Artificial Intelligence Homework 2

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1.1 P(D):

$$
\begin{aligned}
&= \sum_{A,B,C,E} P(A) * P(B|A) * P(C|A) * P(D|B,C) * P(E|C) \\
&= P(A) * P(B|A) * P(C|A) * P(D|B,C) * P(E|C) \\
&+ P(A) * P(B|A) * P(C|A) * P(D|B,C) * P(-E|C) \\
&+ P(A) * P(B|A) * P(-C|A) * P(D|B,-C) * P(E|-C) \\
&+ P(A) * P(B|A) * P(-C|A) * P(D|B,-C) * P(-E|-C) \\
&+ P(A) * P(-B|A) * P(C|A) * P(D|-B,C) * P(E|C) \\
&+ P(A) * P(-B|A) * P(C|A) * P(D|-B,C) * P(-E|C) \\
&+ P(A) * P(-B|A) * P(-C|A) * P(D|-B,-C) * P(E|-C) \\
&+ P(A) * P(-B|A) * P(-C|A) * P(D|-B,-C) * P(-E|-C) \\
&+ P(-A) * P(B|-A) * P(C|-A) * P(D|B,C) * P(E|C) \\
&+ P(-A) * P(B|-A) * P(C|-A) * P(D|B,C) * P(-E|C) \\
&+ P(-A) * P(B|-A) * P(-C|-A) * P(D|B,-C) * P(E|-C) \\
&+ P(-A) * P(B|-A) * P(-C|-A) * P(D|B,-C) * P(-E|-C) \\
&+ P(-A) * P(-B|-A) * P(C|-A) * P(D|-B,C) * P(E|C) \\
&+ P(-A) * P(-B|-A) * P(C|-A) * P(D|-B,C) * P(-E|C) \\
&+ P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B,-C) * P(E|-C) \\
&+ P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B,-C) * P(-E|-C)
\end{aligned}
$$

After the calculations, the only thing that is left is putting the probabilities to the right locations.

$$
\begin{aligned}
&= (0.2) * (0.8) * (0.2) * (0.8) * (0.8) \\
&+ (0.2) * (0.8) * (0.2) * (0.8) * (0.2) \\
&+ (0.2) * (0.8) * (0.8) * (0.8) * (0.6) \\
&+ (0.2) * (0.8) * (0.8) * (0.8) * (0.8) \\
&+ (0.2) * (0.2) * (0.2) * (0.8) * (0.8) \\
&+ (0.2) * (0.2) * (0.2) * (0.8) * (0.2) \\
&+ (0.2) * (0.2) * (0.8) * (0.05) * (0.6) \\
&+ (0.2) * (0.2) * (0.2) * (0.8) * (0.2) \\
&+ (0.8) * (0.2) * (0.8) * (0.05) * (0.6) \\
&+ (0.8) * (0.2) * (0.05) * (0.8) * (0.2) \\
&+ (0.8) * (0.2) * (0.95) * (0.8) * (0.6) \\
&+ (0.8) * (0.2) * (0.95) * (0.8) * (0.4) \\
&+ (0.8) * (0.8) * (0.05) * (0.8) * (0.8) \\
&+ (0.8) * (0.8) * (0.05) * (0.8) * (0.2) \\
&+ (0.8) * (0.8) * (0.95) * (0.05) * (0.6) \\
&+ (0.8) * (0.8) * (0.95) * (0.05) * (0.4) \\
&= \mathbf{0.32}
\end{aligned}
$$

