y \approx -0.003485x(x - 1258)$$

$$y \approx -0.003485(x^2 - 1258x + (\frac{1258}{2})^2 - (\frac{1258}{2})^2)$$

$$y \approx -0.003485(x - 629)^2 + 1379$$

I rounded the y-intercept coordinate to a whole number, due to the large scale that the quadratic is already on. It is also important to recognize that when in vertex form, it is basically the vertex that I had initially calculated, hence I decided to keep it the same as previously. From there, I decided to adjust the vertical translation of the curve, using a slider on Desmos. I noticed that the majority of the mesh was in the curve, when there is a vertical translation 500 units up, thus the *k* in vertex form of the quadratic increases by 500, as *k* also represents the vertical translation of a function. This creates the equation $y \approx -0.003485(x - 629)^2 + 1879$. The graph is shown in *Figure 7*.

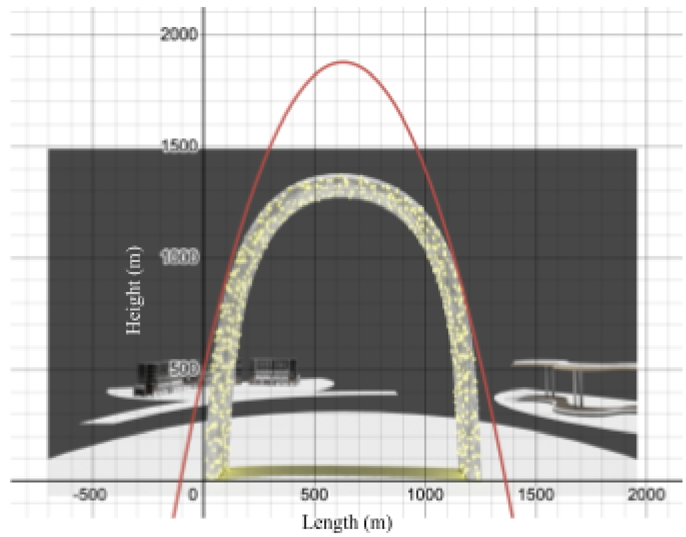

Figure 7. Graph of the Red Quadratic Function, $y \approx -0.003485(x - 629)^2 + 1879$

This seems as though it is a good fit, up until the gap between the arc and the red function at the top. The issue with this is that there will be large inaccuracies when calculating the area under the curve. Therefore, I decided to include another curve that follows the top half of the arc. Through doing this, the integration will be much more accurate and the calculated area will be closer to the actual value. I started with the original function that was determined previously by hand, which was: $y \approx -0.003485(x - 629)^2 + 1879$. I then realized that the curve was compressed too far, thus, I decreased the value of the vertical stretch to -0.001. This allows the function to cover the top portion of the arc. Therefore, the additional curve is $y \approx -0.001(x - 629)^2 + 1379$. The graph is shown in *Figure 8*. Lastly, to find the area of the material used in the arc, another curve has to be present under the arc. I first started by approximating the zeros of the inner portion of the function based on the graph which was x = 63 and x = 1195. These were also rounded to the nearest whole number since we are dealing with a large scale, therefore, the graph will not change significantly if more digits were added. Based on the zeros, I found the equation of the axis, by subtracting the minimum from the maximum and dividing it by 2. This gives the x-coordinate as x =

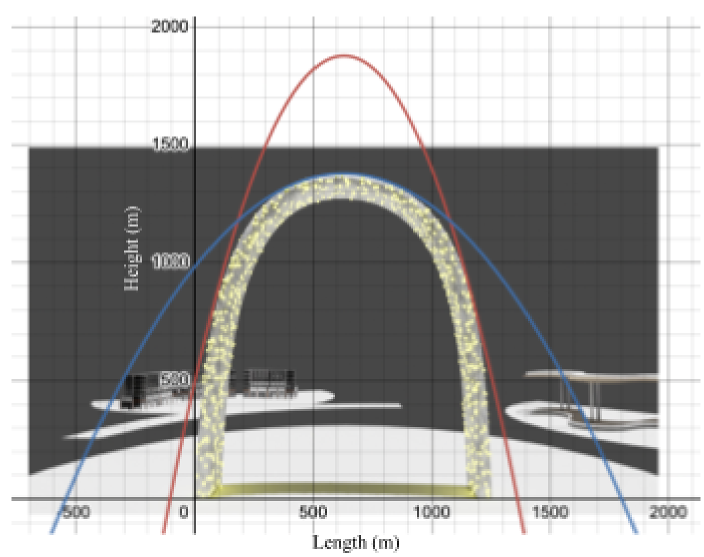

Figure 8. Graph of the Blue Quadratic Function, $y \approx -0.001(x - 629)^2 + 1379$

