# Supplementary Material: Theorem–Lean 4 Identifier Correspondence Table

for "Non-Solvability as a Physical Principle: $A_5$ and the Algebraic Origin of Information Barriers"

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Repository: https://github.com/nuu/A5CosmicNecessity | Toolchain: Lean 4 + Mathlib | Status: sorry = 0, axiom = 0

This document provides the complete correspondence between the theorems, corollaries, definitions, and propositions stated in the main text and the formally verified Lean 4 identifiers in the repository. All listed identifiers have been verified with sorry = 0, axiom = 0. The only unformalized element in the argument chain is Klein's classification of finite rotation groups (cited as a classical theorem; see L7 in Section 7.1).

## Project File Structure

| Lean File | Paper Section | Description |
|---|---|---|
| `A5CosmicNecessity.lean` | — | Root module (imports all sections) |
| `Auxiliary.lean` | — | Core definitions (KleinType, icosahedral constants, group-theoretic lemmas, Sylow theory) |
| `SolvabilityBelow60.lean` | §3.1–3.2 | Exhaustive solvability proofs for all groups of order < 60 |
| `ConjugacyClassGalois.lean` | §4.1.2, Remark 4.1 | Conjugacy class structure of $A_5$, Galois action realization |
| `Section1_Introduction.lean` | §1 | Claims 1–3, epistemic layers, falsifiability framework |
| `Section2_ModelAndPostulates.lean` | §2 | Postulates (H1)–(H3), elimination, independence |
| `Section3_MainTheorem_new.lean` | §3 | Main theorem, five alternative conditions, robustness |
| `Section4_BetaZero.lean` | §4 | Icosahedral data, $\beta_0$ = 11 reconstruction, representation theory |
| `Section5_ScaleProblem.lean` | §5 | RG survival classification (Type I/II/III), falsification protocol |
| `Section6_ProhibitionStructure.lean` | §6 | Triple prohibition (P1)–(P3), five-layer classification, $E_8$ connection |
| `Section7_LimitationsAndOpenProblems.lean` | §7 | Limitations L1–L7, open problems G1′–G7, information barriers |
| `Section8_Conclusion.lean` | §8 | Claims summary, five cosmic constraints, full verification summary |

## §2 Model Class and Postulates

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **Table 2 (H1 candidates)** | `H1_selects_five_families` | (H1) limits candidates to five Klein families | `Section2_...Postulates` |
| **Table 2 (H2 filter)** | `H2_survivors` | (H2) passes only polyhedral groups $\{A_4, S_4, A_5\}$ | `Section2_...Postulates` |
| **— (H2 elimination)** | `H2_eliminates_cyclic_dihedral` | Cyclic and dihedral groups are eliminated by (H2) | `Section2_...Postulates` |
| **— (H2 iff polyhedral)** | `H2_passes_iff_polyhedral` | (H2) passes iff the Klein type is polyhedral | `Section2_...Postulates` |
| **Table 3 (H3 filter)** | `H3_only_A5_passes` | Among polyhedral groups, only $A_5$ passes (H3) | `Section2_...Postulates` |
| **— (H3* equivalence)** | `H3star_equivalence_basis` | (H3) and (H3*) are equivalent on $\{A_4, S_4, A_5\}$ | `Section2_...Postulates` |
| **Table 5, row 1** | `H1_H2_insufficient` | (H1)∧(H2) leaves 3 candidates (no uniqueness) | `Section2_...Postulates` |
| **Table 5, row 2** | `H1_H3_insufficient` | (H1)∧(H3) leaves many candidates | `Section2_...Postulates` |
| **Table 5, row 3** | `H2_H3_selects_A5` | (H2)∧(H3) selects $A_5$ (within finite groups) | `Section2_...Postulates` |
| **§2.5 (non-overlap)** | `elimination_targets_disjoint` | Exclusion targets of (H2) and (H3) are disjoint | `Section2_...Postulates` |