1.2 P(D,-A):

$$
\begin{aligned}
&= \sum_{B,C,E} P(-A) * P(B|-A) * P(C|-A) * P(D|B,C) * P(E|C) \\
&= P(-A) * P(B|-A) * P(C|-A) * P(D|B,C) * P(E|C) \\
&+ P(-A) * P(B|-A) * P(C|-A) * P(D|B,C) * P(-E|C) \\
&+ P(-A) * P(B|-A) * P(-C|-A) * P(D|B,-C) * P(E|-C) \\
&+ P(-A) * P(B|-A) * P(-C|-A) * P(D|B,-C) * P(-E|-C) \\
&+ P(-A) * P(-B|-A) * P(C|-A) * P(D|-B,C) * P(E|C) \\
&+ P(-A) * P(-B|-A) * P(C|-A) * P(D|-B,C) * P(-E|C) \\
&+ P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B,-C) * P(E|-C) \\
&+ P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B,-C) * P(-E|-C)
\end{aligned}
$$

$$
\begin{aligned}
&= (0.8) * (0.2) * (0.8) * (0.05) * (0.6) \\
&+ (0.8) * (0.2) * (0.05) * (0.8) * (0.2) \\
&+ (0.8) * (0.2) * (0.95) * (0.8) * (0.6) \\
&+ (0.8) * (0.2) * (0.95) * (0.8) * (0.4) \\
&+ (0.8) * (0.8) * (0.05) * (0.8) * (0.8) \\
&+ (0.8) * (0.8) * (0.05) * (0.8) * (0.2) \\
&+ (0.8) * (0.8) * (0.95) * (0.05) * (0.6) \\
&+ (0.8) * (0.8) * (0.95) * (0.05) * (0.4) \\
&= \mathbf{0.184}
\end{aligned}
$$

1.3 P(E|-B):

$$
\begin{aligned}
&= \frac{P(E,-B)}{P(-B)} = \frac{P(E,-B)}{P(E,-B)+P(-E,-B)} \\
P(E,-B) &= \sum\nolimits_{A,C,D} P(A)*P(-B|A)*P(C|A)*P(D|-B,C)*P(E|C) \\
&= P(A).P(-B|A).P(C|A).P(D|-B,C).P(E|C) \\
&+ P(A).P(-B|A).P(-C|A).P(D|-B,-C).P(E|-C) \\
&+ P(A).P(-B|A).P(C|A).P(-D|-B,C).P(E|C) \\
&+ P(A).P(-B|A).P(-C|A).P(-D|-B,-C).P(E|-C) \\
&+ P(-A).P(-B|-A).P(C|-A).P(D|-B,C).P(E|C) \\
&+ P(-A).P(-B|-A).P(-C|-A).P(D|-B,-C).P(E|-C) \\
&+ P(-A).P(-B|-A).P(C|-A).P(-D|-B,C).P(E|C) \\
&+ P(-A).P(-B|-A).P(-C|-A).P(-D|-B,-C).P(E|-C) \\
&= (0.2)*(0.2)*(0.2)*(0.8)*(0.8) \\
&+ (0.2)*(0.2)*(0.8)*(0.05)*(0.6) \\
&+ (0.2)*(0.2)*(0.2)*(0.2)*(0.8) \\
&+ (0.2)*(0.2)*(0.8)*(0.95)*(0.6) \\
&+ (0.8)*(0.8)*(0.05)*(0.8)*(0.8) \\
&+ (0.8)*(0.8)*(0.95)*(0.05)*(0.6) \\
&+ (0.8)*(0.8)*(0.05)*(0.2)*(0.8) \\
&+ (0.8)*(0.8)*(0.95)*(0.95)*(0.6) \\
&= \mathbf{0.416} \\
P(-E,-B) &= \sum\nolimits_{A,C,D} P(A)*P(-B|A)*P(C|A)*P(D|-B,C)*P(-E|C) \\
&= P(A)*P(-B|A)*P(C|A)*P(D|-B,C)*P(-E|C) \\
&+ P(A)*P(-B|A)*P(C|A)*P(D|-B,C)*P(-E|C) \\
&+ P(A)*P(-B|A)*P(-C|A)*P(D|-B,-C)*P(-E|-C) \\
&+ P(A)*P(-B|A)*P(-C|A)*P(D|-B,-C)*P(-E|-C) \\
&+ P(-A)*P(-B|-A)*P(C|-A)*P(D|-B,C)*P(-E|C) \\
&+ P(-A)*P(-B|-A)*P(C|-A)*P(D|-B,C)*P(-E|C) \\
&+ P(-A)*P(-B|-A)*P(-C|-A)*P(D|-B,-C)*P(-E|-C) \\
&+ P(-A)*P(-B|-A)*P(-C|-A)*P(D|-B,-C)*P(-E|-C) \\
&= (0.2)*(0.2)*(0.2)*(0.8)*(0.2) \\
&+ (0.2)*(0.2)*(0.2)*(0.2)*(0.2) \\
&+ (0.2)*(0.2)*(0.8)*(0.05)*(0.4) \\
&+ (0.2)*(0.2)*(0.8)*(0.95)*(0.4) \\
&+ (0.8)*(0.8)*(0.05)*(0.8)*(0.2) \\
&+ (0.8)*(0.8)*(0.05)*(0.2)*(0.2) \\
&+ (0.8)*(0.8)*(0.95)*(0.05)*(0.4) \\
&+ (0.8)*(0.8)*(0.95)*(0.95)*(0.4) \\
&= \mathbf{0.264} \\
&= \frac{0.416}{0.416+0.264} = \mathbf{0.612}
\end{aligned}
$$

