Numerical Methods UNIT – III INTERPOLATION

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UNIT – III INTERPOLATION Faculty Name: N V Nagendram

Interpolation: Introduction – Errors in polynomial Interpolation – Finite differences – Forward Differences – Backward Differences – Central Differences – Symbolic relations and separation of symbols – Differences of a polynomial – Newton's formulae for interpolation – Lagrange's Interpolation formula.

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ECE - B Section

UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Planned Topics Lectures

3.1 Introduction

3.2 Errors in polynomial Interpolation

3.3 Finite Differences
- (i) Introduction
- (ii) Forward Differences
- (iii) Forward Difference Table
- (iv) Backward Differences
- (v) Backward Difference Table
- (vi) Central Differences
- (vii) Central Difference Table

3.4 Symbolic relations and Separation of symbols

3.5 Relationship between Δ and E ; operators D and E and some more relations

3.6 Differences of a polynomial

3.7 Interpolation

3.8 Errors in polynomial Interpolation

3.9 Newton's Formula Interpolation Formula

3.10 Newton's Backward Interpolation Formula

3.11 Formula for Error in polynomial Interpolation

3.12 Central Difference Interpolation

3.13 Gauss's Forward Interpolation formula

3.14 Gauss's Backward Interpolation formula

3.15 Interpolation with unevenly spaced points

3.16 Lagrange's Interpolation Formula

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Lecture-1

Introduction:

If we consider the statement $y = f(x)$, $x_0 \le x \le x_n$ we understand that we can find the value of y, corresponding to every value of x in the range $x_0 \le x \le x_n$.

Definition: If the function f(x) is single valued and continuous and is known as explicitly then the values of f(x) for certain values of x like x_0, x_1, x_2, ……, x_n can be calculated. The problem now is if we are given the set of tabular values

X	x_0	x_1	x_2	….	x_{n-2}	x_{n-1}	x_n
Y	y_0	y_1	y_2	….	y_{n-2}	y_{n-1}	y_n

Satisfying the relation $y = f(x)$ and the explicit definition of f(x) is not known, is it possible to find a simple function say $\phi(x)$ such that f(x) and $\phi(x)$ agree at the set of tabulated points. This process of finding $\phi(x)$ is called **"Interpolation"**.

Definition: If $\phi(x)$ is a polynomial then the process is called "**polynomial interpolation"** and $\phi(x)$ is called "**interpolating polynomial"**.

Note: Throughout this chapter we study polynomial interpolation.

3.2 Errors in polynomial Interpolation

Types of Errors: Let x be the value and x^* be an approximation to the value x. Then the difference $x - x^*$ is called an **"Error"** in x^*.

$\boxed{\therefore \text{ exact = Approximation + error}}$

Inherent | Rounding | Truncation | Abs. Relative % Errors

Definition: Inherent Errors:

Errors which are already involved in the statement of a problem before its solution are called "Inherent Errors". These types of errors arise either due to the given data being approximate or due to the limitations of math tables calculations or the digital computer.

Inherent errors can be minimized by taking better date or by using precision computing skills or aids.

Definition: Rounding errors:

Errors which arise from the process of rounding off the number during the calculations are known as "rounding error".

Let two numbers each having n digits be multiplied and the resulting number which will contain 2n digits be rounded to 'n' digits. Indoing so, "a round – off error is introduced".

Similarly, in a decimal system, there may be infinity of digits to the right of the decimal point, and it may not be possible for us to use infinity of digits in a computational problems. So, we use only finite number of digits only in our calculations.

This is the source of the so called "Rounding of Errors". Rounding errors can be reduced by

(i) changing the calculation procedure or division of a small number.

(ii) retaining at least one or more significant figure at each step than that given in the data and rounding off at the last step.

Definition: Truncation Errors:

These errors arise due to approximation of results or on replacing an infinite process by a finite one.

Example: if $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$ in this $\frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$ is replaced by x′ then sin x = x - x′.

Truncation error type is algorithm error.

The magnitude of the error in the value of the function due to cutting (truncation) of its series is equal to the sum of all the discarded terms.

It may be large and may even exceed the sum of the terms retained, thus making calculated result meaningful.

Definition: Absolute error (or) relative (or) % errors: AE / RE / PE:

If x is the value of a quantity and x^* is the its approximate value and then $| x - x^* |$ is called "Absolute error E_0".

Relative error: $\boxed{E_r = \left|\frac{x - x^*}{x}\right|}$.

% Error: $\boxed{E_p = 100\, E_r = 100 \left|\frac{x - x^*}{x}\right|}$.

If $\bar{x}$ such that $| x - x^* | = \bar{x}$ then $\bar{x}$ is an upper limit on the magnitude of absolute error and measures the absolute accuracy.

Note: The relative error, % errors are independent and absolute error is the expression in terms of E_r , E_p of these units.

Note: If a number is correct to n decimal places, then the error is $\boxed{\frac{1}{2}.10^{-n}}$.

Note: if the first significant figure of a number is k and the number is correct to n significant figures, then the relative error $\boxed{E_r < \frac{1}{k.x.10^{n-1}}}$

Definition: Errors in the approximation of a function:

Let $y = f(x_1, x_2)$ for all x_1, x_2.

If $\delta x_1, \delta x_2$ be the errors in x_1, x_2 then the error δy in y is given by

$y + \delta y = \delta(x_1 + \delta x_1 ,x_2 + \delta x_2) = f(x_1, x_2) + \left(\frac{\partial f}{\partial x_1}.\delta x_1 + \frac{\partial f}{\partial x_2}.\delta x_2\right)$ + terms involving high powers of $\delta x_1, \delta x_2$ are infinitesimal their squares and higher powers can be neglected.

Then, $y + \delta y = f(x_1, x_2) + \left(\frac{\partial f}{\partial x_1}.\delta x_1\right) + \left(\frac{\partial f}{\partial x_2}.\delta x_2\right) \Rightarrow \boxed{\delta y = \left(\frac{\partial f}{\partial x_1}.\delta x_1\right) + \left(\frac{\partial f}{\partial x_2}.\delta x_2\right)}$

Ingeneral, δy in $y = f(x_1, x_2, x_3, x_4, \ldots x_n)$ corresponding to errors δx_i in x_i for i = 1,2,3,…,n is given by $\boxed{\delta y = \left(\frac{\partial y}{\partial x_1}.\delta x_1\right) + \left(\frac{\partial y}{\partial x_2}.\delta x_2\right) + \ldots\ldots + \left(\frac{\partial y}{\partial x_n}.\delta x_n\right)}$

Relative error is $E_r = \boxed{\frac{\delta y}{y} = \left(\frac{\partial y}{\partial x_1}.\frac{\delta x_1}{y}\right) + \left(\frac{\partial y}{\partial x_2}.\frac{\delta x_2}{y}\right) + \ldots\ldots + \left(\frac{\partial y}{\partial x_n}.\frac{\delta x_n}{y}\right)}$

Example: Rnd(37.8692) = significant figure (37.87)

Example: Rnd(865200)

$$= E_a = |\ x - x^* \ | = |\ 865250 - 865200 \ | = 50$$

$$E_r = \left| \frac{x - x^*}{x} \right| = \left| \frac{865250 - 865200}{865250} \right| \approx \frac{50}{865250} \approx 6.71\ X\ 10^{-5}$$

$$E_p = E_r X100 = \left| \frac{x - x^*}{x} \right| = 100\ X\ 6.71\ X\ 10^{-5} = 6.71\ X\ 10^{-3}$$

Example: Rnd(37.8692) = significant figure (37.87)

$$= E_a = |\ x - x^* \ | = |\ 37.46235 - 37.4600 \ | = 0.00235$$

$$E_r = \left| \frac{x - x^*}{x} \right| = \left| \frac{37.46235 - 37.4600}{37.464625} \right| \approx \frac{0.00235}{37.4625} \approx 6.27\ X\ 10^{-5}$$

$$E_p = E_r X100 = \left| \frac{x - x^*}{x} \right| = 100\ X\ 6.27\ X\ 10^{-5} = 6.27\ X\ 10^{-3}$$

Lecture-2

Definition: Finite Differences:

The calculus of finite differences deals with the changes that take place in the value of a function due to finite changes in the independent variable.
Let $y_0, y_1, y_2, \ldots \ldots, y_n$ be a set of values of any function $y = f(x)$.

Definition: Forward differences:

The difference $y_1 - y_0, y_2 - y_1, y_3 - y_2, \ldots \ldots, y_n - y_{n-1}$ denoted by $\Delta y_0, \Delta y_1, \Delta y_2, \Delta y_3, \ldots, \Delta y_{n-1}$ respectively are called the "F**irst Forward Differences**" and operator Δ is known as "**Forward difference operator**".

$$\boxed{\therefore \Delta y_n = y_{n+1} - y_n}$$

The differences of the First Forward Differences are called "**Second Forward Differences**" and are denoted by $\Delta^2 y_0, \Delta^2 y_1, \Delta^2 y_2, \Delta^2 y_3, \ldots, \Delta^2 y_n$ etc.,

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$$

$$\boxed{\therefore \Delta^2 y_n = \Delta y_{n+1} - \Delta y_n}$$

For any r, $\boxed{\therefore \Delta^r y_n = \Delta^{r-1} y_{n+1} - \Delta^{r-1} y_n}$ r^{th} forward difference.

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0					
		Δy_0				
x_1	y_1		$\Delta^2 y_0$			
		Δy_1		$\Delta^3 y_0$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		Δy_3		$\Delta^3 y_2$		
x_4	y_4		$\Delta^2 y_3$			
		Δy_4				
x_5	y_5					

This table is called a "**Diagonal Difference Table**". X is called the argument and y the function or the entry. y_0 the first entry is called "**leading term**" and $\Delta y_0, \Delta^2 y_0, \Delta^3 y_0$, ... are called the **"leading differences".**

Note: if Δ is operator

then $\Delta y_0 = y_1 - y_0 \Rightarrow y_1 = \Delta y_0 + y_0 = y_0 + \Delta y_0 = y_0(1+\Delta) = (1+\Delta)\, y_0$

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0 \Rightarrow \Delta y_1 = \Delta^2 y_0 + \Delta y_0 = \Delta^2 y_0 = \Delta y_1, \; \Delta y_0 = y_1$

$\Rightarrow \quad y_2 = \Delta y_1 + y_1 = y_1 + \Delta y_1 = y_1(1+\Delta) = (1+\Delta)\, y_1$

$\Rightarrow \quad y_2 = y_0 + 2.\Delta y_0 + \Delta^2 y_0 = y_0(1+\Delta)^2 = (1+\Delta)^2\, y_0$

$\boxed{\therefore\ y_2 = (1+\Delta)^2\, y_0}$

$\boxed{\therefore\ y_3 = (1+\Delta)^3\, y_0}$

Similarly, $y_k = y_0 + k\,C_1.\Delta y_0 + k\,C_2.\Delta^2 y_0 + ...+ k\,C_k.\Delta^k y_0$

Therefore for k, $\boxed{\therefore\ y_k = (1+\Delta)^k\, y_0}$

Note: $\Delta^2 y_0 = \Delta y_1 - \Delta y_0 = (y_2 - y_1) - (y_1 - y_0) = (y_2 - 2y_1 - y_0)$

$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0 = (y_3 - 2y_2 + y_1) - (y_2 - 2y_1 + y_0) = (y_3 - 3y_2 + 3y_1 - y_0)$

$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0 = (y_4 - 3y_3 + 3y_2 - y_1) - (y_3 - 3y_2 + 3y_1 - y_0)$

$= (y_4 - 4y_3 + 6y_2 - 4y_1 + y_0)$

$\Delta^n y_0 = y_n - nC_1\, y_{n-1} + nC_1\, y_{n-2} - \ldots\ldots + (-1)^n y_0)$

Note: Δ is linear operator if it satisfies below (i) , (ii) axioms:

(i) $\Delta(f(x) + g(x)) = \Delta(f(x)) + \Delta(g(x))$

(ii) $\Delta(c.f(x)) = c.\Delta(f(x))$ for any $c > 0$ and

(iii) $\Delta^m \Delta^n (f(x)) = \Delta^{m+n}(f(x))$ for m, n are positive integers.

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Problems on errors, Diagonal Difference tables **Tutorial-01**

Problem # 01: Construct a diagonal difference table for the following set of values:

x_i	0	2	4	6	8	10	12	14
y_i	625	81	1	1	81	625	2401	65611

[Ans:384]

Problem # 02: Construct a diagonal difference table for the following set of values:

x_i	1	2	3	4	5
y_i	4	13	34	73	136

[Ans:0]

Problem # 03: Find (i) $\Delta\, e^{ax}$ (ii) $\Delta^2\, e^{ax}$ and evaluate (iii) $\Delta \tan^{-1} x$

[Ans: i) $e^{ax}(e^{ah} - 1)$ ii) $(e^h - 1)^2 e^x$ iii) $\tan^{-1}\left[\dfrac{h}{x^2 + hx + 1}\right]$]

Problem # 04: Prove (i) $\Delta.\nabla = \Delta - \nabla$ (ii) $\dfrac{\Delta}{\nabla} - \dfrac{\nabla}{\Delta} = \Delta + \nabla$ (iii) $\nabla E = E\nabla = \nabla$

Problem #05: Evaluate (i) $\Delta \sin(ax+b)$ (ii) $\Delta^2 [3\, e^x]$ (iii) $\Delta [\, e^{ax} \log bx]$

[Ans:i) $2\cos(ax+b+ah/2)\sin(ah/2)$ (ii) $3(e^{x+2h} - 2\, e^{x+h} + e^x$

(iii) $e^{a(x+h)}.\log(b(x+h)) - e^{ax}.\log(bx)$]

Problem # 06: Evaluate $\Delta^2 \left[\dfrac{5x+12}{x^2+5x+6}\right]$ taking the interval of differencing being one unit?

