UNIVERSITY OF GHANA

DEPARTMENT OF MATHEMATICS
SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES
1ST SEMESTER, 2018/2019 ACADEMIC YEAR

COURSE SYLLABUS

Course Code and Title: MATH 223: Calculus II
3 Credits

Prerequisites: **At least a D in MATH 122 - Calculus I**

Course Instructors:

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Mr. J. S. G. Jackson (JSGJ)	Room 23, Mathematics Department	kofigyakye2006@yahoo.com
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Eligibility for Examinations:

Please refer to the University of Ghana Regulations for Junior Members

Learning Outcomes:

At the end of this course, students should be able to

- Understand the Mean Value Theorem (MVT; including that for integrals) and apply it to find solutions to problems.
- Determine the intervals on the domain of a function, on which the function is increasing or decreasing
- Determine when a function has an inverse and find derivatives of the inverse of a function at some point on its domain
- Apply the properties of logarithmic and exponential functions in solving problems
- Perform logarithmic differentiation
- Apply l'Hopitals law to find limits of indeterminate forms
- Define hyperbolic functions and their inverses
- Apply hyperbolic identities to solve problems

- Find derivatives of hyperbolic functions and their inverses
- Define sequences
- Find sums of terms of geometric and arithmetic sequences
- Apply the criteria (root and ratio tests) for establishing convergence of sequences
- Define series
- Apply the criteria (root, ratio and integral tests) for establishing convergence of sequences
- Find the radius of convergence of a series
- Identify series that do not converge
- Define Taylor polynomials and apply the Lagrange remainder theorem in solving problems
- Evaluate Riemann sums
- Use the method of substitution, integration by parts, trigonometric substitutions, partial fractions, and the tables of anti-derivatives to evaluate definite and indefinite integrals.
- Derive reduction formulas to evaluate integrals
- Use integration techniques in the evaluation of integrals involving hyperbolic functions.
- Evaluate improper integrals
- Use the concept of the definite integral to solve problems involving the area between curves, arc length, volumes of surfaces of revolution and area of surface of revolution.
- Apply different techniques in solving first order differential equations

Outline of Topics:

1. The Mean Value Theorem (MVT) and its Applications
    - Proof of the MVT as a generalisation of Rolle's theorem.
        (a) Corollaries (Consequencies of the MVT)
            - Functions with zero derivatives
            - Functions with equal derivatives
            - Increasing and decreasing functions

        (b) Applications of the MVT
            - Use the MVT to establish the validity or otherwise of an inequality
2. Inverse Functions
    - Reflective property of inverse functions
    - The existence of an inverse function
    - The derivative of an inverse function

3. Logarithmic and Exponential Functions
    - The natural logarithmic function defined as an integral

$$\ln x = \int_1^x \frac{1}{t} dt, \qquad x > 1$$

    - The graphs of the natural logarithmic and exponential functions
    - Proofs of the basic properties of the natural logarithmic function using the definition
    - Proofs of the basic properties of the exponential function using the logarithmic properties
    - The definition of the number e
    - Behaviour of the exponential and logarithmic functions, $\exp(x)$ at $\pm\infty$, and $\ln x$ at 0^+ and at $+\infty$
    - General exponential and logarithmic functions
    - Representation of the natural logarithmic function as a limit:

$$\ln y = \lim_{x \to 0} \frac{y^x - 1}{x}, \quad y > 0$$

    - Representation of the natural exponential function as a limit:

$$e^x = \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n, \qquad e^x = \lim_{n \to 0} (1 + x)^{\frac{1}{n}}$$

    - Logarithmic inequalities and limits
    - Scales of infinity: Comparison of order of magnitude of two functions
    - Logarithmic differentiation
4. Indeterminate Forms and l'Hôpital's Rule
5. Hyperbolic Functions
    - Definition of even and odd functions
    - Definition of hyperbolic functions in terms of the exponential functions
    - Sketch of graphs of hyperbolic functions using their definitions
    - Hyperbolic identities and Osborn's Rule
    - Inverses of hyperbolic functions and their domains
    - Inverse hyperbolic functions as natural logarithmic functions
    - Derivatives of hyperbolic functions and their inverses
6. Sequences
    - Definition of a sequence - numerical and recurrent
    - Monotonicity of sequences
    - Geometric and Arithmetic sequences
    - Sums of terms of geometric and arithmetic sequences

- Criteria for convergence of sequences
  - The root and ratio tests

7. Series
   - Definition - numerical and power series
   - Convergence criteria (using ratio, root and integral tests)
   - Radius of convergence
   - Example of a series that does not converge (The Harmonic series)
8. Taylor Polynomials
   - Definition of Taylor polynomials
   - Lagrange Remainder
9. Integration
   (a) Fundamental Theorems of Calculus (FTCs)
   - Intuitive idea of the FTCs
   - MVT for intergrals
   - FTC Part I (with proof) and FTC Part II (without proof).

