INTRODUCTION TO
DIGITAL IMAGE PROCESSING

Lecture #7

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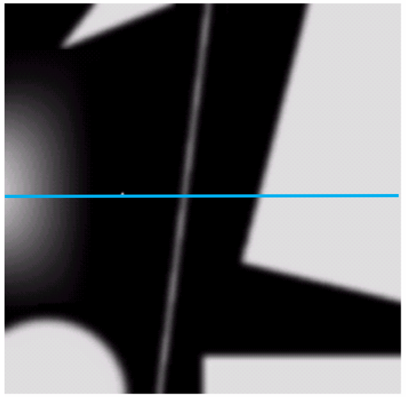
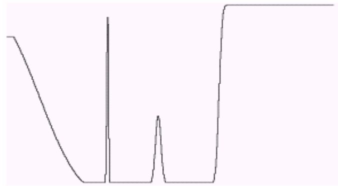
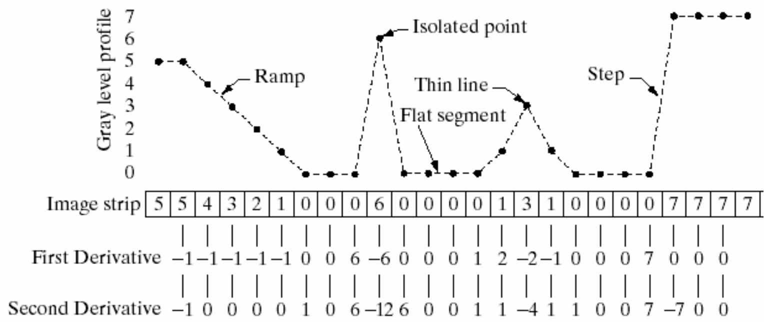
Derivatives Illustration





Derivatives Illustration



$$\frac{\partial f(x)}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f(x)}{\partial x^2} = \partial f(x) - \partial f(x-1) = f(x+1) + f(x-1) - 2f(x)$$

Comparisons

- 1st-order derivatives:
  - produce thicker edges
  - strong response to graylevel steps
- 2nd-order derivatives:
  - strong response to fine detail (thin lines, isolated points)
    → very noise-sensitive
  - double response at step changes in graylevel

Laplacian Operator

- Simplest isotropic derivative operator
- Response independent of direction of the discontinuities.
- *Rotation-invariant:* rotating the image and then applying the filter gives the same result as applying the filter to the image first and then rotating the result.
- Since derivatives of any order are linear operations, the Laplacian is a linear operator.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Discrete Form of Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

where

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\boxed{\begin{aligned}\nabla^2 f = [&f(x+1, y) + f(x-1, y) \\ &+ f(x, y+1) + f(x, y-1) - 4f(x, y)]\end{aligned}}$$

Laplacian Mask

Isotropic result for rotations in increments of 90°

0	1	0
1	−4	1
0	1	0

Isotropic result for rotations in increments of 45°

1	1	1
1	−8	1
1	1	1

Another Rationalization

$1/9^* \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Unweighted Average Smoothing Filter

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Retain Original

$1/9^* \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$ Original – Average

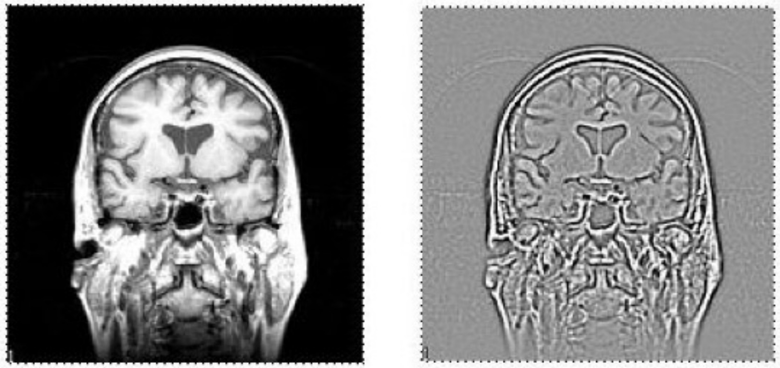
In constant areas: 0 ← **Summation of coefficients in masks equals 0.**