[Ans: $\left[\dfrac{2(5x+16)}{x^3+9x^2+26x+24}\right]$]

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Solutions on errors, Diagonal Difference tables **Tutorial-01**

Problem # 01: Construct a diagonal difference table for the following set of values:

x_i	0	2	4	6	8	10	12	14
y_i	625	81	1	1	81	625	2401	65611

[Ans: 384]

Solution: Forward, Backward and central Differences

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$	$\Delta^5 y_i$
0	625					
		-544				
2	81		464			
		-80		-384		
4	1		80		384	
		0		0		0 Forward
6	1		80		384	
		80		384		0 Central
8	81		464		384	
		544		768		0Backward
10	625		1232		384	
		1776		1152		
12	2401		2384			
		4160				
14	6561					

Problem # 02: Construct a diagonal difference table for the following set of values:

x_i	1	2	3	4	5
y_i	4	13	34	73	136

[Ans:0]

Solution: Diagonal Difference Table is as below

x	Y	Δ y	Δ^2 y	Δ^3 y	Δ^4 y
1	4				
		9			
2	13		12		
		21		6	
3	34		18		0
		39		6	
4	73		24		
		63			
5	136				

Hence the solution is 0.

Problem # 03: Find (i) $\Delta\, e^{ax}$ (ii) $\Delta^2\, e^{ax}$ and evaluate (iii) $\Delta \tan^{-1} x$

[Ans: i) $e^{ax}(e^{ah}-1)$ ii) $(e^h-1)^2 e^x$ iii) $\tan^{-1}\left[\dfrac{h}{x^2+hx+1}\right]$]

Solution: (i) $\Delta\, e^{ax}$

By Definition, $\Delta\, e^{ax} = e^{a(x+h)} - e^{ax} = e^{ah}[e^{ax}] - [e^{ax}] = e^{ax}[e^{ah}-1]$

$$\boxed{\therefore\ \Delta\, e^{ax} = e^{ax}[e^{ah}-1]}$$

To find (ii) Consider $\Delta^2\, e^{ax} = \Delta(\Delta\, e^{ax}) = \Delta(e^{a(x+h)} - e^{ax})$

$= \Delta(e^x[e^h - 1)$

$= [e^h - 1]\,\Delta e^x = (e^h-1)(e^{(x+h)} - e^x) = (e^h-1)^2 e^x$

$$\boxed{\therefore \Delta^2\, e^{ax} = (e^h-1)^2 e^x}$$

To evaluate (iii)

Consider, $\Delta \tan^{-1} x = \tan^{-1}(x+h) - \tan^{-1} x = \tan^{-1}\left[\dfrac{x+h-x}{1+(x+h)x}\right]$

$= \tan^{-1}\left[\dfrac{h}{1+hx+x^2}\right]$

$$\boxed{\therefore\ \Delta \tan^{-1} x = \tan^{-1}\left[\frac{h}{1+hx+x^2}\right]}$$

Hence the solution.

Problem # 04: Prove (i) $\Delta.\nabla = \Delta - \nabla$ (ii) $\dfrac{\Delta}{\nabla} - \dfrac{\nabla}{\Delta} = \Delta + \nabla$ (iii) $\nabla E = E\nabla = \nabla$

Solution: to prove (i) $\Delta.\nabla = \Delta - \nabla$

Consider, $\Delta.\nabla = (1 - E^{-1})(E - 1)$

$= E - 1 - 1 + E^{-1} = E - 1 - (1 - E^{-1}) = \Delta - \nabla$

To prove (ii):

Consider, $\dfrac{\Delta}{\nabla} - \dfrac{\nabla}{\Delta} = \dfrac{\Delta^2 - \nabla^2}{\Delta\nabla} \approx \dfrac{(\Delta+\nabla)(\Delta-\nabla)}{\Delta-\nabla} \approx \Delta + \nabla$

To prove (iii):

Consider, $\nabla E = (1 - E^{-1})\, E = E\nabla = E(1 - E^{-1}) = \Delta$

Thus, $\nabla E = E\nabla = \Delta$

Hence the solution.

Problem #05: Evaluate (i) Δ sin (ax+b) (ii) Δ^2 [3 e^x] (iii) Δ [e^{ax} log bx]

[Ans: i) 2 cos(ax+b+ah/2) sin (ah/2) (ii) 3(e^{x+2h} – 2 e^{x+h} + e^x

(iii) $e^{a(x+h)}$.log(b(x + h)) – e^{ax} .log(bx)]

Solution:

To evaluate (i) Δ sin (ax+b)

Consider, Δ sin (ax+b) = sin (a (ax+h) + b) – sin (ax + b)

= sin (ax + b + ah) – sin (ax + b)

= 2 cos(ax b + ah/2) sin (ah/2)

To evaluate (ii)

Consider, Δ^2 [3 e^x] = 3 Δ [$e^{(x+h)}$ - e^x] = 3 [$\Delta e^{(x+h)}$ - Δe^x] = 3 [e^{x+2h} – 2 e^{x+h} + e^x]

To evaluate (iii)

Consider,

Δ [e^{ax} log bx] = $e^{a(x+h)}$ log (b (x + h)) – e^{ax} log(bx)

Hence the solution.

Problem # 06: Evaluate $\Delta^2 \left[\dfrac{5x+12}{x^2+5x+6}\right]$ taking the interval of differencing being one unit?

[Ans: $\left[\dfrac{2(5x+16)}{x^3+9x^2+26x+24}\right]$]

Solution: let f(x) = $\left[\dfrac{5x+12}{x^2+5x+6}\right]$ and given h = 1

Δ^2 f(x) = Δ (Δ f(x)) = Δ [f (x + 1) – f (x)] = Δ [f (x + 1)] – Δ [f (x)]

= f(x + 2) – 2 (f(x + 1) + f(x)

$$= \frac{5(x+2)+12}{(x+2)^2+5(x+2)+6} - 2\left[\frac{5(x+1)+12}{(x+2)^2+5(x+2)+6}\right] + \frac{5x+12}{x^2+5x+6}$$

= $\left[\dfrac{2(5x+16)}{x^3+9x^2+26x+24}\right]$ Hence the solution.

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Backward Differences **Lecture-3**

Definition: Differences of a polynomial:

"The n^{th} differences of a polynomial of the n^{th} degree are constant when the values of independent variable are at equal intervals and all higher order differences are zero". Converse is true.

Let the polynomial be $y = f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots\ldots + a_{n-1} x' + a_n x^0$….(1)

$y+\delta y = f(x + \Delta x) = a_0 (x + h)^n + a_1 (x + h)^{n-1} + a_2 (x + h)^{n-2} + \ldots\ldots + a_{n-1} (x + h) + a_n$…….(2)

Here let $\Delta x = h$,

(1) – (2) $\Rightarrow$

$$\Delta y = f(x + \Delta x) - f(x) = a_n\, n\, h\, x^{n-1} + [\frac{1}{2}n(n-1)a_0 h^2 + a_1(n-1)h]x^{n-2} + [\frac{1}{6}n(n-1)(n-2)a_0 h^3 + \frac{1}{2}(n-1)(n-2)a_1 h^2 + a_2(n-2)h]x^{n-3} + -$$

Similarly, $\boxed{\Delta^n y = a_0\, n!\, h^n}$ n^{th} difference is constant, all higher difference = 0.

Note: if h = 1, $\Delta^n y = a_0\, n!$ and $\Delta^n(x^n) = n!$.

Definition: Backward Differences:

Let $y_0, y_1, y_2, \ldots\ldots, y_{n-1}, y_n$ denote the values of any function y = f(x). Then The difference $y_1 - y_0$, $y_2 - y_1$, $y_3 - y_2$, ... , $y_n - y_{n-1}$ denoted by ∇y_1, ∇y_2, ∇y_3, ∇y_4, . . . , ∇y_n respectively are called the "F**irst Backward Differences**" and operator ∇ is known as "**Backward difference operator**".

$$\boxed{\therefore\ \nabla y_n = y_n - y_{n-1}}$$

The differences of the First Backward Differences are called "**Second Backward Differences**" and are denoted by $\nabla^2 y_2$, $\nabla^2 y_3$, . . . , $\nabla^2 y_n$ etc.,

$$\nabla^2 y_2 = \nabla y_2 - \nabla y_1 = (y_2 - y_1) - (y_1 - y_0) = y_2 - 2y_1 + y_0$$

$$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2 = (y_3 - 2y_2 + y_1) - (y_2 - 2y_1 + y_0) = y_3 - 3y_2 + 3y_1 - y_0$$

$\boxed{\therefore \nabla^r y_n = \nabla^{r-1} y_n - \nabla^{r-1} y_{n-1}}$, r^{th} Backward Difference.

$\forall$ r, $\boxed{\therefore \nabla^r y_n = \nabla^{r-1} y_n - \nabla^{r-1} y_{n-1}}$ rth Backward/Horizontal difference Table:

x	Y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
x_0	y_0					
		∇y_1				
x_1	y_1		$\nabla^2 y_2$			
		∇y_2		$\nabla^3 y_3$		
x_2	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$	
		∇y_3		$\nabla^3 y_4$		$\nabla^5 y_5$
x_3	y_3		$\nabla^2 y_4$		$\nabla^4 y_5$	
		∇y_4		$\nabla^3 y_5$		
x_4	y_4		$\nabla^2 y_5$			
		∇y_5				
x_5	y_5					

Note: $\therefore \nabla y_n = y_n - y_{n-1} \Rightarrow y_{n-1} = y_n - \nabla y_n = (1 - \nabla) y_n$

$$\nabla^2 y_n = \nabla y_n - \nabla y_{n-1} \; ; \; \nabla y_{n-1} = \nabla y_n - \nabla^2 y_n$$

$$y_{n-2} = y_{n-1} - \nabla y_{n-1} = y_n - \nabla y_n - (\nabla y_n - \nabla^2 y_n)$$

$$= y_n - 2.\nabla y_n + \nabla^2 y_n$$

$$\boxed{\therefore y_{n-2} = y_n - 2.\nabla y_n + \nabla^2 y_n = (1 - \nabla)^2 y_n}$$

So, $\boxed{y_{n-3} = (1 - \nabla)^3 y_n}$

$$\boxed{\therefore y_{n-k} = (1 - \nabla)^n y_n = (1 - {}^kC_1 \nabla + {}^kC_2 \nabla^2 - \ldots..) y_n = (1 - {}^kC_1 \nabla\, y_n + {}^kC_2 \nabla^2\, y_n - \ldots..)}$$

Definition: Error propagation is a finite difference Table :Error E in y_3 values:

y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
y_0				
	∇y_0			
y_1		$\nabla^2 y_0$		
	∇y_1		$\nabla^3 y_0+\in$	
y_2		$\nabla^2 y_1+\in$		
	$\nabla y_3+\in$		$\nabla^3 y_1-3\in$	$\nabla^4 y_0-4\in$
$y_3+\in$		$\nabla^2 y_2-2\in$		
	$\nabla y_3-\in$		$\nabla^3 y_2+3\in$	$\nabla^4 y_1+6\in$
y_4		$\nabla^2 y_3+\in$		
	∇y_4		$\nabla^3 y_3-\in$	$\nabla^4 y_2-4\in$
y_5		$\nabla^2 y_4$		
	∇y_5			
y_6				

Problem: Form the backward Difference Table for the given set of values:

x	1	3	5	7	9
y	8	12	21	36	62

Solution: Backward Difference Table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	8				
		4			
3	12		5		
		9		1	
5	21		6		4
		15		5	
7	36		11		
		26			
9	62				

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Central Differences **Lecture-4**

Definition: Central Differences:

Another system of differences is central differences. In the system, the Central Differences Operator C.D.O. δ is defined by the relations.

Let $y_0, y_1, y_2, \ldots\ldots, y_{n-1}, y_n$ denote the values of any function $y = f(x)$. Then The difference $y_1 - y_0$, $y_2 - y_1$, $y_3 - y_2, \ldots\ldots, y_n - y_{n-1}$ denoted by $\delta y_{1/2}$, $\delta y_{3/2}$, . . . , $\delta y_{n-1/2}$ respectively are called the "F**irst Central Differences**" and operator δ is known as "**Central difference operator**". This is linear operator.