   (b) Riemann Sums and the Definite Integral: Integration as a Sum
   - The existence of the integral
   - The definite integral and its properties
   - Evaluation of basic integrals (with polynomials of at most degree three being the integrand) using the Riemann sum
   - Interpreting infinite limits as definite integrals

   (c) Techniques of Integration

   i. Method of Substitution
   - Trigonometric Integrals
     - Integrals of sines and cosines
     - Integrals of tangents and secants
     - Integrals of cotangents and cosecants
   - Hyperbolic Integrals
     - Some standard hyperbolic integrals (as a result of antiderivatives of the hyperbolic functions)
     - Integrals of hyperbolic sines and cosines
     - Integrals of hyperbolic tangents and secants
     - Integrals of hyperbolic cotangents and cosecants
   - Trigonometric and Hyperbolic Substitutions
     - Substitutions for integrands containing $\sqrt{a^2 - b^2x^2}$, $\sqrt{a^2 + b^2x^2}$ and $\sqrt{b^2x^2 - a^2}$

- Integration of rational functions using partial fractions
  - Integration of proper and improper rational functions
  - Special cases of rational function integrands where partial fractions can be avoided; e.g.

$$\int \frac{x^3+1}{(x-2)^4}dx, \qquad \text{let } y = x-2; \quad \int \frac{dx}{x^4(1-x)}, \qquad \text{expand } (1-x)^{-1};\ \text{etc}$$

- Integrals of the form

$$\int \frac{px+q}{ax^2+bx+c}dx, \qquad \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx, \qquad \int (px+q)\sqrt{ax^2+bx+c}dx$$

  where $a, b, c, p, q \in \mathbb{R}$ and ax^2+bx+c cannot be factored into linear expressions

- Integrals of the form

$$\int \frac{\sin^2 x+\cos x}{\cos^2 x+\sin x}dx, \qquad \int \frac{a\sin x+b\cos x+c}{a_1\sin x+b_1\cos x+c_1}dx, \qquad \int \frac{a\sinh x+b\cosh x+c}{a_1\sinh x+b_1\cosh x+c_1}dx$$

  where the substitutions $t = \tan\frac{\pi}{2}$ and $t = \tanh\frac{\pi}{2}$ are useful and $a, b, c, a_1, b_1, c_1 \in \mathbb{R}$

- Integrals of the form

$$\int \frac{a\sin x+b\cos x}{a_1\sin x+b_1\cos x}dx$$

  where the substitution

$$a\sin x+b\cos x = \lambda\frac{d}{dx}(a_1\sin x+b_1\cos x)+\mu(a_1\sin x+b_1\cos x)$$

  for some scalars λ and μ is useful

ii. Integration by parts

iii. Method of Substitution and integration by parts for definite integrals

iv. Proofs and use of the following properties in simplying and evaluating definite integrals

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx; \qquad \int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$$

10. The Reduction Formulae

(a) Reduction fomulae for indefinite integrals

- Reduction formulae for integrals of the form

$$\int x^n e^x dx, \quad n \in \mathbb{N}$$

- Reduction formulae for integrals of the form

$$\int \frac{dx}{(x^2+a^2)^n}, \quad n \in \mathbb{N}$$

(b) Reduction formulae for trigonometric and hyperbolic integrals

- Reduction formulae for integrals of the forms

$$\int \cosh^n x dsx, \qquad \int \sinh^n x dx, \qquad n \in \mathbb{N}, n \geq 2 \quad \text{etc}$$

- Reduction formulae for integrals of the forms

$$\int \sin^m x \cos^n x dx, \qquad \int \sin^x \cos nx dx, \qquad m, n \in \mathbb{N}, \quad \text{etc}$$

(c) Reduction formulae for defintie integrals

- Reduction formulae for integrals of the forms

$$\int_0^{\frac{\pi}{2}} x \sin^n x dx, \qquad \int_0^{\frac{\pi}{2}} x^n \sqrt{1-x} dx, \quad \text{etc}$$

11. Improper Integrals

    Integrals of the form $\int_a^b f(x)dx$ where

    - f is not defined at a or b or in between a and b
    - the interval of integration is infinite

12. Applications of Integration

    - Area between two curves
    - Volumes of solids of revolution
    - Arc length
    - Area of surface of revolution

13. Ordinary Differential Equations

    - Formation of ordinary differential equations
    - Solutions to first order differential equations
      - Separable equations
      - Using the integrating factor
      - Exact Equations
      - Initial value problems
      - Homogenous equations,
      - Using an integrating factor to make a differential equation exact
      - Bernoulli equations
    - Reduction of orders

Course Delivery

Topic	Allocated Time
The MVT and it Applications & Inverse Functions	5 hours
Logarithmic and Exponential Functions & Indeterminate Forms	4 hours
Hyperbolic Functions	6 hours
Sequences, Series & Taylor Polynomials	9 hours
Integration	10 hours
Reduction Formulae	4 hours
Improper Integrals	2 hours
Applications of Integration	2 hours
Oridinary Differential Equations	8 hours

References:

- *Calculus* by James Stewart
- *Calculus* by Soo T. Tan
- *Calculus and Analytic Geometry* by C.H Edwards Jr and D. E. Penny

Grading:

The course grade consists of three sections: continuous assessment (in-class tests and homeworks), interim assessment tests, and the final exam.

Activity	% of Final Score	Timeline
Final Exam	60	End of semester
Interim Assessment Tests (2)	30	Weeks 7 & 11
Quizzes	10	Weeks 3 & 9