629. I then used Desmos to estimate the y-coordinate of the vertex of the inner portion of the arc, which is y = 1272. Therefore, the vertex is (629, 1272). I then did the same steps that I did for the initial quadratic function, $y \approx -0.003485(x - 629)^2 + 1379$ where I would first put it into factored form, substitute the vertex and find the value. The equation was identified as $y \approx -0.003921(x - 629)^2 + 1272$ (see **Appendix A**). The reason for the number of significant figures is similar to previous reasoning, in that stretches may fluctuate based on more digits being added and translations have less significance, hence rounded to the whole number. However, I noticed that the quadratic equation, once again, wasn't covering the edge of the arc and it was not an accurate representation of the inner portion of the arc. Therefore, I decided to play around with the vertical stretch, as it seemed a lot more compressed to see a better model. In the end, I determined that -0.003 would be the better stretch. *Figure 9* shows the graph for the function. Even though the green curve as shown in *Figure 9* has some area of the arc that is being cut at the ends, this seems as if it is being recovered due to the gaps between the green curve and the inner edge of the arc. Even with these cuts, the gradient of the arc as well as the area of the material used in the arc can be calculated effectively. Architects may need to consider functions, as they are fundamental to determine other useful features such as the gradient and the area of the material that is required. This skill seems to be something that is much needed in the architecture field.

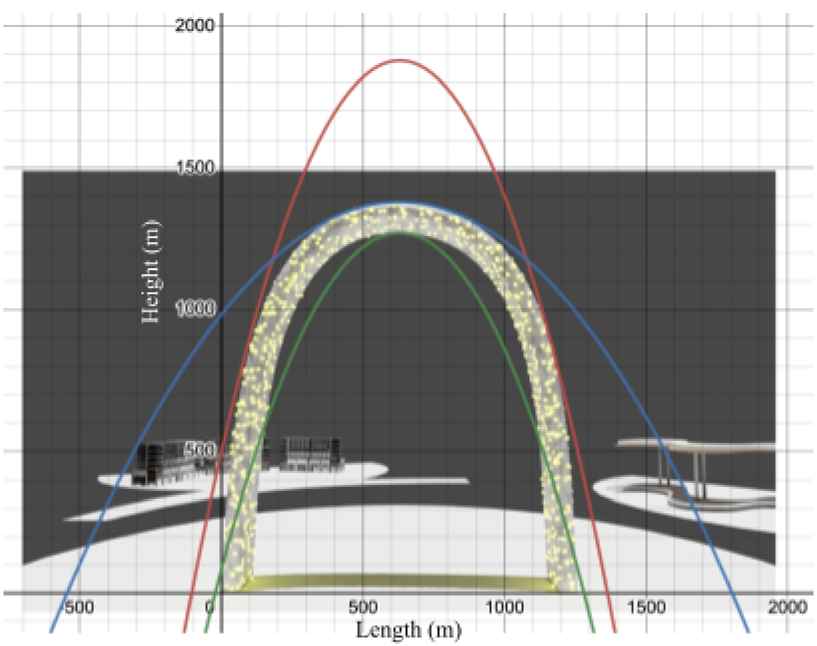

Figure 9. Graph of the Green Quadratic Function, $y \approx -0.003(x - 629)^2 + 1272$

3.3 Gradients of the Arc

The gradient of the arc can be identified based on the curve that has a similar gradient and follows the trend of the arc. Since we are looking at the outer edge of the arc and there are two curves, the red and blue, that tell us the gradient of different sections of the arc, we can differentiate both curves and specify the domain that applies to each derivative to ensure that the most accurate gradient of the arc is calculated. I identified the domain of each function that when differentiated provides the most accurate gradient. As a result of the huge scale, the domain was rounded to the nearest whole number. The red curve's ($y = -0.003485(x - 629)^2 + 1879$) derivative should only be used when x lies in the domain $47 \leq x \leq 180$ and $1077 \leq x \leq 1225$. The blue curve's ($y = -0.001(x - 629)^2 + 1379$), derivative should only be used when x lies in the domain $47 \leq x \leq 1077$. For domains between $0 \leq x \leq 47$ and $1225 \leq x \leq 1258$, it almost seems as though the tangent to the arc is like a vertical line, thus the gradient is undefined as there is an infinite change in y and no change in x. I then began to differentiate the red curve.