1.4 P(A|D,-E):

$$= \frac{P(A, D, -E)}{P(D, -E)} = \frac{P(A, D, -E)}{P(A, D, -E) + P(-A, D, -E)}$$

$$\begin{aligned}
P(A, D, -E) &= \sum\nolimits_{B,C} P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(-E|C) \\
&= P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(-E|C) \\
&+ P(A) * P(B|A) * P(-C|A) * P(D|B, -C) * P(-E| - C) \\
&+ P(A) * P(-B|A) * P(C|A) * P(D| - B, C) * P(-E|C) \\
&+ P(A) * P(-B|A) * P(-C|A) * P(D| - B, -C) * P(-E| - C)
\end{aligned}$$

$$\begin{aligned}
&= (0.2) * (0.8) * (0.2) * (0.8) * (0.2) \\
&+ (0.2) * (0.8) * (0.8) * (0.8) * (0.4) \\
&+ (0.2) * (0.2) * (0.2) * (0.8) * (0.2) \\
&+ (0.2) * (0.2) * (0.8) * (0.05) * (0.4) \\
&= \mathbf{0.048}
\end{aligned}$$

$$\begin{aligned}
P(-A, D, -E) &= \sum\nolimits_{B,C} P(-A) * P(B| - A) * P(C| - A) * P(D|B, C) * P(-E|C) \\
&= P(-A) * P(B| - A) * P(C| - A) * P(D|B, C) * P(-E|C) \\
&+ P(-A) * P(B| - A) * P(-C| - A) * P(D|B, -C) * P(-E| - C) \\
&+ P(-A) * P(-B| - A) * P(C| - A) * P(D| - B, C) * P(-E|C) \\
&+ P(-A) * P(-B| - A) * P(-C| - A) * P(D| - B, -C) * P(-E| - C)
\end{aligned}$$

$$\begin{aligned}
&= (0.8) * (0.2) * (0.05) * (0.8) * (0.2) \\
&+ (0.8) * (0.2) * (0.95) * (0.8) * (0.4) \\
&+ (0.8) * (0.8) * (0.05) * (0.8) * (0.2) \\
&+ (0.8) * (0.8) * (0.95) * (0.05) * (0.4) \\
&= \mathbf{0.0672}
\end{aligned}$$

$$= \frac{0.048}{0.048 + 0.0672} = \mathbf{0.417}$$

1.5 P(B,-E|A):

$$= \frac{P(B,-E,A)}{P(A)} = \frac{P(B,-E,A)}{P(B,E,A)+P(B,-E,A)+P(-B,E,A)+P(-B,-E,A)}$$

