# Envy

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*To envy is to wish that you had something that another person has dic [2023]. There are no measures for quantifying the collective extent of envy prevenlt in a group (like a country). This paper attempts to define such a metric. As is the case with such cases, there are many assumptions and simplifications. Keywords: Envy, Happiness, Economics, Social, Welfare, Utility, Utility Functions*

## I.   One-to-One Envy

Our goal is to define a metric that quantifies the extent of envy of one person, $i$ for another person $j$.

$$(1) \qquad E_{i,j} \in [0,1]$$

In this paper, we will isolate our definition to one type of envy - say, envy for income-level. And we will ignore all other types of envy - envy for wealth, intelligence, attractiveness etc.

We will denote Person $i$'s income as $I_i$ and Person $j$'s income as $I_j$.

We will denote Person $j$'s income relative to Person $i$ as $R_{i,j}$.

$$(2) \qquad R_{i,j} = \frac{I_j}{I_i}$$

We assume that Income is non-negative.

$$(3) \qquad \forall i, I_i \geq 0$$

We assume that a person can feel envy for another only if the latter has more income than the former.

$$(4) \qquad R_{i,j} > 1 \implies 0 < E_{i,j} < 1$$

$$(5) \qquad R_{i,j} \leq 1 \implies E_{i,j} = 0$$

We assume that envy rises from zero as relative income increases, until it reaches some limit, and then declines to zero. Or in other words once the others income is sufficiently high, the person gradually stops feeling envy.

Let's denote the maximum limit with $\alpha$. For symmetry, let's assume that Envy has completely declined to zero by $\alpha^2$.

There are many ways in which the increase and decline could happen, but we will assume the simplest. A simple linear increase followed by a simple linear decline.

$$
(6) \qquad E_{i,j} = \begin{cases} 0, \beta_{i,j} \leq 0 \\ \beta_{i,j}, 0 \leq \beta_{i,j} \leq 1 \\ 2 - \beta_{i,j}, 1 \leq \beta_{i,j} \leq 2, \\ 0, \beta_{i,j} \geq 2 \end{cases}
$$

where

$$
(7) \qquad \beta_{i,j} = log(\frac{R_{i,j}}{\alpha})
$$

## II. One-to-Many Envy

We will use $E_{i,.}$ to denote the collective envy felt by Person $i$ for all other people in the population. We will assume that it is the normalize sum of Envy across the population. I.e.,

$$
(8) \qquad E_{i,.} = \frac{\sum_j E_{i,j}}{N}
$$

where $N$ is the total number of people in the population.

## III. Many-to-Many Envy

Similarly we can define the collective envy felt by all people in the population as $E_{.,.}$ or simply $E$ as follows:

$$
(9) \qquad E = \frac{\sum_i E_{i,.}}{N} = \frac{\sum_i \sum_j E_{i,j}}{N^2}
$$

## REFERENCES

Cambridge Dictionaries Online. `https://dictionary.cambridge.org/dictionary/english/envy/`, 2023. [Online; accessed 29-July-2023].