Course 11

Tracking: Beyond 15 Minutes of Thought

SIGGRAPH 2001 EXPLORE INTERACTION AND DIGITAL IMAGES

Introductions

Danette Allen

Greg Welch

Gary Bishop

Why thinking about tracking is so fun

It’s a simple problem to state

It has a little of everything

- A little physics
- A little electronics
- A little math
- A little signal processing
- A little programming

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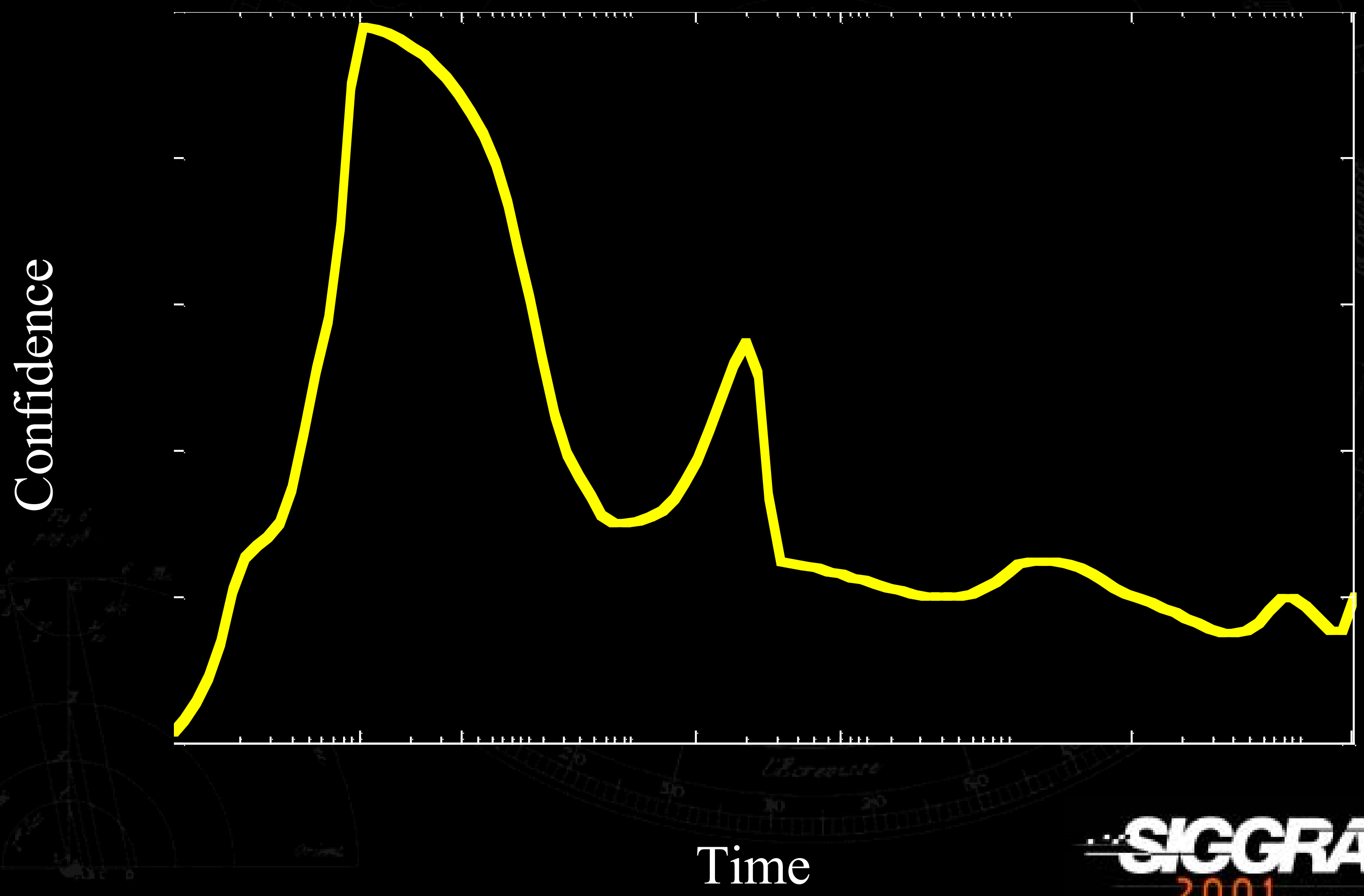
Why don't you just…

- Mount TV cameras on the walls?
- Use GPS?
- Use MEMS accelerometers?
- Use carbon nanotubes?

The 15 minute effect...



Goals

- To get you to the 15 minute point and beyond…
- To equip you to evaluate the various offerings and understand the strengths and weaknesses of each.

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Tracking Technologies

(Danette Allen)

Many ways to slice!

Configuration

- Outside-in vs. Inside-out

Type of measurement

- Absolute vs. Relative
- Range vs. angle

Physical medium

- Five categories

Five (Six) Technologies

Inertial

Acoustic

Magnetic

Mechanical

Optical

Radio (GPS) is the sixth...

- typically used outdoors
- not addressed in this course

Inertial Tracking

Passive

Newton's 2nd Law of Motion

- `F = ma` (linear)
- $\tau = I\alpha$ (rotational)

No physical limits on working volume

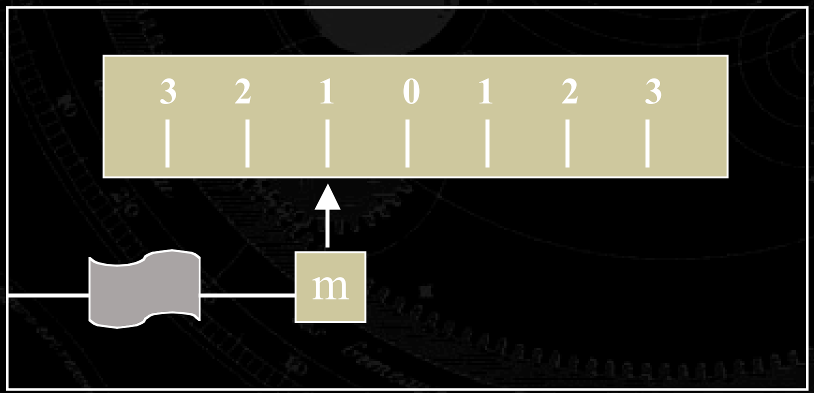
Accelerometers and gyroscopes

- Derivative measurements

Inertial Tracking

Accelerometers

- Measure force exerted on a mass since we cannot measure acceleration directly.
- Proof-mass and damped spring
  - Displacement proportional to acceleration



- Potentiometric and piezoelectric transducers

Inertial Tracking

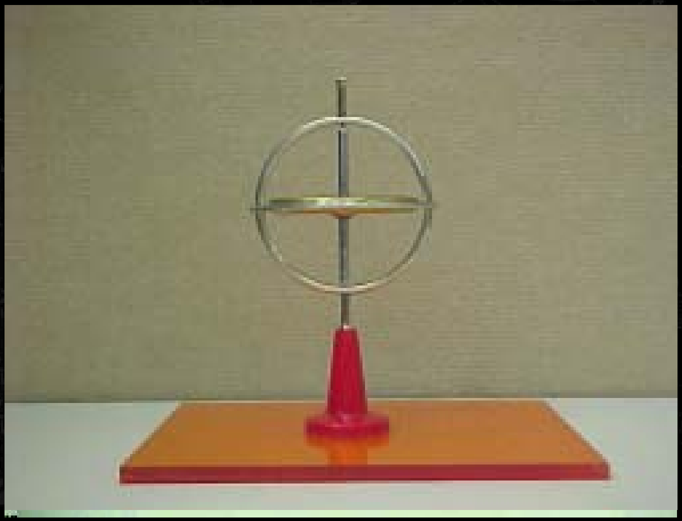
Gyroscopes

- Inertia

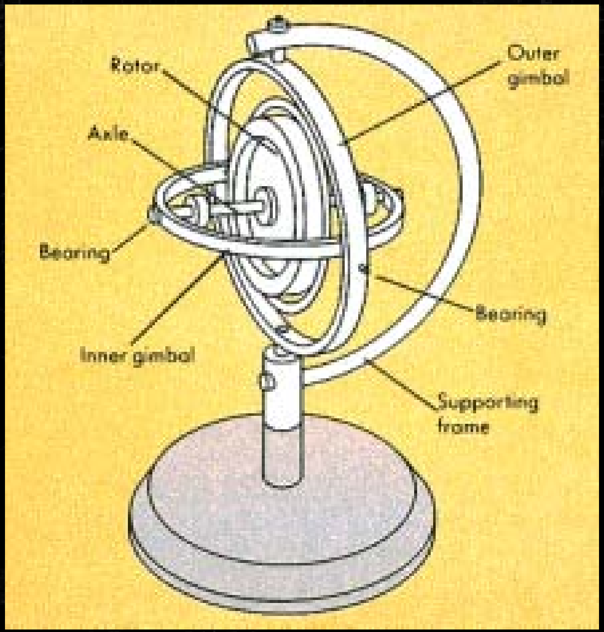
  rigidity in space

- Precession

  a comparatively slow gyration of the rotation axis of a spinning body about another line intersecting it so as to describe a cone (Mirriam-Webster)



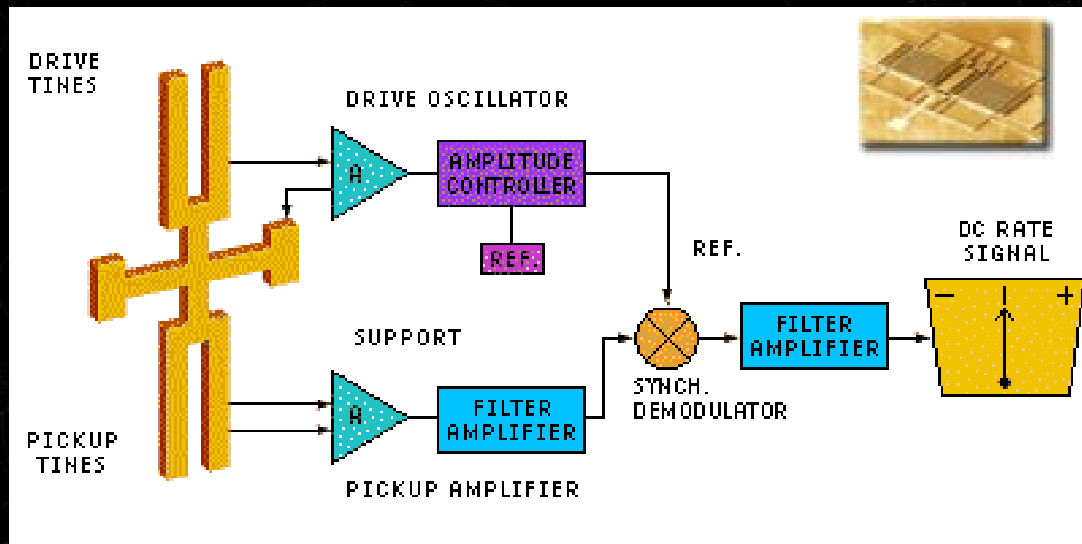
- Gimbal deflection



(Discovery, 2001)

Microgyroscope

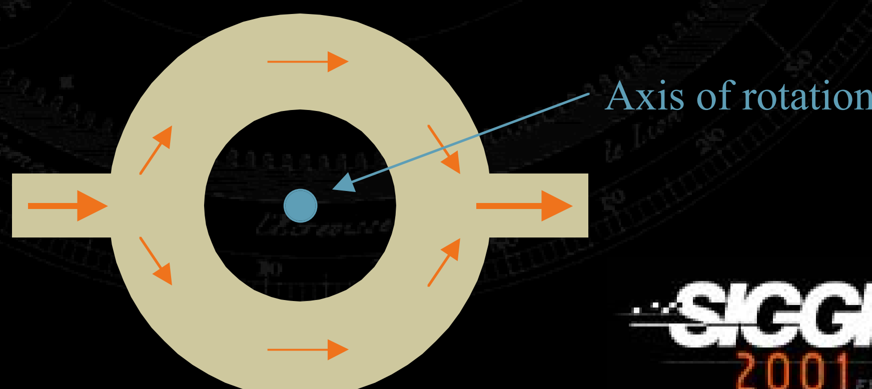
MEMS
Tuning fork



(Systron Donner, 2001)

