

UNIVERSITY OF GHANA

(All rights reserved)

DEPARTMENT OF MATHEMATICS

MATH 223: CALCULUS II (3 credits)

EXERCISE 1

1. Use the Mean Value Theorem to establish the following inequalities
 - (a) $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$ for $0 < x < 1$.
 - (b) $e^a(x-a) < e^x - e^a < e^x(x-a)$ if $a < x$.
 - (c) $e^x > x + 1 \forall x > 0$.
2.
 - (a) Does the function $f(x) = x^3$ satisfy the hypothesis of the Mean Value Theorem on the interval $[-10, 10]$? Justify your answer.
 - (b) If so, determine the point c which is guaranteed by the Mean Value Theorem.
3. Use the sign of the derivative to establish the inequality $\ln(1+x) > x - \frac{x^2}{2}, \forall x > 0$
4. Find the values for which the function $y = 2x^3 - 9x^2 + 12x$ is
 - (a) increasing;
 - (b) decreasing.
5. By considering the derivative of the function $f : [-1, -1] \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x^2-x+1}$, show that f^{-1} exists and find $(f^{-1})'(\frac{3}{7})$
6. By considering the derivative of the function $f : [-1, 1] \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x}{x^2+1}$, show that f^{-1} exists and find $(f^{-1})'(\frac{4}{5})$.
7.
 - (a) Show that the function $f(x) = x^3 - x^2 + 1, x > \frac{2}{3}$ has an inverse.
 - (b) If g is the inverse of f , find $g'(1)$.
8. If g is the inverse of $f(x) = \ln x + \tan^{-1} x$, find $g'(\frac{\pi}{4})$