$$\begin{aligned}
P(B,E,A) &= \sum\nolimits_{C,D} P(A)*P(B|A)*P(C|A)*P(D|B,C)*P(E|C) \\
&= P(A)*P(B|A)*P(C|A)*P(D|B,C)*P(E|C) \\
&+ P(A)*P(B|A)*P(C|A)*P(-D|B,C)*P(E|C) \\
&+ P(A)*P(B|A)*P(-C|A)*P(D|B,-C)*P(E|-C) \\
&+ P(A)*P(B|A)*P(-C|A)*P(-D|B,-C)*P(E|-C) \\
&= (0.2)*(0.8)*(0.2)*(0.8)*(0.8) \\
&+ (0.2)*(0.8)*(0.2)*(0.2)*(0.8) \\
&+ (0.2)*(0.8)*(0.8)*(0.8)*(0.6) \\
&+ (0.2)*(0.8)*(0.8)*(0.2)*(0.6) \\
&= \mathbf{0.1024}
\end{aligned}$$

$$\begin{aligned}
P(B,-E,A) &= \sum\nolimits_{B,C} P(A)*P(B|A)*P(C|A)*P(D|B,C)*P(-E|C) \\
&= P(A)*P(B|A)*P(C|A)*P(D|B,C)*P(-E|C) \\
&+ P(A)*P(B|A)*P(C|A)*P(-D|B,C)*P(-E|C) \\
&+ P(A)*P(B|A)*P(-C|A)*P(D|B,-C)*P(-E|-C) \\
&+ P(A)*P(B|A)*P(-C|A)*P(-D|B,-C)*P(-E|-C) \\
&= (0.2)*(0.8)*(0.2)*(0.8)*(0.2) \\
&+ (0.2)*(0.8)*(0.2)*(0.2)*(0.2) \\
&+ (0.2)*(0.8)*(0.8)*(0.8)*(0.4) \\
&+ (0.2)*(0.8)*(0.8)*(0.2)*(0.4) \\
&= \mathbf{0.0576}
\end{aligned}$$

$$\begin{aligned}
P(-B,E,A) &= \sum\nolimits_{B,C} P(A)*P(-B|A)*P(C|A)*P(D|-B,C)*P(E|C) \\
&= P(A)*P(-B|A)*P(C|A)*P(D|-B,C)*P(E|C) \\
&+ P(A)*P(-B|A)*P(C|A)*P(-D|-B,C)*P(E|C) \\
&+ P(A)*P(-B|A)*P(-C|A)*P(D|-B,-C)*P(E|-C) \\
&+ P(A)*P(-B|A)*P(-C|A)*P(-D|-B,-C)*P(E|-C) \\
&= (0.2)*(0.2)*(0.2)*(0.8)*(0.8) \\
&+ (0.2)*(0.2)*(0.2)*(0.2)*(0.8) \\
&+ (0.2)*(0.2)*(0.8)*(0.05)*(0.6) \\
&+ (0.2)*(0.2)*(0.8)*(0.95)*(0.6) \\
&= \mathbf{0.0256}
\end{aligned}$$

$$\begin{aligned}
P(-B,-E,A) &= \sum\nolimits_{B,C} P(A)*P(-B|A)*P(C|A)*P(D|-B,C)*P(-E|C) \\
&= P(A)*P(-B|A)*P(C|A)*P(D|-B,C)*P(-E|C) \\
&+ P(A)*P(-B|A)*P(C|A)*P(-D|-B,C)*P(-E|C) \\
&+ P(A)*P(-B|A)*P(-C|A)*P(D|-B,-C)*P(-E|-C) \\
&+ P(A)*P(-B|A)*P(-C|A)*P(-D|-B,-C)*P(-E|-C) \\
&= (0.2)*(0.2)*(0.2)*(0.8)*(0.2) \\
&+ (0.2)*(0.2)*(0.2)*(0.2)*(0.2) \\
&+ (0.2)*(0.2)*(0.8)*(0.05)*(0.4) \\
&+ (0.2)*(0.2)*(0.8)*(0.95)*(0.4) \\
&= \mathbf{0.0144}
\end{aligned}$$

