04: Combinational & Sequential Logic Circuits

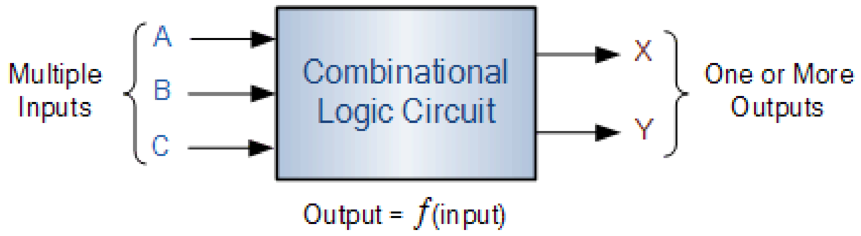
IT1206 – Computer Systems

Level I - Semester 1

4. Combinational & Sequential Logic Circuits

Combinational Logic

- Boolean algebra is a natural way to represent digital information. It helps to represents operations of a computer and understand how a computer makes decisions.
- Adders, decoders, multiplexers are just a few examples of Combinational Logic circuits.
- Combinational Logic Circuits are memoryless digital logic circuits whose output at any instant in time depends only on the combination of its inputs.
- These are made up from basic logic **NAND, NOR** or **NOT** gates that are "combined" or connected to produce more complex switching circuits.



Combinational Logic cont….

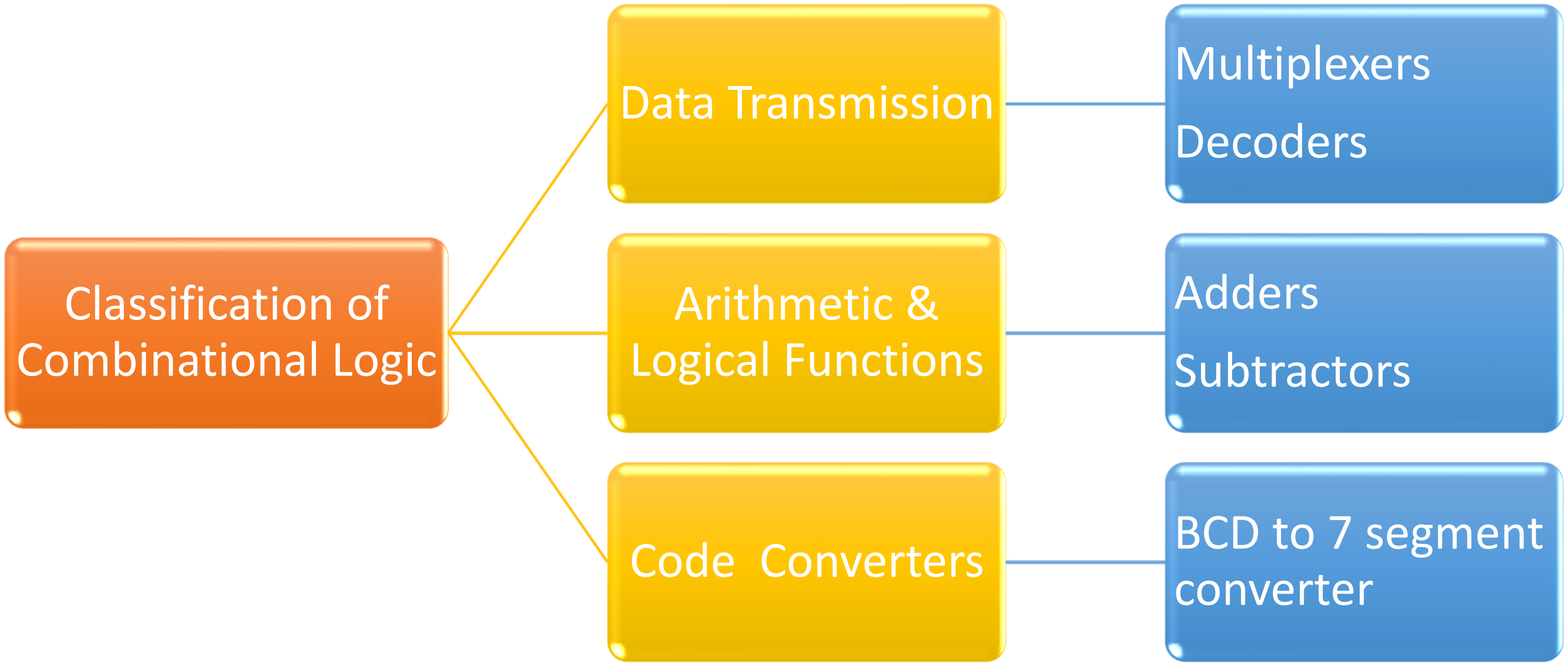
- The three main ways of specifying the function of a combinational logic circuit are:

1. ***Boolean Algebra*** – This forms the algebraic expression showing the operation of the logic circuit for each input variable either Logical True or False that results in a Logical True output.

2. ***Truth Table*** – A truth table defines the function of a logic circuit by providing a concise list that shows all the output states in a tabular form for each possible combination of input variable that the circuit could encounter.

3. ***Logic Diagram*** – This is a graphical representation of a logic circuit that shows the wiring and connections of each individual logic gate, represented by a specific graphical symbol, that implements the logic circuit.

Classification of Combinational Logic


Classification of Combinational Logic
Data Transmission
Arithmetic & Logical Functions
Code Converters
Multiplexers
Decoders
Adders
Subtractors
BCD to 7 segment converter

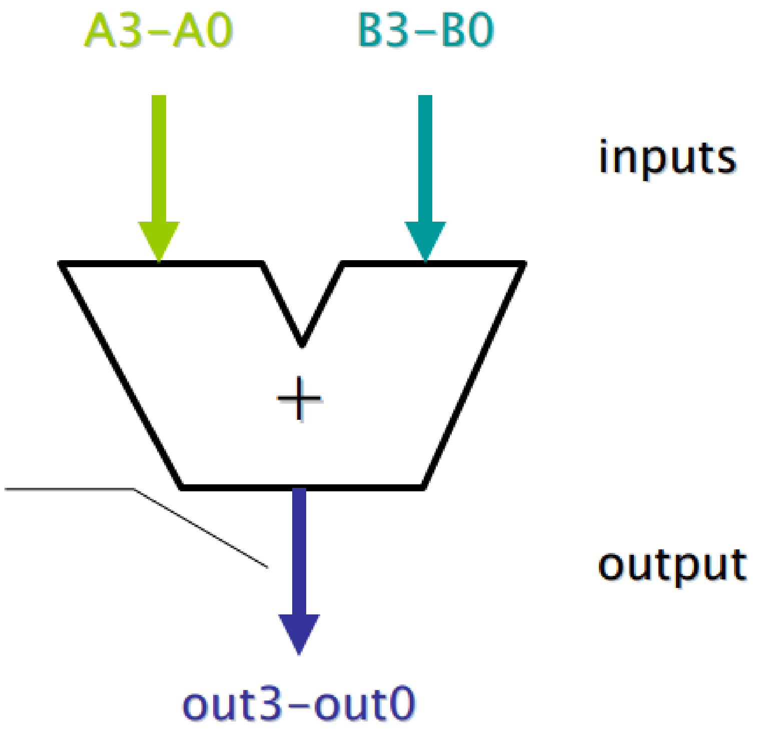
4.1 Adder

- An adder is a digital logic circuit in electronics that implements addition of numbers.
- In many computers and other kinds of processors, adders are used not only in the arithmetic logic units, but also in other parts of the processor, where they are used to calculate addresses, increment and decrement operators, and similar operations.
- There are two main types of Adders:

  1)half adder.

  2) full adder

Adders (and other arithmetic circuits) are usually drawn like this in block diagrams

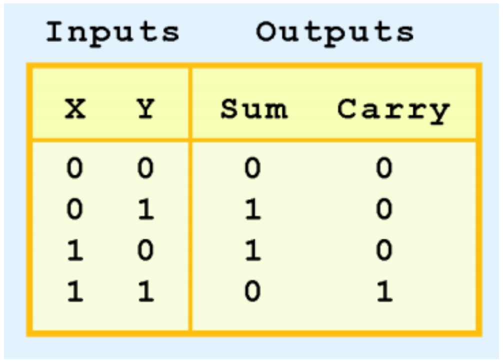


collections of parallel, related wires like this are known as <u>buses</u>; they carry multi-bit values between components

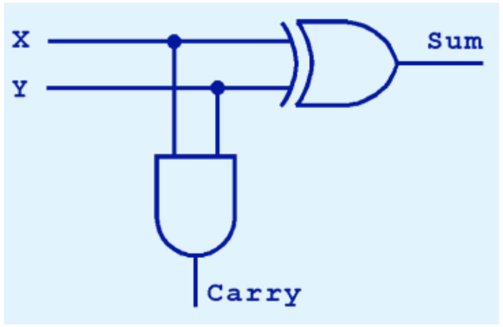
4.1.1 Half Adder

- Finds the sum of two bits

- The sum can be found using the XOR operation and the carry using the AND operation.

