UNIVERSITY OF GHANA
MATH 223-CALCULUS II
SEQUENCES AND SERIES

1- a) For each of the following sequences, determine if it is increasing or decreasing and find the limit.

$$u_n = 2 - \frac{5}{n^2+1}; \; v_k = \frac{e^{-k}}{k}; \; w_m = \frac{2m+3}{m+1}$$

b) Let $(u_n)_{n\geq 0}$ be an arithmetic sequence such that $u_5 = 125$ and $u_{16} = 48$.

(i) Find the common difference and the first term. Deduce the expression of u_n as a function of n.

(ii) Find n such that $u_n = -127$.

iii) From which value of n do we have $u_n \leq -250$?

c) Let $(v_k)_{k\geq 1}$ be an arithmetic sequence such that $v_1 = -3$. Find the integer n and the common difference d so that $v_n = 6$ and $S_n = v_1 + v_2 + \ldots + v_n = 28.5$.

d) Let $(v_k)_{k\geq 1}$ be an arithmetic sequence. Find v_1 and v_n given that $n = 54$, $S_n = v_1 + v_2 + \ldots + v_n = 270$, and the common difference $d = 4$.

2- a) Let $(u_n)_{n\geq 0}$ be a geometric sequence such that $u_3 = 162$, $u_5 = 32$. Find its common ratio and the term u_0

b) Let $(u_n)_{n\geq 0}$ be a geometric sequence such that $u_0 = \frac{1}{2}$ and $u_5 \times u_7 = 1024$. What is its common ratio?

c) Let $(a_k)_{k\geq 1}$ be a geometric sequence. Find the integer k and the common ratio r so that $v_1 = 48$, $v_k = 243$ and $S_k = v_1+v_2+\ldots+v_k = 633$.

d) Consider the sequence $(u_n)_{n\geq 0}$ defined by $u_0 = 1$ and for any $n \geq 0$, $u_{n+1} = \frac{1}{2}u_n + 2n - 1$. Put $v_n = u_n - 4n + 10$.

i) Prove that (v_n) is a geometric sequence and give its common ratio. Hence write v_n as a function of n.

ii) Deduce u_n in terms of n. What is the limit of (u_n)?

iii) Put $S_n = u_0+u_1+\ldots+u_n$. Find S_n in terms of n and determine its limit.

3- a- Consider the sequence (u_n) defined by $u_0 = 1$ and $u_{n+1} = \frac{2}{3}u_n - \frac{1}{3}$.

i) Give the 6 first terms of the above sequence and make a conjecture on the behavior of the sequence.

ii) Put $v_n = u_n + 1$. Prove that (v_n) is a geometric sequence.

iii) find v_n and hence u_n as a function of n. Deduce the limit of u_n.

iv) Consider $s_n = v_0 + v_1 + \ldots + v_n$ and $S_n = u_0 + u_1 + \ldots + u_n$. Find s_n and S_n in terms of n and give their limit when $n \to \infty$.

b- Let $(u_n)_{n\geq 0}$ be a sequence defined by $u_{n+1} = \frac{2u_n+1}{u_n-1}$.

1) For which values of u_0 the sequence (u_n) is constant?

2 Put $u_0 = 2$.

i) Find the first five terms of the sequence.

ii) Consider the function $f(x) = \frac{2x+1}{x-1}$. Sketch the graph of f together with the line $y = x$.

iii) Represent graphically some first terms of the sequence and make a conjecture of the behavior of the sequence and its limit.