Near edges: high values

Effect of Laplacian Operator

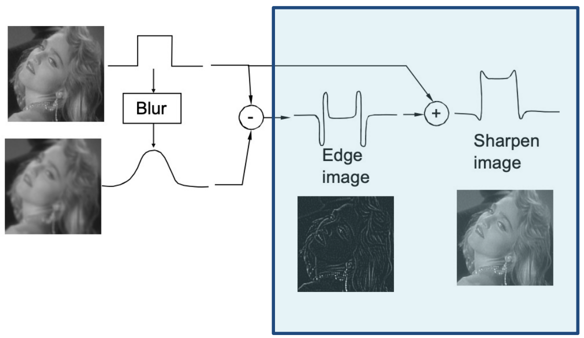
- Since the Laplacian is a derivative operator
  - it highlights graylevel discontinuities in an image
  - it deemphasizes regions with slowly varying gray levels
- The Laplacian tends to produce images that have
  - grayish edge lines and other discontinuities all superimposed on a dark featureless background

Example – MRI Image



Laplacian Sharpening

- Sharpening Recap: Unsharp Masking



Laplacian Sharpening

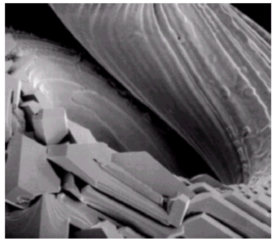
- Sharpening: Addition of Laplacian to image
- Can be combined into one operator

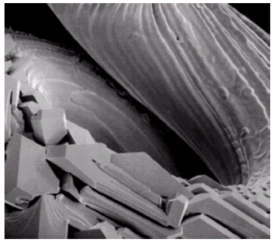
$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 5 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$

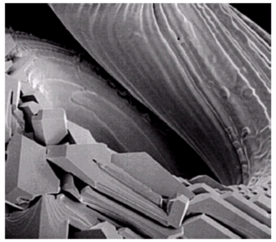
Laplacian Sharpening - Example

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1







Gradient Operator - Summary

- The components of the gradient vector are linear operators, but the magnitude is not (square root).
- The partial derivatives are not rotation invariant (isotropic), but the magnitude is.
- The Laplacian operator yields a scalar: a single number indicating edge strength at point.
- The gradient is actually a vector from which we can compute edge magnitude and direction.

$$f_{mag}(i,j) = \sqrt{f_x^2 + f_y^2} \quad \text{or} \quad f_{mag}(i,j) = |f_x| + |f_y|$$

$$f_{angle}(i,j) = \tan^{-1}\frac{f_y}{f_x} \qquad \text{where} \qquad \boxed{\begin{array}{l} f_x(i,j) = f(i+1,j) - f(i-1,j) \\ f_y(i,j) = f(i,j+1) - f(i,j-1) \end{array}}$$

Common Gradient Operators

1	0
0	-1

0	1
-1	0

Robert Mask

-1	0	1
-1	0	1
-1	0	1

1	1	1
0	0	0
-1	-1	-1

Prewitt Mask

-1	0	1
-2	0	2
-1	0	1

1	2	1
0	0	0
-1	-2	-1

Sobel Mask

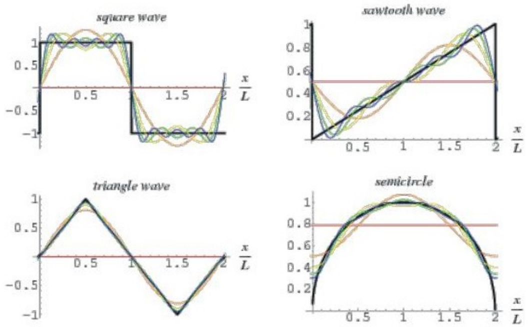
INPUT IMAGE

ROBERT

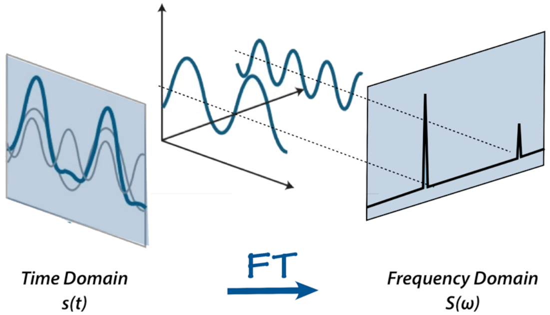
PREWITT

SOBEL

Jean-Baptiste Joseph Fourier

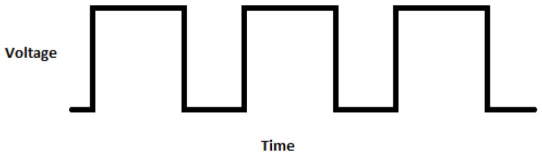


Fourier Transform



Fourier Transform

$$s(t) = \sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \frac{1}{7}\sin(7\omega t) + \cdots$$



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