UNIVERSITY OF GHANA

DEPARTMENT OF MATHEMATICS

MATH 223: CALCULUS II (3 credits)

PRE IA EXERCISES - 1

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1. If $y = \tanh^{-1}\left(\frac{1}{x}\right)$, express y as a natural logarithmic function of x. Sketch the graph of $\tanh^{-1}\left(\frac{1}{x}\right)$ showing it general characteristics.
2. Let $f(x) = \sinh x - (x-1)\cosh x$.
   (a) Find the local maximum and minimum values of f.
   (b) Find the point of inflection and sketch the graph of f.
3. Express $\text{cosech}^{-1}x$ in logarithmic form and hence solve the equation $\text{cosech}^{-1}x + \ln x = 3$.
4. Differentiate the following expressions with respect to x.
   (a) $\ln\sinh(x + \cosh^2 x)$
   (b) $\ln\sinh(x - \cosh^2 x)$
   (c) $\sinh\sqrt{x^2+1}$
   (d) $\sinh(xy) = x + 8$
   (e) $\ln(\coth 3x)$
   (f) $\sqrt{\tanh 3x}$
   (g) $\sinh^{-1}(\cosh 2x)$
5. Solve the following equations
   (a) $10\cosh x - 12\sinh x = 1$.
   (b) $6\tanh x = 4(1 - \text{sech}x)$
   (c) $4\tanh x = \coth x$
   (d) $\cosh x = \frac{13}{5}$
   (e) $\sinh x = \frac{3}{4}$
   (f) $\tanh x = \frac{1}{2}$
   (g) $3\coth^2 x + 4\text{cosech}x = 23$
   (h) $\sinh^{-1} x + \cosh^{-1}(x+2) = 0$
   (i) $2\tanh^{-1}\left(\frac{x-2}{x+1}\right) = \ln 2$
   (j) $4\cosh x + \sinh x = 4$
   (k) $3\sinh x - \cosh x = 1$

(l) $2\cosh 2x + 10\sinh 2x = 5$

6. If $f(x) = \cosh^{-1} x - \sinh^{-1}$ for $x > 1$, prove that $f(x)$ increases with x.
7. Show that the equation $a\cosh x + b\sinh x = 1$ has
   (a) no real solutions if $a^2 - b^2 > 1$
   (b) two solutions if $a^2 - b^2 < 1$
8. Prove that in the range $0 < \theta < \frac{\pi}{2}$, the equation $\cosh^{-1}(\sec\theta) + \ln(\sin 2\theta) = 0$ has just one solution which is $\theta = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$.
9. Prove that
   (a) $\sinh(\ln x) = \dfrac{(x - x^{-1})}{2}$ for $x > 0$
   (b) $\cosh(\ln x) = \dfrac{(x + x^{-1})}{2}$ for $x > 0$
10. Define $\sinh y$ and $\cosh y$ in terms of exponential functions and show that $2y = \ln\left(\dfrac{\cosh y + \sinh y}{\cosh y - \sinh y}\right)$.
    By putting $\tanh y = \frac{1}{3}$, deduce that $\tanh^{-1}\dfrac{1}{3} = \dfrac{1}{2}\ln 2$.
11. Given that $\sinh x = \frac{5}{12}$, find the values of $\cosh x$, $\tanh x$, $\sinh 2x$, $\coth x$, $\text{sech}x$ and $\cosh 2x$.
12. Find the value of x for which the following equations are true.
    (a)
13. Find the possible value of $\sinh x$ for which $12\cosh^2 x + 7\sinh x = 24$.
    [You may use the identity $\cosh^2 x - \sinh^2 x = 1$]
14. Find the equation of the tangent to the curve with equation $y = 3\cosh 2x - \sinh x$ at the point $x = \ln 2$.