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **§2.5 (stepwise)** | `stepwise_elimination_complete` | Complete stepwise elimination from 5 to 1 | `Section2_...Postulates` |
| **§2.2 (n = 2 collapse)** | `dim2_collapse` | n = 2: no non-solvable finite rotation groups | `Section2_...Postulates` |
| **§2.2 (n = 3 privilege)** | `dim3_privilege` | n = 3 is privileged for non-solvable holonomy | `Section2_...Postulates` |
| **§2.2 (n = 4 deviation)** | `dim4_gap_deviation` | n = 4: gap = $1/\phi^4$ gives 38% deviation in $\alpha_s$ | `Section2_...Postulates` |

## §3 Main Theorem: Uniqueness of A■ Holonomy in SO(3)

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **Theorem 3.1** | `solvable_opacity` | Non-solvable G: any solvable probe is non-injective | `Section3_...new` |
| **— (converse)** | `solvable_of_injective_probe` | Injective solvable probe ⇒ G is solvable | `Section3_...new` |
| **Corollary 3.1** | `simple_nonsolvable_total_opacity` | Simple non-solvable: solvable probe has ker = G | `Section3_...new` |
| **— ($A_5$ instance)** | `A5_probe_ker_eq_top` | For $A_5$: $\lvert\ker(\pi)\rvert = 60$ | `Section3_...new` |
| **— ($A_5$ trivializes)** | `A5_probe_trivializes` | Any solvable probe from $A_5$ is trivial | `Section3_...new` |
| **Corollary 3.2** | `A5_probe_cannot_distinguish` | Product of solvable probes is constant on $A_5$ | `Section3_...new` |
| **Theorem 3.2** | `criticality_theorem` | All groups of order < 60 are solvable; $A_5$ is smallest non-solvable | `Section3_...new` |
| **— (supporting)** | `groups_below_60_solvable` | $\lvert G\rvert < 60 \Rightarrow$ G is solvable (exhaustive proof) | `SolvabilityBelow60` |
| **Corollary 3.3** | `minimal_barrier_basis` | $A_5$ is the minimal basis for the information barrier | `Section3_...new` |
| **Theorem 3.3** | `main_theorem` | (H1)∧(H2)∧(H3) ⇒ $H \cong A_5$ | `Section3_...new` |
| **— (converse)** | `main_theorem_converse` | $A_5$ passes both (H2) and (H3) | `Section3_...new` |
| **— (∃! version)** | `main_theorem_unique` | $A_5$ is the unique polyhedral group passing (H3) | `Section3_...new` |
| **Corollary 3.4** | `corollary_H3star` | (H1)∧(H2)∧(H3*) ⇒ $H \cong A_5$ | `Section3_...new` |
| **— (H3↔H3*)** | `H3_iff_H3star_on_polyhedral` | (H3) ⇔ (H3*) on polyhedral candidates | `Section3_...new` |
| **Corollary 3.5** | `corollary_maximality` | (H3′): maximality selects $A_5$ | `Section3_...new` |
| **— (explicit)** | `maximality_selects_A5` | $\lvert A_4\rvert=12 < \lvert S_4\rvert=24 < \lvert A_5\rvert=60$ | `Section3_...new` |
| **Corollary 3.6** | `corollary_nonsolvability` | (H3″): non-solvability selects $A_5$ | `Section3_...new` |
| **— (classification)** | `nonsolvability_classification` | $A_4$, $S_4$ solvable; $A_5$ non-solvable | `Section3_...new` |
| **Corollary 3.7** | `corollary_order5` | (H3‴): 5-fold symmetry selects $A_5$ | `Section3_...new` |
| **— ($A_4$ no ord 5)** | `A4_no_order_5_element` | $A_4$ has no element of order 5 | `Section3_...new` |
| **— ($S_4$ no ord 5)** | `S4_no_order_5_element` | $S_4$ has no element of order 5 | `Section3_...new` |
| **— ($A_5$ has ord 5)** | `A5_has_order_5_element` | $A_5$ contains an element of order 5 | `Section3_...new` |
| **Table 4 (convergence)** | `five_conditions_converge` | All five conditions converge on $A_5$ | `Section3_...new` |
| **Table 5 (§3 version)** | `all_three_conditions_necessary` | All three postulates are necessary for uniqueness | `Section3_...new` |
| **§3.4 (disjointness)** | `elimination_targets_disjoint_s3` | (H2) and (H3) target disjoint sets | `Section3_...new` |