Ring-laser and Fiber

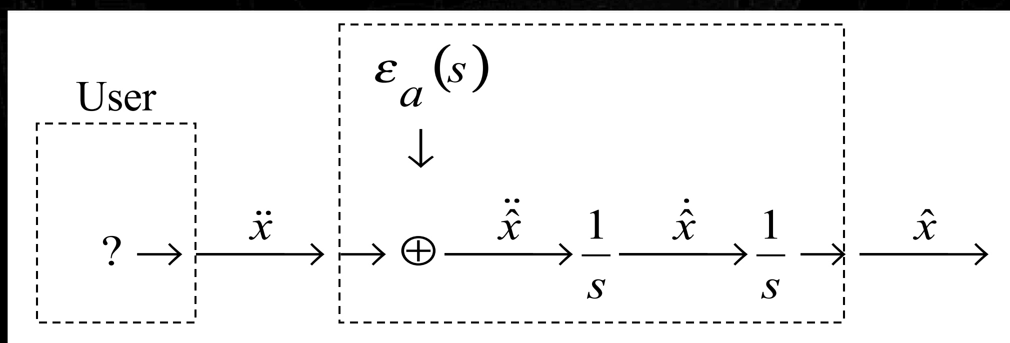
- Doppler effect
- Beat frequency



Inertial Drift

Error accumulation due to integration

- Poor SNR at low frequencies
  - Inverse square weighting of noise



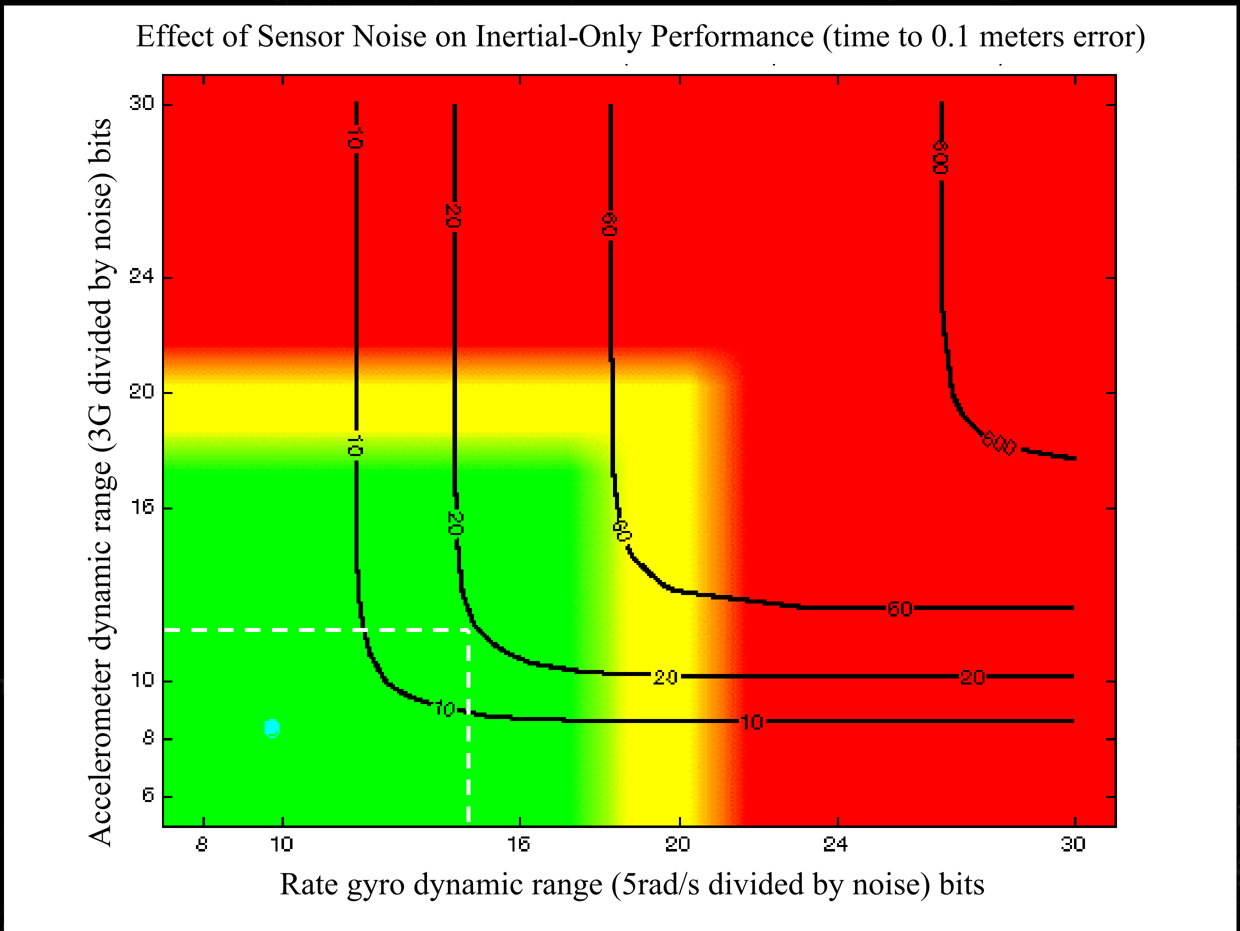
LaPlace Transform
$s = \sigma + j\omega$

- Gravity vector misalignment
  - 1° tilt error over 10 seconds $\Rightarrow$ 9m position error

Periodic recalibration

- hybrid systems typical

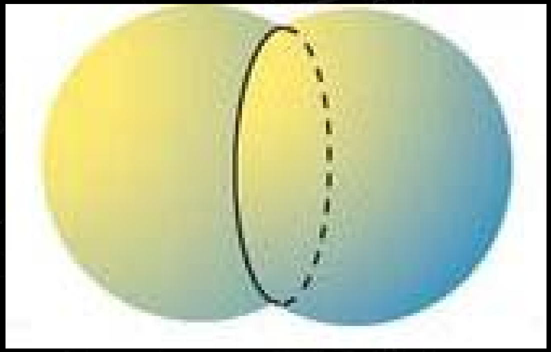
Time [s] to 0.1 [m] Error

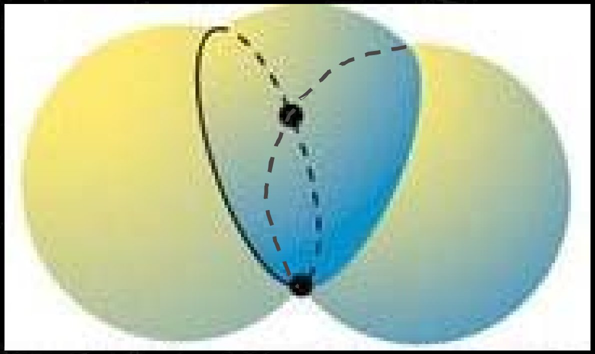


Acoustic Tracking

The Geometry

- The intersection of 2 spheres is a circle.
- The intersection of 3 spheres is 2 points.
  - One of the two points easily eliminated





Speed of Sound

- Varies with temperature and pressure
- ~ 331[m/s] in air at 0° C
  - 1 ft/ms ⇒ SLOW!!

Ultrasonic

- 40 [kHz] typical

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Acoustic Tracking Methods

Time of Flight

- Measures the time required for a sonic pulse or pattern to travel from a transmitter to a receiver.
- d [m] = v [m/s] * t [s], v = speed of sound (c)
- Absolute range measurement

Phase Coherence

- Measures phase difference between transmitted and received sound waves
- Relative to previous measurement
  - still absolute!!

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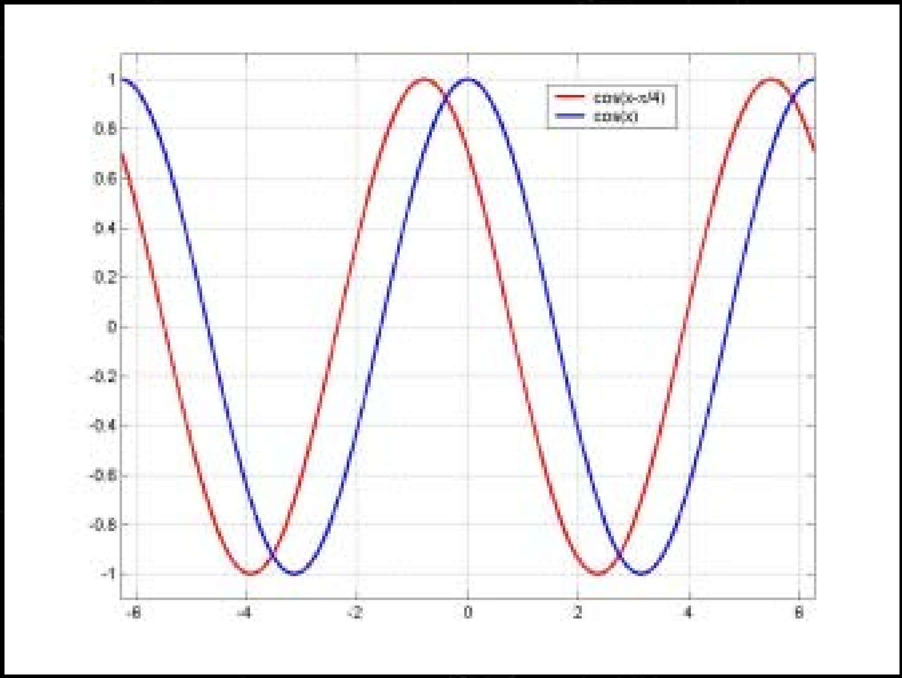
Phase Coherence

Equations

- $A \cos(\omega t - \phi)$
- $c[m/s] = \lambda[m] * f[1/s]$
- $\delta[m] = \lambda[m] * (\phi/2\pi)$



"Relative" Result

- Fractional wavelength
- Need previous range estimate
  - No integration!!!

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Magnetic Tracking

Three mutually-orthogonal coils

$$H_r = \frac{M}{2\pi d^3}\cos\theta \qquad H_\theta = \frac{M}{2\pi d^3}\sin\theta$$

- Each transmitter coil activated serially
  - Three measurements apiece (three receiver coils)
  - Nine-element measurement for 6D position

AC vs. DC

- Ferromagnetic interference

Mechanical Tracking

- Ground-based or body-based
- Used primarily for motion capture
- Provide angle and range measurements
  - gears
  - bend sensors
- Elegant addition of force feedback

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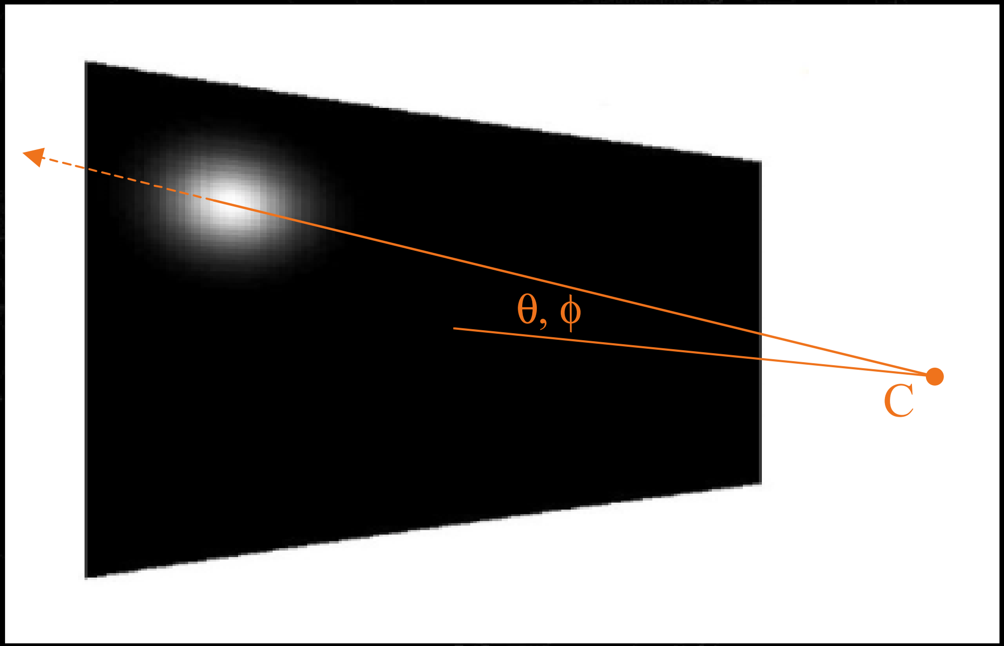
Optical Tracking

Provides angle measurements

- One 2D point defines a ray
- Two 2D points define a point for 3D position
- Additional 2D points required for orientation



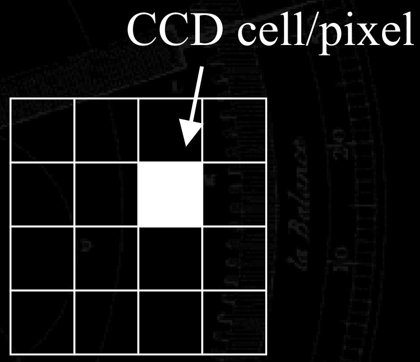
Speed of light

- 2.998 * 10^8 m/s (1 ft/ns)

Active vs. Passive Targets

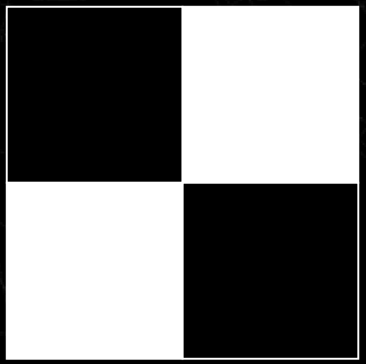
Typical detectors

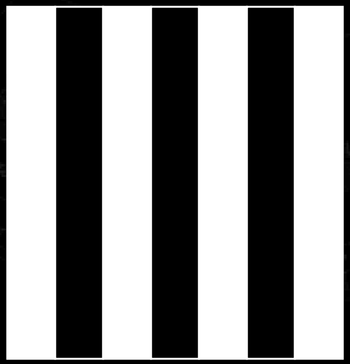
- Video and CCD cameras
  - Computer vision techniques



Passive targets

- Reflective materials, high contrast patterns



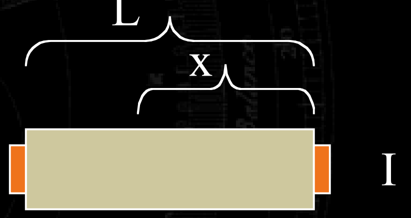


Active vs. Passive Targets

Typical detectors

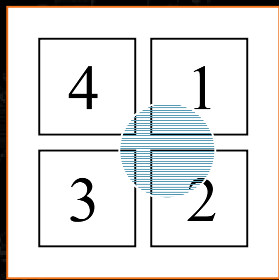
- LEPDs

$$I = I_0\left(\frac{\sinh(\alpha(L-x))}{\sinh(\alpha L)}\right) \quad or \quad I \approx I_0\left(\frac{L-x}{L}\right)$$



- Quad Cells

$$x = \frac{(i_1+i_2)-(i_3+i_4)}{i_1+i_2+i_3+i_4} \qquad y = \frac{(i_1+i_4)-(i_2+i_3)}{i_1+i_2+i_3+i_4}$$



Active targets

- LEDs

Many ways to slice!

