UNIVERSITY OF GHANA

DEPARTMENT OF MATHEMATICS

MATH 223: CALCULUS II (3 credits)
CHAPTER 1: The Mean Value Theorem (MVT) and its applications

At the end of the chapter, learners are expected to be able to:

1. apply the Mean Value Theorem (MVT) and Rolle's Theorem in solving questions,
2. determine functions which have zero derivatives,
3. determine functions which have equal derivatives,
4. determine values for which a function may be increasing or decreasing on its domain,
5. apply the Mean Value Theorem (MVT) to establish the validity or otherwise of an inequality.

LESSON HIGHLIGHTS

The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then $\exists\ p, q \in [a, b]$ such that $f(p) \leq f(x) \leq f(q)$, $\forall x \in [a, b]$.

- $f(p)$ is the extreme minimum and $f(q)$ is the extreme maximum of f on $[a, b]$.

Fermat's Theorem

If f has a relative extremum at c, then $f'(c) = 0$ or $f'(x)$ does not exist.

Rolle's Theorem

If f is continuous on $[a, b]$, differentiable on (a, b) with $f(a) = f(b)$, then $\exists\ c \in [a, b]$ such that $f'(c) = 0$.

Then Mean Value Theorem

If f is continuous on $[a, b[$ and differentiable on (a, b), then $\exists\ c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

The Generalized MVT

Suppose f and g are two continuous functions on $[a, b]$ and differentiable on (a, b). If $\forall\, x \in (a, b)$, $g'(x) \neq 0$, then $\exists\, c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

Functions with Zero Derivative

If f is differentiable on an interval I and $f'(x) = 0\ \forall\, x \in I$, then f is constant on I.

Functions with Equal Deivative

Suppose $f'(x) = g'(x)$, $\forall\, x \in (a, b)$, then f and g differ by a constant.

Definition - Increasing and Decreasing Functions

Let f be a function defined on an interval I.

- If $\forall x_1, x_2 \in I$, $x_1 < x_2$ implies that $f(x_1) \leq f(x_2)$, f is said to be increasing on I.
- If $\forall x_1, x_2 \in I$, $x_1 < x_2$ implies that $f(x_2) \leq f(x_1)$, f is said to be decreasing on I.

If the inequality is strict, then the fuction is said to be strictly increasing or decreasing.

Definition

A function is monotonic if it is either increasing or decreasing.

Theorem

Suppse f is differentiable on an open interval (a, b).

1. If $f'(x) > 0$, $\forall\, x \in (a, b)$, then f is increasing on (a, b).
2. If $f'(x) < 0$, $\forall\, x \in (a, b)$, the f is decreasing on (a, b).
3. If $f'(x) = 0\ \forall\, x \in (a, b)$, then f is constant on (a, b).

IMPORTANT THINGS TO NOTE

- Try your hands on as many exercises as possible.
- Make a point to spend time on your own with your exercises. This would give you an idea of what you truly understand and what you need to work on.
- Make it a point to read over your notes before coming to class.
- Read through what we plan to discuss before coming to class.
- See a teaching assistant or your instructor if you need further clarification.