$$= \frac{0.0576}{0.1024 + 0.0576 + 0.0256 + 0.0144} = \mathbf{0.288}$$

2

2.1 P(D):

$$P(D) = \sum_{A,B,C,E} P(A) * P(B|A) * P(C|A) * P(D|B,C) * P(E|C)$$
$$F_A(B,C) = \sum_A P(A) * P(B|A) * P(C|A)$$

$\mathbf{F}_A$	**P(A)*P(B|A)*P(C|A)**	**P(-A)*P(B|-A)*P(C|-A)**
B,C	0.2*0.8*0.2	0.8*0.2*0.05
B,-C	0.2*0.8*0.8	0.8*0.2*0.95
-B,C	0.2*0.2*0.2	0.8*0.8*0.05
-B,-C	0.2*0.2*0.8	0.8*0.8*0.95

Add columns row by row.

F_A(B,C) = 0.032+0.008 = 0.04
F_A(B,-C) = 0.128+0.152 = 0.28
F_A(-B,C)= 0.008+0.032 = 0.04
F_A(-B,-C)= 0.032+0.608 = 0.64

$$P(D) = \sum_{B,C,E} F_A(B,C) * P(D|B,C) * P(E|C)$$
$$F_B(C) = \sum_B F_A(B,C) * P(D|B,C)$$

$\mathbf{F}_B$	**$\mathbf{F}_A$(B,C)*P(D|B,C)**	**$\mathbf{F}_A$(-B,C)*P(D|-B,C)**
C	0.04*0.8	0.04*0.8
-C	0.28*0.8	0.64*0.05

Add columns row by row.

F_B(C) = 0.032+0.032 = 0.064
F_B(-C) = 0.224+0.032 = 0.256

$$P(D) = \sum_{C,E} F_B(C) * P(E|C)$$
$$F_C(E) = \sum_C F_B(C) * P(E|C)$$

$\mathbf{F}_C$	**$\mathbf{F}_B$(C)*P(E|C)**	**$\mathbf{F}_B$(-C)*P(E|-C)**
E	0.064*0.8	0.256*0.6
-E	0.064*0.2	0.256*0.4

Add columns row by row.

F_C(E) = 0.0512+0.1536 = 0.2048
F_C(-E) = 0.0128+0.1024 = 0.1152

$$P(D) = \sum_E F_C(E) = \mathbf{0.32}$$

2.2 P(D,-A):

$$P(D,-A) = \sum_{B,C,E} P(-A) * P(B|-A) * P(C|-A) * P(D|B,C) * P(E|C)$$
$$F_{B}(C) = \sum_{B} P(-A) * P(B|-A) * P(D|B,C)$$

As I showed the steps above in the previous question, for this time and the later ones, I will not show all the summation steps and getting probability parts for making the answer shorter. The tables and the specific summations of F_{C}, F_{B} etc. will not be showed. There will just be the results of the summations.

$$F_{B}(C) = 0.128 + 0.512 = 0.64$$
$$F_{B}(-C) = 0.128 + 0.032 = 0.16$$

$$P(D,-A) = F_{B}(C) * P(C|-A) * P(E|C)$$
$$F_{C}(E) = \sum_{C} F_{B}(C) * P(C|-A) * P(E|C)$$
$$F_{C}(E) = 0.0256 + 0.0912 = 0.1168$$
$$F_{C}(-E) = 0.0064 + 0.0608 = 0.0692$$