Configuration

- Outside-in vs. Inside-out

next

Type of measurement

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Physical medium

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Source/Sensor Configurations

(Gary Bishop)

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Sensor Configurations

Geometric arrangement of sensors and sources impacts:

- accuracy
- usability
- algorithms

for example CODA mpx30

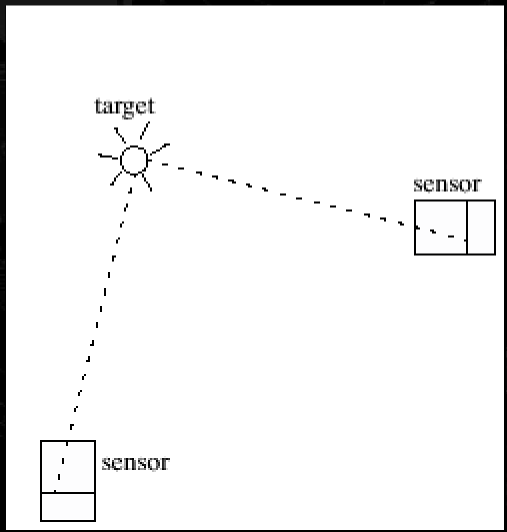
3 1-D CCDs are stationary

LED targets move

Very interesting optics and sensing

CODA mpx30

- Measures angles in lab coordinate frame
- Angle determines a plane
- Intersecting 3 planes determines a point



"Flatland"

CODA mpx30

Such "outside-looking-in" systems

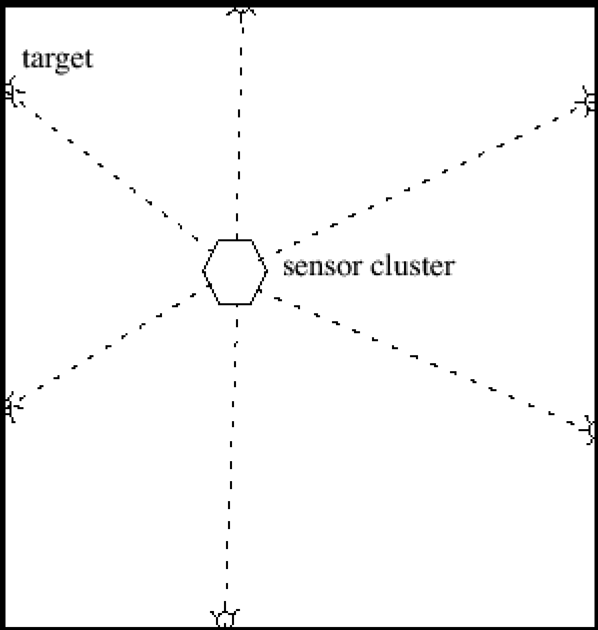
- measure position very well
- allow many small moving targets
- use multiple targets to get orientation
- trade off accuracy and working volume
- provide larger volume / more accuracy with more sensors
- use really simple math

HiBall

6 2-D sensors and 6 lenses in dodecahedron
1000's of LEDs fixed on the ceiling
Calibration gives effectively 26 cameras

HiBall

- Measures angles in user coordinate frame
- Angles determine a constraint relating
  - position
  - orientation
  - view
  - led location



“Flatland”

HiBall

Such "inside-looking out" systems

- directly measure orientation
- allow large working volume with accuracy
- are larger than LED targets
- and thus harder to use for hands, feet, etc.

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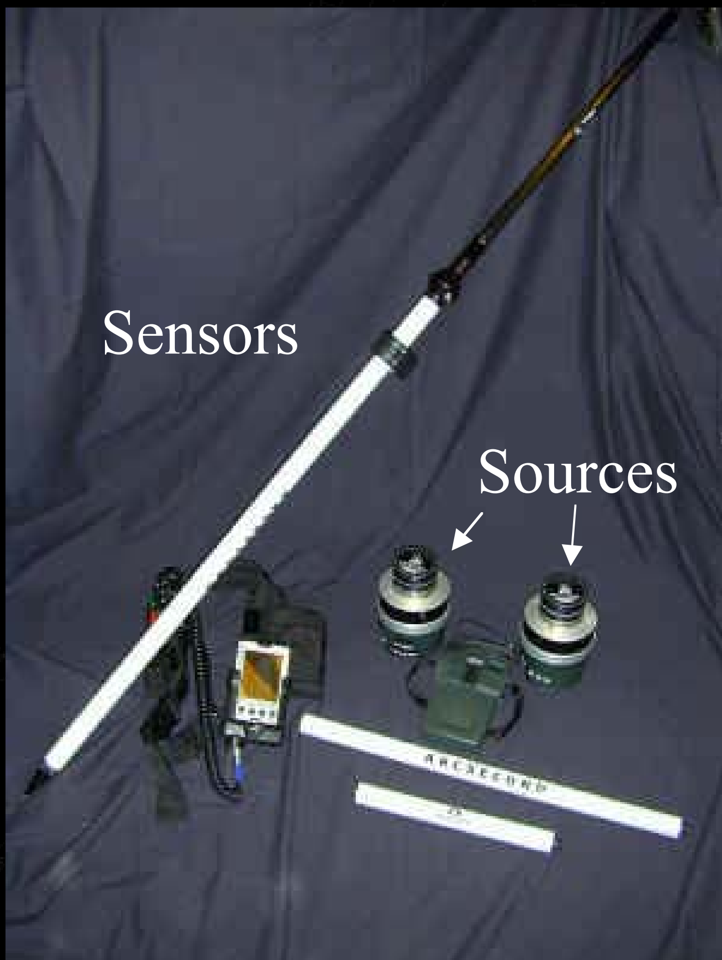
Arc Second Vulcan



- Sources scan “planes of light” through space
- Sensors on target detect passing plane

Arc Second Vulcan

- Time of plane passing converts to angle at the source
- Measures angles in world frame
- Thus like CODA and other “outside-looking-in” systems
- Direction of “looking” really isn’t the issue but coordinate frame of measurement

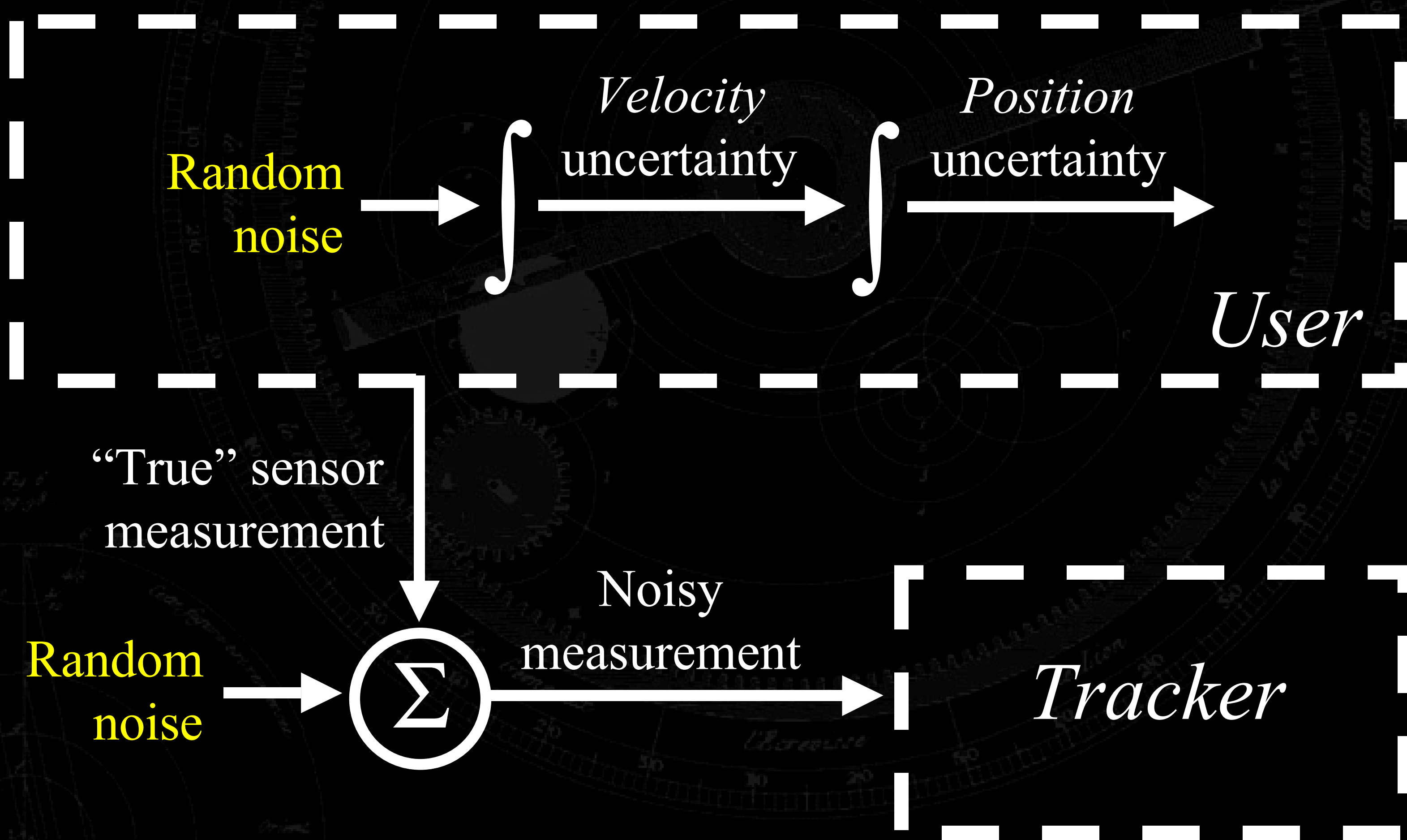
User and Sensor Uncertainty/Information

(Greg Welch)

Pose Uncertainty

- **Measurement uncertainty**
  - Pose estimates from *noisy* sensor measurements
- **User pose uncertainty**
  - Noisy and *temporally-discrete* measurements
  - Modeling user motion is difficult [Weber]
  - Modeling pose *uncertainty* is less difficult

Noise-Driven Processes



Random Variables and Signals

- **Map sample space → real numbers**
  - For example, time to voltage
- **Random Signals**
  - For example, electrical signals
  - Continuous random variables
  - Probability over a *region* of sample space
  - Spatial (statistical) and temporal (spectral) aspects

Cumulative Distribution Function

$$F_X(x) = P(-\infty, x]$$

1. $F_X(x) \to 0$ as $x \to -\infty$
2. $F_X(x) \to 1$ as $x \to +\infty$
3. $F_X(x)$ is a non-decreasing function of x

Probability Density Function

$$f_x(x) = \frac{d}{dx} F_X(x)$$

1. $f_X(x)$ is a non-negative function

2. $\int_{-\infty}^{\infty} f_X(x)dx = 1$

Probability (Continuous)

$$P_x[a,b] = \int_a^b f_x(x)\,dx$$

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Statistical Moments

$$\mu_m = E\left[X^m\right] =$$

Continuous:

$$\int_{-\infty}^{\infty} x^m f_X(x)\,dx$$

Discrete:

$$\sum_x x^m p_X(x)$$

1st Moment or *Mean*

$$\mu = E[X] =$$

Continuous:

$$\int_{-\infty}^{\infty} x f_X(x)\,dx$$

Discrete:

$$\sum_x x p_X(x)$$

Central Moments

$$c_m = E\left[(X-\mu)^m\right] =$$

Continuous:

$$\int_{-\infty}^{\infty} (x-\mu)^m f_X(x)dx$$

Discrete:

$$\sum_x (x-\mu)^m p_X(x)$$

2nd Central Moment or *Variance*

$$V[X] = E\left[(X-\mu)^2\right]$$
$$= E\left[X^2\right] - \mu^2$$

"Mean of square minus square of mean"