<u>Red Curve</u>

$$y \approx -0.003485(x - 629)^2 + 1879$$

$$\frac{dy}{dx} \approx -0.003485(2)(x - 629)^1(1)$$

$$\frac{dy}{dx} \approx -0.00697(x - 629), \{47 \leq x \leq 180, 1077 \leq x \leq 1225\}$$

I then calculated the gradient of the blue curve, which was given to be $\frac{dy}{dx} \approx -0.002(x - 629), \{47 \leq x \leq 1077\}$ (see **Appendix B**). Finding the gradient is quite useful for architects because certain cuts must be made with accurate precision and slope. If architects do not consider gradients in their cuts, they might potentially ruin the shape of the structure. Thus, architects must consider gradient as an important factor. It is important to note that the gradients are estimated, as the curves are not precisely in line with the arc, there are some gaps and spaces, and the arc is not a perfect quadratic.

3.3 Area of the Material Used in the Arc

I further wanted to find the area of the material that is visible. Architects may want to use this information to find out how much material is required for the structure. I first found the domain of the curves that accurately represent the area under the top of the arc. This was similar to the domains that I identified for the differentiation portion, as it was based on the curve that followed the trend of the arc. For the red curve, this was found as $0 \leq x \leq 180$ or $1077 \leq x \leq 1258$ and the blue as $180 \leq x \leq 1077$. Through definite integration, we can figure out the area under the curve.

$$A_1 \approx \int_0^{180} (-0.003485(x - 629)^2 + 1879)\, dx$$

$$A_1 \approx -0.003485\left(\frac{(x-629)^3}{3}\right) + \frac{1879x^1}{1}\Big|_0^{180}$$

$$A_1 \approx -0.001162(x - 629)^3 + 1879x\Big|_0^{180}$$

$$A_1 \approx (-0.001162(180 - 629)^3 + 1879(180)) - (-0.001162(0 - 629)^3)$$

$$A_1 \approx 154282m^2$$

Therefore the area below the red curve and between the ranges of $0 \leq x \leq 180$, is approximately 154282m^2. I then proceeded to find the area below the curve for the ranges $180 \leq x \leq 1077$, using the equation $y = -0.001(x - 629)^2 + 1379$, referred to as the blue curve. This was done using a similar calculation above, which turned out to be 1176824m^2 (see **Appendix C**). Furthermore, I then decided to find the area under the red curve once again, but with the range of $1077 \leq x \leq 1258$, which was identified as approximately 155435m^2, similar to the calculation above (see **Appendix C**). Now that I found the top portion of the curve, I then proceeded to find the area below the arc in an attempt to find the area of the material used. The calculations done to identify the area below the green curve under the arc are done below.

$$A_4 \approx \int_0^{1280} (-0.003(x - 629)^2 + 1272)\, dx$$

$$A_4 \approx \int_0^{1280} -0.003(\frac{(x-629)^3}{3}) + \frac{1272x^1}{1}|_0^{1280}$$

$$A_4 \approx -0.001(x - 629)^3 + 1272x\,|_0^{1280}$$

$$A_4 \approx (-0.001(1280 - 629)^3 + 1272(1280)) - (-0.001(0 - 629)^3 + 1272(0))$$

$$A_4 \approx 1103407m^2$$

Figure 10 shows the different function areas below each of the individual curves. It is important to note that even though the green curve cuts out portions of the arc, the gaps between the green curve and the bottom edge of the curve account for the loss. I then added the areas above the arc and subtracted the area below the arc to find the area of the surface area. The formula is given by Total Area $= (A_1 + A_2 + A_3) - A_4$, where $A_1 \approx$154282m^2, $A_2 \approx$ 1176824m^2, $A_3 \approx$ 155435m^2, $A_4 \approx$ 1103407m^2.

