MSBD 5011

Assignment - 1

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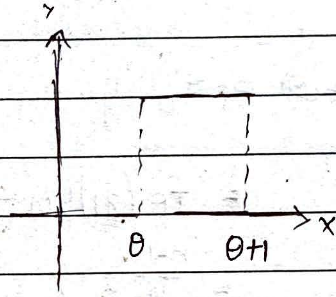
(1) $x_1, \dots, x_n \overset{iid}{\sim} U[\theta, \theta+1]$

and,

$$f_\theta(x) = I\{\theta \leq x \leq \theta+1\}$$



from the figure,

$f_\theta(x_i) = 1$ for all $i = 1, 2, \dots, n$.

The area under the $[\theta, \theta+1]$ must be equal to 1.

$$\therefore L(\theta) = \prod_{1}^{n} f_\theta(x_i)$$

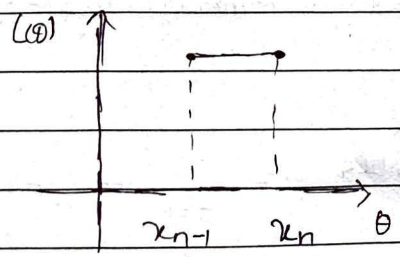
$$= 1^n \prod_{1}^{n} I\{x_i \in [\theta, \theta+1]\}$$

$$= I\{\theta \leq x_1 \leq \dots \leq x_n \leq \theta+1\}$$

$\therefore$ we can assume,

for all $i = 1, 2, \dots, n$

$$x_{n-1} \leq \theta \leq x_n$$



$\therefore$ MLE of $\hat{\theta}$ is interval of $[x_{n-1}, x_1]$

② AR(1) Model: $y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$ $\quad t = 1, \ldots n$

where $\varepsilon_t \overset{iid}{\sim} N(0, \sigma^2)$ i.e.;

$$y_t | y_{t-1} \overset{indep.}{\sim} N(\beta_0 + \beta_1 y_{t-1}, \sigma^2)$$

MLE of $\theta = (\beta_0, \beta_1, \sigma^2)$

Ans:- $L(\theta) = f_\theta(y_1 \ldots y_n)$

$$= f_\theta(y_1|y_0)\, f_\theta(y_2|y_1) \ldots f_\theta(y_n|y_{n-1})$$

$$= \prod_{t=1}^{n-1} f_\theta(y_t|y_{t-1})$$

$$= \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} e^{-\frac{1}{2\sigma^2}\underbrace{\sum (y_t - \beta_0 - \beta_1 y_{t-1})^2}_{\Sigma_t}}$$

$$l(\theta) = -\frac{n}{2}\ln 2\pi\sigma^2 - \frac{1}{2\sigma^2}\sum (y_t - \beta_0 - \beta_1 y_{t-1})^2$$

$x\beta = \beta_0 + \beta_1 y_{t-1}$

$$\hat{\theta} = \underset{\theta}{\arg\max}\, l(\theta)$$

$\Rightarrow$ we need to minimize $\|y_t - \beta_0 - \beta_1 y_{t-1}\|^2$ (L_2-norm)

$$\hat{\beta}_{OLS} = \frac{\sum_{t=1}^{n} y_{t-1} \cdot y_t}{\sum_{t=1}^{n} y_{t-1}^2}, \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{t=1}^{n} \hat{\varepsilon}_t^2,$$

$\cancel{\varepsilon_t = y_t - \hat{\beta} y_t}$

$\varepsilon_t = y_t - \hat{\beta} y_t$