UNIVERSITY OF GHANA

DEPARTMENT OF MATHEMATICS

MATH 223: CALCULUS II (3 credits)

EXERCISE 3 (Logarithmic and Exponential Functions)

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1. Using the properties of the natural logarithmic function, expand $\ln \dfrac{x^3 \cos^2 \pi x}{\sqrt{x^2+1}}$.

2. Find the derivatives of the following functions;
   (a) $f(x) = \ln(2x^2+1)$
   (b) $g(x) = x^2 \ln 2x$
   (c) $y = \ln|\cos x|$
   (d) $y = x^x$
   (e) $y = \ln(e^{2x} + e^{-2x})$
   (f) $f(x) = e^{-x^2}$
   (g) $y = e^{\sqrt{x+1}}$
   (h) $f(x) = 2^2$
   (i) $g(x) = 3^{\sqrt{x}}$
   (j) $y = 10^{\cos 2x}$
   (k) $f(x) = \log_3 x$
   (l) $y = \log_2|\tan x|$
   (m) $f(x) = x^2 \log(e^{2x}+1)$

3. Find the rate of change of $f(x) = \ln \dfrac{x^2(2x^2+1)^3}{\sqrt{5-x^2}}$ when $x = 1$.

4. Solve the following;
   (a) $e^{2-3x} = 6$
   (b) $\ln(2x+5) = 4$
   (c) $\int_0^3 2^x dx$

5. Find $\lim\limits_{t\to\infty} \dfrac{e^{2t}+1}{e^{2t}-1}$

6. By using the definition $\ln(1+x) = \displaystyle\int_1^{1+x} \frac{1}{t} dt$, show that for a certain range of values of x, $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots$

7. Find the solution set of the following
   (a) $2\log_{49}(2x+1) - 1 \leq 0$
   (b) $16(\frac{1}{2})^{5x+1} - 2 > 0$
8. Find the following;
   (a) $\lim\limits_{x\to\infty} \dfrac{2x^2 - 1}{5x^2 - x}$
   (b) $\lim\limits_{x\to-\infty} \dfrac{2x^4 - x^3}{5x^3 - x}$
   (c) $\lim\limits_{x\to\infty} \dfrac{x - 1}{x^2 + 2x + 1}$
9. Find the following;
   (a) $\lim\limits_{x\to\infty} \dfrac{e^x - 12x + x}{x^4 + 1}$
   (b) $\lim\limits_{x\to\infty} \dfrac{e^{-x} + 1}{x - 2}$
   (c) $\lim\limits_{x\to\infty} \sin\left(\dfrac{1}{x}\right)$