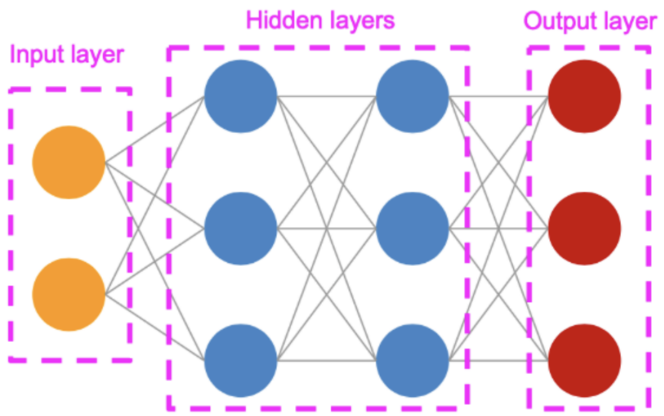
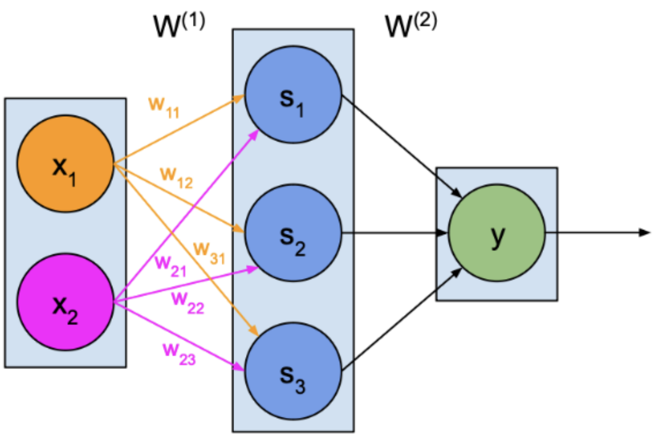
Neural Network



MLP



- Weights
- Net inputs
- Activation

$$\mathbf{h}^{(2)} = \mathbf{x}W^{(1)} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} x_1 w_{11} + x_2 w_{21} & x_1 w_{12} + x_2 w_{22} & x_1 w_{13} + x_2 w_{23} \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix}}$$

다음과 같은 벡터곱으로 연산됨

Implementing a NN from scratch

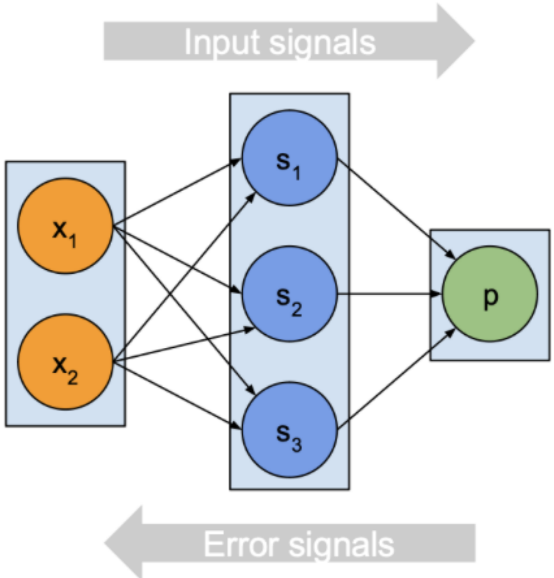
```
import numpy as np


class MLP(object):

    """A Multilayer Perceptron class.
    """

    def __init__(self, num_inputs=3, hidden_layers=[3, 3], num_outputs=2):
        """Constructor for the MLP. Takes the number of inputs,
            a variable number of hidden layers, and number of outputs
        Args:
            num_inputs (int): Number of inputs
            hidden_layers (list): A list of ints for the hidden layers
            num_outputs (int): Number of outputs
        """

        self.num_inputs = num_inputs
        self.hidden_layers = hidden_layers
        self.num_outputs = num_outputs

# create a generic representation of the layers
        layers = [num_inputs] + hidden_layers + [num_outputs]

# create random connection weights for the layers
        weights = []
        for i in range(len(layers)-1):
            w = np.random.rand(layers[i], layers[i+1])
            weights.append(w)
        self.weights = weights


    def forward_propagate(self, inputs):
        """Computes forward propagation of the network based on input signals.
        Args:
            inputs (ndarray): Input signals
        Returns:
```

```
            activations (ndarray): Output values
        """

# the input layer activation is just the input itself
        activations = inputs

# iterate through the network layersfor w in self.weights:

# calculate matrix multiplication between previous activation and weight matrix
            net_inputs = np.dot(activations, w)

# apply sigmoid activation function
            activations = self._sigmoid(net_inputs)

# return output layer activationreturn activations


    def _sigmoid(self, x):
        """Sigmoid activation function
        Args:
            x (float): Value to be processed
        Returns:
            y (float): Output
        """

        y = 1.0 / (1 + np.exp(-x))
        return y


if __name__ == "__main__":

# create a Multilayer Perceptron
    mlp = MLP()

# set random values for network's input
    inputs = np.random.rand(mlp.num_inputs)

# perform forward propagation
    output = mlp.forward_propagate(inputs)

    print("Network activation: {}".format(output))
```

