Probability and Statistics - 2023

1)

Type 01

i) Minimum = $\underline{84}$

$Q_1 = \frac{1}{4}(n+1)$ th value (lower Quartile)

$= \frac{1}{4}(21)$ th

$= 5.25$ th

$= 94 + 0.25(94-94)$

$= \underline{\underline{94}}$

$Q_2 = \frac{1}{2}(n+1)$ th value

$= \frac{1}{2} \times 21$ th

$= 10.5$ th

$= 100 + 0.5 \times (100-100)$

$= \underline{\underline{100}}$

Maximum = $\underline{\underline{115}}$

$Q_3 = \frac{3}{4} \times (n+1)$ th value (upper Quartile)

$= \frac{3}{4} \times 21$ th value

$= 15.75$ th

$= 104 + 0.75(106-104)$

$= \underline{\underline{105.5}}$

Type 02

Minimum = $\underline{\underline{97}}$

$Q_1 = \frac{1}{4} \times (n+1)$ th value

$= 5.25$ th

$= 98 + 0.25(99-98)$

$= \underline{\underline{98.25}}$

$Q_2 = \frac{1}{2}(n+1)$ th valu

$= 10.5$ th

$= \underline{\underline{100}}$

$Q_3 = \frac{3}{4} \times (n+1)$ th

$= 15.75$ th

$= \underline{\underline{101}}$

Maximum = $\underline{\underline{108}}$

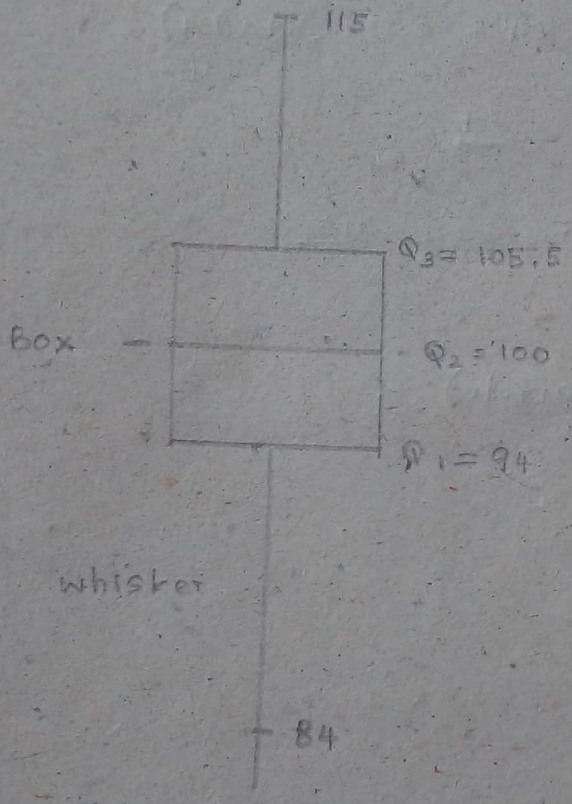
For type 01

$IQR = Q_3 - Q_1 = 105.5 - 94$

$= 11.5$

$1.5IQR = 1.5 \times 11.5$

$= 17.25$



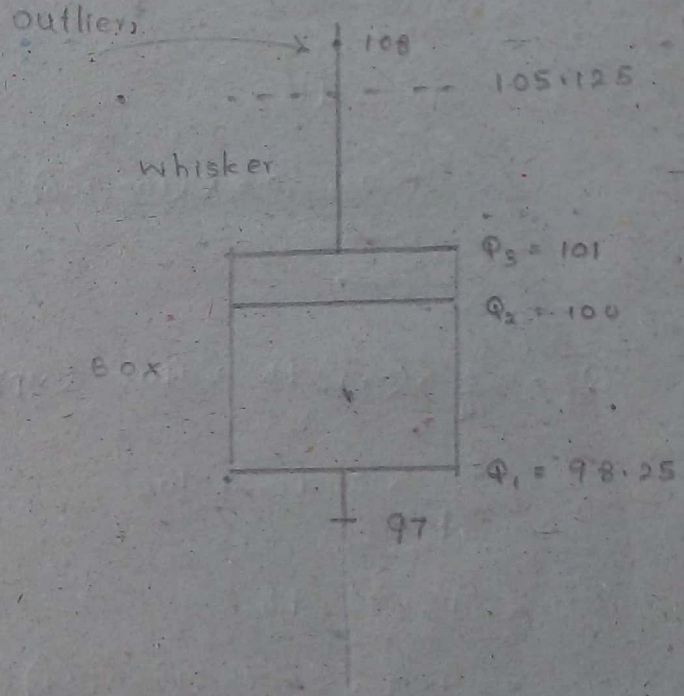
For type 02

$IQR = Q_3 - Q_1 = 101 - 98.25$

$= 2.75$

$1.5\ IQR = 1.5 \times 2.75$

$= 4.125$



43 46 52 53 55 56 58 60 62 63 64 66 66

72 74 74 75 77 77 78 83 85 85 87 88 90

91 94

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n} = 70.57$$

$$Sd = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} = 14.35$$

Stem	Leaf
4	3, 6
5	2, 5, 5, 6, 8
6	0, 2, 3, 4, 6, 6
7	2, 4, 4, 5, 7, 7, 8
8	3, 5, 5, 7, 8
9	0, 1, 4