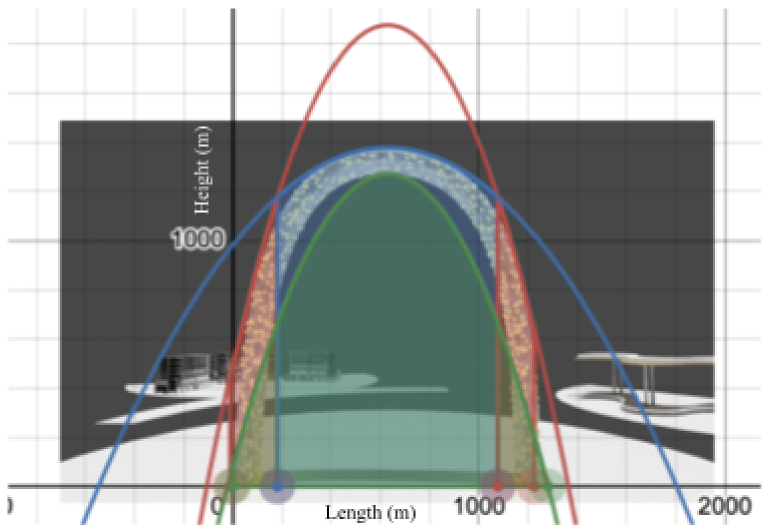

Figure 10. Graphical Representation of Integration Calculations

$$\text{Total Area } = (A_1 + A_2 + A_3) - A_4$$

$$\text{Total Area } \approx (154282 + 1176824 + 155435) - 1103407$$

$$\text{Total Area} \approx 383134m^2$$

Therefore, the area of the arc is approximately 383134m^2. All of the areas were also rounded to the nearest whole number due to us dealing with a large scale, hence I decided not to include decimal points, as it does not change the area.

3.4 Reflection on Quadratic Arc

The calculations and mathematics analyzed were accurate to identify the properties of the arc. However, it is important to note that the functions modelled are not a fully accurate representation of the trend of the arc. It is also important to note that the model assumes that the spaces that are lost are recuperated through the gaps between the function and the arc. Hence, the calculation is an approximation. As far as the effectiveness of the arc in the sustainable community, I believe since the community has something that defines it, it makes an effective principle in the sustainable community.

4. Sinusoidal Recreational Center

4.1 Description of the Architecture

Figure 11. Close-Up View of Sinusoidal Recreational Center

In a sustainable community, it is important to remember that individuals may need to find a place to be physically active and find some sort of entertainment. With this in mind, I decided to create a sine curve to demonstrate the walkway to a recreation center, which is the dome to the right. This is in an attempt to allow individuals to gain easier access to necessities in the community and it further promotes healthy living and balance in the community. The image of the pathway to the recreation center is shown in *Figure 11*.

4.2 Determining The Equation of the Pathway

For this section, I decided to only look at the white portion. When first putting the image of the function, I decided to use the dimensions of the curve that was given in Blender and stretch it out accordingly. The length of the curve was 1524m and the height of the curve was 680m. Once I graphed the function, I identified that there were two functions, one at the top edge of the structure and one at the bottom edge of the structure. Since the structure seemed continuous and there were multiple points where the maximum and minimum exist, I decided to model both edges using a sine function. I first began with modelling a function for the top edge of the structure by determining the maximum y-coordinate of the function, which was y = 679 and the minimum y-coordinate of the function, which was when y = 283. To determine the equation of the axis, which is the line at the center of the function, which is also the shift in the y-axis of the function, I decided to find the average between the maximum and minimum.

$$x = \frac{max + min}{2}$$

$$x = \frac{679 + 283}{2}$$

$$x = 481$$

Hence, the vertical shift and equation of the axis is x = 481. I then identified the amplitude by finding the difference between the maximum point and the equation of the axis, which is 679 - 481. This was identified as x = 198, hence the vertical stretch is 198, as the amplitude is the same as the vertical stretch. Then, I identified the period of the function, which is the length of one cycle, by finding the difference between the two different x-coordinates that are at the same maximum y-value. The two x-coordinates of the maximum points are 240 and 950. It is important to note that there are multiple maximums as the function is continuous. The difference between the coordinates is 710, hence the period is 710. To identify the horizontal stretch, I divided the period by 2π, thus, the horizontal stretch is $\frac{2\pi}{710}$ which can be reduced into $\frac{\pi}{355}$. Lastly, to identify the horizontal shift of the function, I used a slider on Desmos to determine the most fitting shift, which was 60. Using these properties, I identified the equation that can be used to describe the top edge using the form $y = a \sin(k(x - c)) + d$, where *a* is the amplitude or vertical stretch, *k* is horizontal strength or $\frac{2\pi}{period}$, **c** is the horizontal shift, and *d* is the vertical shift or also the equation of the axis. An equation was then identified below as