## Auxiliary: Core Group-Theoretic Facts

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **$\|A_5\| = 60$** | `A5_card` | Fintype.card (alternatingGroup (Fin 5)) = 60 | `Auxiliary` |
| **$\|A_4\| = 12$** | `A4_card` | Fintype.card (alternatingGroup (Fin 4)) = 12 | `Auxiliary` |
| **$\|S_4\| = 24$** | `S4_card` | Fintype.card (Equiv.Perm (Fin 4)) = 24 | `Auxiliary` |
| **$A_5$ is simple** | `A5_is_simple` | IsSimpleGroup (alternatingGroup (Fin 5)) | `Auxiliary` |
| **$A_5$ is non-solvable** | `A5_not_solvable` | ¬ IsSolvable (alternatingGroup (Fin 5)) | `Auxiliary` |
| **$A_5$ is perfect** | `A5_perfect` | commutator subgroup = ■ | `Auxiliary` |
| **$S_4$ not perfect** | `S4_not_perfect` | $S_4^{ab} \cong C_2 \neq 1$ | `SolvabilityBelow60` |
| **$A_4$ not perfect** | `A4_not_perfect` | $A_4^{ab} \cong C_3 \neq 1$ | `SolvabilityBelow60` |
| **$A_4$ solvable** | `A4_solvable` | IsSolvable (alternatingGroup (Fin 4)) | `SolvabilityBelow60` |
| **$S_4$ solvable** | `S4_solvable` | IsSolvable (Equiv.Perm (Fin 4)) | `SolvabilityBelow60` |
| **$\|G\| < 60 \Rightarrow$ solvable** | `groups_below_60_solvable` | Exhaustive: all groups of order < 60 are solvable | `SolvabilityBelow60` |
| **— (order bound)** | `nonsolvable_order_ge_60` | ¬IsSolvable G ⇒ Fintype.card G ≥ 60 | `SolvabilityBelow60` |
| **Subgroup inheritance** | `solvable_of_injective_to_solvable` | Injective hom to solvable ⇒ solvable | `Auxiliary` |

## §4 Proof-of-Concept Bridge: Reconstructing β■ = 11

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **Table 6 (orbit–stab.)** | `orbit_stabilizer_equations` | 60/3=20, 60/2=30, 60/5=12 | `Section4_BetaZero` |
| **— (stabilizers)** | `stabilizers_distinct_primes` | Stabilizer orders {3,2,5} are distinct primes | `Section4_BetaZero` |
| **Euler formula** | `euler_formula` | $F + V - E = \chi = 2$ for the icosahedron | `Section4_BetaZero` |
| **— (3F = 2E)** | `edge_face_relation` | $n \cdot F = 2 \cdot E$ (face–edge relation) | `Section4_BetaZero` |
| **Triple lock** | `triple_lock` | $E/n = F/2 = \|A_5\|/6 = 10$ | `Section4_BetaZero` |
| **Eq. (5): $\phi$ from $A_5$** | `golden_ratio_minimal_polynomial` | $\phi$ is root of $x^2 - x - 1 = 0$ from character table | `Section4_BetaZero` |
| **Eq. (9): 3-coincidence** | `three_coincidence_algebraic` | $\phi^2 + 1/\phi^2 = 3 = \dim(\rho_3)$ | `Section4_BetaZero` |
| **Table 7 ($\lambda_\phi$ unique)** | `lambda_phi_unique_subunitary` | $\lambda_\phi = 1/\phi$ is the unique sub-unitary character value | `Section4_BetaZero` |
| **Remark 4.1 (Galois)** | `lambda_phi_galois_connection` | $\lambda_\phi$ is a trace of the Galois exchange on $\mathbb{Q}(\sqrt{5})/\mathbb{Q}$ | `Section4_BetaZero` |
| **— (Galois realization)** | `galois_action_realization` | Squaring exchanges $C_5^+$ and $C_5^-$; inversion preserves them | `ConjugacyClassGalois` |
| **— (inversion preserves)** | `inverse_preserves_conjugacy_class` | $g \in C_5 \Rightarrow g^{-1} \in C_5$ | `ConjugacyClassGalois` |
| **— (squaring crosses)** | `squaring_maps_C5plus_to_C5minus` | $g \in C_5^+ \Rightarrow g^2 \in C_5^-$ | `ConjugacyClassGalois` |
| **Eq. (8): gap** | `gap_representation_theoretic` | $\text{gap} = \lambda_\phi^3 = 1/\phi^3$ | `Section4_BetaZero` |
| **Table 8 (index collapse)** | `alternative_exponent_collapse` | Only k=3 gives $\alpha_s$ consistent with experiment | `Section4_BetaZero` |
| **Table 9 (character table)** | `burnside_formula` | Burnside: $\sum \dim^2 = \|A_5\| = 60$ | `Section4_BetaZero` |
| **— ($\rho_4$ unique)** | `rho4_unique` | $\rho_4$ is the only non-SO(3) irrep | `Section4_BetaZero` |
| **— ($\rho_4 \otimes \rho_4$)** | `rho4_tensor_self` | $\rho_4 \otimes \rho_4 = \rho_1 \oplus \rho_3 \oplus \rho'_3 \oplus \rho_4 \oplus \rho_5$ | `Section4_BetaZero` |
| **Eq. (10): $\beta_0^{ICO}$** | `Ico_beta0_val` | $E/n + \chi/2 = 30/3 + 1 = 11$ | `Section4_BetaZero` |
| **Eq. (11): V−1 identity** | `Ico_beta0_as_V_minus_one` | $V - 1 = E/n + \chi/2$ (identity) | `Section4_BetaZero` |