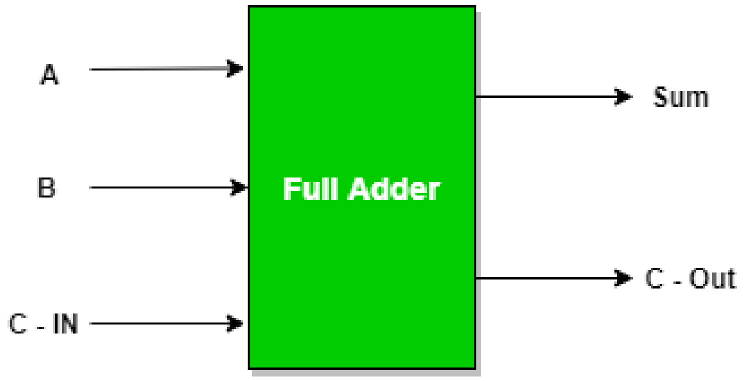


Inputs		Outputs	
X	Y	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



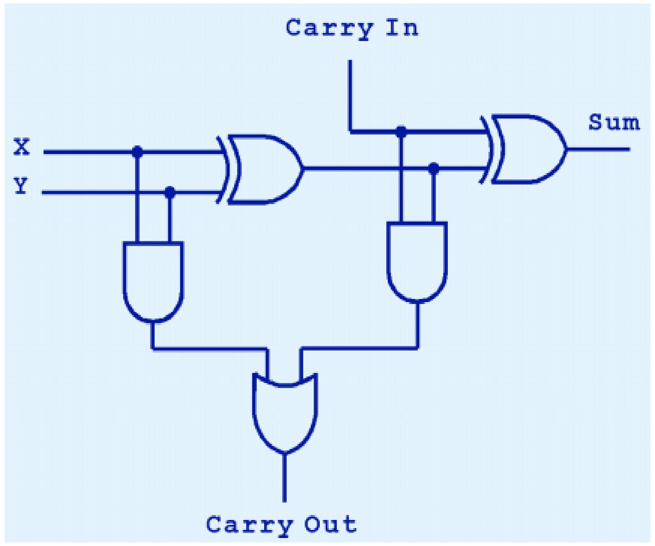
4.1.2 Full Adder

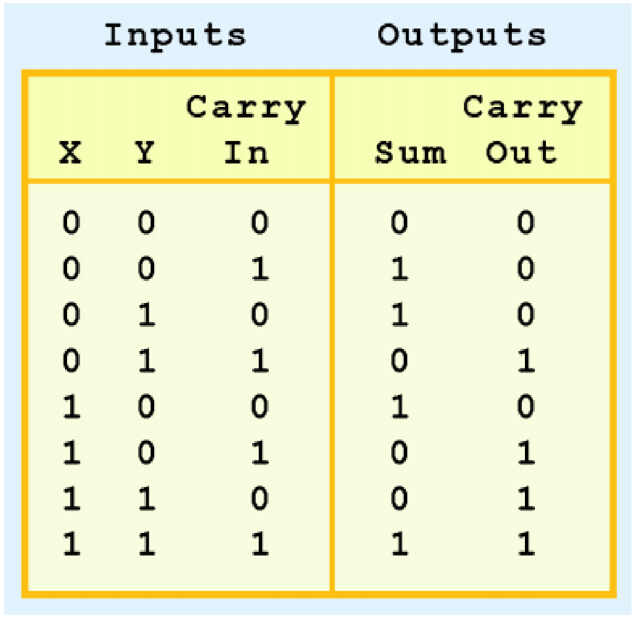
- *Full Adder* is the adder which adds three inputs and produces two outputs. The first two inputs are A and B and the third input is an input carry (as C-IN). The output carry is designated as C-OUT and the normal output is designated as S which is SUM.



4.1.2 Full Adder cont….

- The truth table for a full adder is:

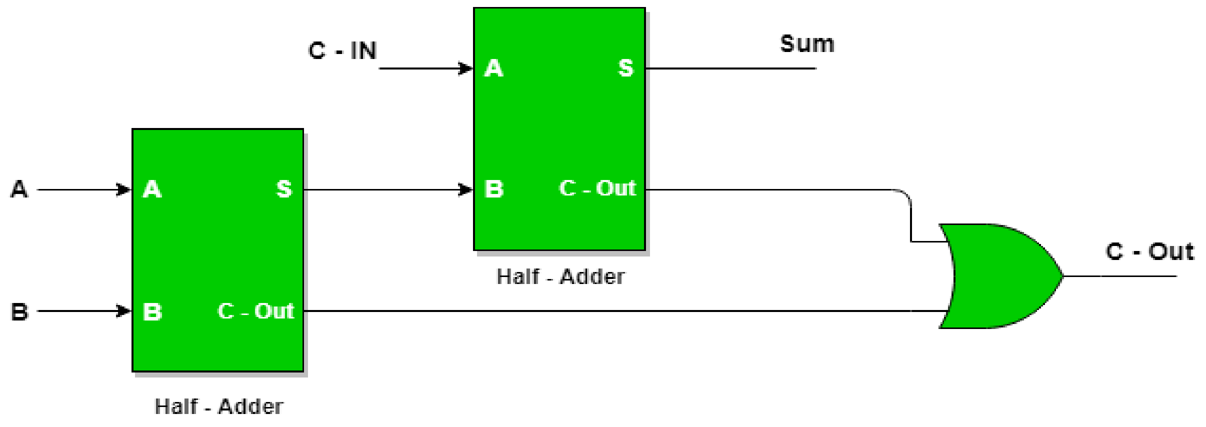




Inputs			Outputs	
X	Y	Carry In	Sum	Carry Out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

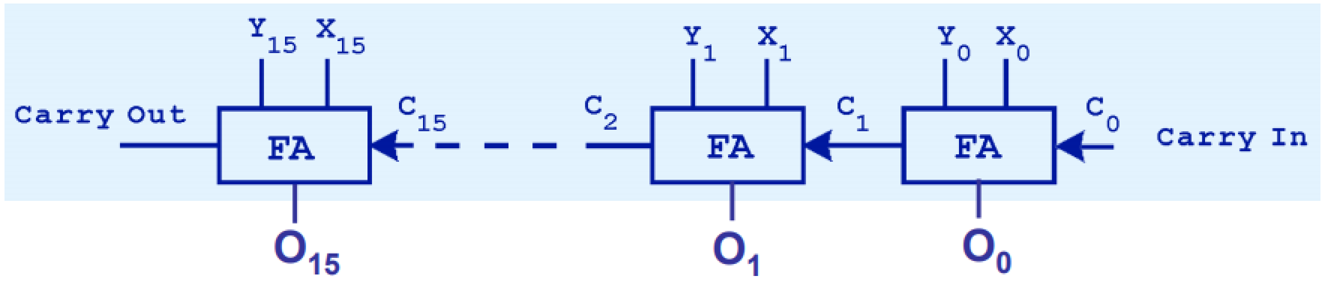
4.1.3 Full Adder using Half Adders

- We can use half adders to construct a full adder by including gates for processing the carry bit.
- 2 Half Adders and a OR gate is required to implement a Full Adder.



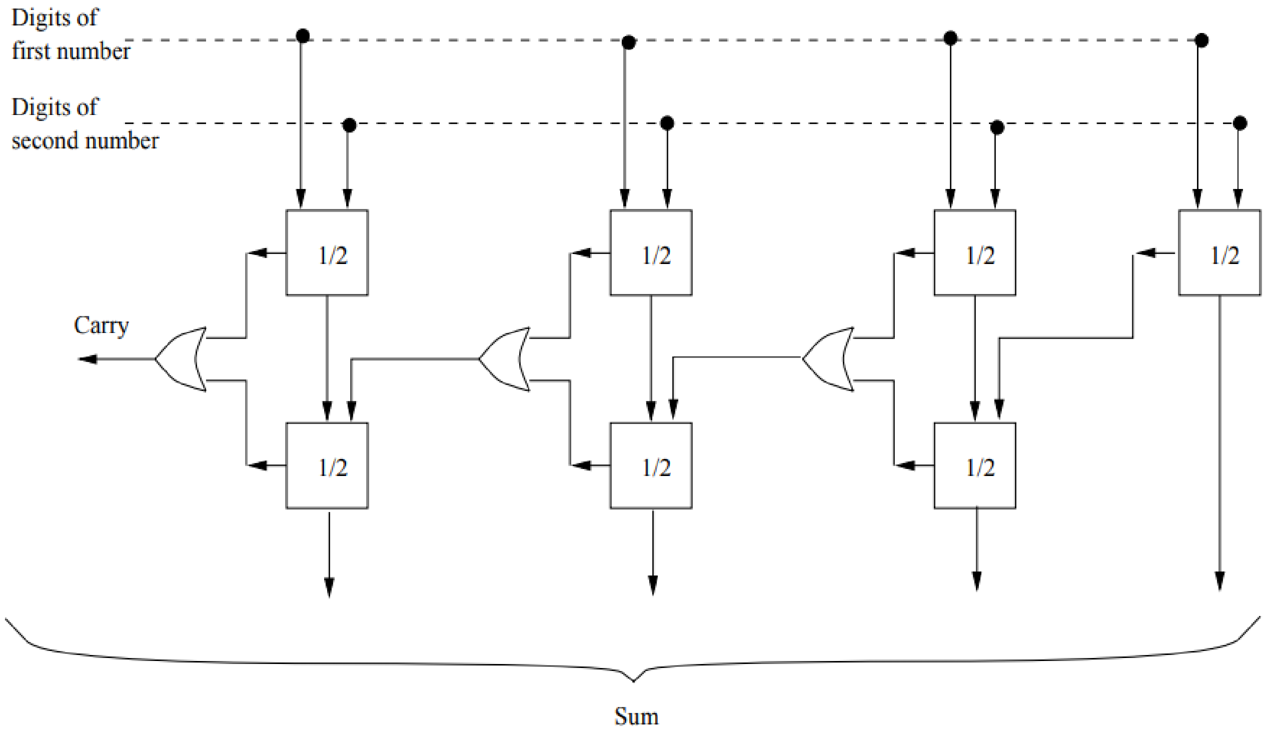
4.1.4 Ripple-carry Adder (I)

- A single adder can add only two single bit numbers.
- To add multiple bit numbers, we can construct a circuit by combining a series of full adders (see the figure at the bottom).
- The carry bit "ripples" from one adder to the next; hence, this configuration is called a ripple-carry adder.
- This will help to create Adders which can add two numbers with multiple bits.



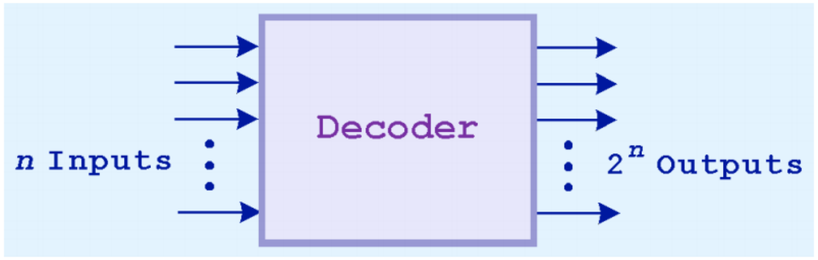
$$\begin{array}{r} Y_{15} \ldots Y_1\ Y_0 \\ +\ X_{15} \ldots X_1\ X_0 \\ \hline O_{15} \ldots O_1\ O_0 \end{array}$$