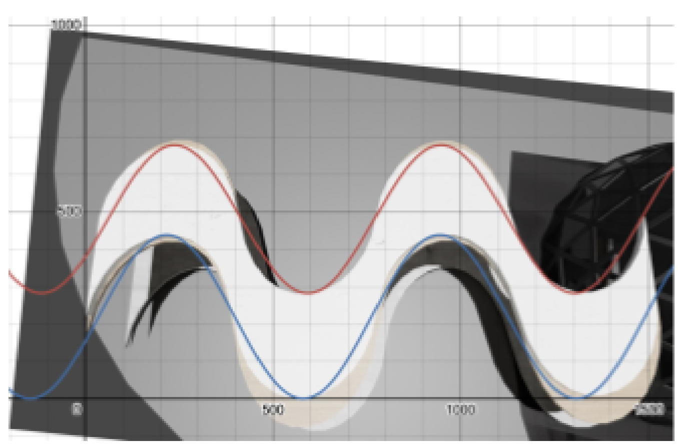

Figure 12. Model Functions for Recreational Center

$y = 198\, sin\, (\frac{π}{355}(x - 60)) + 481$ .A similar procedure was done to identify the sine curve of the bottom edge, which was $y = 219\, sin\, (\frac{π}{364}(x - 34)) + 219$ (see **Appendix D**). The graph of the architecture is given in *Figure 12*, where $y = 198\, sin\, (\frac{π}{355}(x - 60)) + 481$ is the red curve and $y = 219\, sin\, (\frac{π}{364}(x - 34)) + 219$ is the blue curve. It is important to note that the functions determined are not completely accurate to the overall trend in the structure. Some of the curve is cut off, but I felt that some of the gaps between the curve and the mesh that were included accounts for the losses.

4.3 Gradient of the Curve

I first started off by calculating the gradient of the top curve. Below is the calculation that I completed to identify the equation that can be used to find the gradient of the curve.

$$y = 198\, sin\, (\frac{π}{355}(x - 60)) + 481$$

$$y = 198\, sin\, (\frac{πx - 60π}{355}) + 481$$

$$\frac{dy}{dx} = 198\, (\frac{π}{355}) cos\, (\frac{πx - 60π}{355}) + 481(0)$$

$$\frac{dy}{dx} = 198\, (\frac{π}{355}) cos\, (\frac{πx - 60π}{355}) + 481(0)$$

$$\frac{dy}{dx} = \frac{198π}{355} cos\, (\frac{π(x - 60)}{355})$$

I did a similar calculation to identify the gradient of the bottom curve, which was given as the equation of $\frac{dy}{dx} = \frac{219π}{364} cos\, (\frac{π(x - 34)}{364})$ (see **Appendix E)**. This gradient ensures that architects know the specific cuts that need to be done for the material to be accurately shaped, thus differentiation may come in handy for architects. It is also important to mention that the gradient may not be accurate, as the function seems to be quite steeper at points compared to the actual mesh, which seems to be quite flat. There was no rounding involved as exact numbers were used.

4.4 Area of the Material Used in the Curve

As mentioned previously, since engineers may need to identify the amount of area required for the structure to be made, integration must be used to figure out the areas. This can be used by subtracting the area of the bottom curve from the area beneath the top curve. This will be a definite integral, as there is a limit on the length of the structure. In this case, the length is 1524m, hence the boundaries of the integral will be between 0 and 1524. The calculations done to find the area under the function of the top edge are shown below.