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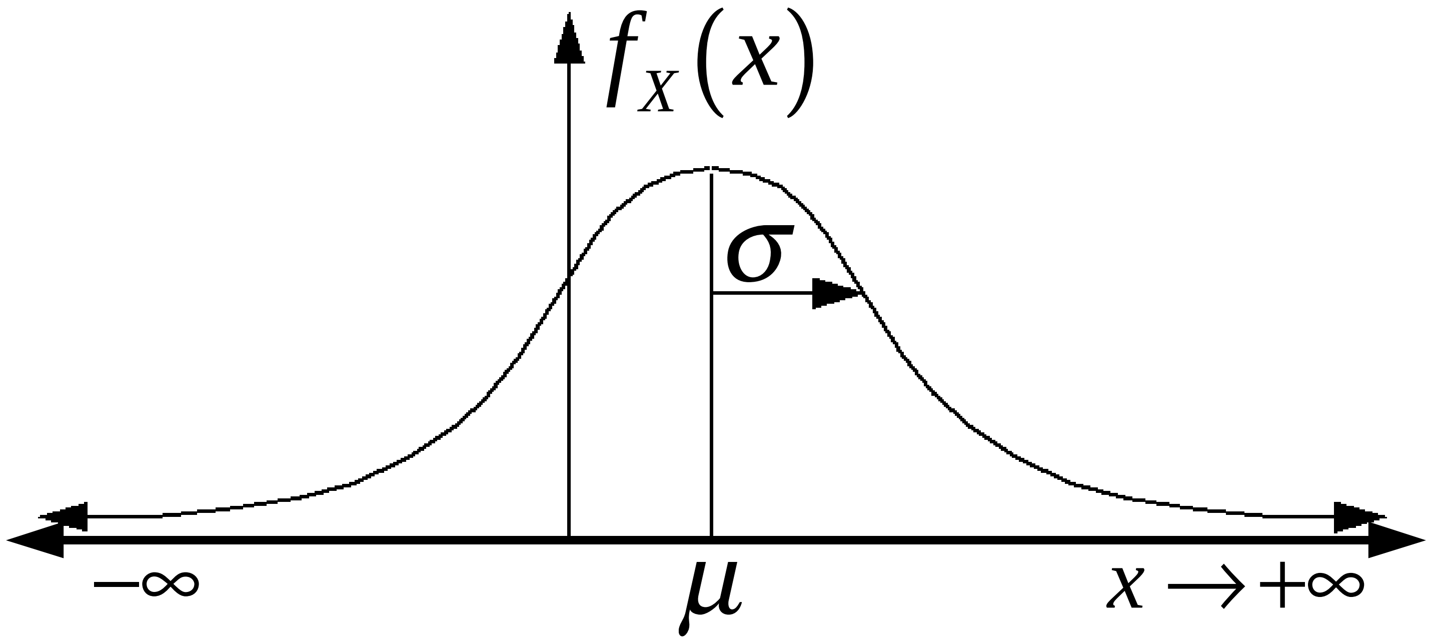
Standard Deviation

$$\sigma = \sqrt{V[X]}$$

Gaussian/Normal Distribution

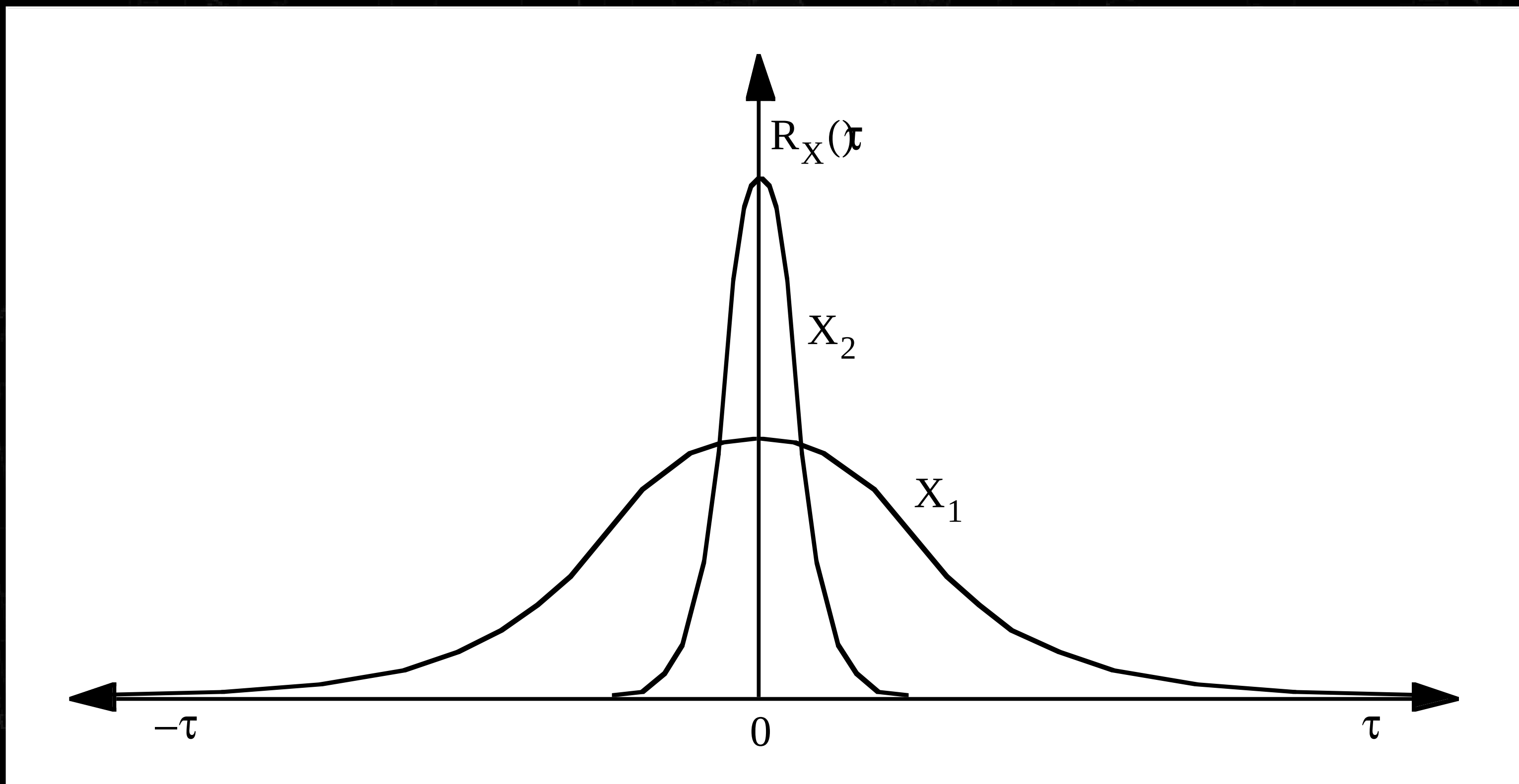
$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

$$X \sim \mathrm{N}\left(\mu, \sigma^2\right)$$



Autocorrelation (Time Domain)

$$R_X(\tau) = E[X(t)X(t+\tau)]$$



Spectral Density (Frequency Domain)

The Wiener-Khinchine relationship

$$S_X(j\omega) = F[R_X(\tau)]$$

$$= \int_{-\infty}^{\infty} R_X(\tau) e^{-j\omega\tau} d\tau$$

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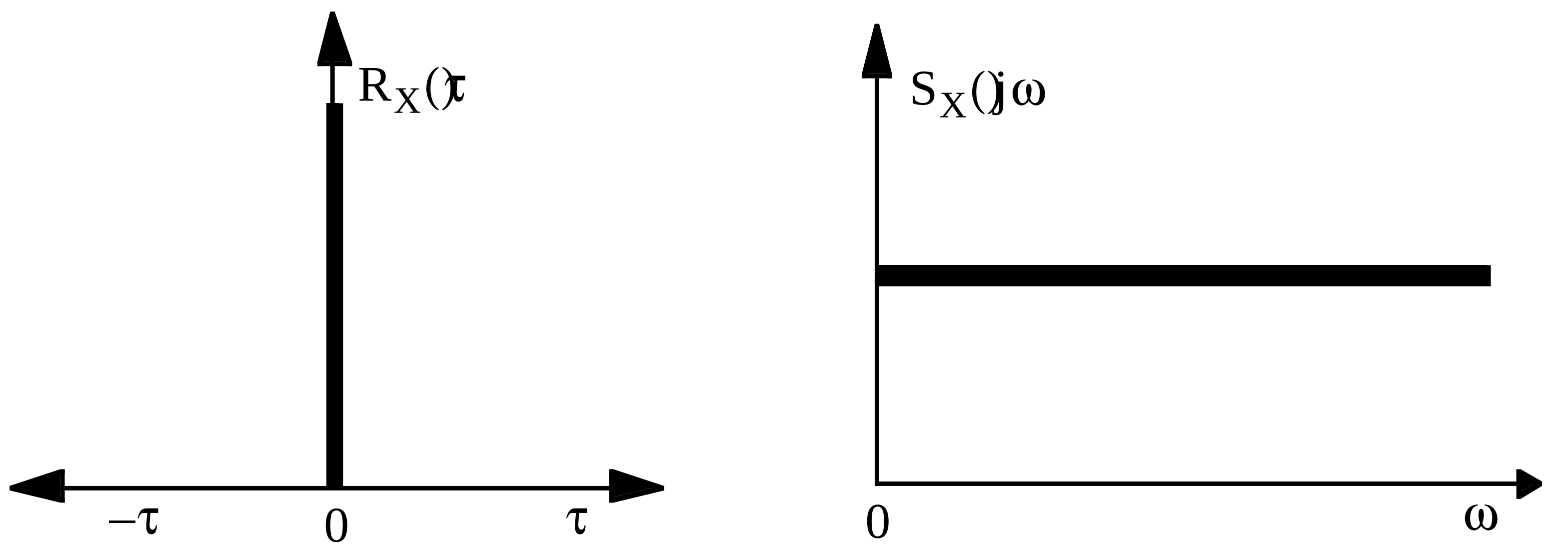
White Noise Process

$$R_X(\tau) = \begin{cases} \text{if } \tau = 0 \text{ then } A \\ \text{else } 0 \end{cases}$$

