



**UNIVERSITY OF GHANA**

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**DEPARTMENT OF MATHEMATICS**

**MATH 223: CALCULUS II (3 credits)**

**CHAPTER 2: Inverse Functions**

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At the end of the chapter, learners are expected to be able to determine:

1. the conditions under which a function may have an inverse,
2. the inverse of a given function,
3. the derivative of the inverse of a function.

**LESSON HIGHLIGHTS**

**Definition**

A function  $g$  is the inverse of the function  $f$  if

$$f(g(x)) = x, \quad \forall x \in \text{dom } g$$

and

$$g(f(x)) = x \quad \forall x \in \text{dom } f$$

**Theorem**

A function that is mapped onto its codomain has an inverse  $\Leftrightarrow$  it is one-to-one.

**Remarks**

- If  $f$  is the inverse of  $g$ , then  $g$  is the inverse of  $f$ .
- If  $f$  has an inverse  $g$ , then  $g$  is unique.
- If  $f$  is the inverse of  $g$ , then domain of  $f$  = range of  $g$  and the range of  $f$  = domain of  $g$ .

**Theorem**

If  $f$  is strictly monotonic on its entire domain, then it is one-to-one and therefore has an inverse function.

## Guidelines for finding an inverse

1. Write  $y = f(x)$
2. Solve this equation for  $x$  in terms of  $y$  (if possible)
3. Interchange  $x$  and  $y$  to obtain  $y = f^{-1}(x)$
4. Define the domain of  $f^{-1}$  to be the range of  $f$
5. Verify that  $f(f^{-1}(x)) = x$  and  $f^{-1}(f(x)) = x$ .

## Theorem

The graph of  $f$  contains the point  $(a, b) \Leftrightarrow$  the graph of  $f^{-1}$  contains the point  $(b, a)$ .

## Theorem

Let  $f$  be differentiable on its domain with an inverse  $g = f^{-1}$ . The derivative of  $g$  is given by

$$g'(x) = \frac{1}{f'(g(x))} \quad \text{where } f'(g(x)) \neq 0$$

## IMPORTANT THINGS TO NOTE

- Spend time trying the exercises on your own. This would give you an idea of what you truly understand and what you need to work on.
- Revise your notes before class and make an effort to read ahead of each class.
- Seek help before it is too late.