

Maths Bootcamp

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Overleaf

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The School of Athens by Raphael



What is Geometry?

- Branch of mathematics studying shapes, sizes, and spatial relationships.
- Derived from Greek: "*geo*" (earth) + "*metron*" (measure).
- Originally focused on measuring the earth, now much broader.

Key Concepts in Geometry

- **Points, Lines, and Angles:** Basic building blocks.
- **Shapes and Figures:** Circles, triangles, polygons.
- **Solids:** 3D objects like cubes, spheres, pyramids.
- **Theorems and Proofs:** Logical reasoning based on axioms and postulates.

Definition of Percentage

- A **percentage** is a way of expressing a number as a fraction of 100.
- The formula to calculate a percentage is:

$$\text{Percentage} = \left(\frac{\text{Part}}{\text{Whole}} \right) \times 100$$

- Example: If you score 45 out of 60 on a test, the percentage is:

$$\left(\frac{45}{60} \right) \times 100 = 75\%$$

Ratio vs Rate

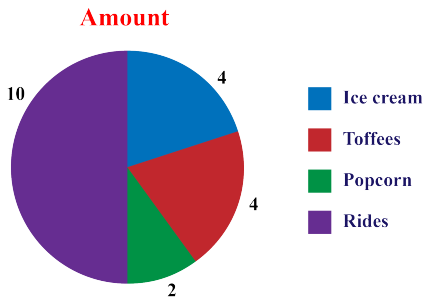


Figure: ratio

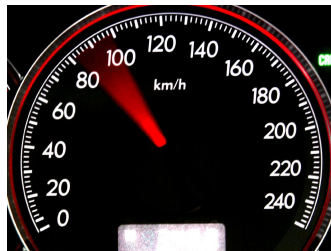


Figure: rate

Key Distinction: Ratio vs Rate

Ratio

- **Definition:** Comparison of two similar units.
- **Units:** Unitless (dimensionless).
- **Form:** Written as $a : b$, $\frac{a}{b}$, or "a to b".
- **Example:** Boys to girls in a classroom is 2 : 3.
- **Key Feature:** Static relationships.

Rate

- **Definition:** Comparison of two different units.
- **Units:** Includes units (e.g., miles per hour).
- **Form:** Written as $\frac{a \text{ unit}_1}{b \text{ unit}_2}$ or "a per b".
- **Example:** A car travels 60 mph.
- **Key Feature:** Dynamic relationships (e.g., over time or space).

Proportional Relationship

Definition

A proportional relationship is a relationship between two quantities where the ratio between them remains constant. If two variables are proportional, it means they can be expressed in the form:

$$y = kx$$

where k is the constant of proportionality and it can be an integer or a fraction or an irrational number.

Proportionality Problem: Mixing Chemicals

Problem

A person mixes 15mL of bleach with 3.75L of water for sanitizing solution for a daycare. What are the possible combinations

- **A.** 12 mL bleach and 3L water
- **B.** 6 mL bleach and 1.5L water
- **C.** 3 mL leach and 0.75L water
- **D.** 20 mL bleach and 5.5L water

Problem

Is the area of square is propotional to side length ?

Proportionality vs. Linearity

- A **proportional relationship** always passes through the origin $(0, 0)$.
- The general form of a proportional relationship is:

$$y = kx$$

where k is the constant of proportionality.

- A **linear relationship** can pass through any point, not necessarily the origin.
- The general form of a linear relationship is:

$$y = mx + b$$

where m is the slope and b is the y-intercept.

- Key Difference:
 - In a proportional relationship, $b = 0$, so the line always passes through $(0, 0)$.
 - In a linear relationship, b can be any value, so the line does not need to pass through the origin.





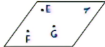
Main Types of Geometry

- **Euclidean Geometry:** Deals with flat, 2D spaces.
- **Non-Euclidean Geometry:** Studies curved spaces (spherical, hyperbolic).
- **Analytic Geometry:** Combines algebra and geometry using coordinates.
- **Differential Geometry:** Uses calculus to study curves and surfaces.