$\delta y_{1/2} = \Delta y_0 = \nabla y_1$; $\delta y_{3/2} = \Delta y_1 = \nabla y_2$; $\delta y_{5/2} = \Delta y_2 = \nabla y_3$;…… $\delta y_{n+1/2} = \Delta y_n = \nabla y_{n+1}$

For n = 0,1,2,…..

The first central differences of the first central differences are called the second central differences and are denoted by $\delta^2 y_1$, $\delta^2 y_2$, $\delta^2 y_3$, ……Thus, $\delta^2 y_1 = \delta y_{3/2} - \delta y_{1/2}$, $\delta^2 y_2 = \delta y_{5/2} - \delta y_{3/2}$, ….., $\delta^2 y_n = \delta y_{n+1/2} - \delta y_{n-1/2}$. Similarly, Higher Order Central Differences are defined as $\delta^2 y_2 - \delta^2 y_1 = \delta^2 y_{3/2}$ and so on.

Central Differences Table as below:

X	y	1st diff	2nd diff	3rd diff	4th diff	5th diff
x_0	δ_0					
		$\delta y_{1/2}$				
x_1	δ_1		$\delta^2 y_1$			
		$\delta y_{3/2}$		$\delta^3 y_{3/2}$		
x_2	δ_2		$\delta^2 y_2$		$\delta^4 y_2$	
		$\delta y_{5/2}$		$\delta^3 y_{5/2}$		$\delta^5 y_{5/2}$
x_3	δ_3		$\delta^2 y_3$		$\delta^4 y_3$	
		$\delta y_{7/2}$		$\delta^3 y_{7/2}$		
x_4	δ_4		$\delta^2 y_4$			
		$\delta y_{9/2}$				
x_5	δ_5					

Difference Operator E and μ:

01. Shift Operator E:

The operator E increases the argument x by h so that

$E f(x) = f(x + h)$,

$E^2 f(x) = f(x + 2h)$,

$E^3 f(x) = f(x + 3h)$,

………………………,

$E^n f(x) = f(x + nh)$

The operator E^{-1} is an inverse operator is defined by $E^{-1} f(x) = f(x - h)$
if $f(x) = y_x$, then $E y_x = y_{x+h}$

$E^{-1} y_x = y_{x-h}$

$E^n y_x = y_{x+nh}$, for any real number y.

02. **Averaging Operator μ:**

μ is defined by, $\mu y_x = \frac{1}{2}\left[y_{x\pm 1/2h} - y_{x\mp 1/2h}\right]$

In the difference calculus, E is regarded as the fundamental operator and

Δ - Delta operator

∇ - Del operator

δ - small delta operator

μ - mue expressed in terms of E.

Some Important Formulae: Relations between Operations & Identities

01. $\Delta = E - 1$ or $E = 1 + \Delta$

Proof: $\Delta y_x = y_{x+h} - y_x = Ey_x - y_x = (E - 1)y_x$

Where Δ and E connected as $\boxed{\Delta = E - 1 \text{ or } E = 1 + \Delta}$

02. $\nabla = 1 - E^{-1}$

Proof: $\nabla y_x = y_x - y_{x-h} = y_x - E^{-1}y_x = (1 - E^{-1})y_x$

Where Δ and E connected as $\boxed{\nabla = 1 - E^{-1} \text{ or } E^{-1} = 1 - \nabla}$

03. $\delta = E^{1/2} - E^{-1/2}$

Proof: $\delta y_x = y_{x+1/2.h} - y_{x-1/2.h} = E^{1/2}y_x - E^{-1/2}y_x = (E^{1/2} - E^{-1/2})y_x$

Where δ and E connected as $\boxed{\delta = E^{1/2} - E^{-1/2}}$

04. $\mu = \frac{1}{2}\left[E^{1/2} - E^{-1/2}\right]$

Proof: $\mu y_x = \frac{1}{2}\left[y_{x+1/2.h} - y_{x-1/2.h}\right] = \frac{1}{2}\left[E^{1/2} - E^{-1/2}\right]y_x$

Where μ and $E^{1/2}$ connected as $\boxed{\mu = \frac{1}{2}\left[E^{1/2} - E^{-1/2}\right]}$

05. $\Delta = E\nabla = \nabla E = \delta E^{1/2}$

Proof: by making use of 01, 02 and 03 formulae

$\Delta = E - 1$ or $E = 1 + \Delta$; $\nabla = 1 - E^{-1}$ and $\delta = E^{1/2} - E^{-1/2}$

Consider, $E - 1 = \delta.E^{1/2} = \nabla E = \Delta$

Where Δ , E,∇ and δ connected as $\boxed{\therefore \Delta = E\nabla = \nabla E = \delta E^{1/2}}$

06. $(1 + \Delta)(1 - \Delta) = I$

Proof: $\Delta f(x) = f(x + h) - f(x) = (E - 1) f(x)$

$\Delta = (E - 1)$ or $E = 1 + \Delta$

$\nabla f(x) = f(x) - f(x - h) = (1 - E^{-1}) f(x)$

$\therefore \nabla = (1 - E^{-1})$ or $E^{-1} = 1 - \nabla$

$\boxed{(1 + \Delta)(1 - \Delta) = EE^{-1} = I}$

07. $\mu^2 = 1 + \frac{1}{4}\delta^2$

By 03, we have $\delta = E^{1/2} - E^{-1/2}$

Squaring both sides, $\delta^2 = E + E^{-} - 2$

$$\mu = \frac{1}{2}\left[E^{1/2} - E^{-1/2}\right]$$

$$\mu^2 = \frac{1}{4}\left[E \; - E^{-1} + 2\right]$$

$$= \frac{1}{4}\left[\delta^2 + 2 + 2\right]$$

$$= \frac{1}{4}\delta^2 + 1$$

$$\boxed{\therefore \mu^2 = \left(\frac{1}{4}\delta^2 + 1\right)}$$

08. $E = e^{hD}$

Proof: $Ef(x) = f(x + h)$

$$= f(x) + h.\, f'(x) + \frac{h^2}{2!} f''(x) + \ldots\ldots\ldots.$$

$$= f(x) + h.\, D\, f(x) + \frac{h^2}{2!} D^2 (x) + \ldots\ldots\ldots.$$

$$= f(x)\,[\,1 + hd + h^2d^2 + \ldots\ldots\ldots\ldots\ldots]$$

$$Ef(x) = f(x).e^{hD} \Rightarrow \boxed{E = e^{hD}}$$

Factorial Powers: Functions:

$\Delta x^n = h\ (x - h)^{[n]} - x^{[n]} = x^{[n-1]}.xh$

$x^{[0]} = 1$

$x^{[n]} = x^n$

$\Delta x^{[n]} = n.h.\ \Delta x^{[n-1]}$

$\Delta^2 x^{[n]} = n(n-1).x^{(n-2)}.\ \Delta x^{[n-1]}\ h^2$

……………………

……………………

$\Delta^m x^{[n]} = n(n-1).....[n-(m-1)]\ x^{(n-m)}.\ h^m$

$\Delta^m x^{[n]} = 0$ for all $m > n$.

Formula for finite Summation:

$S_m = \sum_{i\approx 0}^{m-1} x_i^{[m]}$

$\Delta x^n = n.h.\ \Delta x^{[n+1]}$………………(1)

$\therefore\ x^{[n]} = \dfrac{\Delta x^{[n+1]}}{(n+1).h}$ ....(2)

$(1), (2) \Rightarrow S_m = \sum_{i\approx 0}^{m-1} x_i^{[n+1]}$

$$= \frac{1}{n(n+1)}\left[x_m^{[n+1]} - x_0^{[n+1]}\right]$$

Similar to Newton – Leibnitz formula for a positive integral.

Inverse Operator Δ^{-1} for finite Integration:

If $\Delta y_x = u_x$ then $y_x = \Delta^{-1} u_x$

Here Δ^{-1} is called finite Integration operator or inverse of operator. His operator Δ^{-1} or $\frac{1}{\Delta}$ ordinary integration may be denoted by D^{-1} or $\frac{1}{D}$.

Formulae:

01. $\Delta^{-1}(e^{ax+b}) = \frac{e^{ax+b}}{e^{ab}-1}$ if a = 1, b = 0 then $\Delta^{-1}e^x = \frac{e^x}{e^h-1}$

02. $\Delta^{-1}(a^x) = \frac{a^x}{a^h-1}$ $a \neq 1$

03. $\Delta^{-1}(u_x + v_x) = \Delta^{-1}u_x + \Delta^{-1}v_x$

04. $\Delta^{-1}(cu_x) = c.\Delta^{-1}u_x$

05. $\Delta^{-1}(ax+b)^{(n)} = \frac{(a+bx)^{(n+1)}}{(n+1).hb}$, $n \neq -1$

06. $\Delta^{-1}(x^n) = \frac{x^{(n+1)}}{n+1}$ $n \neq -1$, $h = 1$

07. $\Delta^{-1}\frac{1}{(ax+b)^n} = \frac{-1}{b(n-1)(ax+b)^{n-1}}$, $n \neq 1$

08. $\Delta^{-1}\frac{1}{x^{|n|}} = \frac{-1}{(n-1)x^{(n-1)}}$

Summation series:

Let $S_n = v_1 + v_2 + v_3 + v_4 + ... + v_n = \sum v_i$ for i= 1,2,...,n

Let $v_i = \Delta u_i \Rightarrow u_i = \Delta^{-1}v_i$

$v_i = \Delta u_i = u_{i+1} - u_i$ (n = 1)

$v_1 = \Delta u_1 = u_2 - u_1$

$v_2 = \Delta u_2 = u_3 - u_2$

$v_n = \Delta u_n = u_{n+1} - u_n$

$\boxed{\therefore S_n = v_1 + v_2 + v_3 + v_4 + ... + v_n = \sum v_i = u_{n+1} - u_n = u_{n+1}\ \Delta^{-1}u_1 = [\Delta^{-1}v_x]_1^{n+1}}$

Monmorts theorem result:

$u_0 + u_0x + u_2 x^2 + \ldots\ldots\ldots\ldots = \dfrac{u_0}{1-x} + \dfrac{x\Delta u_0}{(1-x)^2} + \dfrac{x^2\Delta^2 u_0}{(1-x)^3} + \ldots\ldots\ldots$

$u_0 + u_1x + u_2 x^2 + \ldots\ldots\ldots\ldots = u_0 + x\, E\, u_0 + x^2 E^2 u_0 + \ldots\ldots\ldots\ldots$

$= [1 + x\, E + x^2 E^2 + \ldots\ldots\ldots] u_0$

$= [1 - xE]^{-1} u_0$

$= \dfrac{1}{1-xE} u0$

$= \dfrac{1}{1-x(1+\Delta)} u0 \qquad (\text{since, } E = 1 + \Delta)$

$= \dfrac{1}{1-x-x\Delta} u0$

$= \dfrac{1}{(1-x)\left[1-\dfrac{x\Delta}{1-x}\right]} u0$

$= \dfrac{1}{(1-x)}\left[1-\dfrac{x\Delta}{1-x}\right]^{-1} u0$

$= \dfrac{1}{(1-x)}\left(1+\dfrac{x\Delta}{1-x}+\dfrac{x^2\Delta^2}{(1-x)^2}+\ldots\ldots\right) u_0$

$$\boxed{\therefore \sum_{i\approx 0}^{\infty} u_i\, x^i = \frac{u0}{(1-x)} + \frac{x\Delta u0}{1-x} + \frac{x^2\Delta^2 u0}{(1-x)^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots}$$

Lakireddy Bali Reddy College of Engineering, Mylavaram (Autonomous)

B.Tech I Year (II-Semester) May/ June 2014. **T 264- Numerical Methods**

UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Tutorial-2

Problem # 01: Prove that $\Delta^3 y_2 = \Delta^3 y_5$.

Problem # 02: Find the cubic polynomial in x for the given data below:

x	0	1	2	3	4	5
y	-3	3	11	27	57	107

[Ans. $x^3 - 2x^2 + 7x - 3$]

Problem # 03: Show that $\nabla.\Delta = \Delta.\nabla$

Problem # 04: Express $f(x) = 2x^3 - 3x^2 - 3x - 10$ and its differences in L factorial notation h = 1?

[**Ans.** $f(x) = 2x^{[3]} + 3x^{[2]} + 2x^{[1]} - 10$]

Problem # 05: Find the 7th term of 2, 9, 28, 65, 126, 217 and general term?

[**Ans.** 7th term = 344, $y_n = (n+1)^3 + 1$, $y_6 = (6+1)^3 + 1 = 344$.]