$$S_X(j\omega) = A$$



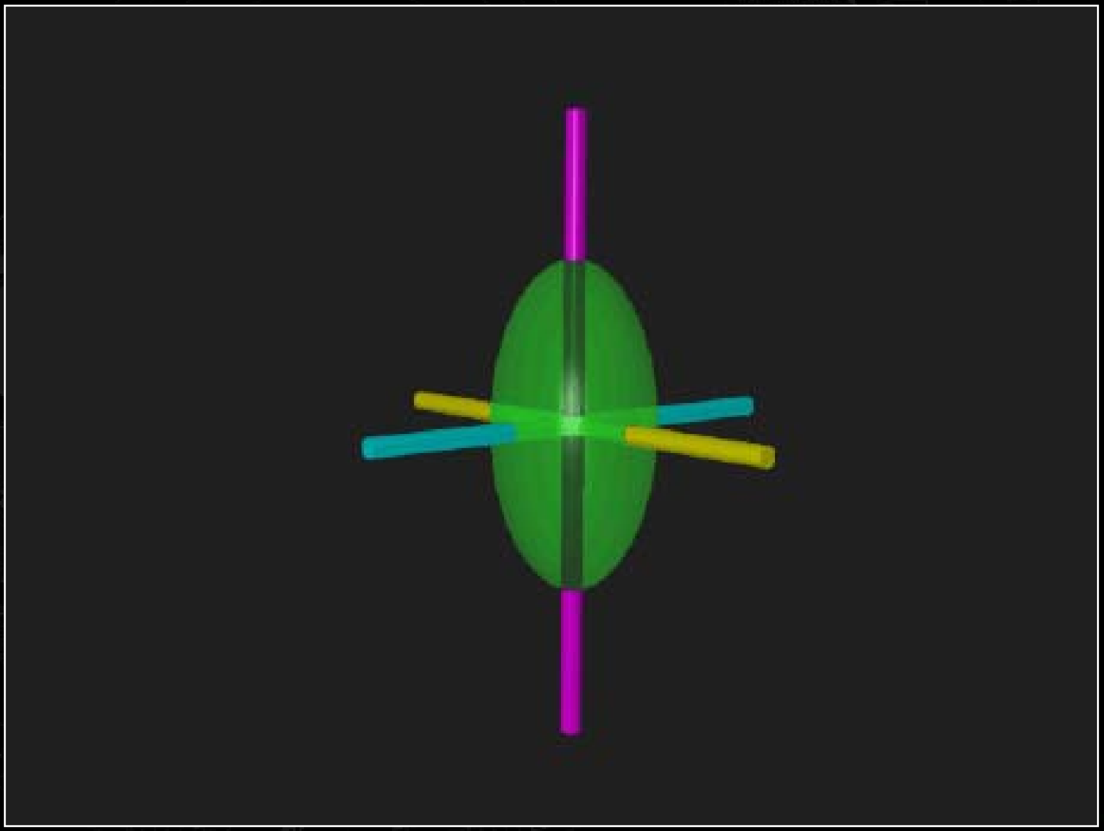
Growth in Pose Uncertainty

$$V[X] = \int_0^{dt} w$$

where

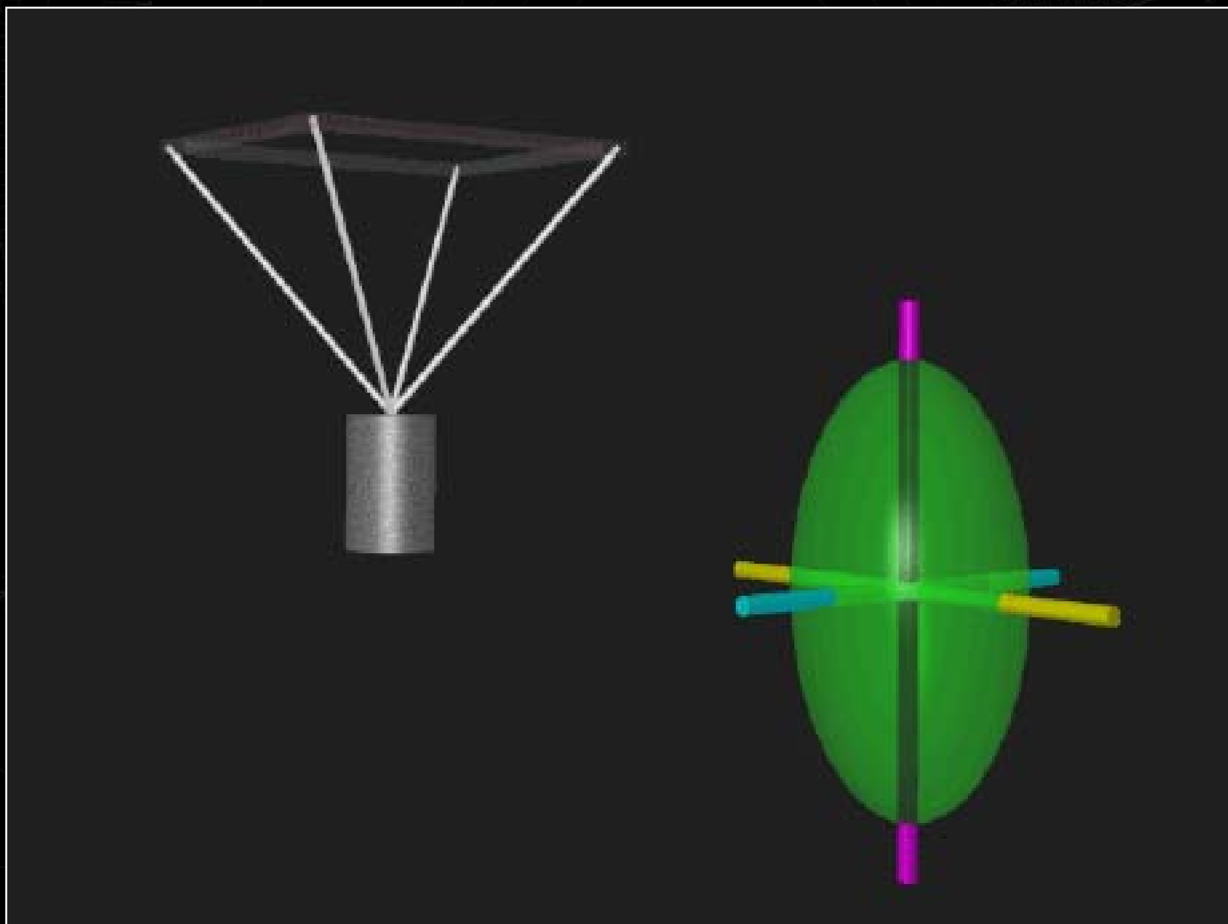
$$w \sim \mathrm{N}(0, q)$$

and “white.”

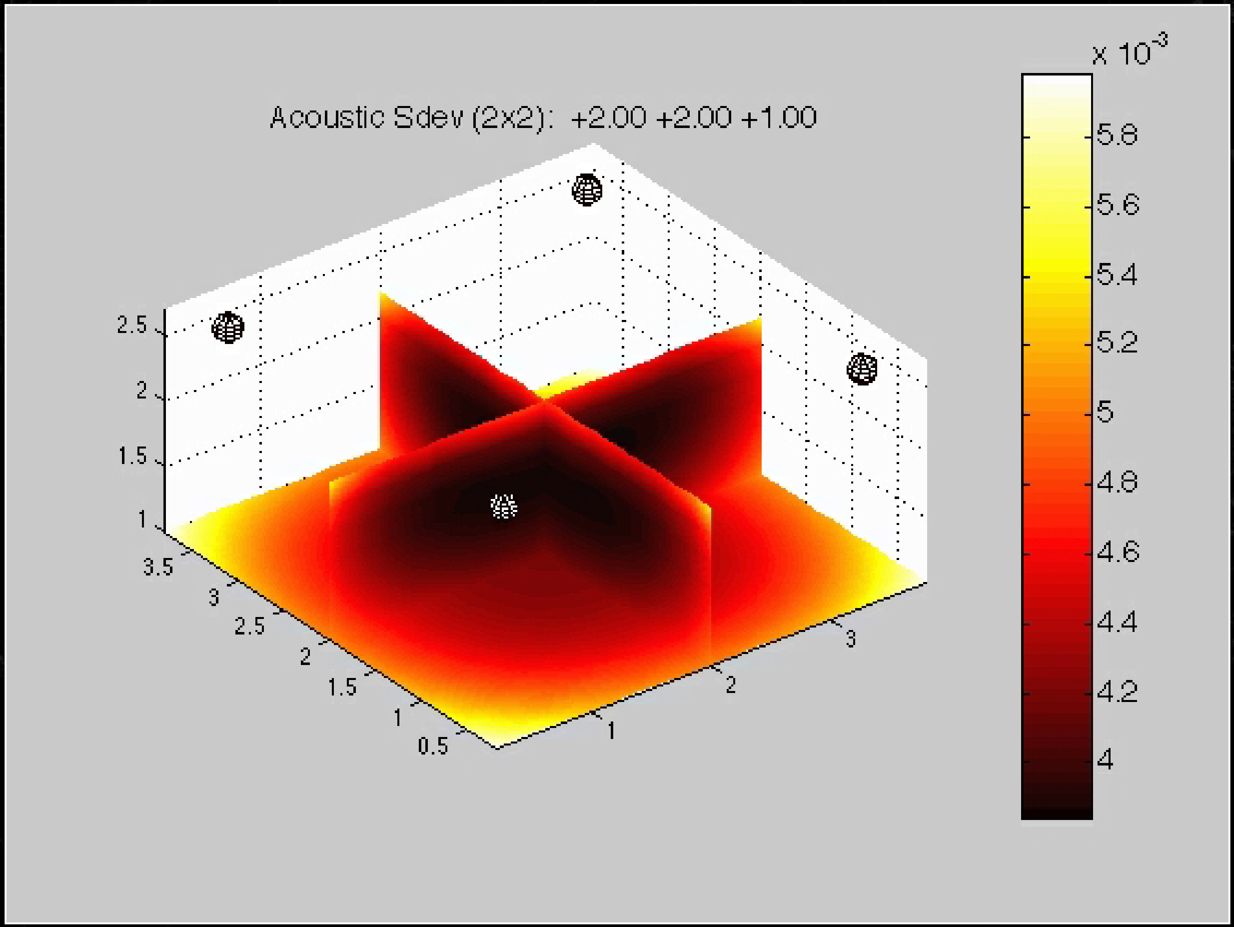


Control of Pose Uncertainty

Measurements $\Rightarrow$ pose information



Sensor Measurements



Break

(15 Minutes)

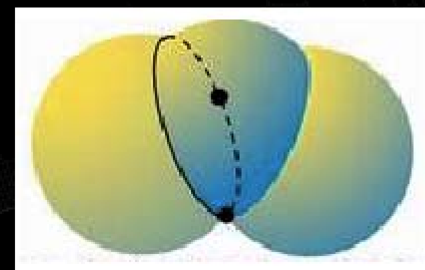
Traditional Approaches

(Gary Bishop)

Traditional Solution Methods

- Simple problem: Determine pose given sensor readings.
- Linear algebra taught us about N equations in N unknowns
- Each equation is a *constraint*
- 3 DOF→3 constraints & 6 DOF→6 constraints
- Unfortunately often non-linear constraints often with multiple solutions

Range Tracker



Intersect 3 spheres

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r_0^2$$

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r_1^2$$

$$(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 = r_2^2$$

Unfortunately, the solution is ugly...

Simplify

a. put mike 0 at origin

b. put mike 1 out X axis 1 unit

c. put mike 2 out Y axis 1 unit

d. all 3 mikes in Z=0 plane

Usual coordinate transform to convert to "lab" coordinates

Simpler Range Equations

$$x^2 + y^2 + z^2 = r_0^2$$

$$(x-1)^2 + y^2 + z^2 = r_1^2$$

$$x^2 + (y-1)^2 + z^2 = r_2^2$$

$$x = \frac{r_0^2 - r_1^2 + 1}{2}$$

$$y = \frac{r_0^2 - r_2^2 + 1}{2}$$

Note ambiguity →

$$z = \pm\sqrt{r_0^2 - x^2 - y^2}$$

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Optical with fixed 1D sensors

For example, 1D CCD with razor blade casting a shadow

- Calibrate to determine 3D plane equation from sensor reading (non-trivial)
- For each sensor reading, write a linear equation relating unknown x,y,z to plane
- Solve the system of equations for x,y,z

Solve...

$$A_i x + B_i y + C_i z = D_i$$

$$M = \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}$$

$$M \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

Optical with fixed 2D sensors

For example, two video cameras looking at LEDs on the user.

- Could treat as four 1-D sensors

OR

- Calibrate to get ray equation from u,v
- Rays won't intersect!
- Minimize distance between them

Set up equations

Ray equations

$$A_1 = C_1 + s_1 D_1$$
$$A_2 = C_2 + s_2 D_2$$

Distance

$$\left\| (C_2 + s_2 D_2) - (C_1 + s_1 D_1) \right\|$$

Minimum distance line must be perpendicular to both rays, so...

$$[(C_2 + s_2 D_2) - (C_1 + s_1 D_1)] \bullet D_1 = 0$$
$$[(C_2 + s_2 D_2) - (C_1 + s_1 D_1)] \bullet D_2 = 0$$

Solve

Distance out each ray to closest point

$$s_1 = \frac{(B \bullet D_1) - (D_2 \bullet D_1)(B \bullet D_2)}{1 - (D_1 \bullet D_2)^2}$$

$$s_2 = \frac{(D_1 \bullet D_2)(B \bullet D_1) - (B \bullet D_2)}{1 - (D_1 \bullet D_2)^2}$$

Halfway between

$$\tilde{P} = \frac{(C_1 + s_1 D_1) + (C_2 + s_2 D_2)}{2}$$

B is the baseline

Stochastic Approaches

(Greg Welch)

Motivation

- **Take into account**
  - Stochastic nature of sensor signals
  - Varying amounts of sensor information
  - Model of user motion
- **Combine sensor/measurement information**
  - Combat (otherwise growing) pose uncertainty
  - Fuse information from heterogeneous sensors

State-Space Models

Begin with difference equation for process

$$y_{k+1} = a_{0,k} y_k + \ldots + a_{n-1,k} y_{k-n+1} + u_k$$

Re-write as

$$\bar{x}_{k+1} \equiv \begin{bmatrix} y_{k+1} \\ y_k \\ y_{k-1} \\ \vdots \\ y_{k-n+2} \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & \cdots & a_{n-2} & a_{n-1} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ \vdots & & \ddots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_k \\ y_{k-1} \\ y_{k-2} \\ \vdots \\ y_{k-n+1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u$$

State-Space Models

$$\bar{x}_{k+1} \equiv \begin{bmatrix} y_{k+1} \\ y_k \\ y_{k-1} \\ \vdots \\ y_{k-n+2} \end{bmatrix} = \underbrace{\begin{bmatrix} a_0 & a_1 & \cdots & a_{n-2} & a_{n-1} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ \vdots & & \ddots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_k \\ y_{k-1} \\ y_{k-2} \\ \vdots \\ y_{k-n+1} \end{bmatrix}}_{\bar{x}_i} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}}_{G} u$$

$$\bar{x}_{k+1} = A\bar{x}_k + Gu_k$$

$$\bar{y}_k = H\bar{x}_k$$

Observer Design Problem

$$\bar{x}_k = A\bar{x}_{k-1} + G\bar{u}_{k-1}$$

$$\bar{z}_k = H\bar{x}_k + \bar{v}_k$$

Measurement noise

Process noise

Optimal Estimation

$$c = \int_0^T \text{cost}\left(\bar{a}(t), \bar{b}(t), t\right) dt$$

Integral of Absolute Value of Error (IAE)

$$\text{cost} = \left|\bar{a} - \bar{b}\right|$$

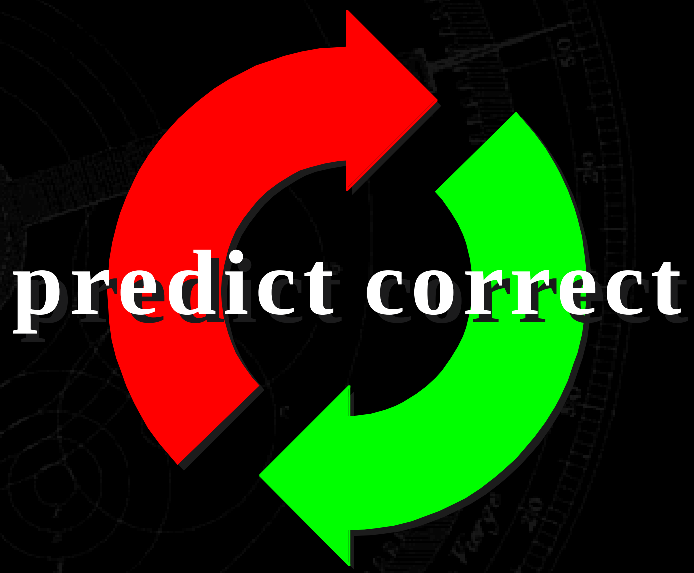
Integral of Square of Error (ISE)

$$\text{cost} = \left(\bar{a} - \bar{b}\right)^2$$

The Kalman Filter

R.E. Kalman, 1960

- Recursive optimal estimator
  - Minimum *variance* of error
- Versatile & robust
  - Estimation
  - Sensor fusion
- Robotics, navigation, computer vision, economics, …
- http://www.cs.unc.edu/~welch/kalman/
  - *Java-Based Learning Tool*, books, papers, etc.
- ACM SIGGRAPH 2001 tutorial (earlier today)