$$A_1 = \int_0^{1524} (198\,sin\,(\frac{\pi}{355}(x - 60)) + 481)\,dx$$

$$A_1 = -198\,(\frac{1}{\frac{\pi}{710}})\,cos\,(\frac{\pi}{355}(x - 60)) + \frac{481x}{1}\,\Big|_0^{1524}$$

$$A_1 = -198\,(\frac{1}{\frac{\pi}{355}})\,cos\,(\frac{\pi}{355}(x - 60)) + \frac{481x}{1}\,\Big|_0^{1524}$$

$$A_1 = -\frac{70290}{\pi}cos\,(\frac{\pi}{355}(x - 60)) + 481x\,\Big|_0^{1524}$$

$$A_1 = (-\frac{70290}{\pi}cos\,(\frac{\pi}{355}(1524 - 60)) + 481(1524)) - (-\frac{70290}{\pi}cos\,(\frac{\pi}{355}(0 - 60)) + 481(0))$$

$$A_1 \approx 633614\,m^2$$

A similar process was used to find the area below the function of the bottom edge with the area being approximately 333756m^2 (see **Appendix F)**. I then subtracted the two areas which gives me an approximate estimate of what the area might be. A graphical representation of the integration is shown in *Figure 13*. Examining the image, we notice that the blue area can be subtracted from the red area to identify the area of the white section.

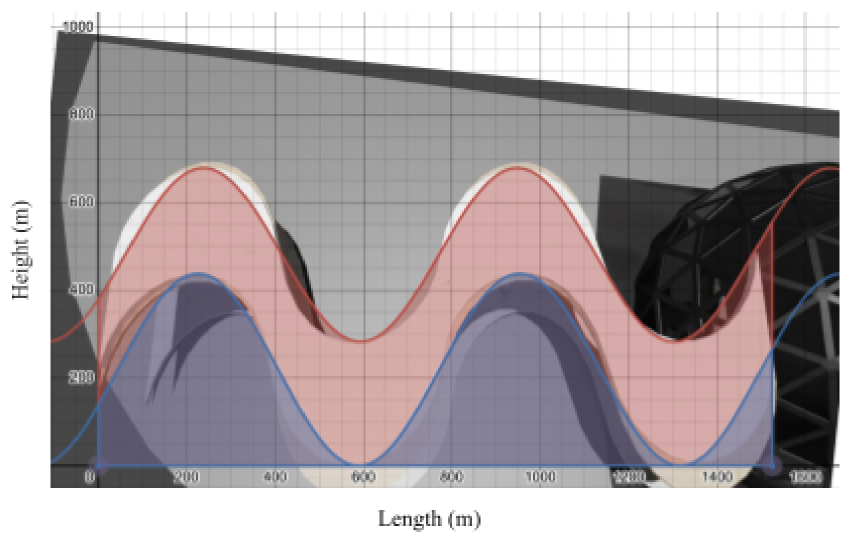

Figure 13. Graphical Representation of Integration Calculations

Below is the calculation for the subtraction process. Note that $A_1 \approx 633614m^2$ and $A_2 \approx 333756m^2$

$$\text{Area} = A_1 - A_2$$

$$\text{Area} \approx 633614 - 333756$$

$$\text{Area} \approx 299858\text{m}^2$$

Therefore, the area of the white portion of the recreational facility is approximately 299858m^2. All of the areas were rounded to the nearest whole number, since we are working with large scales, similar to previous explanations.

4.5 Reflections on Sinusoidal Recreational Center

Similar to the quadratic arc, it is important to recognize that some areas are cut off, hence some areas may be lost. Moreover, the functions do represent an approximate area but do not follow the trend of the functions as well. When modelling the function, the idea I had was to create functions that prioritize the area under the curve. The gradient of the sine function is not accurate, as there are different rates of change between the architecture and the model, but the area is a good approximation since some of the areas that are cut off in the curve are brought back due to the gaps between the curve and the actual architecture itself.

5. Conclusion

5.1 Justification of Models

The models of the architecture accurately demonstrate the area of the material being used. Even though each architecture cannot be perfectly modelled using a singular function, the area of the material was an accurate approximation. This was because logical assumptions were made in the models that determined whether it was a good fit. One major assumption was the fact that areas that were cut off were brought back by gaps between the curve and the mesh. Furthermore, the general function that was used, in this case, a quadratic for the arc and the sine function for the second curved structure, was quite accurate for the overall model of the architecture. However, it is important to note that the first model used demonstrates a more accurate model than the second model. Initially, I decided to use the vertex and factored form to create the equation, however, I felt that it was not as accurate as I would have liked it to be. I then decided to create a few functions to ensure that the area and the gradients were accurate and similar to the overall shape of the architecture. The gradients were heavily justified and quite accurate as they followed the trend of the arc. This was not the case for the second model which uses the sine function, as the gradients seem as though they were not as accurate to the overall trend of the architecture. Hence the first model is much stronger than the second model.