$$P(D,-A) = \sum_{E} F_{C}(E) = \mathbf{0.184}$$

2.3 P(E|-B):

$$P(E|-B) = \sum_{A,C,D,E} P(A) * P(-B|A) * P(C|A) * P(D|-B,C) * P(E|C)$$
$$F_{A}(C) = \sum_{A} P(A) * P(-B|A) * P(C|A)$$
$$F_{A}(C) = 0.008 + 0.032 = 0.04$$
$$F_{A}(-C) = 0.032 + 0.608 = 0.64$$

$$P(E|-B) = \sum_{C,D,E} F_{A}(C) * P(D|-B,C) * P(E|C)$$
$$F_{C}(D,E) = \sum_{C} F_{A}(C) * P(D|-B,C) * P(E|C)$$
$$F_{C}(D,E) = 0.2752$$
$$F_{C}(D,-E) = 0.0192$$
$$F_{C}(-D,E) = 0.3712$$
$$F_{C}(-D,-E) = 0.2448$$

$$P(E|-B) = \sum_{D,E} F_{C}(D,E)$$
$$F_{D}(E) = \sum_{D} F_{C}(D,E)$$
$$F_{D}(E) = 0.416$$
$$F_{D}(-E) = 0.264$$

$$= \frac{0.416}{0.264 + 0.416} = \mathbf{0.612}$$

2.4 P(A|D,-E):

$$P(A|D,-E) = \sum_{B,C} P(B|A) * P(D|B,C)$$
$$F_{B}(C) = \sum_{B} P(A) * P(-B|A) * P(C|A)$$
$$F_{B}(C) = 0.64 + 0.16 = 0.8$$
$$F_{B}(-C) = 0.64 + 0.01 = 0.65$$

$$F_{C}() = \sum_{C} F_{B}(C) * P(C|A) * P(-E|C)$$
$$F_{C}() = 0.032 + 0.208 = 0.24$$
$$P(A,D,-E) = P(A) * F_{C}() = (0.2) * (0.24) = 0.048$$

$$F_{B}(C) = 0.16 + 0.64 = 0.8$$
$$F_{B}(-C) = 0.16 + 0.04 = 0.2$$

$$F_{C}() = 0.008 + 0.076 = 0.084$$
$$P(-A,D,-E) = P(-A) * F_{C}() = (0.8) * (0.084) = 0.0672$$
$$= \frac{0.048}{0.048 + 0.0672} = \mathbf{0.417}$$

2.5 P(B,-E|A):

$$= \frac{P(B,-E,A)}{P(A)} = \frac{P(B,-E,A)}{P(B,E,A)+P(B,-E,A)+P(-B,E,A)+P(-B,-E,A)}$$

$$P(B,E,A) = \sum_{C,D} P(A)*P(B|A)*P(C|A)*P(D|B,C)*P(E|C)$$

$$F_{C}(D) = \sum_{C} P(C|A)*P(D|B,C)*P(-E|C)$$

$$F_{C}(D) = 0.032 + 0.256 = 0.288$$

$$F_{C}(-D) = 0.008 + 0.064 = 0.072$$

$$F_{C}(D) = 0.128 + 0.384 = 0.512$$

$$F_{C}(-D) = 0.032 + 0.096 = 0.128$$

$$P(A,B,E) = (0.2)*(0.8)*(0.64) = 0.1024$$

$$F_{C}(D) = 0.128 + 0.024 = 0.152$$

$$F_{C}(-D) = 0.032 + 0.456 = 0.488$$

$$P(A,-B,E) = (0.2)*(0.2)*(0.64) = 0.0256$$

$$F_{C}(D) = 0.032 + 0.016 = 0.048$$

$$F_{C}(-D) = 0.008 + 0.304 = 0.312$$

$$P(A,-B,-E) = (0.2)*(0.2)*(0.36) = 0.0144$$

$$= \frac{0.0576}{0.0576+0.1024+0.0256+0.0144} = \mathbf{0.288}$$