Key "5|2" means 52

$\bar{x} + s = 70.57 + 14.35$
$= 84.92$

$\bar{x} - s = 70.57 - 14.35$
$= 56.22$

Therefore values within $\bar{x} + s$ and $\bar{x} - s = 15$

$\therefore$ Percentage $= \frac{15}{28} \times 100\%$

$= \underline{\underline{53.57\%}}$

Q2) a) $P(A) = 0.1$

$P(B) = 0.15$

$P(C) = 0.75$

$P(D) = P(B \cap D) + P(C \cap D)$

$= 0.15 + 0.75$

$= 0.9$

D - Not immediately fail

Using Conditional Probability

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{0.75}{0.9} = \underline{\underline{0.833}}$$

i) X - Floor accidents in each month

X = {2, 3, 1, 5, 4, 3, 2, 8, 9, 4, 10, 5}

) Y - The diameter of a metal cylinder in mm

(y can take any real number between 49.5 and 50.5 mm)

x	0	1	2	3	4	5	6
$P(x)$	0.05	0.1	0.15	0.25	0.20	0.15	0.1

$$P_r(x \le 2) = \sum_{i=0}^{2} x_i\, P(x_i)$$

$$= (0 \times 0.05) + (1 \times 0.1) + (2 \times 0.15)$$

$$= \underline{\underline{0.4}}$$

ii) $Pr(x \ge 3) = 1 - P_r(x < 3)$

$= 1 - \sum_{i=0}^{2} x_i\, P(x_i)$

$= 1 - 0.4$

$= 0.6$

1) $$P(x) = \begin{cases} \frac{1}{n} ; & x = 1, 2, 3, \ldots n \\ 0 ; & \text{otherwise} \end{cases}$$

$$E(x) = \sum_x x\, p(x) = \sum_{x=1}^{n} x \cdot p(x) = \sum_{x=1}^{n} x \cdot \frac{1}{n} = \frac{1}{n} \sum_{x=1}^{n} x$$

$$E(x) = \frac{1}{n} \times \frac{n(n+1)}{2} = \frac{n+1}{2}$$

$$V(x) \Rightarrow Var(x) = \sigma^2 = E\left[(x-\mu)^2\right]$$

$$E(x^2) = \sum_x x^2 p(x)$$

$$= \sum_{x=1}^{n} x^2 \cdot \frac{1}{n}$$

$$= \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)(2n+1)}{6}$$

$$= E\left[x^2 - 2\mu x + \mu^2\right]$$

$$= \sum \left(x^2 - 2\mu x + \mu^2\right) p(x)$$

$$= \sum x^2 p(x) - \sum 2\mu x\, p(x) + \sum \mu^2 p(x)$$

$$= E(x^2) - 2\mu^2 + \mu^2 \cdot 1$$

$$= E(x^2) - \mu^2 = E(x^2) - \left[E(x)\right]^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{2(2n^2+n+2n+1)}{12} - \frac{3(n^2+2n+}{12}$$

$$= \frac{4n^2 + \cancel{2n} + \cancel{4n} + 2 - 3n^2 - \cancel{6n} -}{12}$$

$$V(x) = \frac{n^2 - 1}{12} \;//$$

~~P(A) ⇒ P(F | A) = 0.1~~

~~P(B) = P(F | B) = 0.15~~

~~P(C) =~~

~~P(C) = P~~

~~P(D) = P(B) + P(C)~~

~~= 0.15 + 0.75~~

~~= 0.9~~

~~P(E | D) = P(E∩D) / P(D) = 0.75 / 0.9~~ =

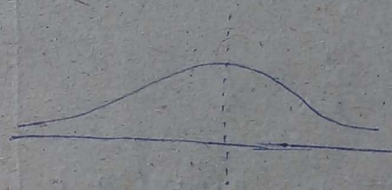
Q3)

7.4, 5.8, 6.5, 8.4, 9.3, 10, 5.9, 7.3, 6.3, 8.1, 7, 7.6, 6.5, 9, 8.2, 8.7

7.8, 9.7, 11.6, 11.3

i)