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **Eq. (12): full identity** | `Ico_beta0_identity` | $\beta_0^{ICO} = E/n + \chi/2 = V-1 = 11$ | `Section4_BetaZero` |
| **Eq. (13): quark coeff.** | `Ico_quark_coeff` | $\lvert A_5\rvert/(E\cdot n) = 60/90 = 2/3$ | `Section4_BetaZero` |
| **Table 10 (10+1)** | `dynamical_decomposition` | $E/n = 10$ (dynamical) + $\chi/2 = 1$ (topological) | `Section4_BetaZero` |
| **Table 11 (alt. groups)** | `alternative_collapse` | Only icosahedron reproduces $\beta_0 = 11$ | `Section4_BetaZero` |
| **Table 12 (acyclicity)** | `noncircularity` | Postulates are not derived from the result | `Section4_BetaZero` |

## §5 The Scale Problem (G2)

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **Table 13 (3 types)** | `three_types_distinct` | Types I, II, III are mutually distinct | `Section5_ScaleProblem` |
| **§5.4 (Type I: $\beta_0$)** | `typeI_beta0` | $\beta_0$ = 11 is RG-independent (Type I) | `Section5_ScaleProblem` |
| **§5.4 (Type I identity)** | `typeI_beta0_identity` | $\beta_0 = E/n + \chi/2 = V-1$ is topological | `Section5_ScaleProblem` |
| **Table 14 (quark)** | `typeI_quark_coeff` | $2/3 = \lvert A_5\rvert/(E\cdot n)$ is Type I | `Section5_ScaleProblem` |
| **Table 14 ($m_t/m_b$)** | `typeI_top_bottom_ratio` | E + V = 42 is an integer (Type I candidate) | `Section5_ScaleProblem` |
| **Table 15 ($m_\mu/m_e$)** | `typeII_muon_exponent` | $\phi^{11} = E/n + \chi/2 = \beta_0$ (Type II) | `Section5_ScaleProblem` |
| **Table 15 ($m_\tau/m_e$)** | `typeII_tau_exponent` | $\phi^{17}$: 17 = F – n (Type II) | `Section5_ScaleProblem` |
| **— (lepton relation)** | `lepton_exponent_relations` | $17 - 11 = 6 = \dim(\Lambda^2\rho_4)$ | `Section5_ScaleProblem` |
| **§5.7 (falsif. logic)** | `falsification_logic` | Rejection of all three types ⇒ G2 negative solution | `Section5_ScaleProblem` |
| **— (conservative)** | `conservative_sufficiency` | Type I alone suffices for proof of concept | `Section5_ScaleProblem` |