Applications of Geometry

- Essential in fields like architecture, physics, engineering.
- Used in navigation, astronomy, and computer graphics.
- Provides tools to understand spatial relationships in various disciplines.

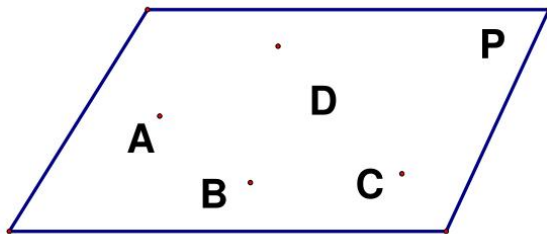
Geometry Terminology

	Description	Figure	Symbol
Point	A geometric element that has zero dimensions.		P or Point P
Line	A line is a collection of points along a straight path with no end points.		\overleftrightarrow{AB} or \overleftrightarrow{BA}
Line segment	A line segment is a part of a line that contains every point on the line between its end points.		\overline{XY} or \overline{YX}
Ray	A ray is a line with a single end point that goes on and on in one direction.		\overrightarrow{PQ}
Plane	A plane is a flat surface that extends to infinity.		Plane EFG or Plane τ

Geometry Terminology: Coplanar & Collinear

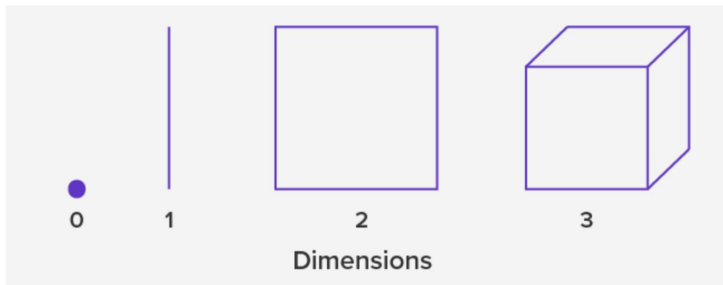


Collinear Points are points on the same line.



Coplanar Points are points that lie in the same plane.

Geometry Terminology: Dimensions



Parallel & Perpendicular Lines

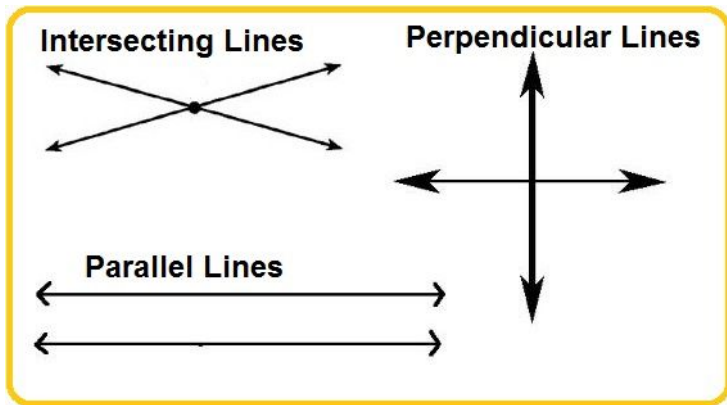
Parallel

Two lines are said to be **parallel** if they never intersect, no matter how far they are extended, and remain the same distance apart at all points.

Perpendicular

Two lines are said to be **perpendicular** if they intersect at a right angle (90 degrees).

Parallel & Perpendicular Lines



Angles

- **Angle:** Formed by two rays with a common endpoint.
- **Acute Angle:** Less than 90° .
- **Right Angle:** Exactly 90° .
- **Obtuse Angle:** Greater than 90° but less than 180° .
- **Straight Angle:** Exactly 180° .