PREDICT

$$\bar{x}_k^- = A\bar{x}_{k-1}$$

$$P_k^- = AP_{k-1}A^T + Q$$

transition

uncertainty

CORRECT

$$\bar{x}_k = \bar{x}_k^- + K\left(\underbrace{\bar{z}_k}_{\text{actual}} - \underbrace{H\bar{x}_k^-}_{\text{predicted}}\right)$$

$$P_k = (I - KH)P_k^-$$

$$K = P_k^- H^T \Big(\underbrace{HP_k^- H^T + R}_{\text{“denominator” (measurement space)}}\Big)^{-1}$$

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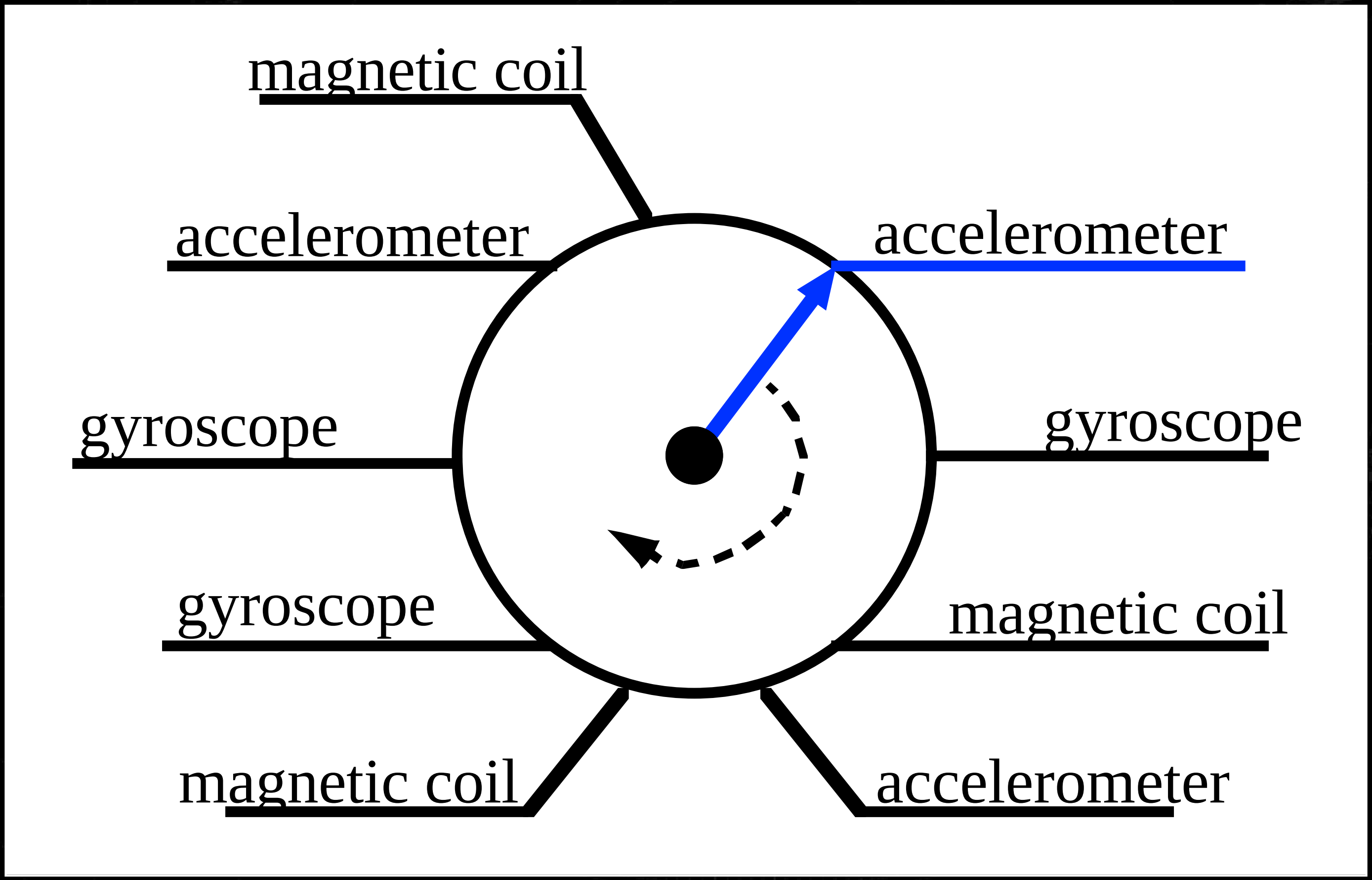
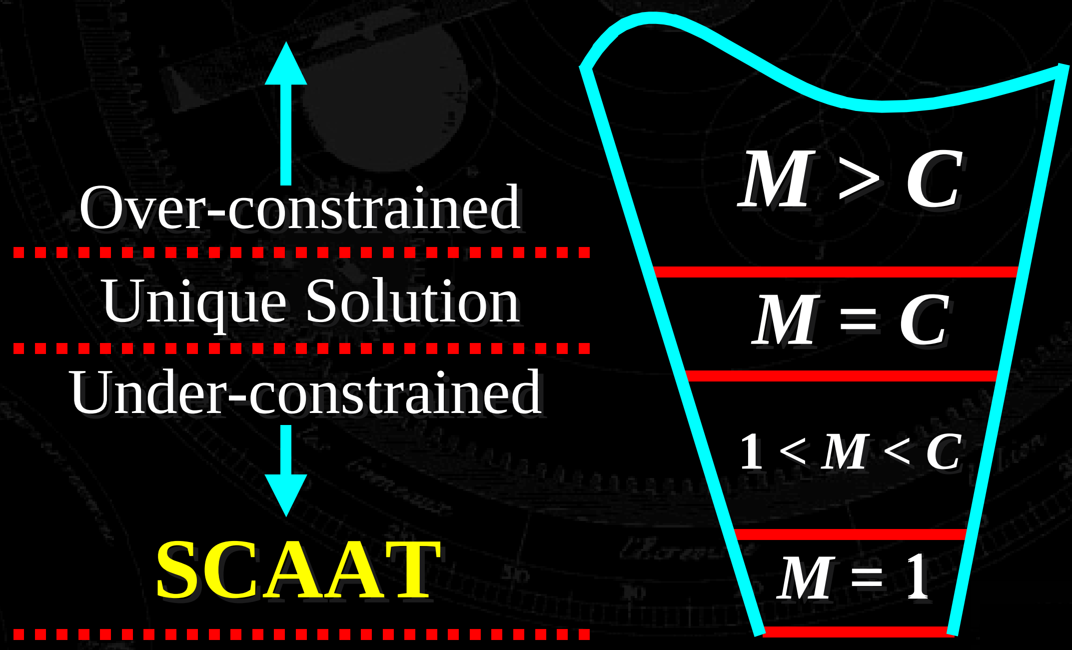
Hybrid Systems and Multi-Sensor Fusion



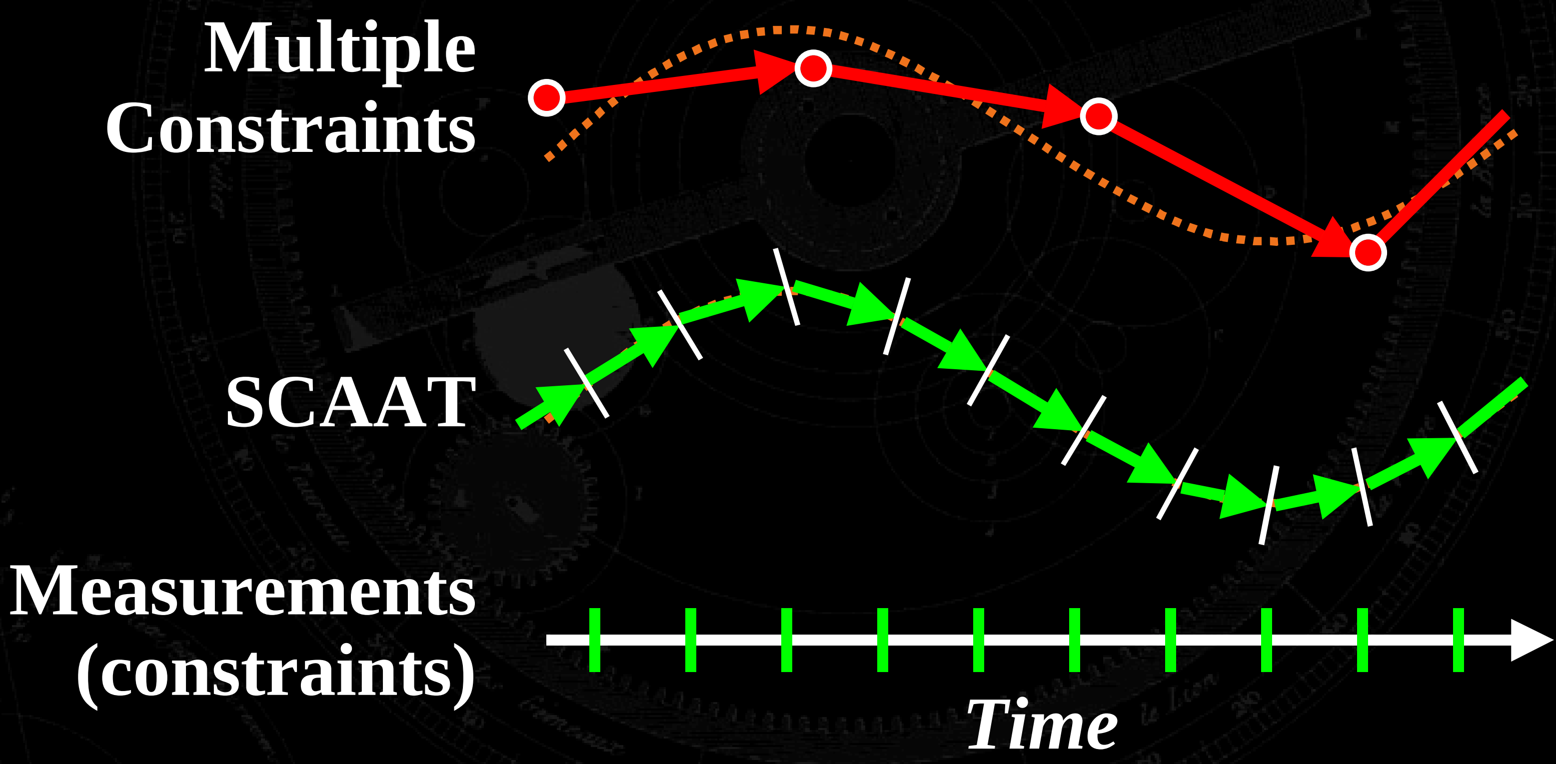
Incremental Estimation
A *Single Constraint at a Time*

C = constraints needed for a unique solution

M = constraints used per estimate update



Inter-Estimate Summary



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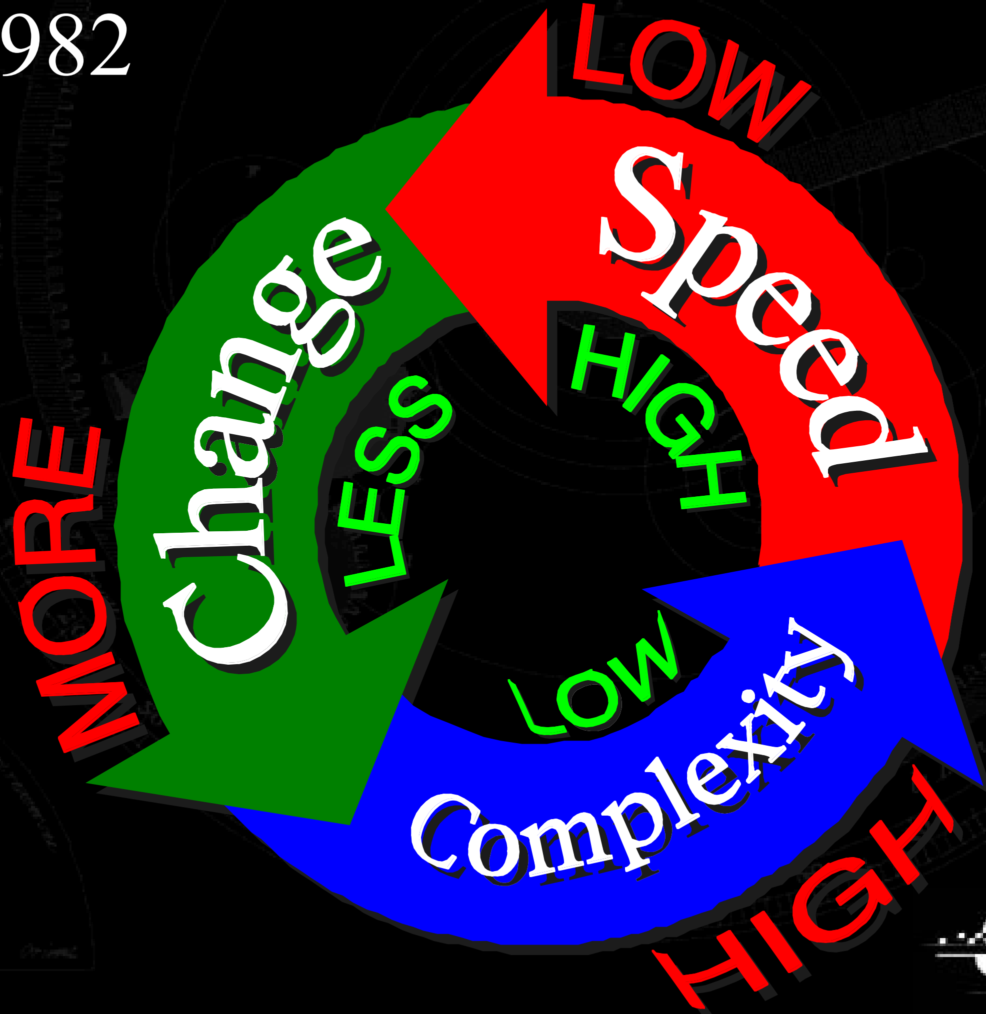
Benefits of SCAAT Approach