4.1.4 Ripple-carry Adder (II)...

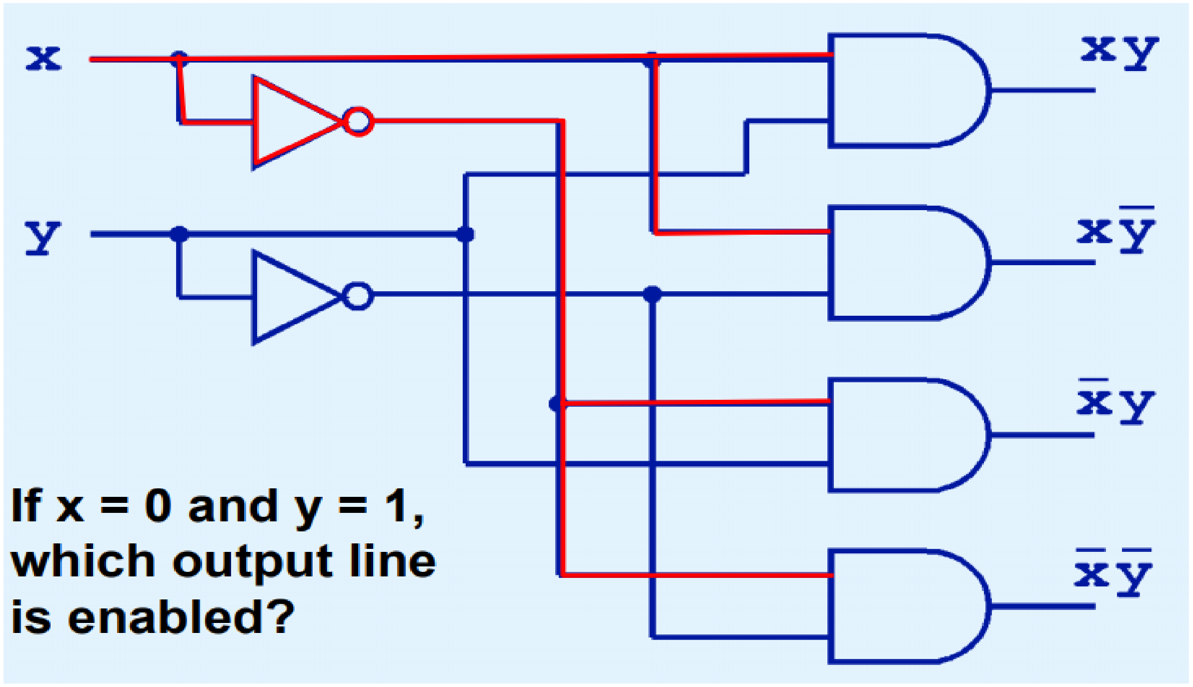


4.2 Decoder

- Binary code of N digits can be used to store 2^N distinct elements of coded information. This is what encoders and decoders are used for. **Encoders** convert 2^N lines of input into a code of N bits(inputs) and **Decoders** decode the N bits into 2^N lines.
- Ex. Decoder can be used to select a memory location according a binary value placed on the address lines of a memory bus.



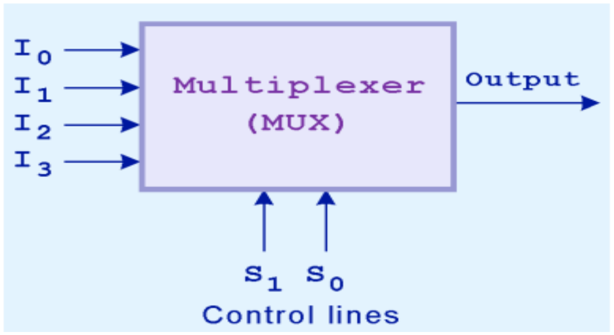
2-to-4 Decoder



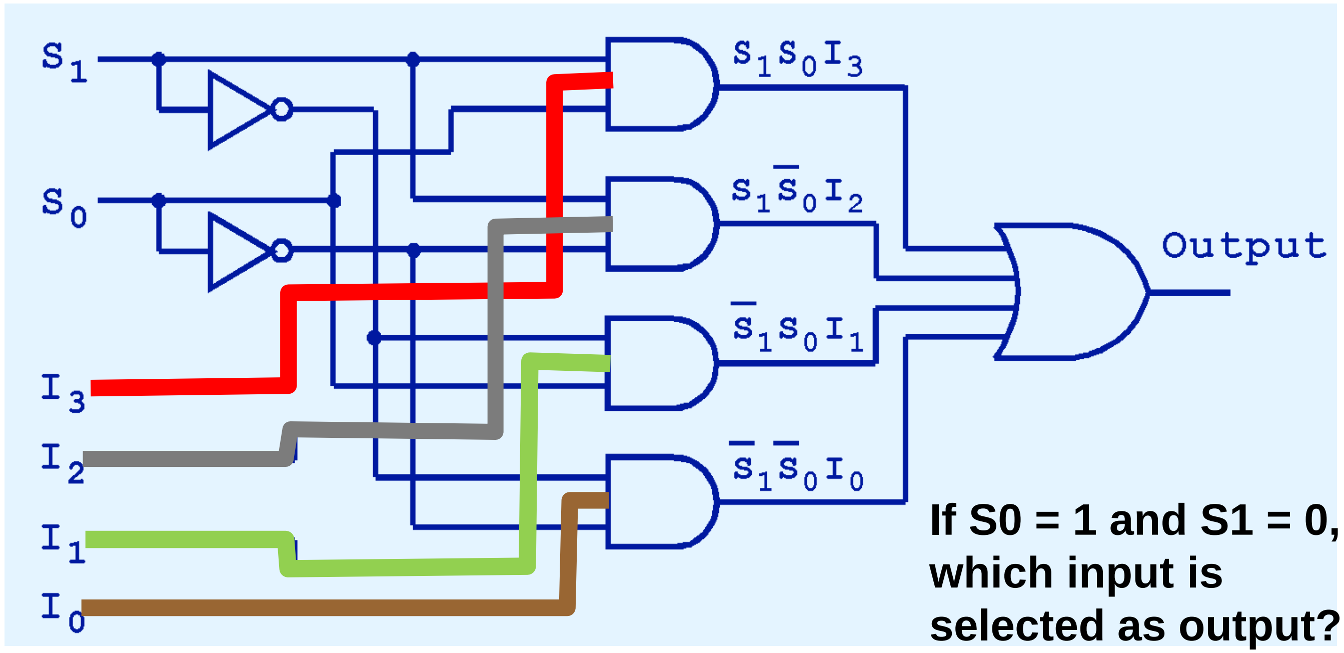
If x = 0 and y = 1, which output line is enabled?

4.3 Multiplexer

- It is a combinational circuit which have many inputs and a single output depending on the control signal to select which input to be on output.
    - For N input lines, $\log_n$(base2) selection lines, or we can say that for 2^n input lines, n selection lines are required.
    - Multiplexers are also known as “**Data n selector, parallel to serial convertor, many to one circuit, universal logic circuit**”.
    - Multiplexers are mainly used to increase amount of the data that can be sent over the network within certain amount of time and bandwidth.



Activity 01 : 4-to-1 Multiplexer

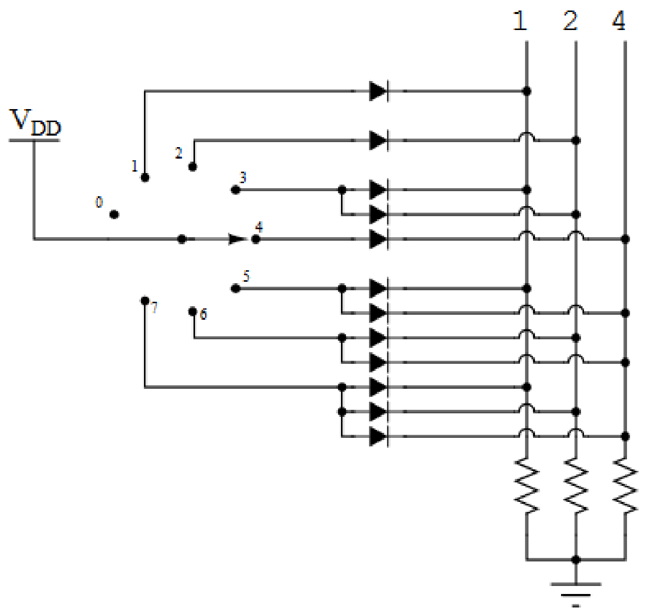


If S0 = 1 and S1 = 0, which input is selected as output?

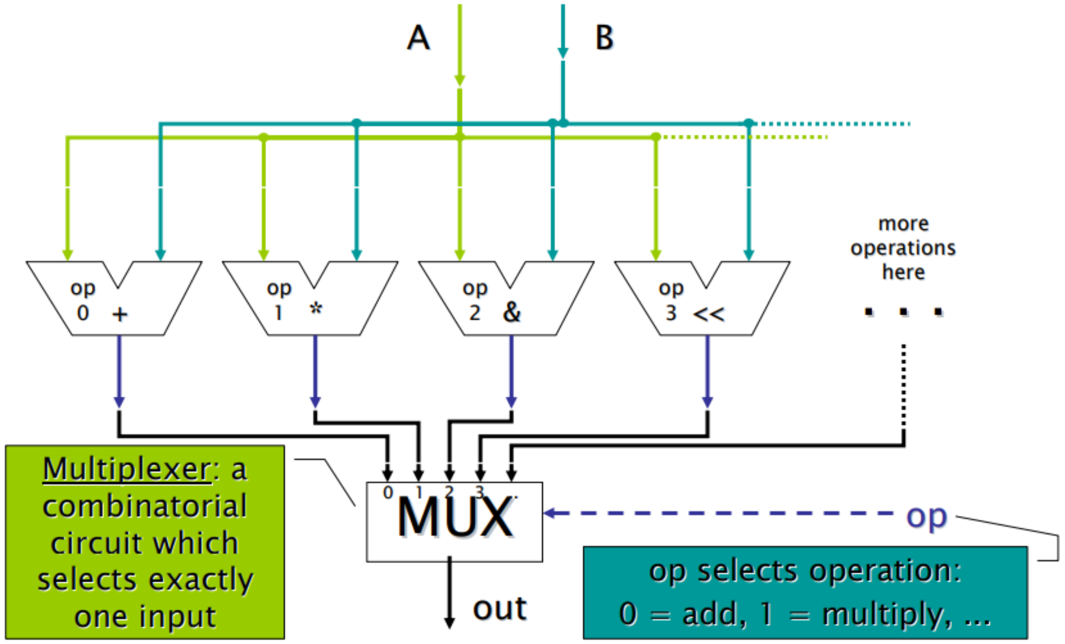
Activity 02

- What does it mean, in general terms, to *encode* something? Conversely, what does it mean to *decode* something?
- Implement 4:1 Multiplexers using truth table for following gates.
  i. NOT gate
  ii. AND gate
  iii. NOR gate
  iv. NAND gate
  v. XOR gate
  vi. XNOR gate

3.The simple switch-and-diode circuit shown here is an example of a digital encoder. Explain what this circuit does, as the switch is moved from position to position:



4.4 Arithmetic Logic Unit (ALU)

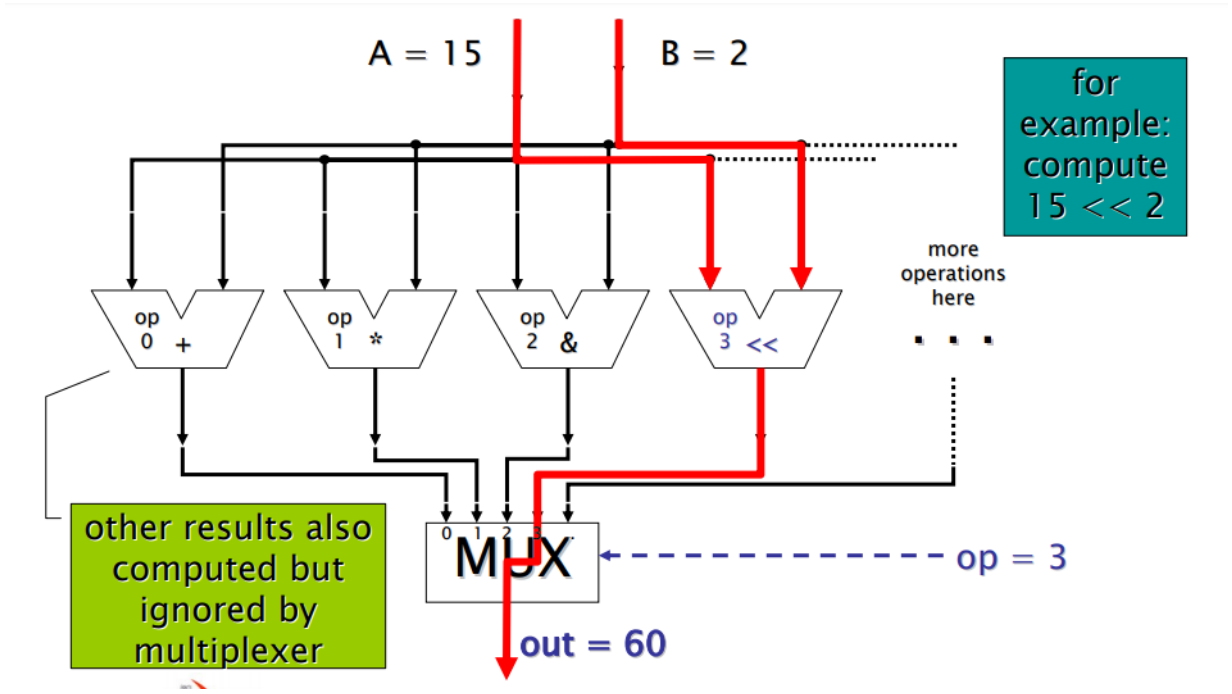


Multiplexer: a combinatorial circuit which selects exactly one input

op selects operation:
0 = add, 1 = multiply, ...

- Computers need to do more than just addition
    - arithmetic operations: + – * / %
    - logical operations: & | ~ << >>
- Need a circuit that can select operation to perform
- ALUs routinely perform the following operations:
    - **Logical Operations:** These include AND, OR, NOT, XOR, NOR, NAND, etc.
    - **Bit-Shifting Operations:** This pertains to shifting the positions of the bits by a certain number of places to the right or left, which is considered a multiplication operation.
    - **Arithmetic Operations:** This refers to bit addition and subtraction. Although multiplication and division are sometimes used, these operations are more expensive to make. Addition can be used to substitute for multiplication and subtraction for division.

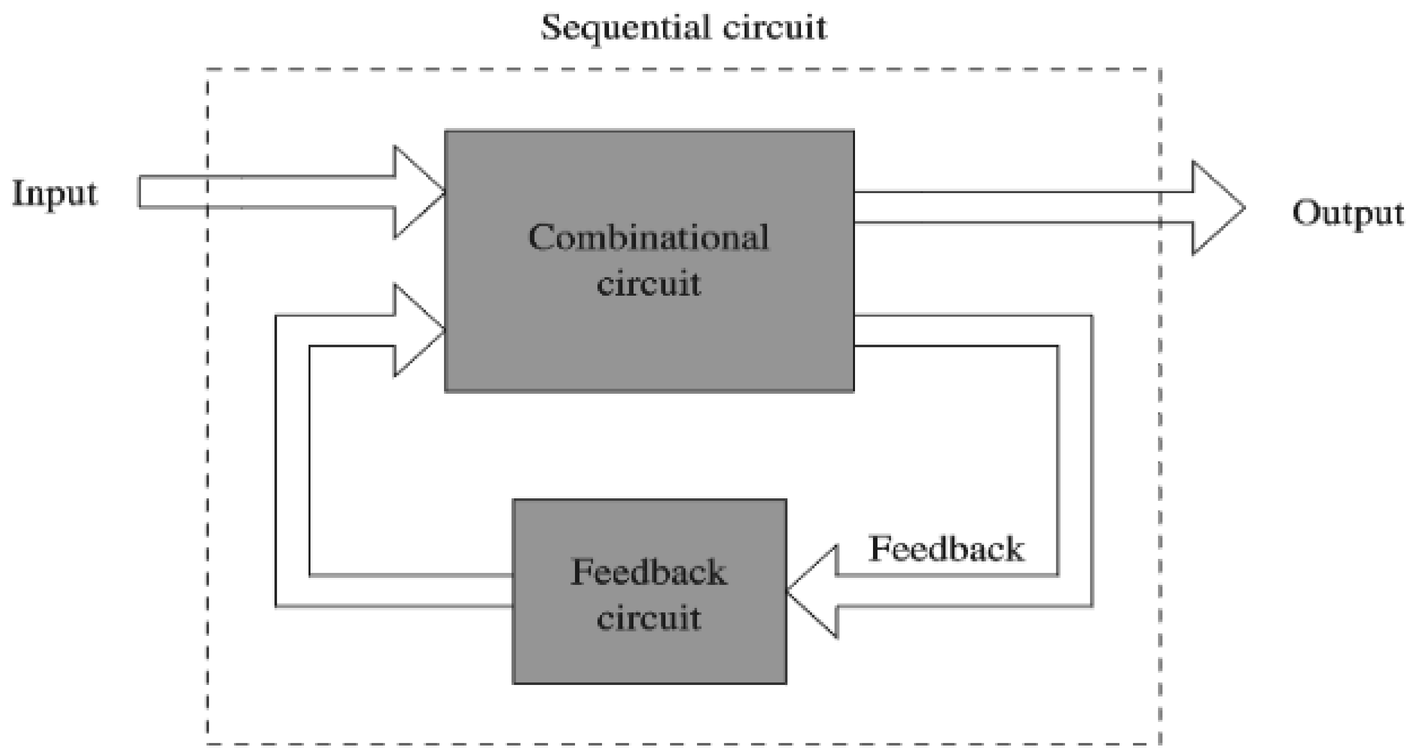
Arithmetic Logic Unit (ALU) - Example



Sequential Logic Circuits

- **Combinational logic** circuits are perfect for situations which require the immediate application of a Boolean function to a set of inputs
- But, there are times when we need a circuit to change its value with consideration to its current state as well as its inputs
  - Therefore, these circuits must “remember” their current state
- Sequential Logic Circuits use flip-flops as memory elements and in which their output is dependent on the input state

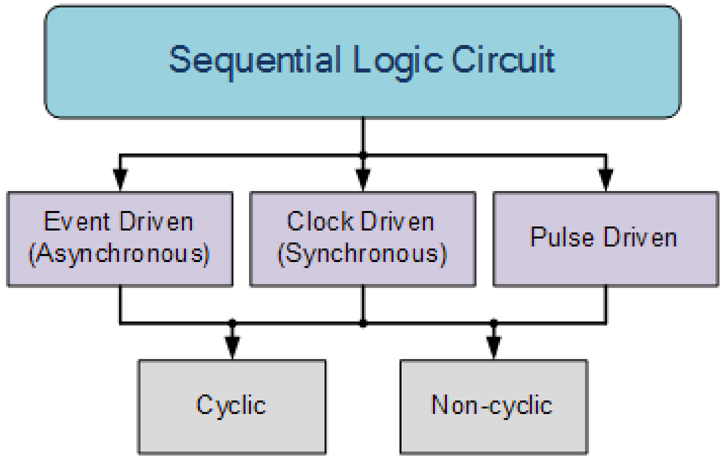
Main components of a sequential circuit



Sequential Logic Circuits cont....

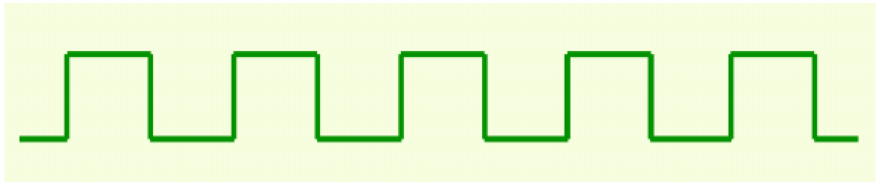
sequential logic circuits can be divided into the following three main categories:

1. **Event Driven** – asynchronous circuits that change state immediately when enabled.
2. **Clock Driven** – synchronous circuits that are synchronized to a specific clock signal.
3. **Pulse Driven** – which is a combination of the two that responds to triggering pulses.



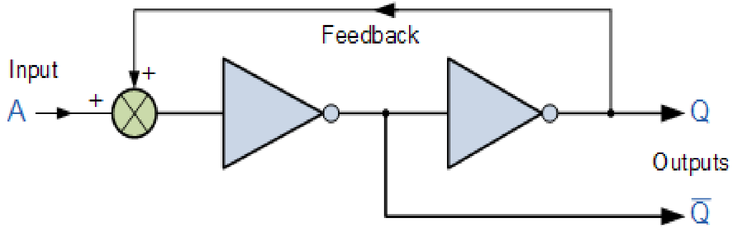
Sequencing Events

- Sequential logic circuits require a means by which events can be sequenced
- State changes are controlled by clocks
    - A "clock" is a special circuit that sends electrical pulses through a circuit
- Clocks produce electrical waveforms such as this one



Feedback in Sequential Logic Circuits

- Sequential circuits rely on feedback to retain their state values
- Feedback in digital circuits occurs when an output is looped back to the input
  - Example,



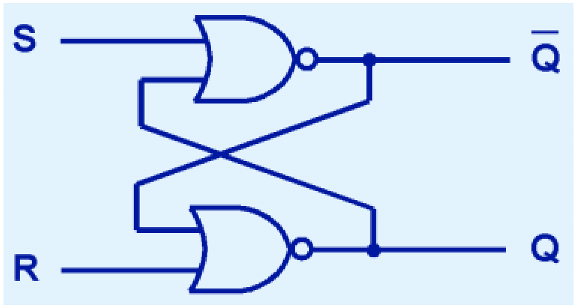
- If Q is 0 it will always be 0, if it is 1, it will always be 1

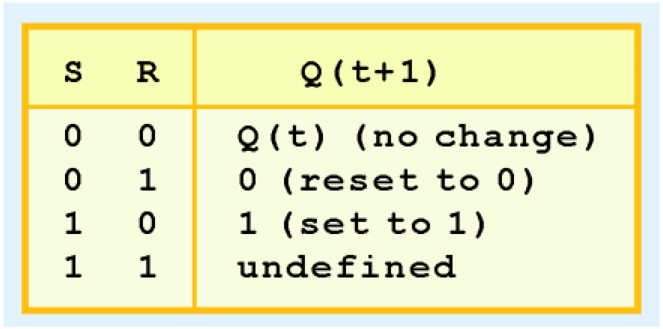
4.5 Flip Flop

- Flip flop is a sequential circuit which generally samples its inputs and changes its outputs only at particular instants of time (not continuously).
- Flip flop is said to be edge sensitive or edge triggered rather than being level triggered like latches.
- The most common types of flip flops are:
  - **SR flip-flop:** Is similar to an SR latch. Besides the CLOCK input, an SR flip-flop has two inputs, labeled SET and RESET.
  - **D flip-flop:** Has just one input in addition to the CLOCK input. This input is called the DATA input.
  - **JK flip-flop:** A JK flip-flop has two inputs, labeled *J* and *K.* The J input corresponds to the SET input in an SR flip-flop, and the K input corresponds to the RESET input.