Shapes and Figures

- **Polygon:** A closed figure formed by line segments.
- **Triangle:** A polygon with three sides.
 - **Equilateral Triangle:** All sides and angles are equal.
 - **Isosceles Triangle:** Two sides and angles are equal.
 - **Scalene Triangle:** All sides and angles are different.
- **Quadrilateral:** A polygon with four sides (e.g., square, rectangle).
- **Circle:** A set of points equidistant from the center.

Properties of Shapes

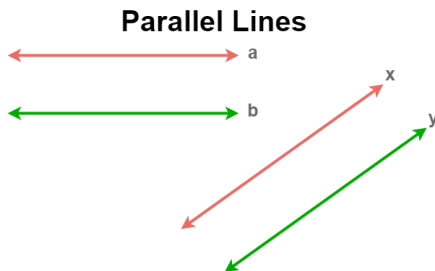
- **Perimeter:** Total distance around a shape.
- **Area:** The measure of space inside a two-dimensional shape.
- **Volume:** The measure of space inside a three-dimensional object.

Transformations

- **Translation:** Moving a shape without rotating or flipping it.
- **Rotation:** Turning a shape around a fixed point.
- **Reflection:** Flipping a shape over a line to create a mirror image.
- **Dilation:** Resizing a shape while maintaining its proportions.

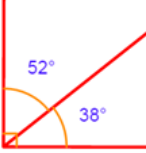
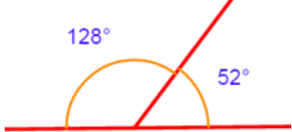
Parallel Lines

- **Definition:** Two or more lines that are always the same distance apart and never meet, no matter how far they are extended.
- They run in the same direction and have the same slope.
- Example: Think of train tracks running side by side—they never cross each other.

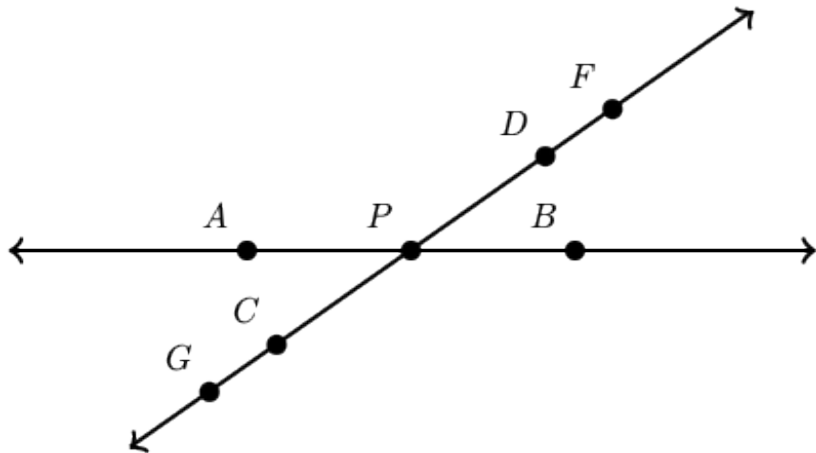


Angles

Supplementary and Complementary Angles

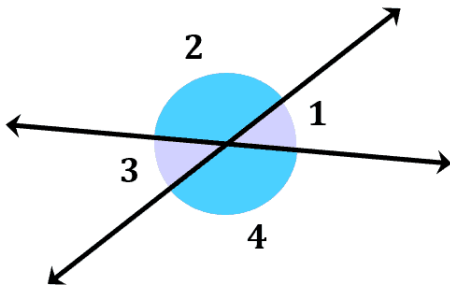
Type of Angles	Description	Example
Complementary Angles	Angles that add up to 90°	 A diagram showing a right angle (90 degrees) divided into two adjacent angles by a red ray. The top angle is labeled 52 degrees and the bottom angle is labeled 38 degrees. A small square at the vertex indicates the total angle is 90 degrees.
Supplementary Angles	Angles that add up to 180°	 A diagram showing a straight line (180 degrees) divided into two adjacent angles by a red ray. The left angle is labeled 128 degrees and the right angle is labeled 52 degrees. A semi-circular arc at the vertex indicates the total angle is 180 degrees.