5.2 Importance of Mathematics in Relation To Architecture

Throughout the exploration, I noticed that there is a common trend between mathematics and the value it has towards architecture. Each structure has some sort of mathematical properties which allow architects to be precise when they are designing and building. Without mathematics, architects and engineers may struggle with creating an accurate real-life structure that resembles the initial design. This is demonstrated through the immense mathematics that was done, such as modelling a function to find the area of the material used through integration and the precision of the cuts needed to be made through differentiation. All these mathematical concepts plus many more ensure that architects have the fundamental knowledge and understanding to plan and create effective buildings that can be brought to real life. It is also important to note that it is highly unlikely that the functions that you see in mathematics will perfectly apply to a building. Each building is unique, and the probability that the building will exactly follow the trend of the function is highly unlikely. As seen in this exploration, we notice that there is not a perfect function that will model the structure. But, imperfections are what give the uniqueness of a structure.

Moreover, the mathematics that has been done here may not apply to the real world to a certain extent. With technology becoming a significant part of our everyday lives, calculations can be done simply with the press of a button; there is no need to do these extensive calculations, hence the accuracy of what an architect would do may differ. As shown through these reflections, my aim was achieved, as I was able to connect the value that mathematics has to the context of architecture as well as the feasibility of such math being used in real-world situations.

5.3 Limitations and Weaknesses

There are a variety of limitations and weaknesses that can be identified in this exploration. First of all, when we are calculating the area of the material, it is important to note that there may be some depth involved in the material itself. The exploration assumes that all of the images are 2D, which may not be the case in the real world. Furthermore, when the photo is taken, different points of view may altercate the calculations. For example, when the image of the arc was taken, I tried to make sure that it was at the front and not slanted. This was in an attempt to make sure that there were no accuracy issues. If the image were taken on the side, there could have been a different area calculated. In terms of the application of the community to real-world standards, it is important to note that the scale of the community was significantly larger than what you would normally see. For example, the quadratic arc is approximately 1 km in length, which is not feasible in the real world. Hence, that provides some inaccuracies if this were compared to real-life situations. Furthermore, the overall scale of a mesh in relation to other meshes is also quite unrealistic. From the bird's eye view, it almost seems as though the length of the tree is similar to the length of a building which is unreasonable. Hence, further consideration could have been made to ensure proportionality among other buildings is met.

5.4 Further Exploration

As emphasized through the exploration, mathematics has a significant impact on the architecture of the world. In some instances, mathematics can also have some impact in terms of historical significance, as different functions and shapes in buildings may have more value and meaning behind them. For that reason, as a further exploration, I want to try to see how mathematical properties in architecture relate to historical events and their significance. Furthermore, as discussed in the limitations, there was a lack of consideration in terms of the z-axis, which is a depth axis in the context of architecture. It would be interesting to see how depth plays a role in the calculation of the area of the building. To expand on the existing topic, looking at a 3D perspective rather than a 2D perspective would be beneficial as we would come closer to answering the question of how engineers use mathematics in real-world scenarios. The exploration can also encompass surface area and volume, which are 3D calculations, and can be useful in the architecture process.

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Appendix A

Determining the Quadratic Equation For The Inner Portion of the Arc

$$y = a(x - 63)(x - 1195)$$

$$1272 = a(629 - 63)(629 - 1195)$$

$$1272 = a(629 - 63)(629 - 1195)$$

$$a \approx -0.003929\ (4\ s.f)$$

$$y \approx -0.003929(x - 63)(x - 1195)$$

$$y \approx -0.003929(x^2 - 1257x + 70490)$$

$$y \approx -0.003929x^2 + 4.939 - 276.95$$

$$y \approx -0.003929(x^2 - 1257x) - 276.95$$

$$y \approx -0.003921(x^2 - 1257x + (\frac{1257}{2})^2 - (\frac{1257}{2})^2) - 276.95$$