Training a neural network

Training a neural network

- Tweak weights of the connections
- Feed training data (input + target) to the network
- Iterative adjustments

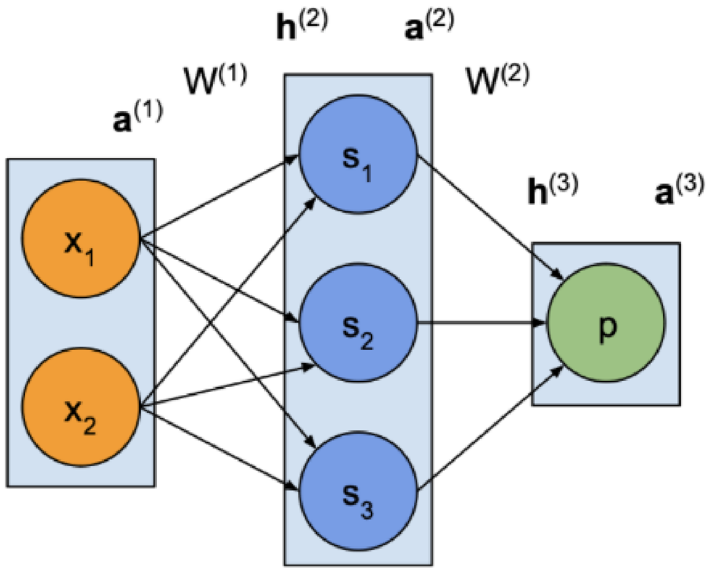


Input signals

1. y 값 예측
2. error 계산

Error signals

1. gradient 계산
2. 파라미터 업데이트



$$\frac{\partial E}{\partial W^{(2)}} = \frac{\partial E}{\partial a^{(3)}} \frac{\partial a^{(3)}}{\partial h^{(3)}} \boxed{\frac{\partial h^{(3)}}{\partial W^{(2)}}}$$

