Envy

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To envy is to wish that you had something that another person has dic [2023]. There are no measures for quantifying the collective extent of envy prevenlt in a group (like a country). This paper attempts to define such a metric. As is the case with such cases, there are many assumptions and simplifications. *Keywords: Envy, Happiness, Economics, Social, Welfare, Utility, Utility Functions*

I. One-to-One Envy

Our goal is to define a metric that quantifies the extent of envy of one person, i for another person j.

$$E_{i,j} \in [0, 1] \tag{1}$$

In this paper, we will isolate our definition to one type of envy - say, envy for income-level. And we will ignore all other types of envy - envy for wealth, intelligence, attractiveness etc.

We will denote Person i's income as I_i and Person j's income as I_j.

We will denote Person j's income relative to Person i as $R_{i,j}$. See also Wikipedia contributors [2023].

$$R_{i,j} = \frac{I_j}{I_i} \tag{2}$$

We assume that Income is non-negative.

$$\forall i, I_i \geq 0 \tag{3}$$

We assume that a person can feel envy for another only if the latter has more income than the former.

$$R_{i,j} > 1 \implies 0 < E_{i,j} < 1 \tag{4}$$

$$R_{i,j} \leq 1 \implies E_{i,j} = 0 \tag{5}$$

We assume that envy rises from zero as relative income increases, until it reaches some limit, and then declines to zero. Or in other words once the others income is sufficiently high, the person gradually stops feeling envy.

Let's denote the maximum limit with α. For symmetry, let's assume that Envy has completely declined to zero by α^2.

There are many ways in which the increase and decline could happen, but we will assume the simplest. A simple linear increase followed by a simple linear decline.

$$E_{i,j} = \begin{cases} 0, \beta_{i,j} \leq 0 \\ \beta_{i,j}, 0 \leq \beta_{i,j} \leq 1 \\ 2 - \beta_{i,j}, 1 \leq \beta_{i,j} \leq 2, \\ 0, \beta_{i,j} \geq 2 \end{cases} \tag{6}$$

[1]

where

$$\beta_{i,j} = log(\frac{R_{i,j}}{\alpha}) \tag{7}$$

II. One-to-Many Envy

We will use $E_{i,\cdot}$ to denote the collective envy felt by Person i for all other people in the population. We will assume that it is the normalize sum of Envy across the population. I.e.,

$$E_{i,\cdot} = \frac{\sum_j E_{i,j}}{N} \tag{8}$$

where N is the total number of people in the population.

III. Many-to-Many Envy

Similarly we can define the collective envy felt by all people in the population as $E_{\cdot,\cdot}$ or simply E as follows:

[1]Add image version of this equation.

$$E = \frac{\sum_i E_{i,\cdot}}{N} = \frac{\sum_i \sum_j E_{i,j}}{N^2} \tag{9}$$

Let p_I denote the proportion of people in the population with income I, and let $E_{I,J}$ denote the Envy felt by a person with income I for a person with income J.

Then we can write the above as:

$$E = \frac{\sum_I \sum_J N p_I \cdot N p_J \cdot E_{I,J}}{N^2} = \sum_I \sum_J p_I \cdot p_J \cdot E_{I,J} \tag{10}$$

The last representation implies that our definition is invariant to scaling the population. I.e., if we double the population proportionally, the Envy remains the same.

Going forward, we will use this representation of *groups of people with the same income*, rather than *individuals*.

IV. Minimizing and Maximizing Envy

A. Perfect Equality

Trivially, when the entire population has the same income, there is no Envy.

$$I_i = k \forall i \implies E = 0 \tag{11}$$

B. Isolated Classes

Also, if the population is split into groups where within each groups individuals have the same income, and the relative income across two groups is more than α^2, then there is no Envy.

For example, if these incomes are $k, k\gamma^2, k\gamma^4 ...,$, where $\gamma \geq \alpha$, then there is no Envy.

$$E = 0 \tag{12}$$

C. Simillar Classes

Conversely, if $\gamma < \alpha$, the classes have an overlap in Envy and the E is no longer zero.

D. Progressive Classes

When $\gamma = \alpha$, the classes are progressive and the Envy is maximized. [2]

REFERENCES

Cambridge Dictionaries Online. https://dictionary.cambridge.org/dictionary/english/envy/, 2023. [Online; accessed 29-July-2023].

Wikipedia contributors. Relative income hypothesis — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Relative_income_hypothesis&oldid=1161843979, 2023. [Online; accessed 29-July-2023].

[2]This needs to be proved.