$$y \approx -0.003921(x^2 - 1257x + 395012.25) + 1548.84 - 276.95$$

$$y \approx -0.003921(x - 629)^2 + 1272$$

Appendix B

Differentiation Calculations For The Blue Curve for Quadratic Arc

$$y \approx -0.001(x - 629)^2 + 1379$$

$$\frac{dy}{dx} \approx -0.001(2)(x - 629)(1)$$

$$\frac{dy}{dx} \approx -0.002(x - 629),\ \{47 \leq x \leq 1077\}$$

Appendix C

Integration Calculations For Other 3 Domains and Curves for Quadratic Arc

Calculation Description	Calculation
Definite Integration of y = -0.001(x - 629)^2 + 1379 (blue curve) when the bounds are 180 and 1077.	$A_2 = \int_{180}^{1077} (-0.001(x - 629)^2 + 1379)\,dx$ $A_2 = -0.001(\frac{(x-629)^3}{3}) + \frac{1379x^1}{1} \|_{180}^{1077}$ $A_2 = -0.0003333(x - 629)^3 + 1379x \|_{180}^{1077}$ $A_2 = (-0.0003333(1077 - 629)^3 + 1379(1077)) - (-0.0003333(180 - 629)^3 + 1379(180))$ $A_2 = 1176824m^2$
Definite Integration of y = -0.003485(x - 629)^2 + 1879 (red curve) when the bounds are 1077 and 1258.	$A_2 = \int_{180}^{1077} (-0.001(x - 629)^2 + 1379)\,dx$ $A_2 = -0.001(\frac{(x-629)^3}{3}) + \frac{1379x^1}{1} \|_{180}^{1077}$ $A_2 = -0.0003333(x - 629)^3 + 1379x \|_{180}^{1077}$ $A_2 = (-0.0003333(1077 - 629)^3 + 1379(1077)) - (-0.0003333(180 - 629)^3 + 1379(180))$ $A_2 = 1176824m^2$

Appendix D

Determining the Equation of the Bottom Edge of the Structure

Maximum Point: (232, 438) and (960, 438)
Minimum Point: (570, 0)

Amplitude = $\frac{0+438}{2} = 219$

Equation Of The Axis = $\frac{0+438}{2}$ = 219

Period = 960 - 232 = 728

Horizontal Stretch = $\frac{2\pi}{728} = \frac{\pi}{364}$

<u>Equation</u>: $y = 219\,sin\,(\frac{\pi}{364}(x - 34)) + 219$

Note that the horizontal shift was calculated using a slider in Desmos.

Appendix E

Determining the Derivative of the Bottom Edge of the Structure

$$y = 219\,sin\,(\frac{\pi}{364}(x - 34)) + 219$$
$$y = 219\,sin\,(\frac{\pi x - 34\pi}{364}) + 219$$

$$\frac{dy}{dx} = 219\,(\frac{\pi}{364})cos\,(\frac{\pi x - 34\pi}{364}) + 219(0)$$

$$\frac{dy}{dx} = 219\,(\frac{\pi}{364})cos\,(\frac{\pi x - 34\pi}{364}) + 219(0)$$

$$\frac{dy}{dx} = \frac{219\pi}{364}cos\,(\frac{\pi x - 34\pi}{364})$$

$$\frac{dy}{dx} = \frac{219\pi}{364}cos\,(\frac{\pi(x - 34)}{364})$$

Appendix F

Determining the Definite Integral of the Bottom Edge of the Structure With Bounds 0 and 1524

$$A_2 = \int_0^{1524} 219\,sin\,(\frac{\pi}{364}(x - 34)) + 219$$
$$A_2 = -219\,(\frac{1}{\frac{\pi}{364}})\,cos\,(\frac{\pi}{364}(x - 34)) + \frac{219x}{1}\Big|_0^{1524}$$

$$A_2 = -\frac{79716}{\pi}\,cos\,(\frac{\pi}{364}(x - 34)) + 219x\Big|_0^{1524}$$

$$A_2 = -\frac{79716}{\pi}\,cos\,(\frac{\pi}{364}(x - 34)) + 219x$$

$$A_2 = (-\frac{79716}{\pi}\,cos\,(\frac{\pi}{364}(1524 - 34)) + 219(1524) - (-\frac{79716}{\pi}\,cos\,(\frac{\pi}{364}(0 - 34)) + 219(0))$$

$$A_2 = 333756m^2$$