Purposefully using minimal constraints

- Avoid *simultaneity assumption*
- Temporal improvements
- Simplicity and flexibility
- Online source/sensor *autocalibration*
- Can be applied to *virtually any tracking system*
  - Measurement model for each type of sensor
  - Dynamic model for user motion (possibly trivial)

Bishop, 1982



Error Sources

(Greg Welch)

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Error in Head Pose

- **Hard to fool "mother nature"**
  - Lifetime of visual experience and expectations
  - Visual-proprioceptive conflicts
  - Virtual-real misregistration
- **What to do?**
  - Some amount of error is unavoidable
  - Understand sources and seek to minimize

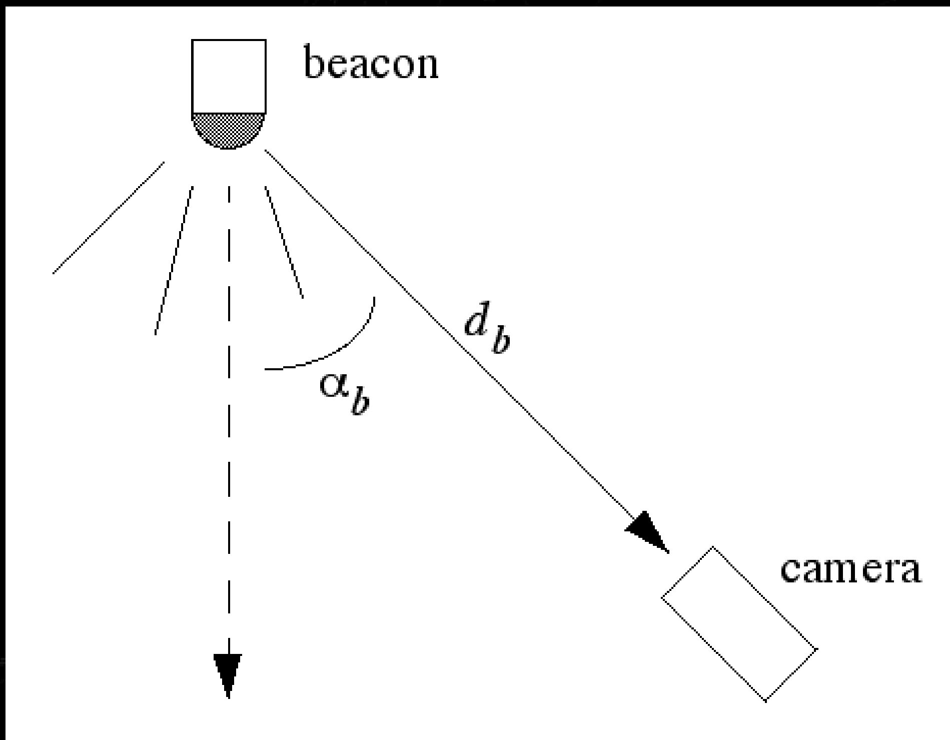
Error Classification

- **Pose estimate life cycle**
  - Noisy sensor measurement → Estimate → Transport → Transform → Display
- **Two primary classes of error**
  - Static (spatial)
  - Delay-induced (temporal)

Static Measurement Error

- **Static field distortion**
  - Repeatable error in the measurement data
  - “Bias” that might be corrected via calibration
- **Random noise or jitter**
  - Non-repeatable error
  - Random (electrical) noise such as described earlier
  - Often dependent on the current pose

Pose-Dependent Noise (Example)



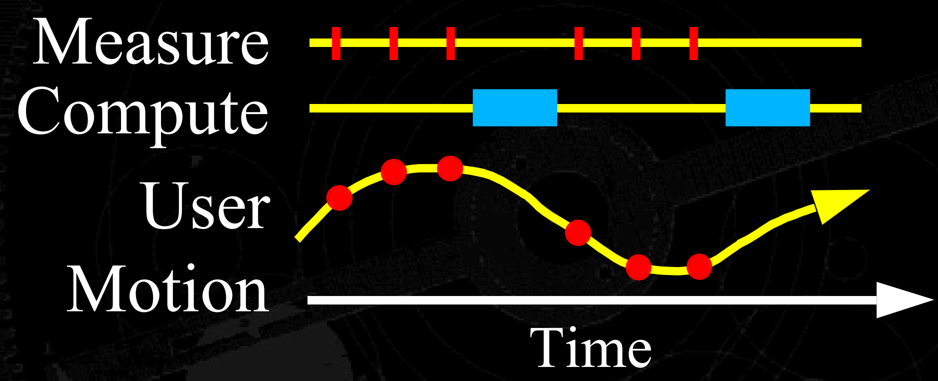
Baseline noise ξ_0 and coefficients a, b, and c were determined off line.

$$\sqrt{\xi_c} = \frac{\sqrt{\xi_0}\, d_b^2}{a\alpha_b^3 + b\alpha_b^2 + c\alpha_b + 1}$$

Delay-Induced Error

- **Measurement validity**
  - Good at sample time, then old (aging)
  - Finite, non-zero sample time
  - Old sample ⇒ misregistration
- **Motion predicition**
  - Measure where you *are*, but *want* where you *will be*
  - Later w/ Bishop

The Simultaneity Assumption



Moderate arm & wrist translation

1/2 [s] • 3 [m/s] • 20-80 [ms] ⇒ 1-10 [cm]

Moderate head rotation

1/2 [s] • 180 [°/s] • 20-80 [ms] ⇒ 6-25 [cm] (at arm's length)

First-Order Dynamic Error

$$\varepsilon_{\mathrm{dyn},\theta} = \dot{\theta}\Delta t$$

$$\varepsilon_{\mathrm{dyn},x} = \dot{x}\Delta t$$

Instantaneous velocities

Tracker + graphics pipeline latency

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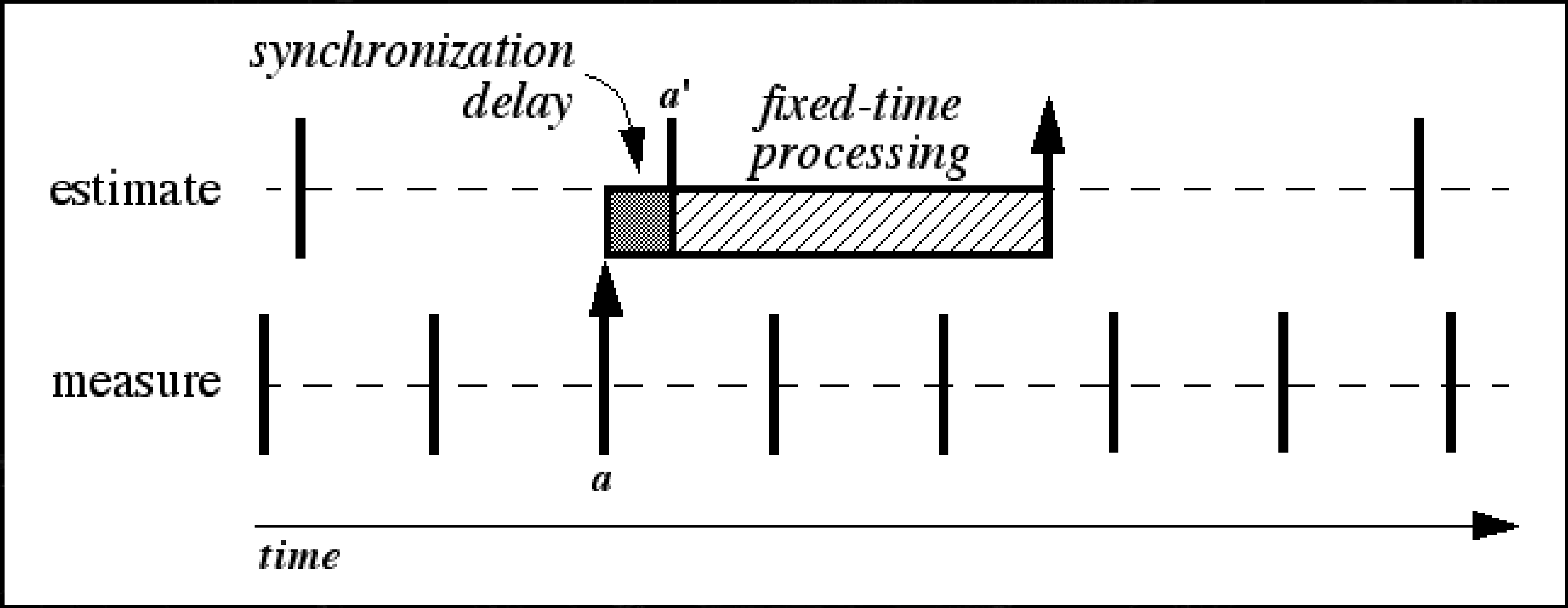
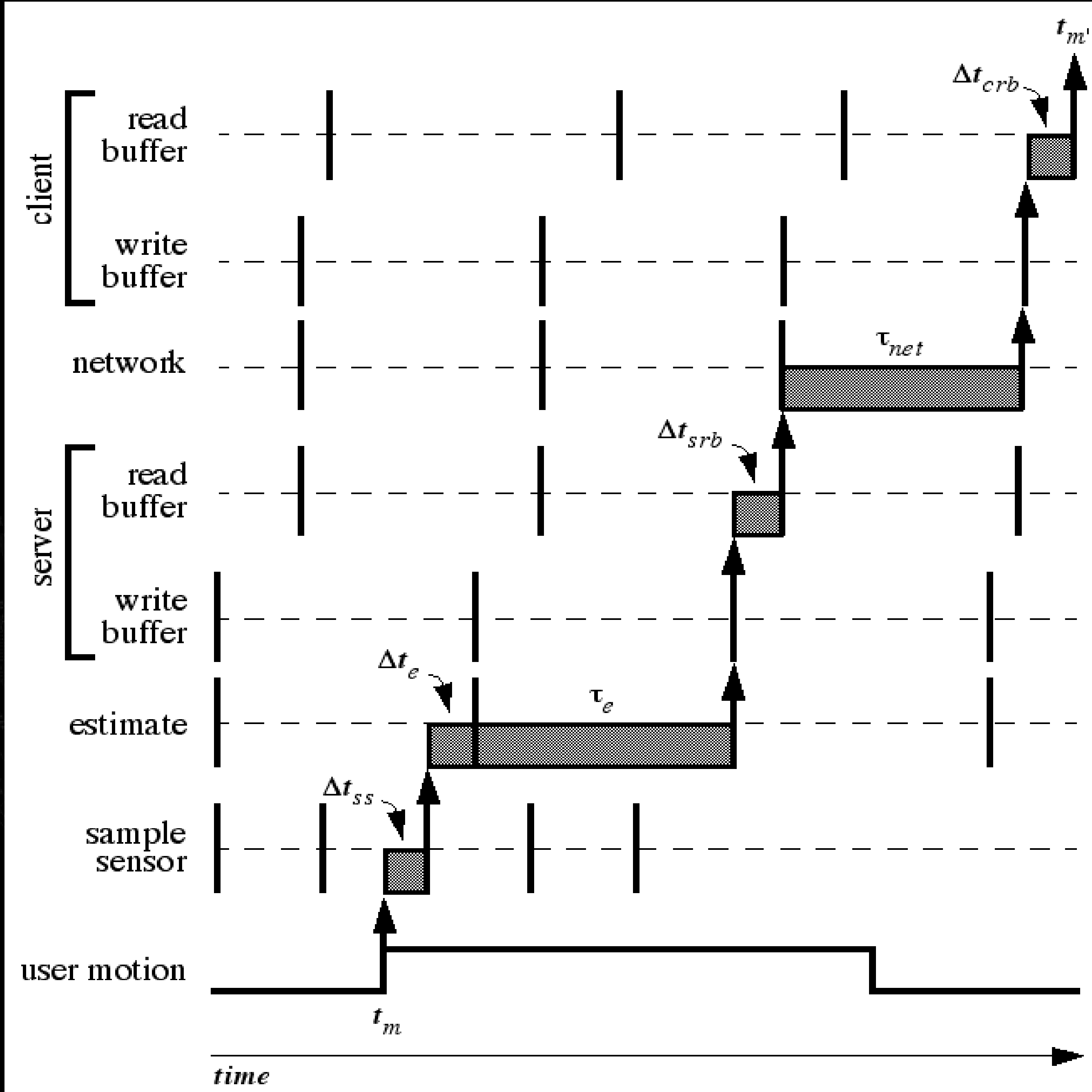
Synchronization Delay