SR Flip-flop (Set-Reset)

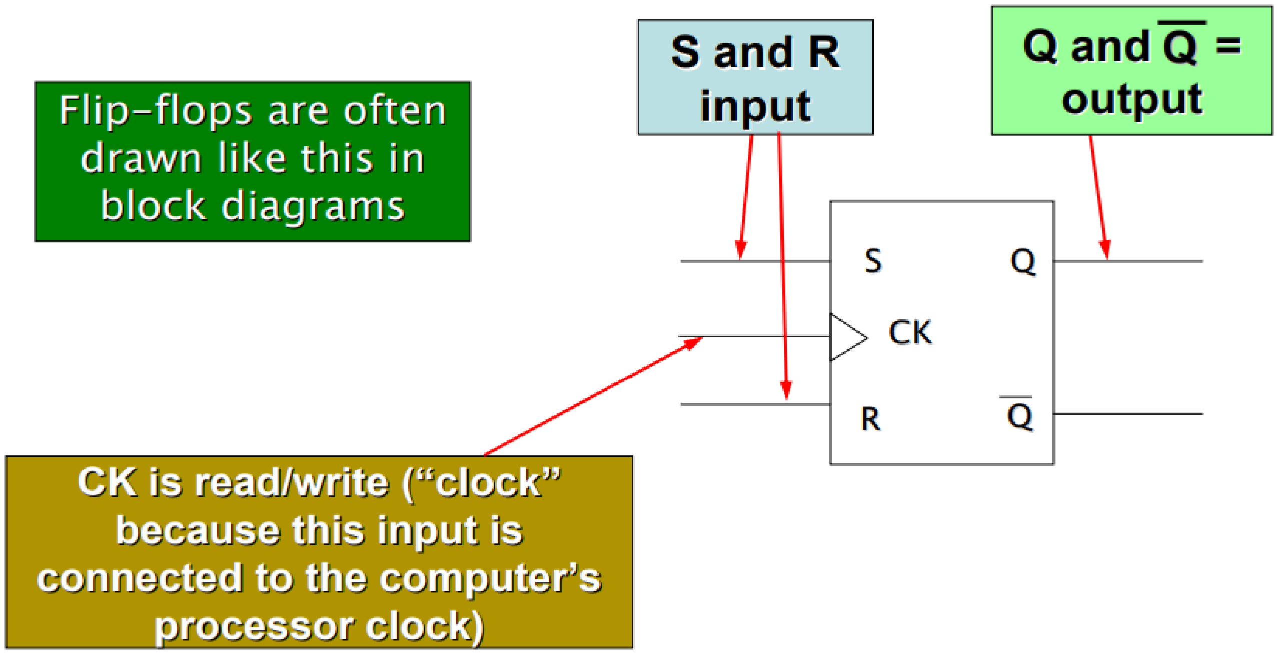
- The **SR flip-flop**, also known as a *SR Latch*.
- This has two inputs, one which will "**SET**" the device (meaning the output = "1"), and is labelled **S** and one which will "**RESET**" the device (meaning the output = "0"), labelled **R**.
- The behavior of an SR flip flop is described by a characteristic table
  - Q(t) output at time t
  - Q(t+1) output after the next clock pulse



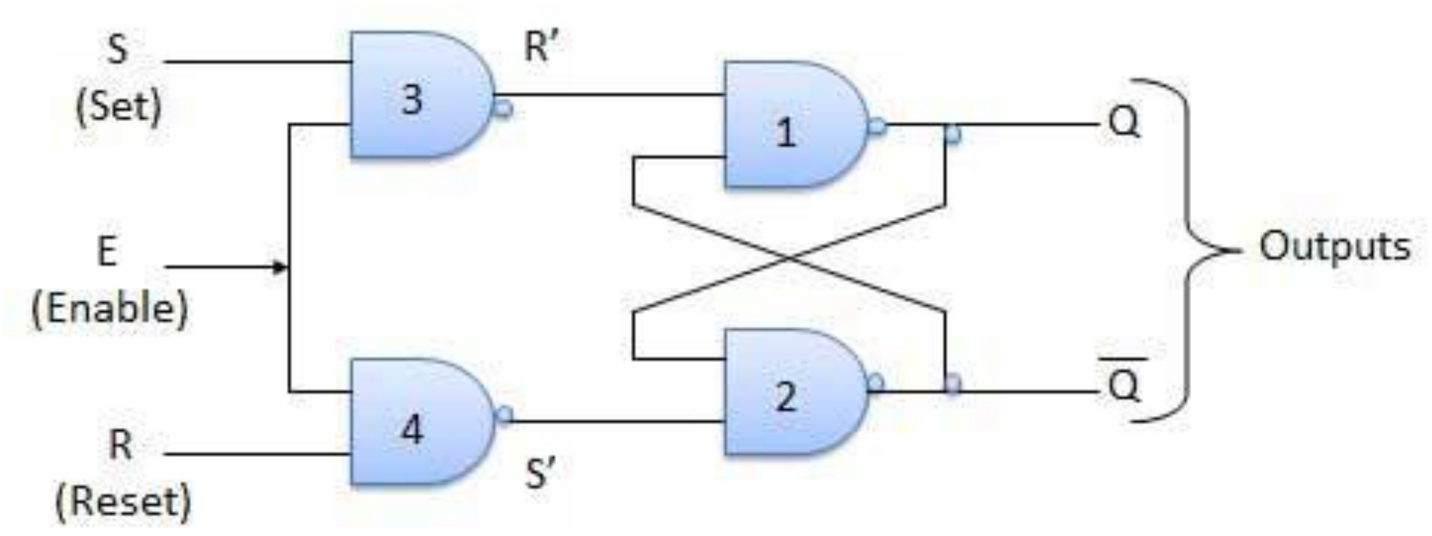


S	R	Q(t+1)
0	0	Q(t) (no change)
0	1	0 (reset to 0)
1	0	1 (set to 1)
1	1	undefined

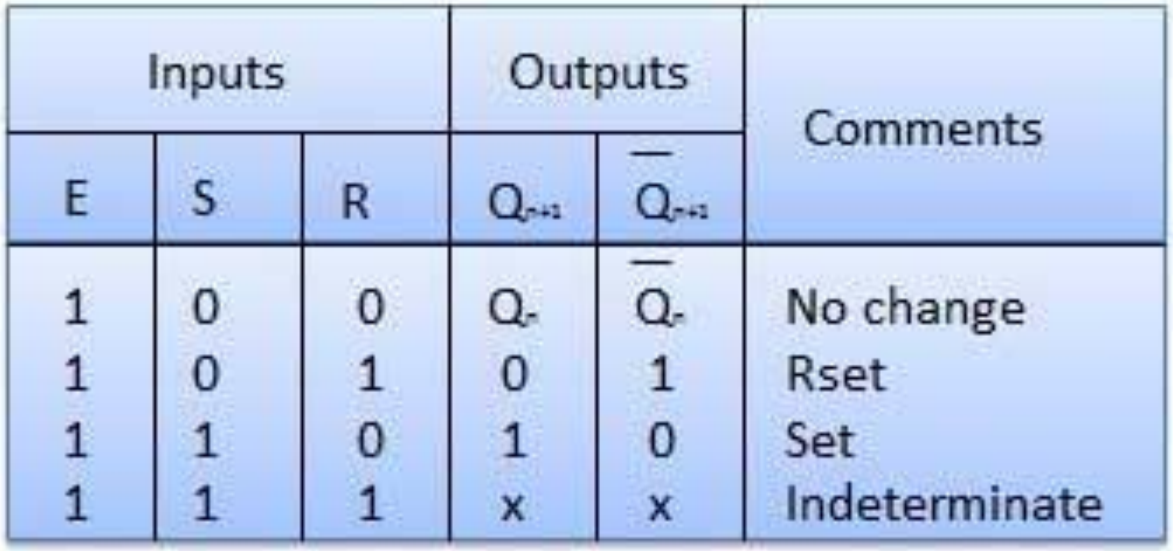
SR Flip-flop: Block Diagram


Flip-flops are often drawn like this in block diagrams
S and R input
Q and $\overline{Q}$ = output
S
Q
CK
R
$\overline{Q}$
CK is read/write ("clock" because this input is connected to the computer's processor clock)

SR Flip-flop: Circuit Diagram



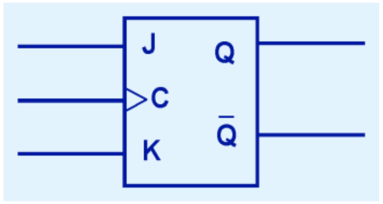
SR Flip-flop: Truth Table

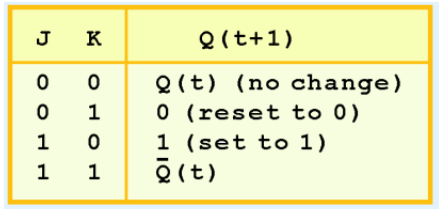


Inputs			Outputs		Comments
E	S	R	Q_{n+1}	$\overline{Q}_{n+1}$	
1	0	0	Q_n	$\overline{Q}_n$	No change
1	0	1	0	1	Rset
1	1	0	1	0	Set
1	1	1	x	x	Indeterminate

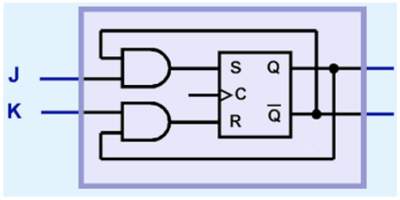
JK Flip-flop (Jack Kilby)

- Modified version of the SR flip-flop to provide a stable state when both inputs are 1.
- **Master Slave JK FF** is a cascade of two SR FF with feedback from the output of second to input of first. Master is a positive level triggered. But due to the presence of the inverter in the clock line, the slave will respond to the negative level. Hence when the clock = 1 (positive level) the master is active and the slave is inactive. Whereas when clock = 0 (low level) the slave is active and master is inactive