위 식과 같이 연쇄법칙으로 사용해서 gradient를 계산함

gradient를 낮추는 방향으로 leaning rate만큼 step을 진행 -> error 최소화

Training a neural network: Implementing back propagation from scratch

```
import numpy as np
from random import random


class MLP(object):
    """A Multilayer Perceptron class.
    """

    def __init__(self, num_inputs=3, hidden_layers=[3, 3], num_outputs=2):
        """Constructor for the MLP. Takes the number of inputs,
            a variable number of hidden layers, and number of outputs
        Args:
```

```
            num_inputs (int): Number of inputs
            hidden_layers (list): A list of ints for the hidden layers
            num_outputs (int): Number of outputs
        """

        self.num_inputs = num_inputs
        self.hidden_layers = hidden_layers
        self.num_outputs = num_outputs

# create a generic representation of the layers
        layers = [num_inputs] + hidden_layers + [num_outputs]

# create random connection weights for the layers
        weights = []
        for i in range(len(layers) - 1):
            w = np.random.rand(layers[i], layers[i + 1])
            weights.append(w)
        self.weights = weights

# save derivatives per layer
        derivatives = []
        for i in range(len(layers) - 1):
            d = np.zeros((layers[i], layers[i + 1]))
            derivatives.append(d)
        self.derivatives = derivatives

# save activations per layer
        activations = []
        for i in range(len(layers)):
            a = np.zeros(layers[i])
            activations.append(a)
        self.activations = activations


    def forward_propagate(self, inputs):
        """Computes forward propagation of the network based on input signals.
        Args:
            inputs (ndarray): Input signals
        Returns:
            activations (ndarray): Output values
        """

# the input layer activation is just the input itself
        activations = inputs

# save the activations for backpropogation
        self.activations[0] = activations

# iterate through the network layersfor i, w in enumerate(self.weights):
# calculate matrix multiplication between previous activation and weight matrix
            net_inputs = np.dot(activations, w)

# apply sigmoid activation function
            activations = self._sigmoid(net_inputs)

# save the activations for backpropogation
            self.activations[i + 1] = activations
```

```
# return output layer activationreturn activations


    def back_propagate(self, error):
        """Backpropogates an error signal.
        Args:
            error (ndarray): The error to backprop.
        Returns:
            error (ndarray): The final error of the input
        """

# iterate backwards through the network layersfor i in reversed(range(len(self.derivat
ives))):

# get activation for previous layer
            activations = self.activations[i+1]

# apply sigmoid derivative function
            delta = error * self._sigmoid_derivative(activations)

# reshape delta as to have it as a 2d array
            delta_re = delta.reshape(delta.shape[0], -1).T

# get activations for current layer
            current_activations = self.activations[i]

# reshape activations as to have them as a 2d column matrix
            current_activations = current_activations.reshape(current_activations.shap
e[0],-1)

# save derivative after applying matrix multiplication
            self.derivatives[i] = np.dot(current_activations, delta_re)

# backpropogate the next error
            error = np.dot(delta, self.weights[i].T)


    def train(self, inputs, targets, epochs, learning_rate):
        """Trains model running forward prop and backprop
        Args:
            inputs (ndarray): X
            targets (ndarray): Y
            epochs (int): Num. epochs we want to train the network for
            learning_rate (float): Step to apply to gradient descent
        """
# now enter the training loopfor i in range(epochs):
            sum_errors = 0

# iterate through all the training datafor j, input in enumerate(inputs):
                target = targets[j]

# activate the network!
                output = self.forward_propagate(input)

                error = target - output

                self.back_propagate(error)
```

```
# now perform gradient descent on the derivatives# (this will update the weights
                self.gradient_descent(learning_rate)

# keep track of the MSE for reporting later
                sum_errors += self._mse(target, output)

# Epoch complete, report the training errorprint("Error: {} at epoch {}".format(sum_er
rors / len(items), i+1))

        print("Training complete!")
        print("=====")


    def gradient_descent(self, learningRate=1):
        """Learns by descending the gradient
        Args:
            learningRate (float): How fast to learn.
        """
# update the weights by stepping down the gradientfor i in range(len(self.weights)):
            weights = self.weights[i]
            derivatives = self.derivatives[i]
            weights += derivatives * learningRate


    def _sigmoid(self, x):
        """Sigmoid activation function
        Args:
            x (float): Value to be processed
        Returns:
            y (float): Output
        """

        y = 1.0 / (1 + np.exp(-x))
        return y


    def _sigmoid_derivative(self, x):
        """Sigmoid derivative function
        Args:
            x (float): Value to be processed
        Returns:
            y (float): Output
        """
        return x * (1.0 - x)


    def _mse(self, target, output):
        """Mean Squared Error loss function
        Args:
            target (ndarray): The ground trut
            output (ndarray): The predicted values
        Returns:
            (float): Output
        """
        return np.average((target - output) ** 2)


if __name__ == "__main__":
```

```
# create a dataset to train a network for the sum operation
    items = np.array([[random()/2 for _ in range(2)] for _ in range(1000)])
    targets = np.array([[i[0] + i[1]] for i in items])

# create a Multilayer Perceptron with one hidden layer
    mlp = MLP(2, [5], 1)

# train network
    mlp.train(items, targets, 50, 0.1)

# create dummy data
    input = np.array([0.3, 0.1])
    target = np.array([0.4])

# get a prediction
    output = mlp.forward_propagate(input)

    print()
    print("Our network believes that {} + {} is equal to {}".format(input[0], input
[1], output[0]))
```