A.k.a. *phase delay* or *rendezvous delay*



Pose Estimate Timeline



Total Tracker Latency

$$
\begin{aligned}
\Delta t_m &= t_{m'} - t_m \\
&= \Delta t_{ss} + \Delta t_e + \tau_e + \Delta t_{srb} + \tau_{net} + \Delta t_{crb} \\
&= \frac{1}{2r_{ss}} + \frac{1}{2r_e} + \tau_e + \frac{1}{2r_{srb}} + \tau_{net} + \frac{1}{2r_{crb}}
\end{aligned}
$$

Sample latency

Estimate the sensor(s) synchronization

Estimate the post

Server buffer synchronization

Network transport

Client buffer synchronization

Total Tracker Error

$$\varepsilon_{\theta} \approx \varepsilon_{\text{stat},\,\theta} + \varepsilon_{\text{sa},\,\theta} + \dot{\theta}(\Delta t_m + \Delta t_g)$$

$$\varepsilon_{x} \approx \varepsilon_{\text{stat},\,x} + \varepsilon_{\text{sa},\,x} + \dot{x}(\Delta t_m + \Delta t_g)$$

Static error

Simultaneity assumption

Total tracker latency

Rendering delay

Closing (Error Sources)

- Did I mention error magnification?
- Consider the technology
  - Understand its limitations
  - Stay within the envelope
- Prediction (next)

Motion Prediction

(Gary Bishop)

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Motion Prediction

End-to-end delay

- hurts in VR / hurts worse in AR
- sources
  - time to measure pose
  - delay in communicating pose
  - application response to change
  - graphics update
  - display refresh

What to do about delay?

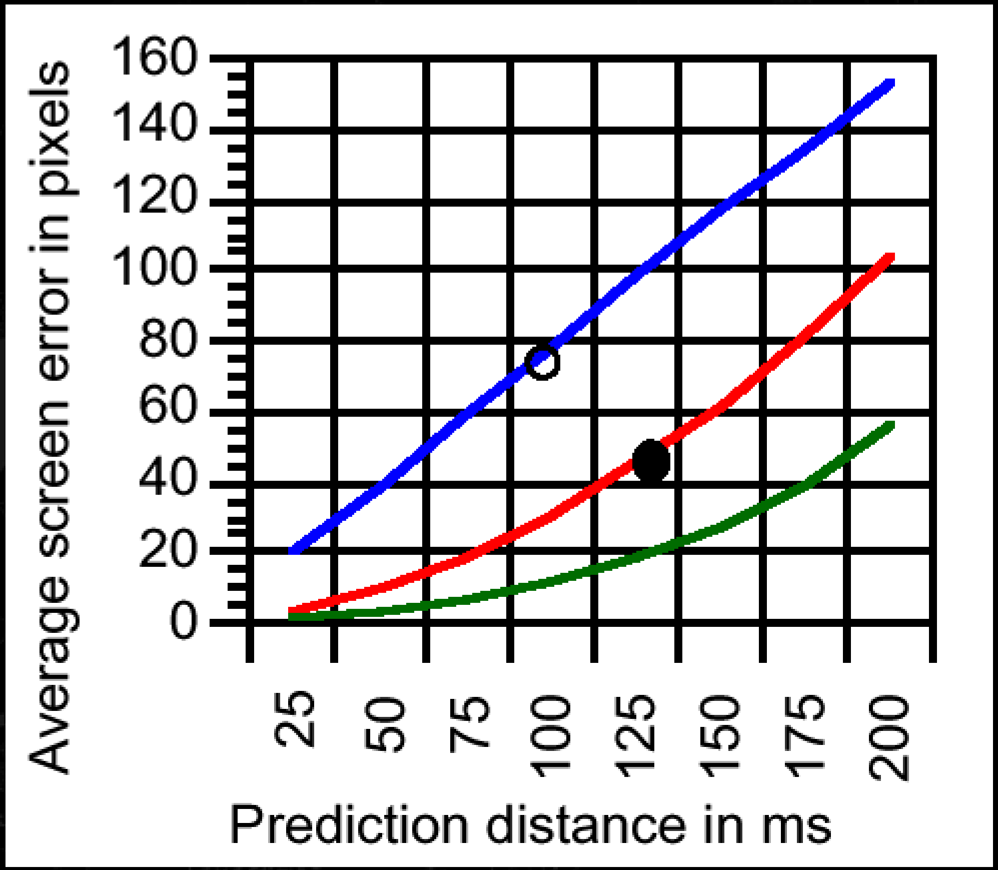
1. Monitor
2. Minimize
3. Mitigate

Latency is not *only* a tracker problem.

But mitigation is best handled at the tracker.

Can prediction help?

Blue→no prediction
Red→w/out inertial
Green→w/ inertial



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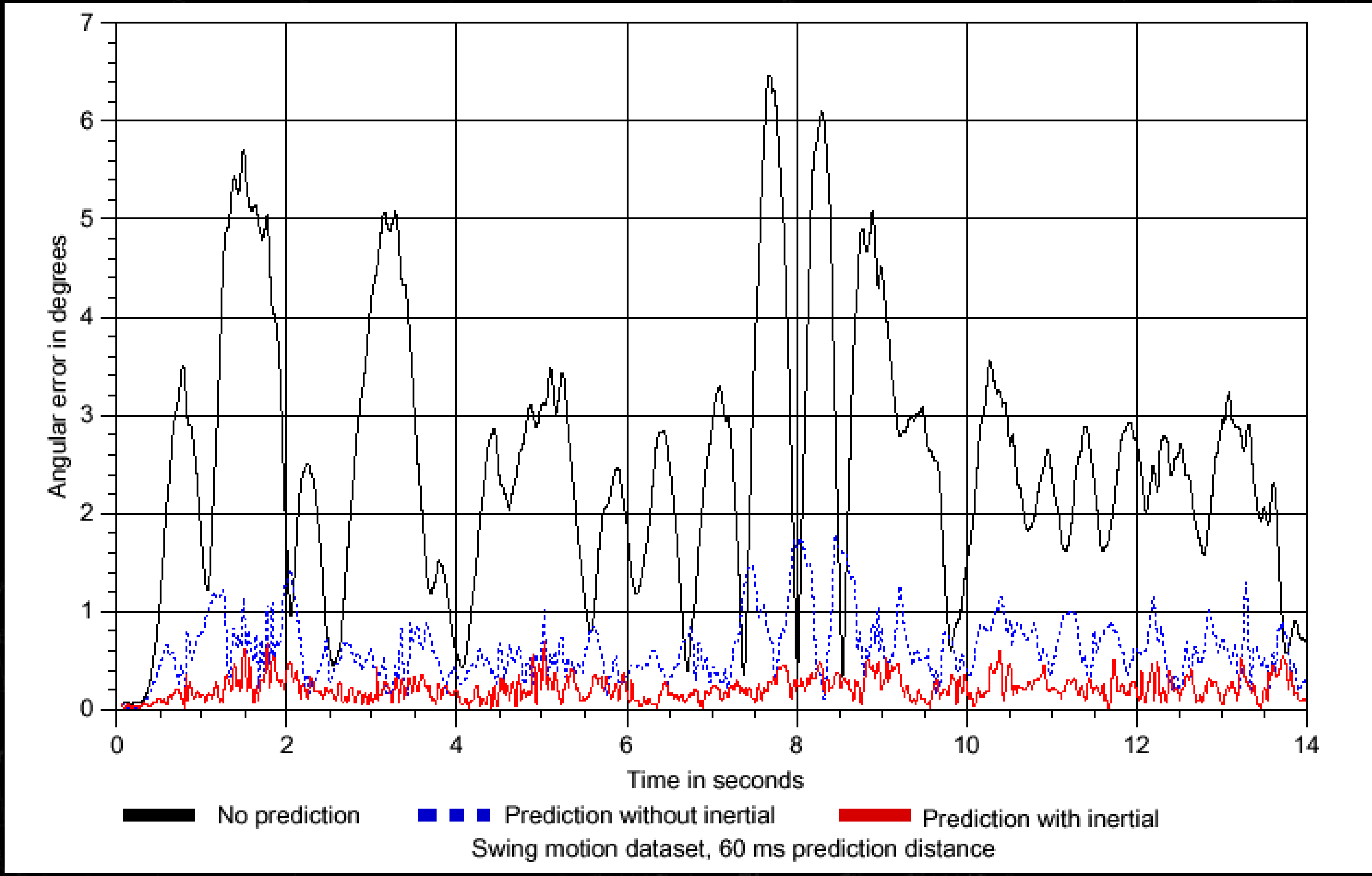
Can prediction help?



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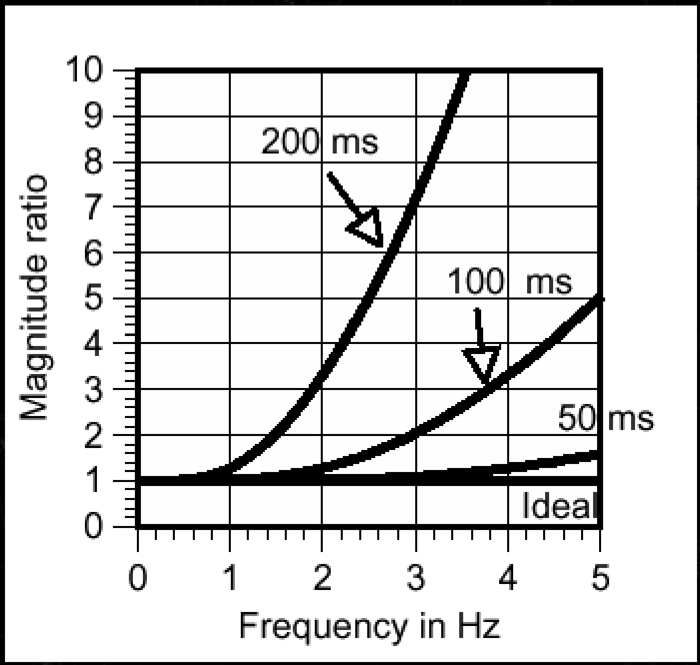
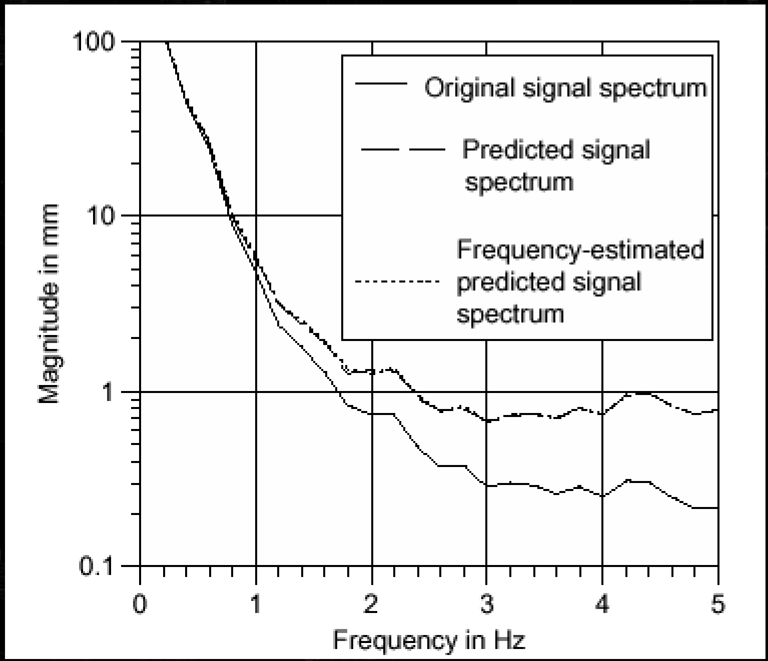
Limits to prediction

Prediction error grows quadratically with motion bandwidth and prediction interval





Prediction ideas

- Extrapolate past behavior to the future
- The more history the better
- Correlations in the *users* coordinate frame
- Inertial sensors help
- Monitor |predicted – actual| for tuning
- Use image shifting to reduce jitter

Conclusions

(Gary Bishop)

Final Thoughts

- **No silver bullet**
  - Tracking anywhere for any purpose is a dream
- **No free lunch, only tradeoffs**
  - Energy / Accuracy / Bandwidth / Latency / Noise
- **No end in sight**
  - Lots of possibilities for interesting work
  - ReActor not based on any of the principles described here

Resources

- http://www.cs.unc.edu/~welch/kalman
- check out Course 8 notes
- Dozens of books on KF, here are a few
  - "Optimal Estimation with an ..." by Lewis
  - "Introduction to Random Signals..." by Brown
  - "Kalman Filtering Theory and Practice" by Grewal
- Beginnings of a tracking bibliography

Exhibits we're going to check out

- 3rd Tech (cool demo)
- 5DT
- Ascension (ReActor is a new method)
- InterSense
- Measurand
- MetaMotion / PhoeniX Technologies / Vicon
- Polhemus

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End