DEPARTMENT OF MATHEMATICS
(2014/2015)SEMESTER 1

MATH 223–CALCULUS II

Exercise 5

1. Evaluate the given integral by computing $\lim_{n\to\infty}\sum_{i=1}^{n} f(t_i)\Delta x$ for a regular partition of the interval of integration. Take $t_i = x_i$ (i.e. the right end-point of each subinterval in the partition) in each case.
(a) $\int_1^5 (4-3x)dx$ (b)$\int_0^3 \left(3x^2+1\right)dx$ (c) $\int_0^b \left(x^3+4x\right)dx$ (d) $\int_0^1 \left(x^3-5x^4\right)dx$.

2. By evaluating appropriate integrals, prove that:

   (a) $\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\ldots+\frac{n}{(2n-1)^2}\to\frac{1}{2}$ as $n\to\infty$.

   (b) $\lim_{n\to\infty}\left\{\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+4}}+\ldots+\frac{1}{\sqrt{2n^2}}\right\}=\ln\left(1+\sqrt{2}\right)$.

   (c) $\lim_{n\to\infty}\sum_{r=2n}^{3n}\frac{2r}{n^2+r^2}=\ln 2$.

   (d) $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\pi}{4n}\tan\left(\frac{r\pi}{4n}\right)=\ln\sqrt{2}$.

   (e) $\lim_{n\to\infty}\left\{\frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\ldots+\sqrt{n}}{n\sqrt{n}}\right\}=\frac{2}{3}$.

3. Show that $G'(x)=\phi(h(x))h'(x)$ if $G(x)=\int_a^{h(x)}\phi(t)dt$.

4. Differentiate the function f by first writing $f(x)$ in the form $g(u)$, where u is the upper limit of the integral
(a) $f(x)=\int_1^{3x}\sin(t^2)dt$ (b) $f(x)=\int_0^{\sin x}\sqrt{1-t^2}dt$ (c) $f(x)=\int_i^{x^2+1}\frac{1}{t}dt$

5. Find the derivative of each function
(a) $f(x)=\int_{\sqrt{x}}^{x^3}\sqrt{t}\sin t dt$ (b) $\int_{\cos x}^{5x}\cos\left(u^2\right)du$

6. If $f(x)=\int_2^x\frac{1}{\sqrt{1+t^4}}dt$, find $\left(f^{-1}\right)'(0)$.

7. If f is a continuous function such that $\int_0^x f(t)dt = xe^{2x}+\int_o^x e^{-t}f(t)dt$ for all x, find an explicit expression for $f(x)$.

8. If $F(x)=\int_1^x f(t)dt$, where $f(t)=\int_1^{t^2}\frac{\sqrt{1+u^4}}{u}du$, find $F''(2)$.

9. Find the interval on which the curve $y=\int_0^x\frac{1}{1+t+t^2}dt$ is concave upward.

10. By finding a suitable substitution for each of the following integrals, evaluate:
(a) $\int\frac{x}{(1-9x^2)}dx$ (b)$\int\frac{e^x}{(1+e^x)^2}dx$ (c)$\int\frac{1+\ln x}{x}dx$ (d) $\int\frac{x-2}{1+x+\sqrt{1+x}}dx$
(e) $\int(x-1)e^{\left(x^2-2x+4\right)}dx$ (f) $\int e^x(e^x+2)^4dx$ (g) $\int x\sin\left(x^2\right)dx$ (h) $\int \text{sech}x dx$

11. By finding a suitable substitution for each of the following integrals, evaluate:
(a) $\int(4+\cos x)^3\sin x dx$ (b) $\int\cos x\sin^5 x dx$ (c) $\int\sec^3\theta\tan\theta d\theta$ (d) $\int\tan^2 x\sec^2 x dx$
(e) $\int\cos^2 x\sin^3 x dx$ (f) $\int\sin^4 x\cos^3 x dx$ (g) $\int\cot^3 x\text{cosec}^4 x dx$ (h)$\int\cot^4 x\text{cosec}^4 x dx$

12. By finding a suitable substitution for each of the following integrals, evaluate:
(a) $\int_0^{\pi/4}\frac{\sin 2x}{1+\cos^2 x}dx$ (b) $\int_0^{\pi/3}\tan^5 x\sec^3 x dx$ (c) $\int_0^{\pi/3}\sin^5 x\cos^2 x dx$
(d) $\int_{\ln 3}^{\ln 8}\frac{1}{\sqrt{e^x+1}}dx$

13. Use appropriate trigonometric identities to evaluate the following trigonometric integrals:
(a) $\int\sin^2 x\cos^2 x dx$ (b) $\int\sin^4 3x\cos^3 3x dx$ (c) $\int\cos^6 4x dx$ (d) $\int\tan^2 2x\sec^4 2x dx$
(e) $\int\cot^4 3x dx$ (f) $\int\sin 3x\cos 5x dx$ (g) $\int\sin 2x\sin 4x dx$ (h) $\int\cos x\cos 4x dx$

14. By writing each integral as a sum of two integrals (i.e splitting the integrals into two), evaluate

(a) $\int \frac{\tan x+\sin x}{\sec x} dx$ (b) $\int \frac{\cot x+\text{cosec}x}{\sin x} dx$ (c) $\int \frac{\cot x+\text{cosec}^2 x}{1-\cos^2 x} dx$ (d) $\int \frac{1+x}{1+x^2} dx$.

15. Integrate at sight the following (i.e for each integral, find, by inspection, an expression whose derivative is the integrand)

(a) $\int \frac{2\cos x-3\sin x}{3\cos x+2\sin x} dx$ (b) $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$ (c) $\int \frac{1}{x\ln x} dx$ (d) $\int \frac{\sec^2 x}{1-\tan x} dx$

(e) $\int \tan x\sqrt{\sec x} dx$ (f) $\int \left(1-2x^2\right) e^{-x^2} dx$.