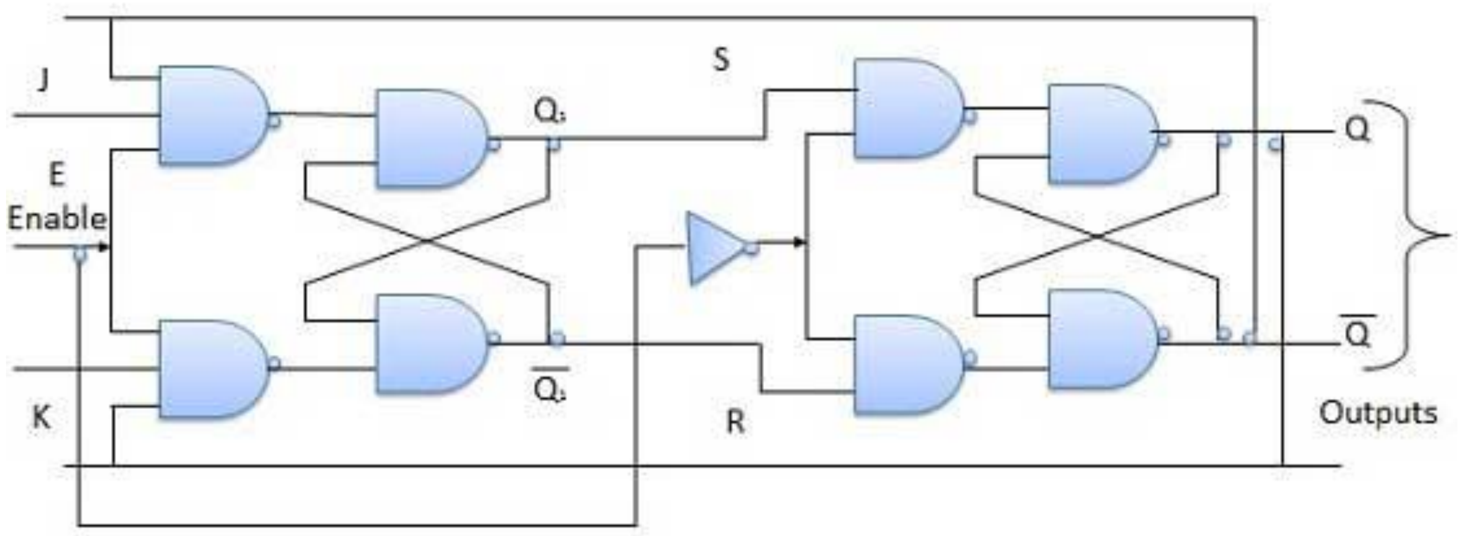




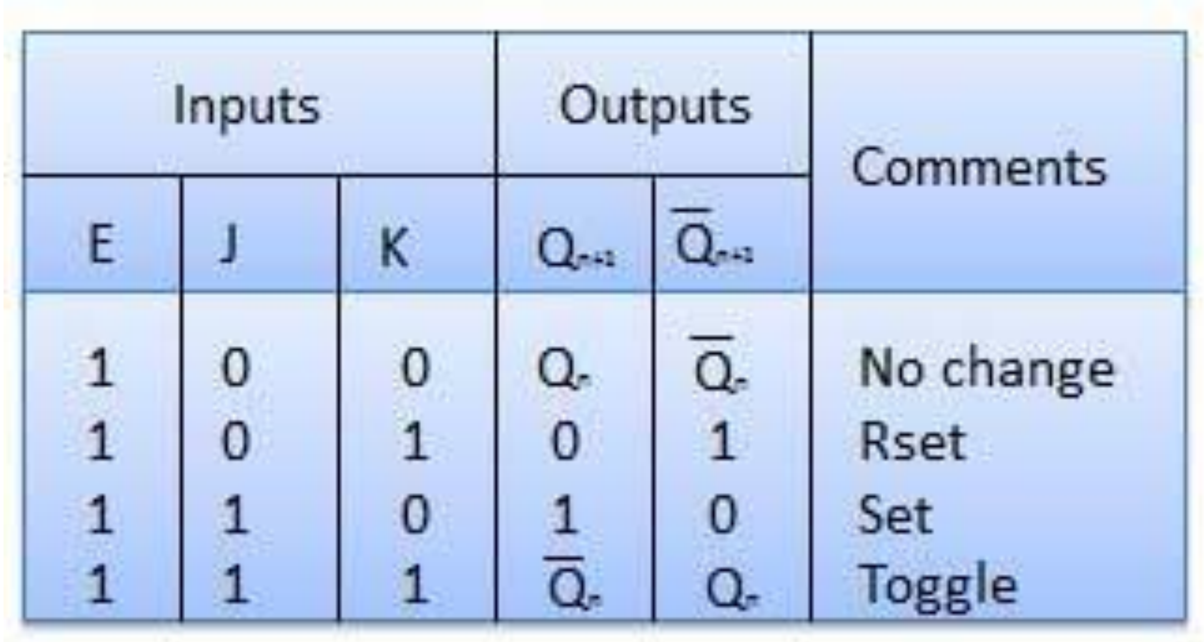
J	K	Q(t+1)
0	0	Q(t) (no change)
0	1	0 (reset to 0)
1	0	1 (set to 1)
1	1	$\bar{Q}$(t)



JK Flip-flop: Circuit Diagram



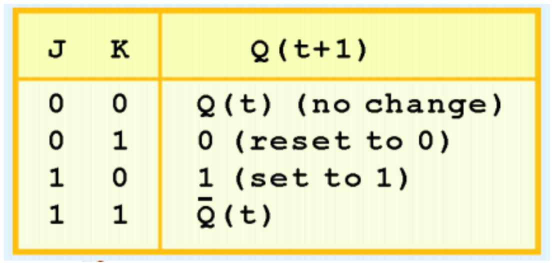
JK Flip-flop: Truth Table



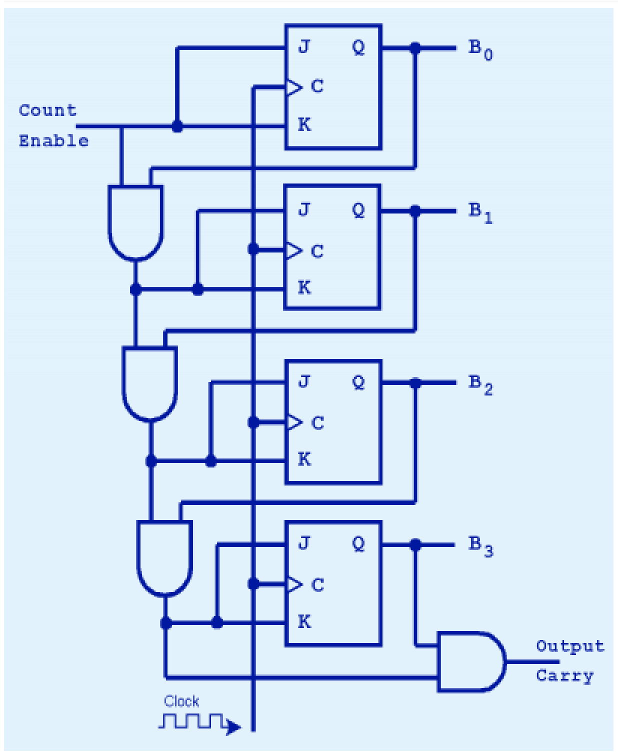
Inputs			Outputs		Comments
E	J	K	Q_{n+1}	$\overline{Q}_{n+1}$	
1	0	0	Q_n	$\overline{Q}_n$	No change
1	0	1	0	1	Rset
1	1	0	1	0	Set
1	1	1	$\overline{Q}_n$	Q_n	Toggle

JK Flip-flop: Binary Counter

- The low-order bit is complemented at each clock pulse
- Whenever it changes from 1 to 0, the next bit is complemented, and so on through the other flip-flops

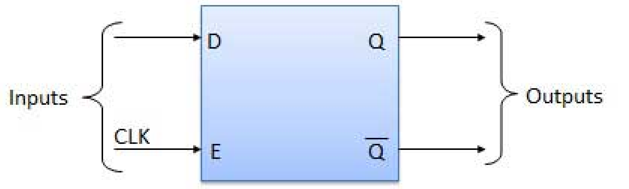


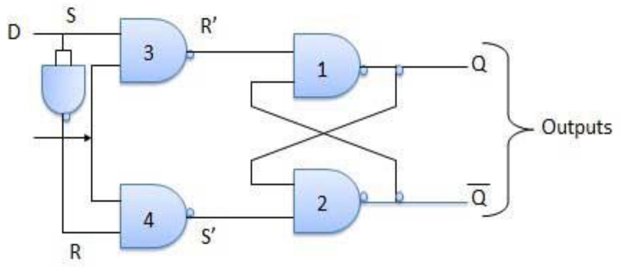
J	K	Q(t+1)
0	0	Q(t) (no change)
0	1	0 (reset to 0)
1	0	1 (set to 1)
1	1	$\bar{Q}(t)$

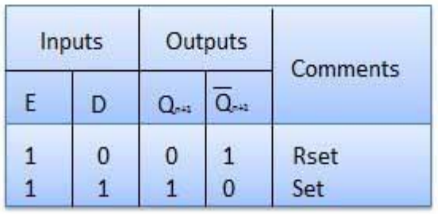


D Flip-flop (Data)

- Fundamental circuit of computer memory.
- Used to store 1 bit.
- Can be implemented with gates.
- Also known as *Data Latch, Delay flip flop, D-type Bistable, D-type Flip Flop* or just simply a **D Flip Flop**
- Not combinatorial logic
  - because current output may depend on previous state
- Example of sequential logic
  - current output depends on inputs and prior output

