UNIVERSITY OF GHANA
MATH 223-CALCULUS II
Duration:1 hour 30 minutes

1- a) Let $(v_k)_{k\geq 1}$ be an arithmetic sequence such that $v_1 = -3$. Find the integer n and the common difference d so that $v_n = 6$ and $S_n = v_1 + v_2 + \ldots + v_n = 28.5$.

b) Let $(a_k)_{k\geq 1}$ be a geometric sequence. Find the integer k and the common ratio r so that $v_1 = 48$, $v_k = 243$ and $S_k = v_1 + v_2 + \ldots + v_k = 633$.

c) Consider the sequences (u_n) defined by $u_0 = 1$ and $u_{n+1} = \frac{2}{3}u_n - \frac{1}{3}$ and $v_n = u_n + 1$.
Prove that (v_n) is a geometric sequence.
Find v_n and hence u_n as a function of n. Deduce the limit of u_n.

2- a) Find if the following series converge or not.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\sqrt{5}}}; \quad \sum_{n=0}^{\infty} \frac{4n-1}{n^3+1}; \quad \sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{3\sqrt{n}+5}; \quad \sum_{n=0}^{\infty} \frac{3}{n+2}.$$

Justify all your answers.

b) Find the partial sum S_n and hence evaluate the series $\sum_{n=0}^{\infty} 5\left(\frac{1}{2}\right)^n$.

c) Consider the series $S = \sum_{n=1}^{\infty} \frac{-5}{n^2+4n+3}$.

i) Why is S convergent?

ii) Find the partial sum S_n and hence determine the value of S.

3- a) Determine the radius and the interval of convergence of the following power series

$$\sum_{n=1}^{\infty} \frac{2^n}{n+3}(x+2)^n.$$

b) Find the Taylor's series about $a = 0$ of $f(x) = \ln(1-x)$. Give an expression of the Taylor's polynomial or order 4 with error term (Lagrange reminder) and estimate the error for $x \in (-1, 1)$.

c) Recall for $f : [a, b] \to \mathbb{R}$, the Riemann sum is given by

$$R_n = \frac{b-a}{n} \sum_{k=1}^{n} f(a + k\frac{b-a}{n}).$$

Using the Riemann sum, evaluate the Riemann integral $\int_3^5 (2x - 3x^2)dx$.