# DEPARTMENT OF MATHEMATICS
# (2014/2015)SEMESTER 1

## MATH 223–CALCULUS II

### Exercise 3

1. Find the domain of each of the hyperbolic functions
2. Prove each identity. When possible, give the analogous identity for ordinary trigonometric functions.
   (a) $\cosh^2 x - \sinh^2 x = 1$
   (b) $\operatorname{sech}^2 x + \tanh^2 x = 1$
   (c) $\coth^2 x + \operatorname{csch}^2 x = 1$
   (d) $\sinh 2x = 2\sinh x \cosh x$
   (e) $\sinh(\ln x) = (x - x^{-1})/2$ for $x > 0$.
   (f) $\cosh(\ln x) = (x + x^{-1})/2$ for $x > 0$.
3. Define $\sinh y$ and $\cosh y$ in terms of exponential functions and show that
$$2y = \ln\left\{\frac{\cosh y + \sinh y}{\cosh y - \sinh y}\right\}$$
   By putting $\tanh y = \frac{1}{3}$ deduce that
$$\tanh^{-1}\frac{1}{3} = \frac{1}{2}\ln 2.$$
4. Evaluate the following derivatives and integrals
   (a) $\frac{d}{dx}\left(\sinh\sqrt{x^2+1}\right)$
   (b) Find $y'(x)$ when $\sinh(xy) = x + 8$.
   (c) $\int x\sinh\left(x^2+4\right)dx$
   (d) $\int_0^{\ln 3} x\sinh\left(x^2+4\right)dx$
   (e) $\frac{d}{dx}(\ln\coth 3x)$
   (f) $\frac{d}{dx}\left(\sqrt{\tanh 3x}\right)$
   (g) $\frac{d}{dx}\left(\sinh^{-1}(\cosh 2x)\right)$
5. Find the values of $x$ for which the following equations are true
   (a) $\cosh x = \frac{13}{5}$
   (b) $\sinh x = \frac{3}{4}$
   (c) $\tanh x = \frac{1}{2}$
   (d) $3\coth^2 x + 4\operatorname{cosech} x = 23$
   (e) $\sinh^{-1} x + \cosh^{-1}(x+2) = 0$
   (f) $2\tanh^{-1}\left(\frac{x-2}{x+1}\right) = \ln 2$.
6. Given that $\sinh x = \frac{5}{12}$, find the values of
   (a) $\cosh x$ (b) $\tanh x$ (c) $\sinh 2x$ (d) $\coth x$ (e) $\operatorname{sech} x$ (f) $\cosh 2x$
   Determine the value of $x$ as a natural logarithm.

7. Solve the following equations giving your answers in terms of natural logarithms.

   (a) $4\cosh x + \sinh x = 4$

   (b) $3\sinh x - \cosh x = 1$

   (c) $2\cosh 2x + 10\sinh 2x = 5$

8. Find the possible values of $\sinh x$ for which

$$12\cosh^2 x + 7\sinh x = 24$$

   (The identity $\cosh^2 x - \sinh^2 x = 1$ may prove useful)

9. Express $25\cosh x - 24\sinh x$ in the form $R\cosh(x - \alpha)$ giving the values of $R$ and $\tanh\alpha$.

10. Solve the following simultaneous equations

$$\begin{aligned} \cosh x - 3\sinh y &= 0 \\ 2\sinh x + 6\cosh y &= 5. \end{aligned}$$

11. Find the equation of the tangent to the curve with equation

$$y = 3\cosh 2x - \sinh x$$

    at the point where $x = \ln 2$.

12. Show that

$$\cosh x + \sinh x = e^x$$

    and deduce that

$$\cosh nx + \sinh nx = \sum_{k=0}^{n} \binom{n}{k} \cosh^{n-k} x \sinh^k x$$

    (verify your answer by the principle of mathematical induction)