Problem 1



Vertical Angles

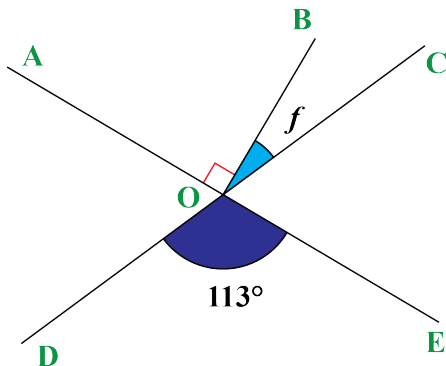
Vertical angles, also known as opposite angles, are the angles formed when two lines intersect



$$\angle 1 \cong \angle 3$$

$$\angle 2 \cong \angle 4$$

Problem 2: Vertical Angles



Transversal & Parallel Lines

A **transversal line** is a line that crosses or intersects two or more other lines at different points

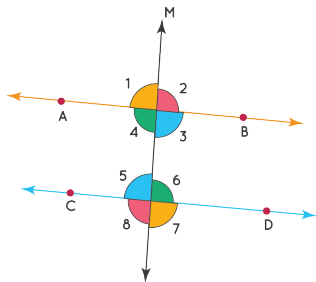
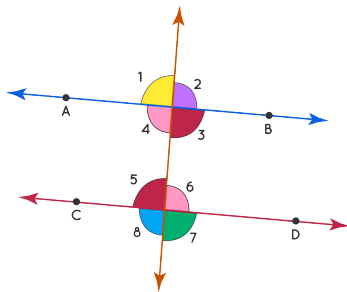


Figure: alternate interior angles Figure: alternate exterior angles

Altitudes, Medians and Centroid

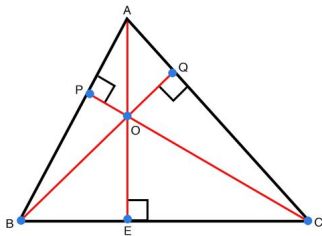


Figure: altitudes

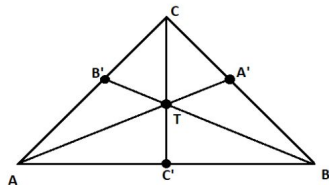
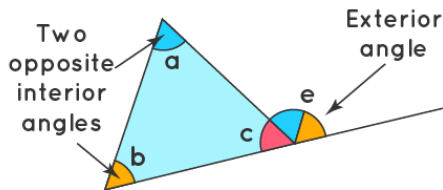


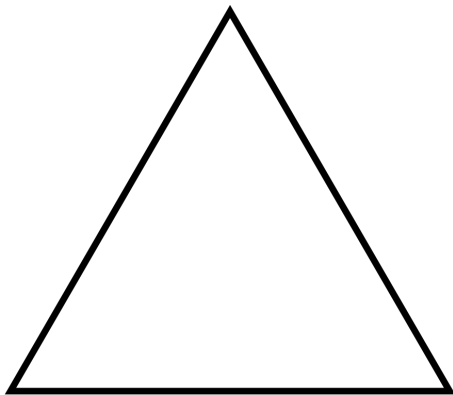
Figure: medians

Exterior Angle Property



$$\angle e = \angle a + \angle b$$

Angles of a Triangle Measure to 180°



Equilateral and Isosceles Triangle

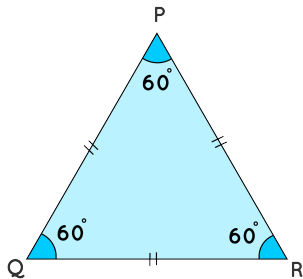


Figure: Equilateral

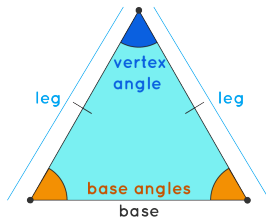
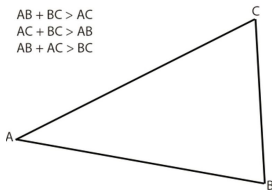


Figure: Isosceles

Triangle Inequality

For any triangle with sides a , b , and c , the following conditions must hold true:

- $a + b > c$
- $a + c > b$
- $b + c > a$



What are Rigid Transformations?