Problem # 06: Prove that $\delta^2 y_5 = y_6 - 2y_5 + y_4$

Problem # 07: Evaluate $\Delta^n(\frac{1}{x})$? [**Ans.** $\frac{(-1)^n}{x(x+1)(x+2)(x+3)\ldots\ldots(x+n)}$]

Problem # 08: Prove that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$

Problem # 09: Find the difference table of f(x) and E f(x) for given data:

X	0.20	0.22	0.24	0.26	0.28	0.30	0.32
f(x)	1.6596	1.6698	1.6804	1.6941	1.7028	1.7146	1.7268

[Ans. x = 0.26 is 1.6941. 0.0027 = 1.6914]

Problem # 10: Find y_6 when $y_0 = 9$, $y_1 = 18$, $y_2 = 20$, $y_3 = 24$ third differences being constant? [**Ans.** $y_6 = E^6 y_0 = 138$]

Lakireddy Bali Reddy College of Engineering, Mylavaram (Autonomous)

B.Tech I Year (II-Semester) May/ June 2014. **T 264- Numerical Methods**

UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Solutions **Tutorial-2**

Problem # 01: Prove that $\Delta^3 y_2 = \Delta^3 y_5$.

Solution: L.H.S. $= \Delta^3 y_2 = (E - 1)^3 y_2 = (E^3 - 3E^2 - 3E - 1) y_2 = y_2 - 3y_4 + 3y_3 - y_2$

R.H.S. $= \Delta^3 y_5 = (E - 1)^3 y_5 = (E^3 - 3E^2 - 3E - 1) y_5 = y_8 - 3y_7 + 3y_6 - y_5$

But, $\Delta^3 y_5 = (1 - E^{-1})^3 y_5 = (1 - 3E^{-1} + 3E^{-2} - E^{-3}) y_5 = y_5 - 3y_4 + 3y_3 - y_2$

Therefore, it implies that $\boxed{\Delta^3 y_2 = \Delta^3 y_5}$.

Hence proved the result.

Problem # 02: Find the cubic polynomial in x for the given data below:

X	0	1	2	3	4	5
Y	-3	3	11	27	57	107

[Ans. $x^3 - 2x^2 + 7x - 3$]

Solution: Function be y_x , y_x be cubic polynomial so, $\nabla^4 y$ is zero.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0	$y_0 = -3$				
		6			
1	$y_1 = 3$		2		
		8		6	
2	$y_2 = 11$		8		0
		16		6	
3	$y_3 = 27$		14		0
		30		6	
4	$y_4 = 57$		20		
		50			
5	$y_5 = 107$				

Now, $y_x = E^x (y_0) = (1 + \Delta)^x y_0 = \left\{ \{1 + x\Delta + \frac{x(x-1)}{2}\Delta^2 + \frac{x(x-1)(x-2)}{3!}\Delta^3 \right\} y_0$

$$= y_0 + x\Delta y_0 + \frac{x(x-1)}{2}\Delta^2 y_0 + \frac{x(x-1)(x-2)}{3!}\Delta^3 y_0$$

$$= -3 + 6x + \frac{x(x-1)}{2}(2) + \frac{x(x-1)(x-2)}{3!}(6)$$

$$= -3 + 6x + x(x-1) + x(x-1)(x-2)$$

$$= x^3 - 2x^2 + 7x - 3$$

$\boxed{\therefore \text{ Cubic polynomial of } y_x = x^3 - 2x^2 + 7x - 3}$.

Problem # 03: Show that $\nabla.\Delta = \Delta.\nabla$

Solution: L.H.S. $\nabla.\Delta y_x = \nabla(y_{x+h} - y_x) = \nabla y_{x+h} - \nabla y_x$

$$= (y_{x+h} - y_x) - (y_x - y_{x-h})$$

$$= y_{x+h} - 2y_x + y_{x-h}$$

= R.H.S.

$\therefore$ L.H.S. = R.H.S. i.e., $\boxed{\nabla.\Delta y_x = \Delta.\nabla y_x}$ For any y_x function.

Hence Proved the result.

Problem # 04: Express $f(x) = 2x^3 - 3x^2 - 3x - 10$ and its differences in L factorial notation h = 1?

[**Ans.** $f(x) = 2 x^{[3]} + 3 x^{[2]} + 2 x^{[1]} - 10$]

Solution: let $f(x) = 2x^3 - 3x^2 - 3x - 10 = Ax(x-1)(x-2)+ Bx(x-1) + C(x + 1)$

put x = 0, D = -10

put x = 1, C + D = -8 or C = 2

put x = 2, 2B + 2C + D = 0 or B = 3

on equating co-efficients of x^3 both sides, A = 2

$\therefore f(x) = 2x^{[3]} + 3x^{[2]} + 2x^{[1]} - 10$

$\Delta f(x) = 2.3.x^{[2]} + 3.2.x^{[1]} + 2.1x^{[0]}$

$\Delta^2 f(x) = 2.3.2.x^{[1]} + 3.2.1.x^{[0]}$

$\Delta^3 f(x) = 2.3.2.1$

Hence the solution.

Problem # 05: Find the 7^{th} term of 2,9,28,65,126,217 and general term?

[Ans. 7^{th} term = 344, $y_n = (n + 1)^3 + 1$, $y_6 = (6 + 1)^3 + 1 = 344$.]

Solution:

X	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0	$y_0 = 2$				
		7			
1	$y_1 = 9$		12		
		19		6	
2	$y_2 = 28$		18		0
		37		6	
3	$y_3 = 65$		24		0
		61		6	
4	$y_4 = 126$		30		
		91			
5	$y_5 = 217$				

7 th term :

$$y_6 = y_0 + 6C_1.\Delta^1 y_0 + 6C_2.\Delta^2 y_0 + 6C_3.\Delta^3 y_0 + 6C_4.\Delta^4 y_0 + 6C_5.\Delta^5 y_0 + 6C_6.\,\Delta^6 y_0$$

$$= 2 + 6(7) + 15(12) + 20(6) + 15(0) = 2 + 42 + 180 + 120 + 0 = 344$$

So, $y_6 = 344$.

Genereal term:

$$y_n = y_0 + {}^nC_1.\Delta^1 y_0 + {}^nC_2.\Delta^2 y_0 + {}^nC_3.\Delta^3 y_0 + {}^nC_4.\Delta^4 y_0 + {}^nC_5.\Delta^5 y_0 + {}^nC_6.\,\Delta^6 y_0 + \ldots\ldots$$

$$= 2 + n(7) + \frac{n(n-1)}{2}(12) + \frac{n(n-1)(n-2)}{6}(6) + 0$$

$$= 2 + 7n + 6n^2 - 6n + n^3 - 3n^2 + 2n$$

$$= n^3 + 3n^2 + 3n + 2$$

$$= (n+1)^3 + 1$$

$\boxed{\therefore\ y_n = (n+1)^3 + 1 \text{ implies } y_6 = (6+1)^3 + 1 = 344.}$

Hence the solution.

Problem # 06: Prove that $\delta^2 y_5 = y_6 - 2\, y_5 + y_4$

Solution: consider, L.H.S. $= \delta^2 y_5 = \delta(\delta y_5)$

$$= \delta(y_{11/2} - y_{9/2}) = \delta y_{11/2} - y_{9/2}$$

$$= (y_6 - y_5) - (y_5 - y_4) = (y_6 - 2y_5 + y_4)$$

$\boxed{\therefore\ \delta^2 y_5 = y_6 - 2y_5 + y_4}$ Hence the solution.

Problem # 07: Evaluate $\Delta^n(\frac{1}{x})$? **[Ans.** $\frac{(-1)^n}{x(x+1)(x+2)(x+3)\ldots\ldots(x+n)}$]

Solution: Consider, L.H.S. $= \Delta^n(\frac{1}{x}) = \Delta^{n-1}\,\Delta(\frac{1}{x})$

Now, $\Delta(\frac{1}{x}) = \frac{1}{x+1} - \frac{1}{x} = -\frac{1}{x(x+1)}$ and Also, $\Delta^2(\frac{1}{x}) = \frac{(-1)^2}{x(x+1)(x+2)}$

On repeating the process, we get $\boxed{\Delta^n(\frac{1}{x}) = \frac{(-1)^n}{x(x+1)(x+2)\ldots\ldots(x+n)}}$

Hence the solution.

Problem # 08: Prove that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$

Solution: let h be the interval differencing ←--------------------------------→

$f(x+h) = E\, f(x)$ **E f (x)**

$= (\Delta+1)\, f(x)$ **x-h** **x** **x+h**

$f(x+h) = (\Delta f(x) + f(x)$ implies that $\frac{f(x+h)}{f(x)} = \frac{\Delta f(x)}{f(x)} + 1$

On taking logarithms both sides, we get $\log\left[\frac{f(x+h)}{f(x)}\right] = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$

$$\log f(x+h) - \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$$

$$\boxed{[\log f(x+h) - \log f(x)] = \Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]}$$

Hence Proved the result.

Problem # 09: Find the difference table of f(x) and E f(x) for given data:

X	0.20	0.22	0.24	0.26	0.28	0.30	0.32
f(x)	1.6596	1.6698	1.6804	1.6941	1.7028	1.7146	1.7268

[Ans. x = 0.26 is 1.6941. 0.0027 = 1.6914]

Solution:

x	**f(x)**	**Δf(x)**	**Δ^2f(x)**	**Δ^3f(x)**
0.20	1.6596			
		102		
0.22	1.6698		4	
		106		27
0.24	1.6804		31	
		137		-81
0.26	1.6941		-50	
		87		81
0.28	1.7028		31	
		118		-27
0.30	1.7146		4	
		122		
0.32	1.7268			

On observation $\sum \Delta^3 f(x) = 27 - 81 + 81 - 27 = 0$

Irregularity of is at 2.6 difference is 1.6941 – 0.0027 = 1.6914 crept in while copying.

Hence the solution.

Problem # 10: Find y_6 when $y_0 = 9$, $y_1 = 18$, $y_2 = 20$, $y_3 = 24$ third differences being constant? **[Ans.** $y_6 = E^6 y_0 = 138$]

Solution: $y_6 = E^6 y_0 = (1 + \Delta)^6 y_0 = (1 + 6\Delta + 15\Delta^2 + 20\Delta^3 + 15\Delta^4 + 6\Delta^5 + \Delta^6) y_0$

Since 3rd difference of constant , sub sequent differences are zero.

$\therefore Y_6 = (1 + 6\Delta + 15\Delta^2 + 20\Delta^3) y_0$

$\therefore Y_6 = y_0 + 6\Delta y_0 + 15\Delta^2 y_0 + 20\Delta^3 y_0$

Table as :

x	**f(x)**	**Δf(x)**	**Δ²f(x)**
9			
	9		
18		-7	
	2		9
20		2	
	4		
24			

$\therefore Y_6 = y_0 + 6\Delta y_0 + 15\Delta^2 y_0 + 20\Delta^3 y_0 = 9 + 6(9) + 15(-7) + 20(9) = 138$

$\boxed{\therefore Y_6 = 138.}$

Hence the solution.

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Try Ur Self student **Tutorial-3**

Problem #11: Express $y = 2x^3 - 3x^2 + 3x - 10$ in factorial &show that $\Delta^2 y = 12$.

[**Ans.** $y = 2x^{[3]} + 3x^{[2]} + 3x^{[1]} - 10$ and $\Delta^2 y = 12$]

Problem # 12: Find the missing data in the following data table

X	0	1	2	3	4
f(x)	1	3	9	-	81

[Ans. λ = 31, $3^3 = 27$]

Problem # 13: Prove that $\Delta^n y_{n-1} = y_x - n\, y_{n-1} + \frac{n(n-1)}{2} y_{n-2} + \text{-----} + (-1)^n y_{x-n}$.

Problem #14: Prove That

$$y_0 + \frac{y_1 x}{1!} + \frac{y_2 x^2}{2!} + \frac{y_3 x^3}{3!} + \ldots\ldots \approx e^x \left(y_0 + x\Delta y_0 + \frac{x^2 \Delta^2 y_0}{2!} + \frac{x^3 \Delta^3 y_0}{3!} + \ldots\ldots\right)$$

Problem # 15: Show that $(u1 - u0) - x(u2 - u1) + x2\,(u3 - u2) + \text{-------}$ to $\infty =$

$$\frac{\Delta u_0}{1+x} - \frac{x\Delta^2 u_0}{(1+x)^2} + \frac{x^2 \Delta^3 u_0}{(1+x)^3} + \ldots\ldots\ldots\infty .$$

Problem # 16: Find $\Delta^{-1}[x(x + 1)]$? [Ans. $\frac{(x+1)x(x-1)}{3}$]

Problem # 17: Find $1^3 + 2^3 + 3^3 + \ldots. + n^3$? [Ans. $\frac{(n+1)n(n+1)}{4} \approx \left[\frac{n(n+1)}{2}\right]^2$]

Problem # 18: Find $2.3 + 3.4 + 4.5 + \text{---------} + (n+1)(n+2)$ where,

$u_n = (x+1)(x+2)$. [Ans. $\sum_{x\approx 1}^{n} (x+1)(x+2) \approx \frac{n(n^2 + 6n + 11)}{3}$]

........Dear student Try Ur Self problem 11 to problem 18............