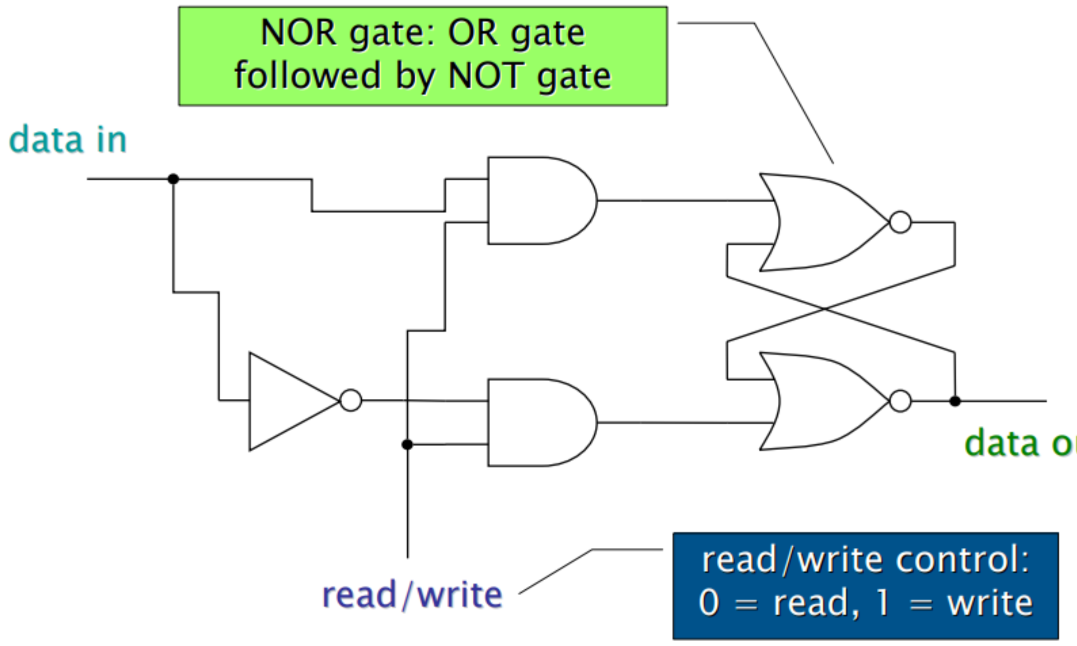




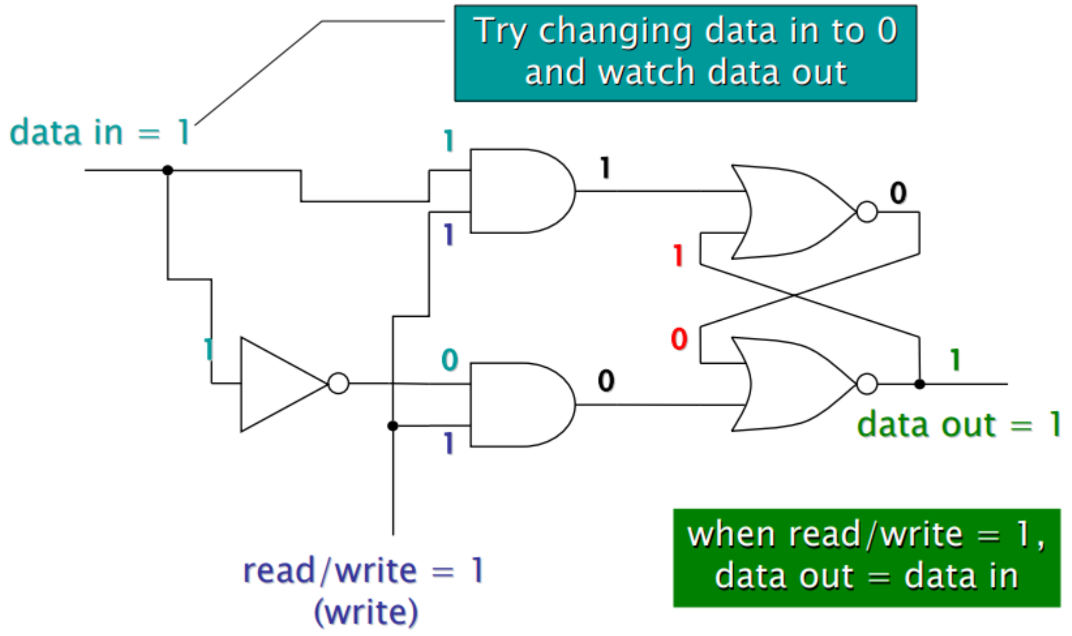


Inputs		Outputs		Comments
E	D	Q_{n+1}	$\overline{Q}_{n+1}$	
1	0	0	1	Rset
1	1	1	0	Set

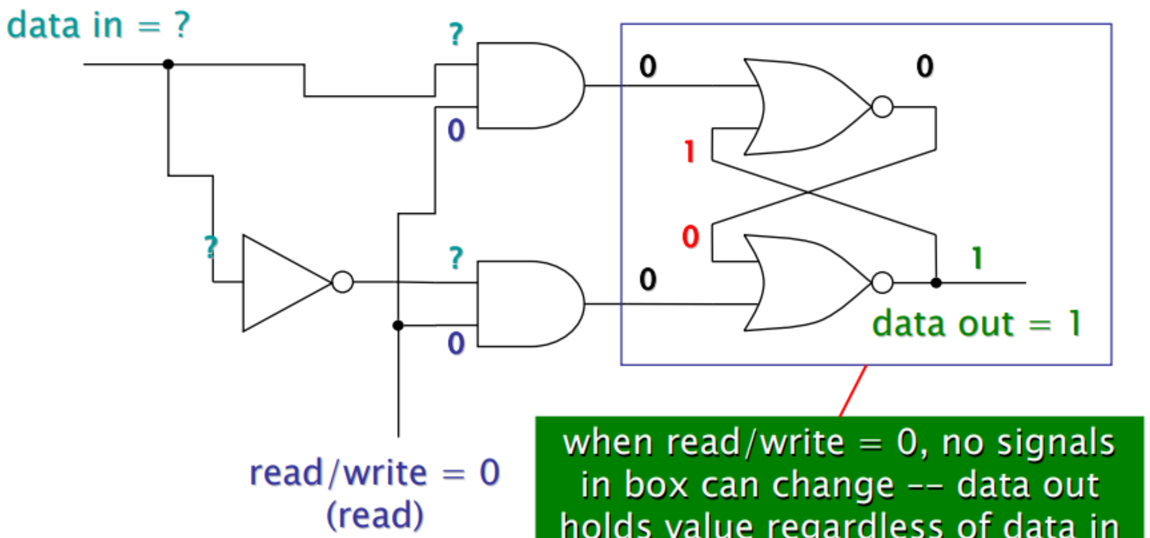
D Flip-flop


NOR gate: OR gate followed by NOT gate
data in
data out
read/write
read/write control: 0 = read, 1 = write

D Flip-flop: Writing

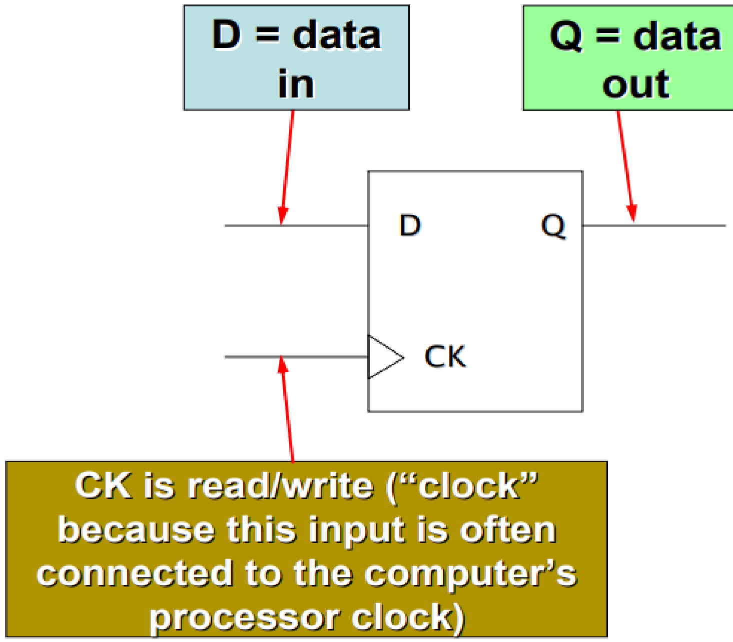

Try changing data in to 0 and watch data out
data in = 1
1
1
1
0
1
0
0
1
1
0
1
data out = 1
read/write = 1
(write)
when read/write = 1, data out = data in

D Flip-flop: Reading

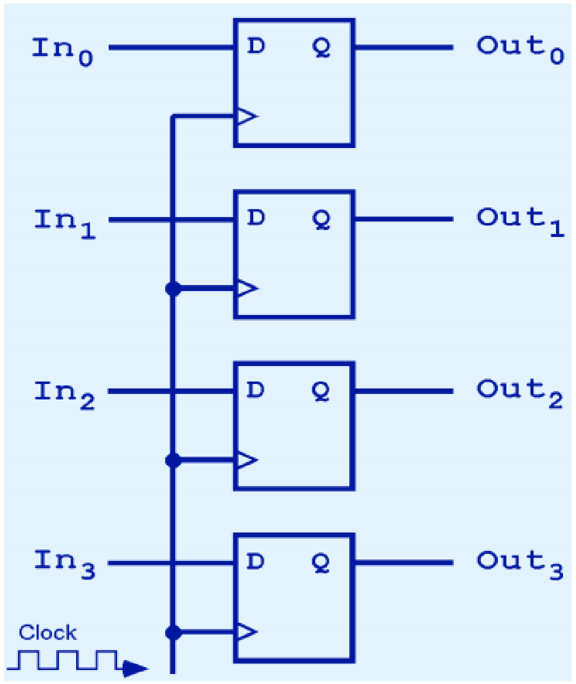


when read/write = 0, no signals in box can change -- data out holds value regardless of data in

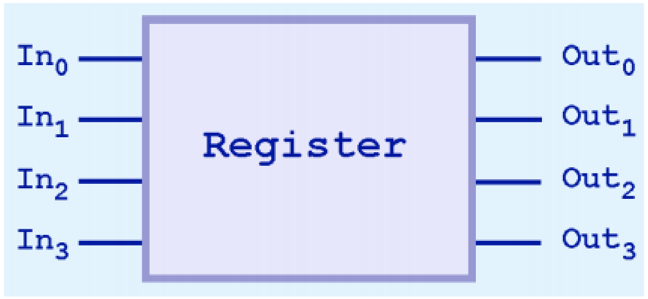
D Flip-flop: Block Diagram


D = data in
Q = data out
D
Q
CK
CK is read/write ("clock" because this input is often connected to the computer's processor clock)

