Real Numbers

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Maveric Systems

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Integers

- The set of integers is denoted by $\mathbb{Z}$.
- Integers include:

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

- Formally, $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
- Common properties:
  - $\mathbb{Z}$ is infinite and unbounded in both the negative and positive directions.
  - Closed under addition, subtraction, and multiplication:

$$\forall a, b \in \mathbb{Z}, \quad a \pm b \in \mathbb{Z}, \quad a \cdot b \in \mathbb{Z}.$$

- The quotient of any two integers is not necessarily an integer. So we need to extend arithmetic to **rational numbers**

Rational Numbers

- The set of rational numbers is denoted by $\mathbb{Q}$.
- Definition:

$$\mathbb{Q} = \left\{ \frac{p}{q} \,\middle|\, p \in \mathbb{Z},\ q \in \mathbb{Z},\ q \neq 0 \right\}.$$

- Every integer is also a rational number (e.g., $5 = \frac{5}{1}$).
- Examples:

$$\frac{1}{2}, \quad -\frac{3}{4}, \quad 0, \quad 7, \quad \frac{11}{5}, \ldots$$

- Properties:
  - Closed under addition, subtraction, multiplication, and division (except division by zero).
  - Densely packed on the number line: between any two rationals, there is another rational.

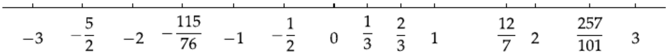
Interesting Facts

- Why division by zero is prohibited ?
  - Division is inverse of multiplication in the sense

$$\frac{m}{n} \cdot n = m$$

  - if $n = 0$ and $m = 1$, we get $\frac{1}{0} \cdot 0 = 1$ which is nonsensical as any number multiplied by zero is zero
- Rational numbers suffice for all actual physical measurements like weight, height and length
- But Geometry, Algebra and Calculus force us to consider **real numbers**

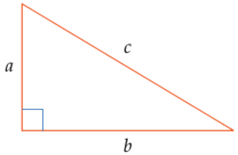
A Real Number Line



Some rational numbers on the real line.

- if n is a positive integer then $\frac{1}{n}$ is to the right of 0 by the length obtained by dividing the segment from $1to0$ in to n segments of equal length

Is every Real Number a Rational ?



- $c^2 = a^2 + b^2$. If $a = 1, b = 1$ then $c^2 = 2$. Then what rational number is c