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UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Problems **Tutorial-4**

Problem #01. The following data give the melting point of an alloy of lead and zinc, θ is the temperature in degrees centigrade x is percent of load.

x:	40	50	60	70	80	90
θ:	184	204	226	250	276	304

Find θ when x = 43 and when x = 84? [**Ans.** 189.79, 286.96]

Problem #02. Find the value of y from the following data at x = 0.47?

x:	0.0	0.1	0.2	0.3	0.4	0.5
y:	1.0000	1.1103	1.2428	1.3997	1.5836	1.7974

[**Ans.** 1.7299]

Problem #03. The following data given, the values of y are consecutive terms of which 23.6 is the 6^{th} term. Find the first and tenth terms of the series.

x:	3	4	5	6	7	8	9
y:	4.8	8.4	14.5	23.6	36.2	52.8	73.9

[**Ans.** 3.1, 100]

Problem #04. Usin the Newton's forward interpolation formula and find the value of sin 52^0 from the following data. Estimate the error

x:	45^0	50^0	55^0	60^0
y=sin x:	0.7071	0.7660	0.8192	0.8660

[**Ans.** 0.0000392]

Problem #05. The following data gives the distance in nautical miles of the visible horizon for the given heights in feet above the earths surface.

Height :x	100	150	200	250	300	350	400
Distance:y	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find y when x = 218 ft. and when x = 410 ft.? [**Ans.** 15.7, 21.53 naut. miles]

Problem #06. The following data of the function f(x) = 1/x find 1/ 2.72 using quadratic interpolation. Find an Estimate Error.

X	2.7	2.8	2.9
f(x)	0.3704	0.3571	0.3448

[**Ans.** 0.5 X 10^{-5}]

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Problems **Tutorial-5**

Problem #01. The following data given , find the number of students whose weight is between 60 and 70.

Wt Kg x:	0-40	40-60	60-80	80-100	100-120
No of Students	250	120	100	70	50

Ans. y(70)- y(60) = 54]

Problem #02. The following data given the population of a town.

Year x:	1941	1951	1961	1971	1981	1991
Pop. y:	20	24	29	36	46	51

Estimate the population increases during the period 1946 – 1976?

[**Ans.** 19.4 lakhs]

Problem #03. Find the cubic polynomial which takes the following values.

X	0	1	2	3
f(x)	1	2	1	10

Hence or otherwise evaluate f (4)? [**Ans.** 41]

Problem #04. The following data given, estimate the number of students who obtained marks between 40 and 45.

Marks:	36-40	40-50	50-60	60-70	70-80
N of stu:	31	42	51	35	31

[**Ans.** 17]

Problem #05. Find a polynomial of degree two which takes the values

x:	0	1	2	3	4	5	6	7
y:	1	2	4	7	11	16	22	29

[**Ans.** $\frac{1}{2}(x^2 + x + 2)$]

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UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Summary **SUM-01**

1. Newton's Forward Interpolation Formula:

$$\phi(x) = \phi(x_0 + ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots\ldots. + \frac{p(p-1)(p-2)\ldots.[p-(n-1)\;]}{n!}\Delta^n y_0$$

2. Error in Newton's Forward Difference Interpolation Formula:

$$E(x) = y(x) - \phi(x) = \frac{(x-x_0)(x-x_1)\ldots..(x-x_n)}{(n-1)!} y^{(n-1}(\xi) \text{ where } x < \xi < xn$$

3. Newton's Backward Interpolation Formula:

$$\phi(x) = \phi(x_n + ph) = y_n + p\Delta y_n + \frac{p(p+1)}{2!}\Delta^2 y_n + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_n + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + \; p(p+1)(p+2)\ldots.[p+(n-1)h]\;\Delta^n y_n$$

4. Error in Newton's Backward Difference Interpolation Formula:

$$E(x) = y(x) - \phi(x) = \frac{(x-x_n)(x-x_{n-1})\ldots..(x-x_0)}{(n+1)!} y^{(n-1}(\xi) \text{ where } x < \xi < xn$$

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UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Summary on Formulae **SUM-02**

NEWTON'S FORWARD INTERPOLATION FORMULA:

Let y = f(x) be a polynomial of degree n and taken in the following form

$$y = f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_3) + \text{--------------------} + b_n(x - x_0)(x - x_1)\ldots\ldots(x - x_{n-1}) \quad \ldots\ldots\ldots\ldots\ldots \text{(A)}$$

This polynomial passes through all the points $[x_i, y_i]$ for i = 0, 1, 2,...,n. therefore, we can obtain the y_i 's by substituting the corresponding x_i 's as:

At $x = x_0$, $y_0 = b_0$

At $x = x_1$, $y_1 = b_0 + b_1(x_1 - x_0)$

At $x = x_2$, $y_2 = b_0 + b_1(x_2 - x_0) + b_2(x_2 - x_0)(x_2 - x_1)$ .... (1)

Let "h" be the length of interval such that x_i 's represent as below:
x_0 , x_0 +1h, x_0 +2h, x_0 +3h, ..., x_0 + n·h.

$\Rightarrow (x_1 - x_0) = h$, $(x_2 - x_0) = 2h$, $(x_3 - x_0) = 3h$, ..., $(x_n - x_0) = n \cdot h$ .... (2)

From (1) and (2) we get,

$y_0 = b_0$

$y_1 = b_0 + b_1 \cdot h$

$y_2 = b_0 + b_1 \cdot 2h + b_2 \cdot 2h \cdot h$

$y_3 = b_0 + b_1 \cdot 3h + b_2 \cdot 3h \cdot 2h \cdot h + b_2 \cdot 3h \cdot 2h \cdot h$
....
....
$y_3 = b_0 + b_1 \cdot nh + b_2 \cdot nh \cdot (n-1)h + \text{-----} + b_n \cdot nh \cdot (n-1) \cdot h \cdot (n-2) \cdot h$ .... (B)

On solving the above equations for $b_0, b_1, b_2, \ldots, b_n$ we get

$b_0 = y_0$

$b_1 = \dfrac{y_1 - b_0}{h} = \dfrac{y_1 - y_0}{h} = \dfrac{\Delta y_0}{h}$

$b_2 =$

$$\frac{y_2 - b_0 - b_1 2h}{2h^2} = \frac{y_2 - y_0 - \left(\frac{y_1 - y_0}{h}\right)2h}{2h^2} = \frac{y_2 - y_0 - 2y_1 - 2y_0}{2h^2} = \frac{y_2 - 2y_1 \pm y_0}{2h^2} = \frac{\Delta^2 y_0}{2h^2}$$

$\therefore\ b_2 = \dfrac{\Delta^2 y_0}{2!\, h^2}$

Similarly, we can see that $b_3 = \dfrac{\Delta^3 y_0}{3!\, h^3}$, $b_4 = \dfrac{\Delta^4 y_0}{4!\, h^4}$, $b_5 = \dfrac{\Delta^5 y_0}{5!\, h^5}$,..., $b_n = \dfrac{\Delta^n y_0}{n!\, h^n}$,

$\therefore\ y = f(x)$

$$= y_0 + \frac{\Delta y_0}{h}(x - x_0) + \frac{\Delta^2 y_0}{2!\, h^2}(x - x_0)(x - x_1) + \frac{\Delta^3 y_0}{3!\, h^3}(x - x_0)(x - x_1)$$

$$(x - x_3) + \text{--------------------} + \frac{\Delta^n y_0}{n!\, h^n}(x - x_0)(x - x_1)\ldots\ldots\ldots(x - x_{n-1}) \quad \ldots (3)$$

If we us the relationship $x = x_0 + ph \Rightarrow x - x_0 = ph$ where $p = 0, 1, 2, \ldots, n$

then $x - x_1 = x - (x_0 + h) = (x - x_0) - h = ph - h = (p - 1)h$

$x - x_2 = x - (x_1 + h) = (x - x_1) - h = (p - 1)h - h = (p - 2)h$

....

....

$x - x_i = (p - i)h$

....

....

$x - x_{n-1} = [p - (n-1)]h$

$\therefore$ Equation (3) becomes,

$\therefore\ y = f(x) = f(x_0 + ph)$

$$= y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \text{-----------------------}$$

$$+ \frac{p(p-1)(p-2)\ldots\ldots[p-(n-1)]\Delta^n y_0}{n!} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

This formula is known as "**Newton's forward interpolation formula**" or "**Newton's Gregory forward interpolation formula".**

Uses: This formula for interpolation near the beginning of a set of tabular values.

NEWTON'S BACKWARD INTERPOLATION FORMULA:

Let y = f(x) be a polynomial of degree n and taken in the following form

$y = f(x) = a_0 + a_1(x - x_n) + a_2(x - x_n)(x - x_{n-1}) + a_3(x - x_n)(x - x_{n-1})(x - x_{n-2})$ + -------------------- $+ a_n(x - x_n)(x - x_{n-1})\ldots\ldots(x - x_1)$ .... (5)

and impose the condition that y and $y_n(x)$ should agree at the tabulated points $x_n, x_{n-1}, x_{n-2}, x_{n-3}, \ldots\ldots, x_2, x_1, x_0$.

we obtain $y_n(x)$ =

$$y_n + p\Delta y_n + \frac{p(p+1)}{2!}\Delta^2 y_n + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_n + \text{------------------------}$$

$$+ \frac{p(p+1)(p+2)\ldots\ldots[p+(n-1)]\Delta^n y_n}{n!} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

Where $p = \frac{x - x_n}{h}$

Uses: This formula is useful for tabular values to the left of y_n and also useful for interpolation near the end of the tabular values.

Formulae for error in polynomial Interpolation:

If y = f(x) is the exact curve and $y = \phi_n(x)$ is the interpolating polynomial curve, then the error in polynomial interpolation is given by

$$\boxed{\text{Error} = f(x) - \phi_n(x) = \frac{(x - x_0)(x - x_1)\ldots.(x - x_n)}{(n+1)!} f^{\,n-1}(\xi) \quad \ldots\ldots\ldots \ \ldots\ldots\ldots\ldots\ldots (7)}$$

For any x, where $x_0 < x < x_n$ and $x_0 < \xi < x_n$.

The error in Newton's Forward Interpolation formula is given

$$\boxed{f(x) - \phi_n(x) = \frac{p(p-1)(p-2)\ldots.(p-n)}{(n+1)!}\Delta^{n+1} f(\xi) \text{ where } p = \frac{x - x_n}{h} \quad \ldots\ldots\ldots.. (8)}$$

The error in Newton's Backward Interpolation formula is given by

$$\boxed{f(x) - \phi_n(x) = \frac{p(p+1)(p+2)\ldots.(p+n)}{(n+1)!} h^{n+1} f^{\,n+1}(\xi) \text{ where } p = \frac{x - x_n}{h} \quad \ldots\ldots. (9)}$$

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Summary on Formulae **SUM-03**

CENTRAL DIFFERENCE INTERPOLATION

As mentioned earlier, Newton's forward interpolation formula is useful to find the value of y = f(x) at a point which is near the beginning value of x and the Newton's Backward Interpolation Formula is useful to find the value of x and the Newton's Backward Interpolation formula is useful to find the value of "y" at a point which is near the terminal value of x. We now derive the Interpolation Formulas that can be employed to find the value of x which is around the middle to the specified values.

For this purpose, we take x_0 as one of the specified values of x that lies around the middle of the difference table and denote $x_0 - rh$ by x_{-r} and the corresponding value of y by y_{-r}. Then the middle part of the forward difference table will appear as shown below:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
….	….					
x_{-4}	y_{-4}					
		Δy_{-4}				
x_{-3}	y_{-3}		$\Delta^2 y_{-4}$			
		Δy_{-3}		$\Delta^3 y_{-4}$		
x_{-2}	y_{-2}		$\Delta^2 y_{-3}$		$\Delta^4 y_{-4}$	
		Δy_{-2}		$\Delta^3 y_{-3}$		$\Delta^5 y_{-4}$
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$	
		Δy_{-1}		$\Delta^3 y_{-2}$		$\Delta^5 y_{-3}$
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$	
		Δy_0		$\Delta^3 y_{-1}$		$\Delta^5 y_{-2}$
x_1	y_1		$\Delta^2 y_0$		$\Delta^4 y_{-1}$	
		Δy_1		$\Delta^3 y_0$		$\Delta^5 y_{-1}$
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		Δy_2		$\Delta^3 y_1$		
x_3	y_3		$\Delta^2 y_2$			
		Δ_3				
x_4	y_4					
…..	…..					

From the table, we note the following:

$\Delta y_0 = \Delta y_{-1} + \Delta^2 y_{-1}$,

$\Delta^2 y_0 = \Delta^2 y_{-1} + \Delta^3 y_{-1}$,

$\Delta^3 y_0 = \Delta^3 y_{-1} + \Delta^4 y_{-1}$,

$\Delta^4 y_0 = \Delta^4 y_{-1} + \Delta^5 y_{-1}$ .... and so on (1).

And

$\Delta y_{-1} = \Delta y_{-2} + \Delta^2 y_{-2}$,

$\Delta^2 y_{-1} = \Delta^2 y_{-2} + \Delta^3 y_{-2}$,

$\Delta^3 y_{-1} = \Delta^3 y_{-2} + \Delta^4 y_{-2}$,

$\Delta^4 y_{-1} = \Delta^4 y_{-2} + \Delta^5 y_{-2}$

$\Delta^5 y_{-1} = \Delta^5 y_{-2} + \Delta^6 y_{-2}$ .... and so on (2).