## §6 The Prohibition Structure as Falsification Target

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **Burnside formula** | `burnside_formula_s6` | 1+9+9+16+25 = 60 ($A_5$ irreps) | `Section6_...Structure` |
| **$\rho_4$ unique non-SO(3)** | `rho4_unique_non_SO3` | $\rho_4$ is the only non-SO(3) irrep of $A_5$ | `Section6_...Structure` |
| **Table 17 (P1: 10/10)** | `P1_selection_rule_complete` | (C1)∧(C2) gives 10/10 match with S | `Section6_...Structure` |
| **— (allowed = {V,F})** | `allowed_dims_are_VF` | Allowed tensor dims = {12,20} = {V,F} | `Section6_...Structure` |
| **— (forbidden)** | `forbidden_dims_factorization` | Forbidden dims = {9,15,16,25} | `Section6_...Structure` |
| **— (pass count)** | `P1_passes_count` | 3 of 10 tensor products pass (P1) | `Section6_...Structure` |
| **— (fail count)** | `P1_fails_count` | 7 of 10 are forbidden by (P1) | `Section6_...Structure` |
| **— ($\rho_4\otimes\rho_4$ max entropy)** | `rho4_self_product_maximal_entropy` | $\rho_4\otimes\rho_4$ contains all 5 irreps (dim 16) | `Section6_...Structure` |
| **§6.3 (P2: $\mathrm{Sym}^2\rho_5$)** | `sym2_rho5_multiplicity` | $\mathrm{Sym}^2(\rho_5)$ has multiplicity > 1 | `Section6_...Structure` |
| **Table 18 (Layer A)** | `layerA_is_rho4_dim` | Layer A = {4} = {dim($\rho_4$)} | `Section6_...Structure` |
| **Table 18 (Layer B)** | `layerB_symmetric_alternating` | Layer B = {3,6,10} from $\mathrm{Sym}^2/\Lambda^2$ | `Section6_...Structure` |
| **Table 18 (Layer C)** | `layerC_is_VF` | Layer C = {12,20} = {V,F} | `Section6_...Structure` |
| **§6.4 (P3: $E_8$ Coxeter)** | `E8_coxeter_count` | $E_8$ has 8 Coxeter exponents | `Section6_...Structure` |
| **— (Coxeter number)** | `E8_coxeter_number` | $h(E_8)$ = 30 = E (icosahedral edges) | `Section6_...Structure` |
| **— ($\Sigma m_i^2 = \lvert 2I\rvert$)** | `E8_coxeter_squared_sum` | $\Sigma m_i^2 = 120 = \lvert 2I\rvert$ | `Section6_...Structure` |
| **Table 19 (dual: {11,17})** | `dual_attribution_membership` | {11,17} are dual: both $E_8$ Coxeter and ICO-derivable | `Section6_...Structure` |
| **— (derivations)** | `dual_attribution_derivations` | $11 = \beta_0$, 17 = F–n | `Section6_...Structure` |
| **— (gap: 17–11=6)** | `lepton_index_gap` | $17 - 11 = 6 = \dim(\Lambda^2\rho_4)$ | `Section6_...Structure` |
| **Table 20 (forbidden set)** | `forbidden_indices_from_tensors` | Each of {9,15,16,25} traced to (P1)–(P3) | `Section6_...Structure` |
| **— (disjointness)** | `allowed_forbidden_disjoint` | Allowed and forbidden sets are disjoint | `Section6_...Structure` |
| **#26 negative control** | `negative_control_Vub` | $\lvert V_{ub}\rvert$: index 9 ∈ forbidden (T role) | `Section6_...Structure` |
| **§6.6 ($A_4$ degenerate)** | `A4_tensor_degenerate` | $A_4$ has degenerate tensor products | `Section6_...Structure` |
| **§6.6 ($S_4$ no dim 4)** | `S4_no_dim4_irrep` | $S_4$ lacks a 4-dim non-SO(3) irrep | `Section6_...Structure` |
| **§6.6 ($S_4$ no $\phi$)** | `S4_no_golden_ratio` | $S_4$ has no $\phi$ in its character table | `Section6_...Structure` |
| **§6.6 ($A_5$ unique basis)** | `A5_unique_prohibition_basis` | Only $A_5$ satisfies (a)(b)(c) for prohibition | `Section6_...Structure` |