- Rigid transformations (or isometries) preserve the shape and size of geometric objects.
- They do not alter:
 - Distances between points.
 - Angles between lines or curves.
- The object remains congruent to itself after the transformation.

Types of Rigid Transformations

- 1 **Translation:** Moves every point by the same distance in a given direction.
- 2 **Rotation:** Rotates an object around a fixed point by a certain angle.
- 3 **Reflection:** Flips an object over a specified line (the "mirror line").
- 4 **Glide Reflection:** Combines a reflection and a translation along the direction of the reflection line.

Definition:

- Moves every point of an object by the same distance in a specific direction.

Mathematical Representation:

$$(x', y') = (x + a, y + b)$$

Properties:

- Preserves distances and angles.
- Does not change the orientation of the object.

Rotation

Definition:

- Rotates an object around a fixed point by a certain angle.

Mathematical Representation: For a rotation by angle θ around the origin:

$$x' = x \cos \theta - y \sin \theta$$

$$y' = x \sin \theta + y \cos \theta$$

Properties:

- Preserves distances and angles.
- Changes the orientation depending on the direction of rotation.

Reflection

Definition:

- Flips an object over a line (the "mirror line").

Examples:

- Reflection across the x -axis, y -axis, or any line $y = mx + c$.

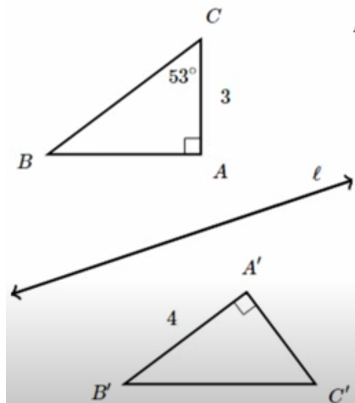
Properties:

- Preserves distances and angles.
- Changes the orientation of the object.

Properties of Rigid Transformations

- **Distance Preservation:** The distance between any two points remains unchanged.
- **Angle Preservation:** Angles between lines or curves are preserved.
- **Parallelism:** Parallel lines remain parallel.
- **Co-ordinates** Co-ordinates are not preserved
- **Congruence:** The original and transformed shapes are congruent.

Exercise: Rigid Transformations



$\triangle ABC$ is reflected across line ℓ to get $\triangle A'B'C'$.

What's $A'C'$?

What's the measure of angle B' ?

What's the area of $\triangle ABC$?

What's the perimeter of $\triangle A'B'C'$?

Definition:

- Dilation involves scaling distances from a point (the center of dilation) by a constant factor k . It changes the size of a figure but not its shape.
- A non rigid transformation where lengths are not preserved
- Dilation will preserve angles

What is Congruence

Definition

Congruence means that two figures or shapes are identical in shape and size. They can be transformed into each other using rigid transformations such as translation, rotation, or reflection

- Vertical angles are congruent
- Alternate interior angles are congruent
- Alternate exterior angles are congruent
- Corresponding angles are congruent

Congruence in Triangles

SSS

if three sides of one triangle are congruent to the three sides of another triangle

SAS

if two sides and an included angle of one triangle is congruent to another

ASA

if two angles and included side of a triangle is congruent to another

AAS

if two angles and non included side of a triangle is congruent to another

Similarity in Triangles

AAA

if three angles of one triangle are congruent to another

SSS

if three sides of a triangle are proportional to another

SAS

if two sides of a triangle are proportional and included angle is congruent

Proportional Relationship

A relationship between two quantities is proportional if the ratio between those quantities is always equivalent. We will look at side length ratios to find out whether triangles are similar or not

Similarity in All Shapes

The concept of similarity applies to any two shapes that have the same **shape** but may differ in **size**. **Conditions for Similarity:**

- **Corresponding angles must be equal:** The angles in one shape must match the angles in the other.
- **Corresponding sides must be proportional:** The lengths of corresponding sides must have the same ratio (scaling factor).