By using the expressions (1) and (2), we now obtain two versions of the following Newton's Forward interpolation Formula:

$$y_p = \left[y_0 + p(\Delta y_0) + \frac{p(p-1)}{2!}(\Delta^2 y_0) + \frac{p(p-1)(p-2)}{3!}(\Delta^3 y_0) + \frac{p(p-1)(p-2)(p-3)}{4!}(\Delta^4 y_0) + ... \right] \quad .... (3)$$

here y_p is the value of y at $x = xp = x_0 + ph$.

GAUSS's FORWARD INTERPOLATION FORMULA:

Substituting for $\Delta^2 y_0$, $\Delta^3 y_0$....... from (1) in the formula (3) we get,

$$y_p = \begin{bmatrix} y_0 + p(\Delta y_0) + \frac{p(p-1)}{2!}(\Delta^2 y_{-1} + \Delta^3 y_{-1}) + \frac{p(p-1)(p-2)}{3!}(\Delta^3 y_{-1} + \Delta^4 y_{-1}) + \\ \frac{p(p-1)(p-2)(p-3)}{4!}(\Delta^4 y_{-1} + \Delta^5 y_{-1}) + \ldots\ldots\ldots \end{bmatrix}$$

$$y_p = \left[y_0 + p(\Delta y_0) + \frac{p(p-1)}{2!}(\Delta^2 y_{-1}) + \frac{p(p-1)(p-2)}{3!}(\Delta^3 y_{-1}) + \frac{p(p-1)(p-2)(p-3)}{4!}(\Delta^4 y_{-1}) + \ldots\ldots\ldots \right]$$

Substituting for $\Delta^4 y_{-1}$ from (2), this becomes

$$y_p = \left[y_0 + p(\Delta y_0) + \frac{p(p-1)}{2!}(\Delta^2 y_{-1}) + \frac{p(p-1)(p-2)}{3!}(\Delta^3 y_{-1}) + \frac{p(p-1)(p-2)(p-3)}{4!}(\Delta^4 y_{-2}) + \ldots\ldots\ldots \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

This version of Newton's Forward Interpolation Formula is known as the Gauss's Forward Interpolation Formula.

We observe that the formula (4) contains y_0 and the even differences $\Delta^2 y_{-1}$, $\Delta^4 y_{-2}$, ... which lie on the line containing x_0 (called the central line) and the odd differences Δy_0, $\Delta^3 y_{-1}$ ...... which lie on the line just below this line, in the difference table.

Note: we observe the following difference table that

$\Delta y_0 = \Delta y_{1/2}$

$\Delta^2 y_{-1} = \Delta^2 y_0$

$\Delta^3 y_{-1} = \Delta^3 y_{1/2}$

$\Delta^4 y_{-2} = \Delta^4 y_0$ and so on.

Accordingly the formula (4) can be re-written in the notation of central differences as given below:

$$y_p = \left[y_0 + p(\delta y_{1/2}) + \frac{p(p-1)}{2!}(\delta^2 y_0) + \frac{p(p-1)(p-2)}{3!}(\delta^3 y_{1/2}) + \frac{p(p-1)(p-2)(p-3)}{4!}(\delta^4 y_0) + \ldots\ldots\ldots \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

GAUSS's BACKWARD INTERPOLATION FORMULA:

Next, let us substitute for Δy_0, $\Delta^2 y_0$, $\Delta^3 y_0$ ...... from (1) in the formula (3). Thus we obtain

$$y_p = \left[y_0 + p(\Delta y_{-1} + \Delta^2 y_{-1}) + \frac{p(p-1)}{2!}(\Delta^2 y_{-1} + \Delta^3 y_{-1}) + \frac{p(p-1)(p-2)}{3!}(\Delta^3 y_{-1} + \Delta^4 y_{-1}) + \frac{p(p-1)(p-2)(p-3)}{4!}(\Delta^4 y_{-1} + \Delta^5 y_{-1}) + \ldots\ldots \right]$$

$$y_p = \left[y_0 + p(\Delta y_{-1}) + \frac{p(p+1)}{2!}(\Delta^2 y_{-1}) + \frac{p(p-1)(p+1)}{3!}(\Delta^3 y_{-1}) + \frac{p(p-1)(p+1)(p-2)}{4!}(\Delta^4 y_{-1}) + \ldots\ldots \right]$$

Substituting for $\Delta^3 y_{-1}$, $\Delta^4 y_{-1}$ from (2), this becomes

$$y_p = \left[y_0 + p(\Delta y_{-1}) + \frac{p(p+1)}{2!}(\Delta^2 y_{-1}) + \frac{p(p-1)(p+1)}{3!}(\Delta^3 y_{-1} + \Delta^4 y_{-2}) + \frac{p(p-1)(p+1)(p-2)}{4!}(\Delta^4 y_{-2} + \Delta^5 y_{-2}) + \ldots\ldots \right]$$

$$y_p = \left[y_0 + p(\Delta y_{-1}) + \frac{p(p+1)}{2!}(\Delta^2 y_{-1}) + \frac{p(p-1)(p+1)}{3!}(\Delta^3 y_{-2}) + \frac{p(p-1)(p+1)(p-2)}{4!}(\Delta^4 y_{-2}) + \ldots\ldots \right]$$

.... (6)

The version of the Newton's Forward Interpolation Formula is known as the Gauss's Backward Interpolation Formula.

Observe the formula (6) contains y0 and the even differences $\Delta^2 y_{-1}$, $\Delta^4 y_{-2}$ .... which lie on the central line and the odd differences Δy_{-1}, $\Delta^3 y_{-2}$... which lie on the line just above this line.

Note: In the notation of central differences the formula reads

$$y_p = \left[y_0 + p(\delta y_{-1/2}) + \frac{p(p+1)}{2!}(\delta^2 y_0) + \frac{p(p-1)(p+1)}{3!}(\delta^3 y_{-1/2}) + \frac{p(p-1)(p+1)(p-2)}{4!}(\delta^4 y_0) + \ldots\ldots \right]$$

.... (7)

INTERPOLATION WITH UNEVENLY SPACED POINTS:

In the previous classes of lectures we have derived interpolation formulae which are of great importance. But in those formulae the disadvantage is that the values of the independent variables are to be equally spaced. We desire to have interpolation formulae with unequally spaced values of the independent variables. We discuss Lagrange's Interpolation Formula which uses only function values.

LAGRANGE'S INTERPOLATION FORMULA:

Let $x_0, x_1, x_2, \ldots, x_n$ be the (n + 1) values of x which are not necessarily equally spaced. Let $y_0, y_1, y_2, \ldots, y_n$ be the (n + 1) values of y = f(x).
Let the polynomial of degree n for the function y = f(x) passing through the (n + 1) points $(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2)), \ldots, (x_n, f(x_n))$ be in the following form

$$\begin{aligned} y = f(x) = {} & a_0 (x - x_1)(x - x_2) \ldots\ldots\ldots\ldots (x - x_n) + \\ & a_1 (x - x_0)(x - x_2) \ldots\ldots\ldots\ldots (x - x_n) + \\ & a_2 (x - x_0)(x - x_1)(x - x_3) \ldots\ldots (x - x_n) + \\ & \text{---} + \\ & a_n (x - x_0)(x - x_1) \ldots\ldots\ldots\ldots (x - x_{n-1}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1) \end{aligned}$$

where $a_0, a_1, a_2, \ldots, a_n$ are constants.

Since the polynomial passes through $(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2)), \ldots, (x_n, f(x_n))$, the constants can be determined by substituting one of the values of $x_0, x_1, x_2, \ldots, x_n$ for x in the above equation (1).

Putting $x = x_0$ in (1) we get, $f(x_0) = a_0 (x_0 - x_1)(x_0 - x_2) \ldots\ldots\ldots\ldots (x_0 - x_n)$

$$\Rightarrow \quad a_0 = \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)\ldots.(x_0 - x_n)}$$

Putting $x = x_1$ in (1) we get, $f(x_1) = a_1 (x_1 - x_0)(x_1 - x_2) \ldots\ldots\ldots\ldots (x_1 - x_n)$

$$\Rightarrow \quad a_1 = \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)\ldots.(x_1 - x_n)}$$

Similarly,
Substituting $x = x_2$ in (1) we get, $f(x_2) = a_2 (x_2 - x_0)(x_2 - x_1) \ldots\ldots (x_2 - x_n)$

$$\Rightarrow \quad a_2 = \frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)\ldots.(x_2 - x_n)}$$

Continuing in this manner and
Putting $x = x_2$ in (1) we get, $f(x_n) = a_n (x_n - x_0)(x_n - x_1) \ldots\ldots (x_n - x_{n-1})$

$$\Rightarrow \quad a_n = \frac{f(x_n)}{(x_n - x_0)(x_n - x_1)\ldots.(x_n - x_{n-1})}$$

Substituting these values of a_0, a_1, a_2, ….., a_n we get,

$$f(x) \approx \frac{(x-x_1)(x-x_2).....(x-x_n)}{(x_c-x_1)(x_0-x_2).....(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2).....(x-x_n)}{(x_1-x_0)(x_1-x_2).....(x_1-x_n)} f(x_1) \; +$$

$$\frac{(x-x_0)(x-x_1).....(x-x_n)}{(x_2-x_0)(x_2-x_1).....(x_2-x_n)} f(x_0) + ... + \frac{(x-x_0)(x-x_1).....(x-x_{n-1})}{(x_n-x_0)(x_n-x_1).....(x_n-x_{n-1})} f(x_n)$$

This is known as Lagrange's Interpolation Formula.

This can be expressed as $$f(x) \approx \sum_{k \approx 0}^{n} f(x_k) \prod_{\substack{j \;\approx\; 0 \\ j \;\neq\; k}}^{n} \frac{(x-x_j)}{(x_k-x_j)}$$

Another form is :

$$f(x) \approx \frac{(x-x_2)(x-x_3).....(x-x_n)}{(x_1-x_2)(x_1-x_3).....(x_1-x_n)} f(x_1) + \frac{(x-x_1)(x-x_3)(x-x_4).....(x-x_n)}{(x_1-x_0)(x_1-x_2).....(x_1-x_n)} f(x_2) \; +$$

$$.........\frac{(x-x_1)(x-x_2).....(x-x_{n-1})}{(x_n-x_1)(x_n-x_2).....(x_n-x_{n-1})} f(x_n)$$

Lakireddy Bali Reddy College of Engineering, Mylavaram
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B.Tech I Year (II-Semester) May/ June 2014. **T 264- Numerical Methods**

UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Solutions **Tutorial-4**

Problem #01. The following data give the melting point of an alloy of lead and zinc, θ is the temperature in degrees centigrade x is percent of load.

x:	40	50	60	70	80	90
θ:	184	204	226	250	276	304

Find θ when x = 43 and when x = 84? [**Ans.** 189.79, 286.96]

Solution: The values of x and θ differences of θ are tabulated below:

x	θ	Δθ	$\Delta^2\theta$
40	184		
		20	
50	204		2
		22	
60	226		2
		24	
70	250		2
		26	
80	276		2
		28	
90	304		

By Newton's Interpolation Formula, $\theta = \theta_0 + p\Delta\theta_0 \; \frac{p(p-1)}{2!}\Delta^2\theta_0$

Here $\theta_0 = 184$, $\Delta\theta_0 = 20$, $\Delta^2\theta_0 = 2$

To find $\theta(_{43})$ $p = \frac{x - x_0}{h} \approx \frac{43-40}{10} \approx 0.3$

$\theta(_{43})$ $p = 184 + (\,0.3\,)\,20 + \frac{(0.3)(0.3-1)}{2\,!} X\, 2.0 \approx 189.79$

Since x = 84 is near the end of the table, we have to use by Newton's Backward Interpolation Formula

$\theta = \theta_n + p\,\Delta\theta_n + \frac{p(p+1)}{2!}\Delta^2\theta_n$ Also – $p = \frac{x_n - x}{h} \approx \frac{90-84}{10} \approx 0.6$, p = - 0.6, x = - 0.6

$\theta(_{84})$ $= 304 + (-0.6)\,28 + \frac{(-0.6)(-0.6\ -1)}{2} X\, 0.2 \approx 286.96$. Hence the solution.

Problem #02. Find the value of y from the following data at x = 0.47?

x:	0.0	0.1	0.2	0.3	0.4	0.5
y:	1.0000	1.1103	1.2428	1.3997	1.5836	1.7974

[**Ans.** 1.7299]

Solution: Since we require the value of corresponding to a value of x near the end of the table, Newton's Backward Formula is to be used.