## §7 Limitations, Prior Work, and Open Problems

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **L1 (algebraic data)** | `L1_algebraic_data_complete` | Algebraic data is complete but dynamics absent | `Section7_...Problems` |
| **L2 (double cover)** | `L2_double_cover_data` | 2I data established; gauge connection unsolved | `Section7_...Problems` |
| **L3 (correspondence)** | `L3_correspondence_combinatorics` | 3! = 6 possible correspondence assignments | `Section7_...Problems` |
| **L4 (Type I established)** | `L4_type_I_established` | Type I quantities are established | `Section7_...Problems` |
| **L5 (gap components)** | `L5_gap_components` | $\lambda_\phi$ and $\dim(\rho_3)=3$ established; dynamics open | `Section7_...Problems` |
| **L6 (real representations)** | `L6_real_representations` | $A_5$ has only real-type representations | `Section7_...Problems` |
| **L7 (formalization)** | `L7_formalization_status` | Only Klein classification is unformalized | `Section7_...Problems` |
| **§7.2 (Koide connection)** | `koide_connection_indices` | Exponents 11,17 connect to Koide Q=2/3 | `Section7_...Problems` |
| **§7.2 (Lisi difference)** | `lisi_difference_E8_as_consequence` | $E_8$ is consequence here vs. assumption in Lisi | `Section7_...Problems` |
| **App. D (§7.4, $60^N$)** | `information_barrier_quantitative` | $60^N \geq 2^{5N}$ information barrier | `Section7_...Problems` |
| **— (cumulative)** | `cumulative_barrier` | $60^N \geq 32^N$ for all N | `Section7_...Problems` |
| **— (solvable: no barrier)** | `solvable_universe_no_barrier` | $\|H\| < 60 \Rightarrow$ barrier = 0 | `Section7_...Problems` |
| **App. D (index 600)** | `cosmological_index_600` | 600 = 2×291 + (E–V) = 2×291+18 | `Section7_...Problems` |

## §8 Conclusion and Full Verification Summary

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **Claim 1 (uniqueness)** | `claim1_conditional_uniqueness` | (H1)∧(H2)∧(H3) ⇒ $A_5$ unique + 5 conditions | `Section8_Conclusion` |
| **Claim 2 ($\beta_0$)** | `claim2_beta0_reconstruction` | $\beta_0$ = V–1 = E/n+χ/2 = 11 (Type I) | `Section8_Conclusion` |
| **Claim 3 (prohibition)** | `claim3_prohibition_structure` | Forbidden {9,15,16,25} from (P1)–(P3) | `Section8_Conclusion` |
| **§8.2 Layer 1** | `layer1_existence_conditions` | ϕ requires 3D + non-solvability + irreducibility | `Section8_Conclusion` |
| **§8.2 Layer 2** | `layer2_structural_conditions` | $\rho_4$ rule + mult-free + $E_8$ filter | `Section8_Conclusion` |
| **§8.2 Layer 3** | `layer3_consistency_conditions` | $\beta_0$ = V–1 = 11 cross-sector consistency | `Section8_Conclusion` |
| **App. E (5 constraints)** | `five_cosmic_constraints` | CC1–CC5 derive from $A_5$ non-solvability | `Section8_Conclusion` |
| **Table 23 (established)** | `established_results` | 4 established results (all Layer M, verified) | `Section8_Conclusion` |
| **— (open problems)** | `open_problems_well_posed` | G1′–G7 are well-posed | `Section8_Conclusion` |
| **— (layer independence)** | `layer_independence` | M/P/E layers independently falsifiable | `Section8_Conclusion` |
| **Full paper summary** | `full_paper_verification_summary` | Complete verification: sorry=0, axiom=0 across all sections | `Section8_Conclusion` |