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = 8.12$$



ii)

$$-z_{\alpha/2} \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < z_{\alpha/2}$$

$$-z_{\alpha/2} \cdot \sigma/\sqrt{n} \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2} \; \sigma/\sqrt{n}$$

$$\bar{X} - z_{\alpha/2} \, \sigma/\sqrt{n} \leq \mu \leq \bar{X} + z_{\alpha/2} \cdot \sigma/\sqrt{n}$$

First arrange the sample in ascending order

5.8, 5.9, 6.3, 6.5, 6.5, 7, 7.3, 7.4, 7.6, 7.8, 8.1, 8.2, 8.4, 8.7, 9.0, 9.3, 9.7,
10.0, 11.3, 11.6

Then, the point estimate for the median is the value from th
that seperates the lowest 50% of the data
highest 50%.

$$\text{Median} = \left(\frac{20+1}{2}\right)\text{th value}$$

$$= 10.5 \text{ th value}$$

$$= \frac{(7.8+8.1)}{2}$$

$$= \underline{\underline{7.95}}$$

iii.) Calculate a point estimate of the population standa
deviation σ

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

b) $n = 17$, $S^2 = 137324.3$ $\alpha = 0.025$

Estimating the variance:

$$\boxed{X^2 = \frac{(n-1)s^2}{\sigma^2}}$$

$$P\left(X^2_{1-\alpha/2} < X^2 < X^2_{\alpha/2}\right) = 1-\alpha$$

$$P\left(X^2_{1-\alpha/2} < X^2 < X^2_{\alpha/2}\right) = 1-\alpha$$

$$X^2_{1-\alpha/2} < \frac{(n-1)s^2}{\sigma^2} < \chi^2_{\alpha/2}$$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$; $n = 17$

DoF = 16

$S^2 = 137324.3$

$$\frac{16 \times 137324.3}{28.845} < \sigma^2 < \frac{16 \times 137324.3}{6.908}$$

$$76172.26 < \sigma^2 < 318064.39$$

$$\underline{\underline{275.99 < \sigma < 563.97}}$$

$H_0: \mu_1 - \mu_2 \le 10$, $H_1: \mu_1 - \mu_2 > 10$

	n	$\bar{x}$	s
② No fusion	40	290.2	277.3
① Fused	35	310.8	205.9

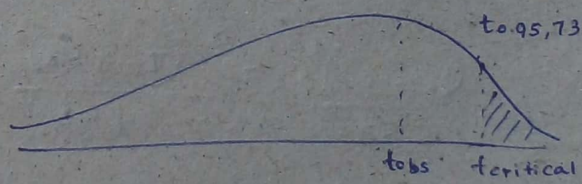
$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1+n_2-2)}}$$

$$= \sqrt{\frac{(39 \times 277.3^2) + (35-1)\,205.9^2}{(40+35-2)}} = 246.63$$

$$t_{obs} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(310.8 - 290.2) - 10}{246.63\sqrt{\frac{1}{40} + \frac{1}{35}}} = 0.1857$$

$$D.o.f = n_1 + n_2 - 2 = 73$$

$$t_{critical} = 1.688$$



$t_{obs} < t_{critical}$

Therefore we accept H_0

* Then fusion process not increase more than 10 units

		Failure Mode			
		1	2	3	
Design	1	16	40	11	67
	2	8	17	7	32
	3	10	31	13	54
	4	9	12	6	27
		43	100	37	180

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - e_i)^2}{e_i}$$

$$e_{11} = \frac{67 \times 43}{180} = 16.01$$

$$e_{12} = \frac{67 \times 100}{180} = 37.22$$

$$e_{13} = \frac{67 \times 37}{180} = 13.77$$

$$e_{21} = \frac{32 \times 43}{180} = 7.64$$

$$e_{22} = \frac{32 \times 100}{180} = 17.7$$

$$e_{23} = \frac{32 \times 37}{180} = 6.58$$

$$e_{31} = \frac{54 \times 43}{180} = 12.9$$

$$e_{32} = \frac{100 \times 54}{180} = 30$$

$$e_{33} = \frac{54 \times 37}{180} = 11.1$$

$$e_{41} = \frac{27 \times 43}{180} = 6.4$$

$$e_{42} = \frac{27 \times 100}{180} = 15$$

$$e_{43} = \frac{27 \times 37}{180} = 5.$$

$$\frac{(O_1 - e_1)^2}{e_1} = \frac{(16 - 16.01)^2}{16.01} = 6.25 \times 10^{-6}$$