The backward table is given as below:

x	Y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0.0	1.0000				
		0.1103			
0.1	1.1103		0.0222		
		0.1325		0.0022	
0.2	1.2428		0.0244		0.0004
		0.1569		0.0026	
0.3	1.3997		0.0270		0.0003
		0.1839		0.0029	
0.4	1.5836		0.0299		
		0.2138			
0.5	1.7974				

Here h = 0.1,

$$- p = \frac{x_n - x}{h} \approx \frac{0.50 - 0.47}{0.1} \approx -\frac{0.03}{0.10}$$

implies p = - 0.3

Newton's Backward Difference Formula is

$$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4\,!}\nabla^4 y_n$$

$$=1.7974 + (-0.3)(0.2138) + \frac{(-0.3)(-0.3+1)}{2\,!}(0.0299) + \frac{(-0.3)(-0.3+1)(-0.3+2)}{3\,!}(0.0029)$$

$$+ \frac{(-0.3)(-0.3+1)(-0.3+2)(-0.3-3)}{4\,!}(0.0003)$$

= 1.7299

Hence the solution.

Problem #03. The following data given, the values of y are consecutive terms of which 23.6 is the 6^{th} term. Find the first and tenth terms of the series.

x:	3	4	5	6	7	8	9
y:	4.8	8.4	14.5	23.6	36.2	52.8	73.9

[**Ans.** 3.1, 100]

Solution:

Since we require the value of corresponding to a value of x near the end of the table, Newton's Forward Formula is to be used.

The Forward table is given as below:

x	Y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
3	4.8				
		3.6			
4	8.4		2.5		
		6.1		0.5	
5	14.5		3.0		0
		9.1		0.5	
6	23.6		3.5		0
		12.6		0.5	
7	36.2		4.0		0
		16.6		0.5	
8	52.8		4.5		
		21.1			
9	73.9				

Here h = 1, x = 1, x_0 = 3 and

$$p = \frac{x - x_0}{h} \approx \frac{1-3}{1} \approx -2$$

implies p = - 2

Newton's Forward Difference Formula is

$$y(1) = y_{-2} = 4.8 + \frac{(-2)}{1} \times 3.6 + \frac{(-2)(-3)}{1\cdot 2} \times 2.5 + \frac{(-2)(-3)(-4)}{1\cdot 2\cdot 3} \times 0.5 = 3.1$$

to find the 10 th term, use Newton's Backward Difference Formula with

$x_n = 9$, x = 10, h = 1 and $p = \frac{x - x_0}{h} \approx \frac{10-9}{1} \approx 1$

$$y(10) = y_1 = 73.9 + \frac{1}{1} \times (21.1) + \frac{1\cdot 2}{2!} \times (4.5) + \frac{1\cdot 2\cdot 3}{3\,!} \times (0.5) = 100$$

Hence the solution.

Problem #04. Usin the Newton's forward interpolation formula and find the value of sin 52^0 from the following data. Estimate the error

x:	45^0	50^0	55^0	60^0
y=sin x:	0.7071	0.7660	0.8192	0.8660

[**Ans.** 0.0000392]

Solution:

x	y = sin x	Δy	$\Delta^2 y$	$\Delta^3 y$
45^0	0.7071			
		0.0589		
50^0	0.7660		-0.0057	
		0.0539		-0.0007
55^0	0.8192		-0.0064	
		0.0468		
60^0	0.8680			

$$p = \frac{x - x_0}{h} \approx \frac{52-45}{50} \approx 1.4$$

From Newton's Forward Formula

$$\phi(x) = \phi(x_0 + ph)$$

$$= y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{p(p-1)(p-2)\ldots.[p-(n-1)]}{n!}\Delta^n y_0$$

$$= 0.7071 + 1.4 \times 0.0589 + \frac{(1.4)(1.4-1.0)}{2} \times (-0.0057) + \frac{(1.4)(1.4-1.0)(1.4-2)}{6} \times (-0.0007)$$

$$= 0.7071 + 0.8246 - 0.001596 + 0.0000392$$

$$= 0.7880032$$

Sin 52^0 = 0.788032

$$\text{Error} = \frac{p(p-1)(p-2)\ldots.(p-n)}{n+1!}\Delta^{n+1} y_0 ; \quad \text{Taking n = 2}$$

$$\text{Error} = \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 = \frac{(1.4)(1.4-1.0)(1.4-2)}{6} \times (-0.0007) = 0.0000392$$

Hence the solution.

Problem #05. The following data gives the distance in nautical miles of the visible horizon for the given heights in feet above the earths surface.

Height :x	100	150	200	250	300	350	400
Distance:y	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find y when x = 218 ft. and when x = 410 ft.? [**Ans.** 15.7, 21.53 naut. miles]

Solution:

X	Y	Δy	Δ2y	Δ3y	Δ4y
100	10.63				
		2.40			
150	13.03		-0.39		
		2.01		0.15	
200	15.04		-0.24		-0.07
		1.77		0.08	
250	16.81		-0.16		-0.05
		1.61		0.03	
300	18.42		-0.13		-0.01
		1.48		0.02	
350	19.90		-0.11		
		1.37			
400	21.27				

Taking x_0 = 200, x = 218, h = 50, p = $\frac{x - x_0}{h} \approx \frac{218-200}{50} \approx 0.36$

Newton's Forward Interpolation Formula is given by,

Y_{218} =

$$y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots + \frac{p(p-1)(p-2)\ldots.[p-(n-1)]}{n!}\Delta^n y_0$$

$$=15.04+0.36\times 1.77+\frac{(0.36)(-0.64)}{2}\times(-0.16)+\frac{(0.36)(-0.64)(-1.64)}{6}\times(0.03)+$$
$$\frac{(0.36)(-0.64)(-1.64)(-2.64)}{24}\times(-0.01)$$

= 15.04 + 0.637 + 0.018 + 0.02 + 0.0004

= 15.697

= 15.7 nautical miles

To find the value of y when x = 410 which is near the end of the table, we use Newton's Backward Interpolation Formula

$x_n = 400$, $p = \frac{x - x_n}{h} \approx \frac{410-400}{50} \approx 0.2$

using the line of Backward Differences

$y_n = 21.27$, $\nabla y_n = 1.37$, $\nabla^2 y_n = -0.11$, $\nabla^3 y_n = 0.02$ etc.,

Newton's backward Formula is given by

$Y_{410} =$

$$= y_{400} + p\nabla y_{400} + \frac{p(p+1)}{2!}\nabla^2 y_{400} + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_{400} + \frac{p(p+1)(p+2)(p+3)}{4\,!}\nabla^4 y_{400}$$

$= 21.27 + 0.274 - 0.0132 + 0.0018 - 0.0007$

$= 21.53$ nautical miles. **Hence the solution.**

Problem #06. The following data of the function f(x) = 1/x find 1/ 2.72 using quadratic interpolation. Find an Estimate Error.

X	2.7	2.8	2.9
f(x)	0.3704	0.3571	0.3448

[**Ans.** $0.5 \text{ X } 10^{-5}$]

Solution:

The difference table is as below:

X	f(x)	Δf (x)	Δ^2 f(x)
2.7	0.3704		
		-0.0133	
2.8	0.3571		0.0010
		-0.0123	
2.9	0.3448		

$x = 2.72$, $p = \frac{2.72 - 2.70}{0.1} \approx 0.2$; $f(2.72) = \frac{1}{2.72}$

$$\frac{1}{2.72} = 0.3704 + 0.2\,(-0.0133) - \frac{0.2 \times 0.8}{2} \times 0.0010$$

$$= 0.3704 - 0.0026 - 0.00008 = 0.3677$$

$$\text{Error} = \frac{h^3}{3\,!}x(x-1)(x-2)(x-3)\,f^{n}(\xi); E(2.72) < \frac{0.2 \times 0.8 \times 1.8}{6} \times (0.1)^3 \times \frac{6}{(2.72)^4}$$

$= 0.5 \times 10^{-5.}$ Hence the solution.

Lakireddy Bali Reddy College of Engineering, Mylavaram (Autonomous)

B.Tech I Year (II-Semester) May/ June 2014. **T 264- Numerical Methods**

UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Solutions **Tutorial-5**

Problem #01. The following data given , find the number of students whose weight is between 60 and 70.

Wt Kg x:	0-40	40-60	60-80	80-100	100-120
No of Students	250	120	100	70	50

Ans. y(70)- y(60) = 54]

Solutions:

The cumulative values and differences table is as below:

Wt: x	Stu: y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
< 40	250				
		120			
< 60	370		-20		
		100		-10	
< 80	470		-30		20
		70		10	
< 100	540		-20		
		50			
< 120	590				

$x_0 = 40,\ x = 70,\ p = \frac{70-40}{20} \approx 1.5$

$y(70) = y0 + p\Delta y_0 + \Delta^2 y_0 + \ldots\ldots\ldots\ldots$

$=250+(1.5)(120)+ \frac{(1.5)(0.5)}{2}(-20) + \frac{(1.5)(0.5)(-0.5)}{6}(-10) + \frac{(1.5)(0.5)(-0.5)(-1.5)}{24}(20)$

$= 250 + 180 - 7.5 + 0.625 + 0.46875$

$= 423.59$

$= 424$

No of students whose weight is between 60 and 70 = y(70) – Y(60)

= 424 – 370 = 54

Hence the solution.

Problem #02. The following data given the population of a town.

Year x:	1941	1951	1961	1971	1981	1991
Pop. y:	20	24	29	36	46	51

Estimate the population increases during the period 1946 – 1976?

[**Ans.** 19.4 lakhs]

Solution:

The difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^4 y$
1941	20					
		4				
1951	24		1			
		5		1		
1961	29		2		0	
		7		1		-9
1971	36		3		-9	
		10		-8		
1981	46		-5			
		5				
1991	51					

When x = 1946, p = $\frac{1946-1941}{10} \approx 0.5$

$y(1946) = y0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{6}\Delta^3 y_0 + \ldots\ldots\ldots\ldots$

$=20+(0.5)(4)+ \frac{(0.5)(-0.5)}{2}\Delta^2 y_0 + \frac{(-1.5)(0.5)(-0.5)}{6}(1) + \frac{(0.5)(-0.5)(-2.5)(-1.5)}{24}(0)$

$+ \frac{(0.5)(-0.5)(-2.5)(-1.5)(-3.5)}{120}(-9)$

= 20 + 2 – 0.125 + 0.0625 - 0.24609

= 21.69

When x = 1976, p = $\frac{1976-1991}{10} \approx -1.5$

$y(1976) = yn + p\nabla y_n + \frac{p(p+1)}{2}\nabla^2 y_n + \frac{p(p+1)(p+2)}{6}\nabla^3 y_n + \ldots\ldots\ldots\ldots$

$=51-(1.5)(5)+ \frac{(-0.5)(-1.5)}{2}(-5) + \frac{(-1.5)(0.5)(-0.5)}{6}(-8) + \frac{(0.5)(-0.5)(1.5)(-1.5)}{24}(-9)$

$+ \frac{(0.5)(-0.5)(1.5)(-1.5)(2.5)}{120}(-9)$

= 51- 7.5 – 1.875 – 6.5 – 0.2109375 – 0.10546875

= 40.8085938

Therefore increase in population during the period 41.089 – 21.69 = 19.4 lakhs. Hence the solution.

Problem #03. Find the cubic polynomial which takes the following values.

X	0	1	2	3
f(x)	1	2	1	10

Hence or otherwise evaluate f (4)? [**Ans.** 41]

Solution:

The difference table is

x	f(x)	Δ f(x)	Δ^2 f(x)	Δ^3 f(x)
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

$$p = \frac{x-0}{h} \approx x$$

Using Newton's Forward Interpolation formula, we get

$$f(x) = f(0) + \frac{x}{1}\Delta f(0) + \frac{x(x-1)}{1\cdot 2}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{1\cdot 2\cdot 3}\Delta^3 f(0) + \ldots\ldots\ldots\ldots$$

$$=1+(x)(1) + \frac{x(x-1)}{1\cdot 2}(-2) + \frac{x(x-1)(x-2)}{1\cdot 2\cdot 3}(12) + \frac{(0.5)(-0.5)(-2.5)(-1.5)(-3.5)}{120}(-9)$$

$= 2x^3 - 7x^2 + 6x + 1$ which is required polynomial

To compute f(4), we take $x_n = 3$, $x = 4$

So that $p = \frac{x - x_n}{h} \approx 1$

Using Newton's Backward Interpolation Formula, we get

$$f(4) = f(3) + p\nabla f(3) + \frac{p(p+1)}{1\cdot 2}\nabla^2 f(3) + \frac{p(p+1)(p+`2)}{1\cdot 2\cdot 3}\Delta^3 f(3) + \ldots\ldots\ldots\ldots$$

$= 10 + 9 + 10 + 12 = 41$

Which is the same value as that obtained by substituting x = 4 in the cbic polynomial $2x^3 - 7x^2 + 6x + 1$.

Hence the solution.

Problem #04. The following data given, estimate the number of students who obtained marks between 40 and 45.