Examples of Similar Shapes:

- **Quadrilaterals:** Squares, rectangles, rhombuses, and parallelograms can be similar if corresponding angles are equal and side lengths are proportional.
- **Polygons:** Any polygons (pentagons, hexagons, etc.) can be similar if corresponding angles and side lengths meet the conditions.
- **Circles:** All circles are similar because they have the same shape. The ratio of their radii, diameters, or circumferences is the scaling factor.
- **3D Shapes:** Cubes, spheres, pyramids, and other 3D shapes can also be similar if angles and sides are proportional.

Key Point: Similarity applies to all shapes, both in 2D and 3D, as long as the conditions of equal angles and proportional sides are met.

Original Triangle and Scaled Version

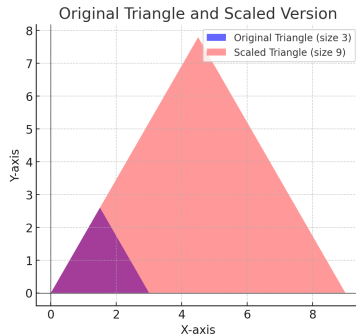


Figure: A triangle of size 3 and its scaled version by a factor of 3

Similar Triangles Postulates

- **Angle-Angle (AA) Similarity Postulate:**

- If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.

- **Side-Angle-Side (SAS) Similarity Postulate:**

- If one angle of a triangle is congruent to one angle of another triangle, and the sides that include these angles are proportional, then the triangles are similar.

- **Side-Side-Side (SSS) Similarity Postulate:**

- If the three sides of one triangle are proportional to the three corresponding sides of another triangle, the triangles are similar.

Variability

Variability refers to how data points differ from one another within a data set. In real-world data, there is almost always some variation because no two measurements, observations, or events are exactly the same.

Variability Problems

- How much does my pet weight ?
- What is the average number of cars in a parking lot on Monday mornings ?
- Am i hungry?
- How often am I hungry after lunch ?
- How much time do you spend on facebook every month?

What is a Unit Circle

The Unit circle

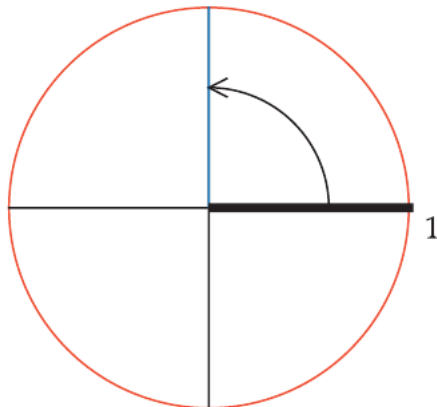
The unit circle is the circle with radius 1 centered at the origin

Equation of unit Circle

The unit circle in the xy -plane is the set of points (x,y) such that

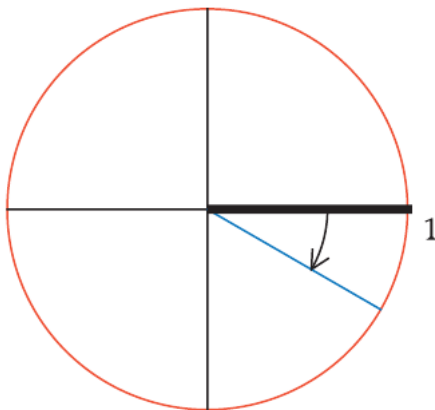
$$x^2 + y^2 = 1$$

Radius corresponding to a positive angle



90°

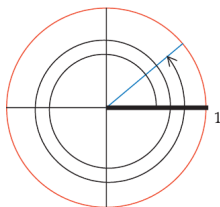
Radius corresponding to a negative angle