$$\frac{(O_2 - e_2)^2}{e_2} = \frac{(40 - 37.22)^2}{37.22} = 0.208$$

$$\frac{(O_3 - e_3)^2}{e_3} = \frac{(11 - 13.77)^2}{13.77} = 0.557$$

$$\frac{(O_4 - e_4)^2}{e_4} = \frac{(8 - 7.64)^2}{7.64} = 0.0169$$

$$\frac{(O_5 - e_5)^2}{e_5} = \frac{(17 - 17.77)^2}{17.77} = 0.033$$

$$\frac{(O_6 - e_6)^2}{e_6} = \frac{(7 - 6.58)^2}{6.58} = 0.027$$

$$\frac{(O_7 - e_7)^2}{e_7} = \frac{(10 - 12.9)^2}{12.9} = 0.652$$

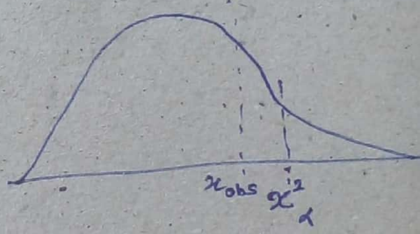
$$\frac{(o_8 - e_8)^2}{e_8} = \frac{(31-30)^2}{30} = 0.033$$

$$\frac{(o_9 - e_9)^2}{e_9} = \frac{(13-11.1)^2}{11.1} = 0.325$$

$$\frac{(o_{10} - e_{10})^2}{e_{10}} = \frac{(9-6.45)^2}{6.45} = 1.01$$

$$\frac{(o_{11} - e_{11})^2}{e_{11}} = \frac{(12-15)^2}{15} = 0.6$$

$$\frac{(o_{12} - e_{12})^2}{e_{12}} = \frac{(6-5.55)^2}{5.55} = 0.036$$



$$x^2_{obs} = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i} = 3.5699$$

H_0 - There is no effect on failure mode by design (Independent)

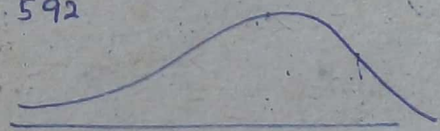
H_A - There is an effect on failure mode by design (Dependent)

$D_f = (r-1)(c-1)$

$= 2 \times 3$

$= 6$

$x^2_{critical}$ (x^2_{α}) $= 12.592$



$X_{obs} < X_{critical}$, H_0 accepted

$H_0: \mu_1 = \mu_2 = \cdots = \mu_t$

$H_A: \mu_i \neq \mu_j$

Location 1	Location 2	Location 3	location 4
5	8	6	7
7	7	8	8
8	6	7	8
9	7	9	6

$y_{1.} = 29$ $y_{2.} = 28$ $y_{3.} = 30$ $y_{4.} = 29$

$$SS_{Tr} = \sum_{i=1}^{n} \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{N}$$

$$= \left(\frac{29^2}{4} + \frac{28^2}{4} + \frac{30^2}{4} + \frac{29^2}{4}\right) - \frac{116^2}{16}$$

$$= \frac{1}{2} = 0.5$$

$=$

$$SS_T = \sum_{i=1}^{n} \sum_{j=1}^{n} y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$= 860 - \frac{116^2}{16}$$

$$= 19$$

$SS_T = SS_{Tr} + SS_E$

$SS_E = 18.5$

Source of variance	Sum of Squares	D.f	Mean square	F value
SS_{Tr}	0.5	$S-1 = 3$	$\alpha = \frac{0.5}{3}$	f value $= \alpha/\beta$ $= 0.1081$
SS_E	18.5	$N-S = 12$	$\beta = \frac{18.5}{12}$	$F_{obs} = 0.1081$
SS_T	19	$N-1 = 15$		

$F_{0.05, 3, 12} = 3.49 > F_{obs}$

Therefore H_0 accept,

$\mu_1 = \mu_2 = \mu_3 = \dots$

$\bar{x} = 56.42$

$\bar{y} = 278.75$

Q5)

$$r = \frac{cov(x,y)}{S_x S_y} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{(n-1) S_x S_y}$$

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{(n-1) \sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}$$

Y	$(y_i - \bar{y})$	X_i	$(x_i - \bar{x})$	$(y_i - \bar{y})(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$
240	-38.75	25	-31.42	1217.525	987.22	1501.56
236	-42.75	31	-25.42	1086.705	646.18	1827.56
290	11.25	45	-11.42	-128.475	130.42	126.56
274	-4.75	60	3.58	-17.005	12.82	22.56
301	22.25	65	8.58	190.905	73.62	495.06
318	37.25	72	15.58	580.355	242.74	1387.56
300	21.25	80	23.58	501.075	556.02	451.56 ~~556.02~~
296	17.25	84	27.58	475.755	760.66	297.56 ~~760.66~~
267	-11.75	75	18.58	-218.315	345.22	138.06
276	-2.75	60	3.58	-9.845	12.82	7.56
288	9.25	50	-6.42	-59.385	41.22	85.56
261	-17.75	88	-18.42	326.955	339.30	315.06
				3946.25	4148.24	6656.2

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{(n-1) \sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}$$