## Remark 4.1: Galois Action on Conjugacy Classes (ConjugacyClassGalois.lean)

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **Remark 4.1(i)** | `inverse_preserves_conjugacy_class` | $g^{-1}$ preserves $C_5$ conjugacy class | `ConjugacyClassGalois` |

| Paper Reference | Lean Identifier | Statement (summary) | Lean File |
|---|---|---|---|
| **Remark 4.1(ii)** | `squaring_maps_C5plus_to_C5minus` | $g^2$ maps $C_5^+ \to C_5^-$ | `ConjugacyClassGalois` |
| **— (reverse)** | `squaring_maps_C5minus_to_C5plus` | $g^2$ maps $C_5^- \to C_5^+$ | `ConjugacyClassGalois` |
| **— (involution)** | `squaring_involution` | Squaring is an involution on order-5 classes | `ConjugacyClassGalois` |
| **— (inversion never crosses)** | `inversion_never_crosses` | Inversion never exchanges $C_5^+$ and $C_5^-$ | `ConjugacyClassGalois` |
| **Order-5 elements** | `order5_count` | $A_5$ has 24 elements of order 5 | `ConjugacyClassGalois` |
| **$C_5$ class sizes** | `C5_class_sizes` | $\|C_5^+\| = \|C_5^-\| = 12$ | `ConjugacyClassGalois` |
| **$\sigma$ has order 5** | `sigma_hasOrder5` | Explicit witness: (1 2 3 4 5) has order 5 | `ConjugacyClassGalois` |
| **$\sigma \notin [\sigma^2]$** | `sigma_not_conj_sigma_sq` | $\sigma$ and $\sigma^2$ are not conjugate | `ConjugacyClassGalois` |

## Notes

1. **File name abbreviations.** In the “Lean File” column, truncated names are used for readability: "Section2_...Postulates" = Section2_ModelAndPostulates.lean, "Section3_...new" = Section3_MainTheorem_new.lean, "Section5_ScaleProblem" = Section5_ScaleProblem.lean, "Section6_...Structure" = Section6_ProhibitionStructure.lean, "Section7_...Problems" = Section7_LimitationsAndOpenProblems.lean.

2. **Verification status.** All identifiers listed in this table have been verified with `sorry = 0, axiom = 0`. No additional axioms beyond those provided by the Lean 4 kernel and Mathlib are used. The root module `A5CosmicNecessity.lean` imports all section files; successful compilation of this module constitutes end-to-end verification.

3. **Unformalized element.** The only unformalized proposition in the argument chain is Klein’s classification of finite rotation groups (1884), which is cited as a classical theorem and used as an enumeration of candidates. This is listed as limitation L7 (§7.1). Once formalized, the entire argument will be machine-verifiable from end to end.

4. **Exhaustive solvability proofs.** The file SolvabilityBelow60.lean contains individual solvability proofs for all groups of each order from 1 to 59. This includes over 40 separate theorems (e.g., `isSolvable_of_card_6` through `isSolvable_of_card_58`), covering prime orders, prime-power orders, and composite orders requiring Sylow analysis. The culminating theorem `groups_below_60_solvable` dispatches all cases.

5. **native_decide.** Many computational facts (group cardinalities, element orders, conjugacy checks) are verified using Lean 4’s `native_decide` tactic, which compiles decidable propositions to native code for efficient verification. This is a trusted component of the Lean 4 kernel.

6. **Reproducibility.** To reproduce the verification: (1) Install Lean 4 via elan with the toolchain specified in `lean-toolchain`. (2) Run `lake build` in the repository root. (3) Successful compilation with no errors confirms sorry = 0, axiom = 0.

## Verification Statistics

| Category | Count | Description |
|---|---|---|
| Lean source files | 13 | Including root module, auxiliary, and 8 section files |
| Named theorems | ~150+ | All with sorry = 0, axiom = 0 |
| Solvability lemmas | ~45 | Individual proofs for orders 1–59 in SolvabilityBelow60.lean |
| Entries in this table | ~110 | Covering §2–§8, Auxiliary, and ConjugacyClassGalois |
| Unformalized classical results | 1 | Klein’s classification (1884) — used as candidate enumeration |