D Flip-flop: 4-bit Register



A register stores data inside the CPU

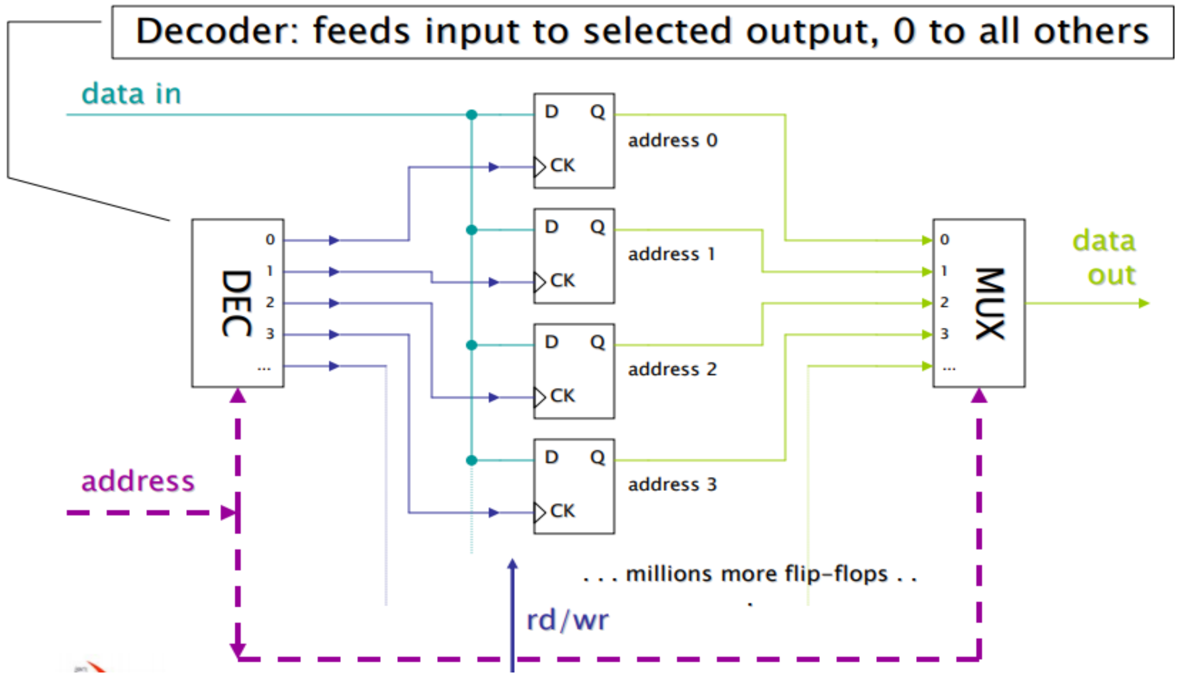


Memory

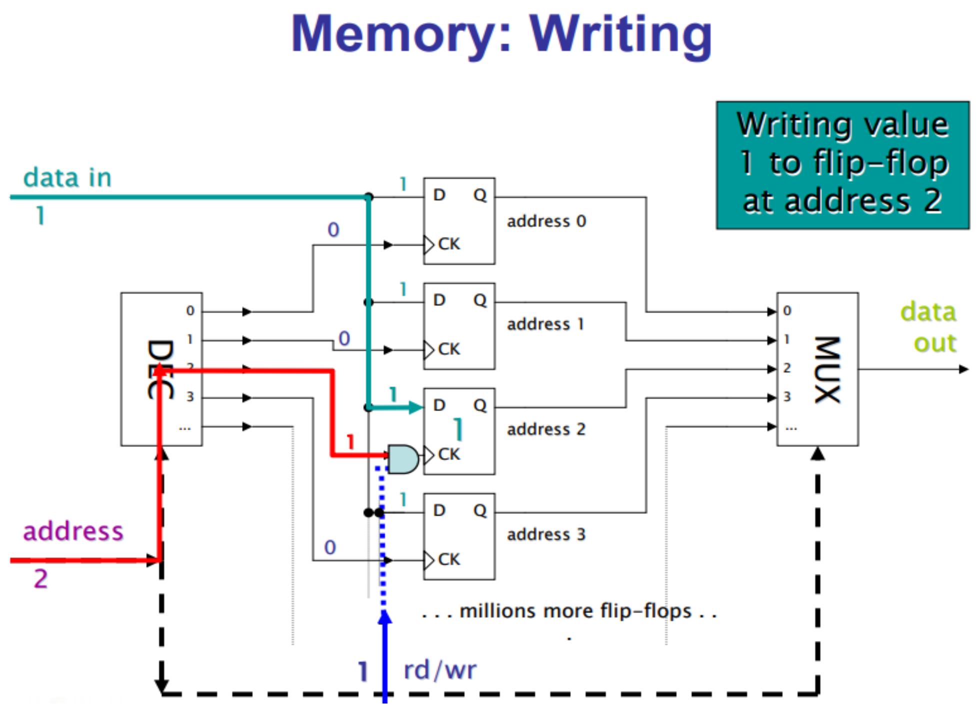
- Memory can store many bits independently
    - register banks contain many flip-flops

- Need to identify which bit (flip-flop) to read or write
- Give each flip-flop a unique number (address)

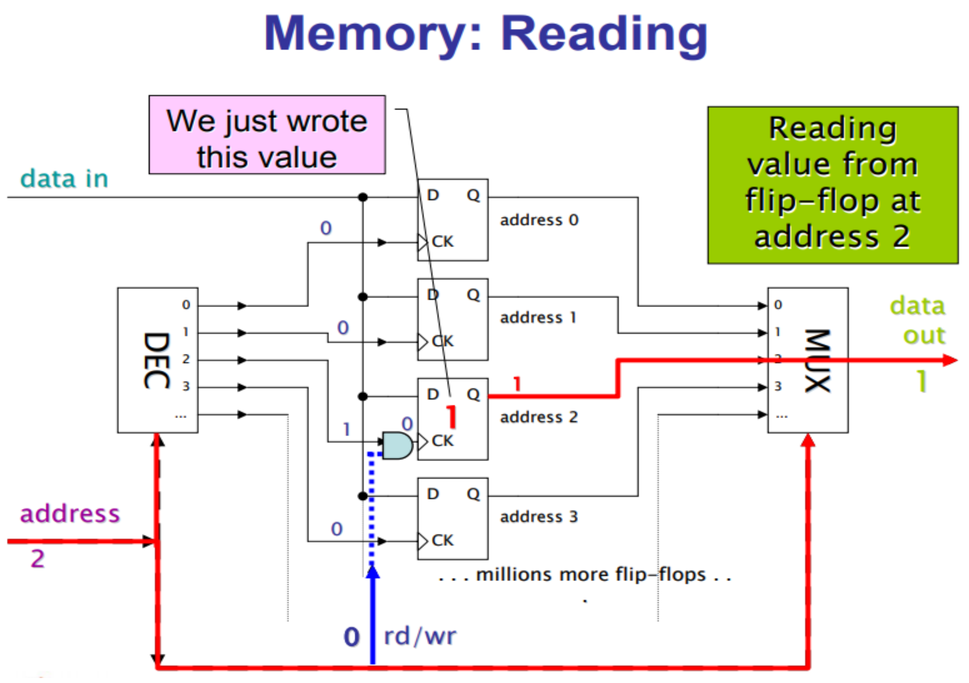
Memory: Circuit Diagram


Decoder: feeds input to selected output, 0 to all others
data in
D Q
CK
address 0
address 1
address 2
address 3
DEC
0
1
2
3
...
MUX
data out
address
rd/wr
. . . millions more flip-flops . .

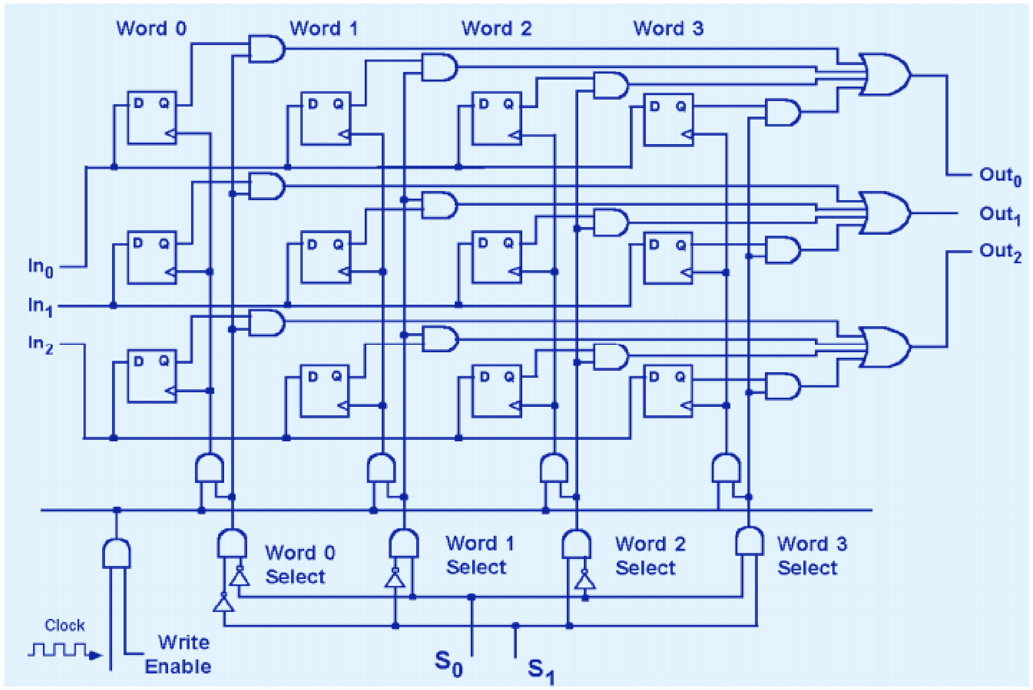
Memory: Writing



Memory: Reading

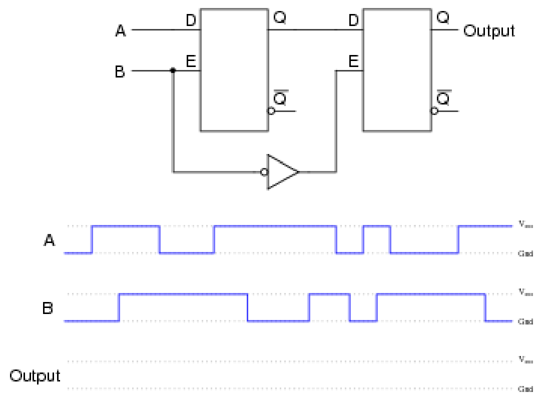


Memory: 4-words, 3 bits/word



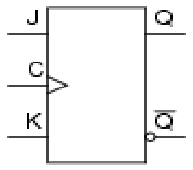
Activity - 03

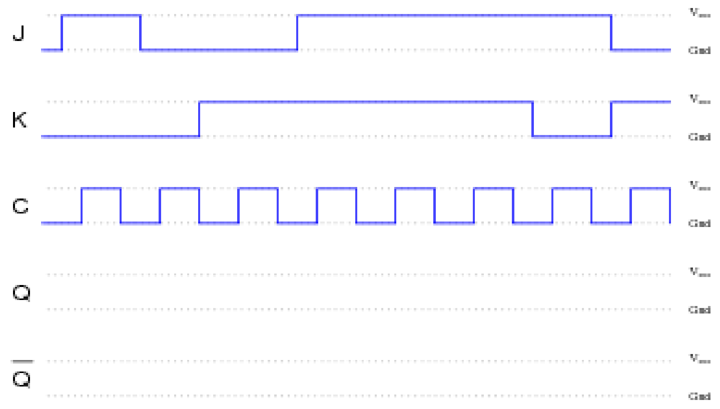
- Determine the final output states over time for the following circuit, built from D-type gated latches:



At what specific times in the pulse diagram does the final output assume the input's state? How does this behavior differ from the normal response of a D-type latch?

2.Determine the output states for this J-K flip-flop, given the pulse inputs shown:





Pre- requites

- Computers are implementations of Boolean logic.
- Boolean functions are completely described by Truth Tables.
- Logic gates are small circuits that implement Boolean operators.
- The basic gates are AND, OR and NOT.
- The “universal gates” are NOR and NAND.

Summary

- Computer circuits consist of combinational logic circuits and sequential logic circuits
- Combinational circuits produce outputs (almost) immediately when their inputs change
- Sequential circuits have internal states as well as combinations of input and output logic
  - The outputs may also depend on the states left behind by previous inputs
  - Sequential circuits may require clocks to control their changes of state
  - The basic sequential circuit unit is the flip-flop