UNIVERSITY OF GHANA
MATH 223-CALCULUS II
SEQUENCES AND SERIES

1- a) Find if the following series converge or not.

$$\sum_{n=1}^{\infty}\frac{2^n e^{-n}}{n};\ \sum_{n=0}^{\infty}\frac{2^n}{4^n+1};\ \sum_{n=0}^{\infty}\frac{(-1)^n}{n!};\ \sum_{n=1}^{\infty}\frac{2}{\sqrt{n}+1};\ \sum_{n=0}^{\infty}\frac{4n-1}{n^3+1};\ \sum_{n=1}^{\infty}\frac{5}{n^n}$$

$$\sum_{n=1}^{\infty}\frac{3\sqrt{n}+1}{n+5};\ \sum_{n=1}^{\infty}\frac{\sqrt{2^n+1}}{n!};\ \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{\sqrt{5}}};\ \sum_{n=2}^{\infty}\frac{5}{n\ln n}.$$

b) For each of the following series, find the partial sum S_n and hence evaluate the series

$$\sum_{n=0}^{\infty}5\left(\frac{1}{2}\right)^n;\ \sum_{n=1}^{\infty}5^{-n-1}2^{n+5};\ \sum_{n=5}^{\infty}-2\left(\frac{2}{3}\right)^n;\ \sum_{n=0}^{\infty}\left(\frac{6}{9}\right)^{n+3}$$

c) For each of the following series, find the partial sum S_n and hence evaluate the series

$$\sum_{n=1}^{\infty}\frac{3}{2n(n+1)};\ \sum_{n=1}^{\infty}\frac{-5}{n^2+4n+3};\ \sum_{n=1}^{\infty}\frac{-5}{n^2+3n+2};\ \sum_{n=2}^{\infty}\frac{-1}{\ln(n)}+\frac{1}{\ln(n+1)}$$

2- a) Determine the radius and the interval of convergence of the following power series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2^n}x^{2n};\ \sum_{n=0}^{\infty}\frac{(x-1)^{n+1}}{n!};\ \sum_{n=1}^{\infty}\frac{(x-3)^n}{n^n};\ \sum_{n=1}^{\infty}\frac{2^n}{n}(2x-3)^n$$

b) Find a power series representation for each of the following functions.

i) $f(x)=\frac{2}{1-x^2}$;
ii) $g(x)=\frac{1}{(1-x)^2}$;
iii) $h(x)=\frac{-x}{(1+x^2)^2}$;
iv) $m(t)=\ln(1+t)$;
v) $k(s)=\ln(3-s)$.

3- a) Find the Taylor's series about $a=0$ of

$$\cos x;\ \sin x;\ \ln(1+x);\ \ln(1+x^2);\ \int\frac{\sin x}{x}dx.$$

Indicate where the Taylor's series coincides with the function.

b) Find the Taylor's series about a of the following functions.

i) $f(x) = -\frac{1}{x}$, $a = -1$.

ii) $g(x) = \ln x$, $a = 2$.

iii) $h(y) = e^{-y}$, $a = -2$.

iv) $m(x) = x^2 e^{-5x^3}$, $a = 0$.

v) $k(x) = \frac{1}{x}\cos x$, $a = \frac{\pi}{2}$.

4- a- Recall the following binomial expansion formula:

$$(a+b)^n = \sum_{i=0}^{n} \mathcal{C}_i^n a^{n-i} b^i$$

where $\mathcal{C}_i^n = \frac{n(n-1)(n-2)\ldots(n-i+1)}{i!}$, $\mathcal{C}_0^n = 1$.
Find the binomial expansion of $(2x+1)^4$ and $(x-3)^5$.

b- For $|x| < 1$, we have the following binomial power series:

$$(1+x)^k = \sum_{n=0}^{\infty} \mathcal{C}_n^k x^k$$

where $\mathcal{C}_n^k = \frac{k(k-1)(k-2)\ldots(k-n+1)}{n!}$, $n = 1, 2, 3 \ldots$, and $\mathcal{C}_0^k = 1$.
Find the Binomial series of the following functions, indicating the domain of validity.

i) $\frac{1}{1-2x}$;

ii) $\sqrt{4-x}$.