-30°

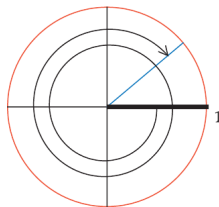
Positive and Negative Angles

- Angle measurements for a radius on the unit circle are made from the positive horizontal axis.
- Positive angles correspond to moving counterclockwise from the positive horizontal axis.
- Negative angles correspond to moving clockwise from the positive horizontal axis.



760°

Figure: +ve angle



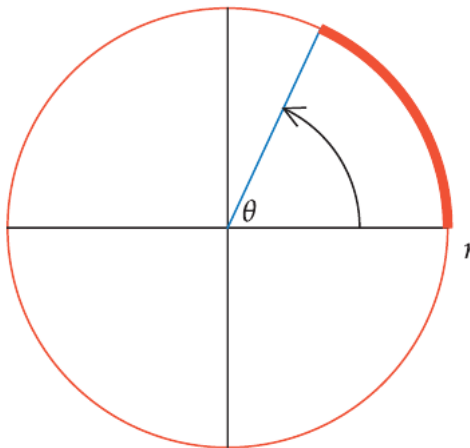
-680°

Figure: -ve angle

cyclic behaviour of angles

A radius of the unit circle corresponding to θ degrees also corresponds to $\theta + 360n$ degrees for every integer n .

Length of a Circular Arc

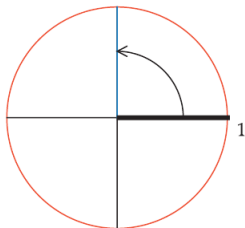


This circular arc has length $\frac{\theta\pi r}{180}$.

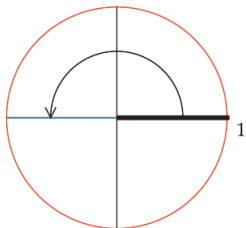
Radians

Radians are a unit of measurement for angles such that 2π radians correspond to a rotation through an entire circle.

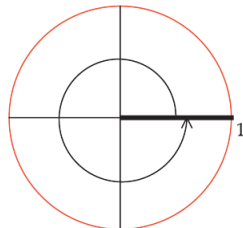
Radians



$\frac{\pi}{2}$ radians



π radians



2π radians

Degree to Radians

$$360^\circ = 2\pi \text{ radians}$$

$$1^\circ = \frac{2\pi}{360} \text{ radians}$$

Arc Length

length of a circular arc

If $0 < \theta \leq 2\pi$, then a circular arc on the unit circle corresponding to θ radians has length θ

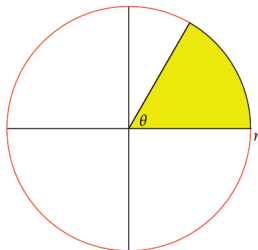


Figure: Area of slice

Area of slice

A slice with angle θ radians inside a circle with radius r has area $\frac{1}{2}\theta r^2$.

Cosine and Sine

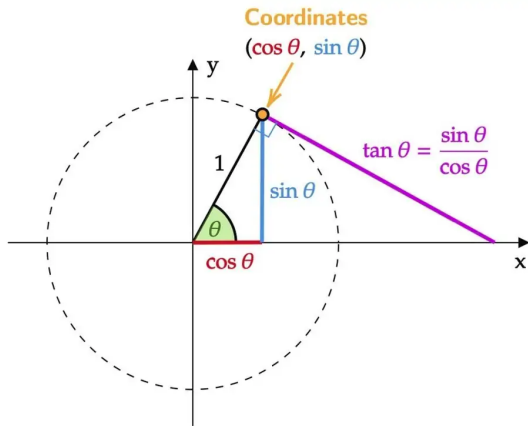


Figure: sin and cos