Marks:	36-40	40-50	50-60	60-70	70-80
N of stu:	31	42	51	35	31

[**Ans.** 17]

Solution:

The difference table is as below:

x	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
< 40	31				
		42			
< 50	73		9		
		51		-25	
< 60	124		-16		37
		35		12	
< 70	159		-4		
		31			
< 80	190				

x = 45, x_0 = 40, p = $\frac{x - x_0}{h} \approx \frac{5}{10} \approx 0.5$

Using Newton's Interpolation Formula we get,

y(45) = y40 + $p\nabla y_{40} + \frac{p(p-1)}{2}\nabla^2 y_{40} + \frac{p(p-1)(p-2)}{6}\nabla^3 y_{40} + \frac{p(p-1)(p-2)(p-3)}{24}\nabla^3 y_{40}$

$=31+(0.5)(42)+\frac{(-0.5)(0.5)}{2}(9) + \frac{(-1.5)(0.5)(-0.5)}{6}(-25) + \frac{(0.5)(-0.5)(-2.5)(-1.5)}{24}(-9)$

= 31+ 21 – 1.125 – 1.5625 – 1.4453

= 47.87

The number of students with marks < 45 is 47.87 i.e., 48.

But the number of students with marks less than 40 is 31.

Hence the number of students getting marks between 40&45 = 48 – 31 = 17.

Hence the solution.

Problem #05. Find a polynomial of degree two which takes the values

x:	0	1	2	3	4	5	6	7
y:	1	2	4	7	11	16	22	29

[**Ans.** ½(x^2 + x + 2)]

Try Urself.............

Lakireddy Bali Reddy College of Engineering, Mylavaram (Autonomous)

B.Tech I Year (II-Semester) May/ June 2014. **T 264- Numerical Methods**

UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Problems **Tutorial-6**

Problem #01: Find f (0.2) if f (0) = 176, f (1) = 185, f (2) = 194, f (3) = 203, f (4) = 212, f (5) = 220, f (6) = 229. [**Ans.** 177.8]

Problem #02: From the following table, find y when x = 1.85 and x = 2.4 by Newton's Interpolation Formulae.

x	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$y= e^x$	5.474	6.050	6.686	7.389	8.166	9.025	9.974

[**Ans. 6.36, 11.02**]

Problem #03: Construct a polynomial for the data given below. Find also y (x = 5).

x:	4	6	8	10
y:	1	3	8	16

[**Ans.** y = $\frac{3}{8}x^2 - \frac{11}{4}x + 6,\ y(5) \approx \frac{13}{8}$]

Problem #04: given sin 45^0 = 0.7071, sin 50^0 = 0.7660, sin 55^0 = 0.8192, sin 60^0 = 0.8660 find sin 52^0 using Newton's Forward Formulae? [**A.0.788**]

Problem #05: Find y (32) if y (10) = 35.3, y (15) = 32.4, y (20) = 29.2, y (25) = 26.1, y (30) = 23.2, y (35) = 20.5. [**Ans. 22.0948**]

Problem #06: Estimate the values of f (22) and f (42) from the following available data:

x:	20	25	30	35	40	45
f(x):	354	332	291	260	231	204

[**Ans. 352, 219**]

Problem # 07: calculate $\sqrt{5.5}$ given $\sqrt{5}$ = 2.236, $\sqrt{6}$ = 2.449, $\sqrt{7}$ = 2.646, $\sqrt{8}$ = 2.828. [**Ans. 2.344**]

Problem # 08: From the following data, estimate the number of persons having incomes between 2000 and 2500.

Incomes :	< 500	500 - 1000	1000-2000	2000-3000	3000-4000
No of persons	6000	4250	3600	1500	650

[**Ans. 14,706**]

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UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Problems **Tutorial-7**

Problem # 01: From the following data, estimate the number of students who secured marks between 40 and 45.

Marks :	30-40	40-50	50-60	60-70	70-80
No of students	31	42	51	35	31

[**Ans. 17**]

Problem # 02: From the following data gives the population of a town during the last six census. Estimate the increase in the population during the period from 1976 to 1978.

Year	1941	1951	1961	1971	1981	1991
Population	12	15	20	27	39	52

[**Ans. 2530**]

Problem # 03: Using a polynomial of third degree, complete the record given below of the export of a certain commodity during five years.

Year	1989	1990	1991	1992	1993
Export Tn	443	384	---	397	467

[**Ans. 369**]

Problem # 04: The area A of a circle of a diameter d is given for the following values.

d:	80	85	90	95	100
A:	5026	5674	6362	7088	7854

Calculate the area of a circle of diameter 105. [**Ans. 369**]

Problem # 05: Construct Newton's Forward Interpolation polynomial for the following data.

x:	4	6	8	10
y:	1	3	8	16

Evaluate y for x = 5. [**Ans. 369**]

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B.Tech I Year (II-Semester) May/ June 2014. **T 264- Numerical Methods**

UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Objective Type Questions **OTQ-01**

01. If $\mu_0 = 1$, $\mu_1 = 5$, $\mu_2 = 8$, $\mu_3 = 3$, $\mu_4 = 7$, $\mu_5 = 0$ then $\Delta^5\mu_6$ =.....
A) -61 B) 61 C) -62 D) 62 [A]

02. $\Delta[f(x)]$ =.....
A) $f(x) – f(x – h)$ B) $f(x + h)$ C) $f(x + h) –f(x)$ D) $f(x – h)$ [C]

03. The relation between Δ and ∇ is....
A) $\nabla = 1 + E^{-1}$ B) $\nabla . 1 + E$ C) $\nabla = 1 - E$ D) $1 - E^{-1}$ [D]

04. If the interval of differencing is unity and $f(x) = ax^2$ find which one of the following choices is wrong...
A) $\Delta[f(x)] = a (2x +1)$ B) $\Delta^2[f(x)] = 2a$ C) $\Delta^3[f(x)] = 2$ D) $\Delta^4[f(x)] = 0$ [C]

05. If x3 – x – 4 = 0, by bisection method first two approximations x_0 and x_1 are 1 and 2 then x_2 is........
A) 1,25 B) 2.0 C) 1.75 D) 1.5 [D]

06. $3y_5$ =
A) $y_6+ 3y_5 + 3y_4 + y_3$ B) $y_5 -3y_4- 3y_3 – y_2$ C) $y_6 - 3y_5 +3y_4 -y_3$ D) $y_5+ 3y_4 + 3y_3 + y_2$
[B]

07. Gauss – forward interpolation formula is used to interpolate the values of y for
A) $0 < p < - \alpha$ B) $-\alpha < p < 0$ C) $– 1 < p < 0$ D) $0 < p < 1$ [D]

08. If first two approximations x0 and x1 are roots of $x^3 – x^2 + 1 = 0$ are 1 and 2 then x2 by regula-falsi method is .....
A) 1.05 B) 1.25 C) 1.15 D) 1.35 [B]

09. If first two approximations x0 and x1 are roots of $x^3 – 5x + 3 = 0$ is x0=0.64 then x1 by Newton Raphson Method (N.R.M) is .....
A) 0.825 B) 0.6565 C) 0.721 D) 0.6724 [B]

10. Given that

X	10	15	20
f(x)	19.97	21.51	22.47

Then $\Delta f(10)$ =......
A) – 0.58 B) 0.96 C) 1.05 D) 1.54 [D]

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B.Tech I Year (II-Semester) May/ June 2014. T 264- Numerical Methods

UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Objective Type Questions **OTQ-02**

01.$\Delta(e^x)$ = ... taking h = 1

A) $(e - 1)e^x$ B) $(e + 1)e^{-1}$ C) ec^x D) $(e + 1)e^x$ [A]

02. If f(x) = x2 + x + 1, then the value of Δf(x) is taking h = 1

A) 2x + 3 B) 2 C) 2x + 2 D) $x^2 + 1$ [B]

03. If $\Delta y = 1 + 2x + 3x^2$ which of the following is not true?

A) $y = x^2 + x^3$ B) $\Delta^3 xy = 6$ C) $\Delta^4 y = 0$ D) $\Delta^2 = 6x + y$[A]

04. Give that

X	1	2	3
f(x)	3	8	15

Then $\Delta^2 f(1)$= .....

A) 1 B) 3 C) 4 D) 2 [D]

05. Newtons iterative formula to find the value of $\sqrt{N}$ is ....

A) $x_{n+1} = \frac{1}{2}(x_n^3 - \frac{N}{x_n})$ B) $x_{n+1} = \frac{1}{2}(x_n - \frac{N}{x_n})$ C) $x_{n+1} = (2x_n - N)$ D) $x_{n+1} = \frac{1}{2}(x_n + \frac{N}{x_n})$

[D]

06. Which one is wrong ?

A) $\Delta[f_1(x)f_2(x)] = \Delta f_1(x).\ \Delta f_2(x)$ B) E 1 +Δ C) $\Delta[f_1 + f_2] = \Delta f_1 + \Delta f_2$ D) $\Delta(5)=0$

[A]

07. Newton's iterative formula to find the value of $\sqrt[3]{N}$ is ....

A)$x_{n+1}=\frac{1}{3}(2x_n - \frac{N}{x_n^2})$ B)$x_{n+1}=\frac{1}{3}(2x_n + \frac{N}{x_n^2})$ C)$x_{n+1} = \frac{1}{3}(x_n - \frac{N}{x_n^2})$ D)$x_{n+1}=(2x_n - \frac{N}{x_n^2})$

[B]

08. A linear version of the lagrange's interpolation formula for f(x) is .....

A) $\left(\frac{x-x_1}{x_0-x_1}\right)f(x_0) - \left(\frac{x-x_0}{x_1-x_0}\right)f(x_1)$ B) $\left(\frac{x-x_0}{x_0-x_1}\right)f(x_0) + \left(\frac{x_1-x}{x_0-x_1}\right)f(x_1)$

C) $\left(\frac{x-x_0}{x_0-x_1}\right)f(x_0) + \left(\frac{x-x_0}{x-x_1}\right)f(x_1)$ D) $\left(\frac{x-x_1}{x_0-x_1}\right)f(x_0) + \left(\frac{x-x_0}{x_1-x_0}\right)f(x_1)$ [D]

09. If $x^3 - 9x + 1 = 0$ the first approximation x_0 and x_1 are 0 and 1 by bisection method is ....

A) 0.5 B) 0.25 C) 0.125 D) 1 [A]

10. The order of convergence in Newton-Raphson Method (N.R.M) is .....

A) 1 B) 3 C) 0 D) 2 [D]

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B.Tech I Year (II-Semester) May/ June 2014. **T 264- Numerical Methods**

UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Problems on central diff, gauss ⇒,⇐ ward **Tutorial-8**

Problem #01: Find f(2.5) using the following data table

X	1	2	3	4
f(x)	1	8	27	64

[**Ans.** 15.625]

Problem #02: From the following table values of x and y = e^x interpolate values of y when x = 1.91.

x	1.7	1.8	1.9	2.0	2.1	2.2
e^x	5.4739	6.0496	6.6859	7.3891	8.1662	9.0250

[**Ans.** 6.7531]

Problem # 03: From the following table find y when x = 38

x	30	35	40	45	50
y	15.9	14.9	14.1	13.3	12.5

[**Ans.14**.4132]

Problem # 04: From the following table find y when x = 38

x	1.0	1.2	1.4	1.6	1.8	2.0
y	0.0	-0.112	-0.016	0.336	0.992	2.0

[**Ans.**-0.062125]

Problem #05: Use Gauss's Forward Interpolation Formula to find f(3.3) from the following table:

x	1	2	3	4	5
y=f(x)	15.30	15.10	15.00	14.50	14.00

[**Ans.14**.9000]

Problem #06: Use Gauss's Forward Interpolation Formula to find f(30) given that f(21) = 18.4708, f(25) = 17.8144, f(29) = 17.1070, f(33) = 16.3432, f(37) = 15.5154. [**Ans.16**.9210]

Problem #07: Find by Gauss's Backward Interpolation Formula the value of y at x = 1936, using the following table:

x	1901	1911	1921	1931	1941	1951
y	12	15	20	27	39	52

[**Ans.**32.3450]

Problem #08: Use Gauss's Backward Interpolation Formula to find f(32) given that f(25) = 0.2707, f(30) = 0.3027, f(35) = 0.3386, f(40) = 0.3794.

[**Ans.0**.3165]

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UNIT – III **INTERPOLATION** **Faculty Name: N V Nagendram**

Problems on central diff, gauss ⇒,⇐ ward **Tutorial-9**

Problem #01: Find f(2.36) from the following table:

x:	1.6	1.8	2.0	2.2	2.4	2.6
y:	4.95	6.05	7.39	9.03	11.02	13.46

[**Ans.** 10.5892]

Problem #02: Find f(22) from the Gauss's Forward formula:

x:	20	25	30	35	40	45
f(x):	354	332	291	260	231	204

[**Ans.**347.9831]

Problem #03: Using Gauss's Backward Formula,Find f(8) from the table:

x:	0	5	10	15	20	25
y:	7	11	14	18	24	32

[**Ans.1**3.9056]

Problem #04: Find y(25) given that $y_{20} = 24$, $y_{24} = 32$, $y_{28} = 35$, $y_{32} = 40$, Using Gauss's Forward difference Formula. [**Ans.**32.9450]

Problem #05: Given that $\sqrt{6500} = 80.6223$, $\sqrt{6510} = 80.6846$, $\sqrt{6520} = 80.7456$, $\sqrt{6530} = 80.8084$, $\sqrt{6526}$ by using Gauss's Backward Formula.

[**Ans.**80.7836]