INTRODUCTION TO
DIGITAL IMAGE PROCESSING

Lecture #13

Niels Volkmann
Professor

ECE Department
Department of Bioengineering
Quantitative Bioscience Program

Morphological processing

- **Morphology**: a branch of biology that deals with the form and structure of animals and plants
- Morphological image processing is used to extract image components for representation and description of region shape, such as boundaries, skeletons, and the convex hull

Preliminaries (Set theory)

- **Reflection**

The reflection of a set B, denoted $\hat{B}$, is defined as

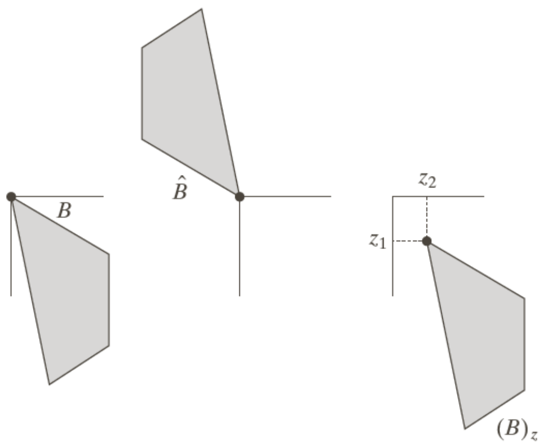
$$\hat{B} = \{w \mid w = -b, \text{for } b \in B\}$$

- **Translation**

The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_Z$, is defined as

$$(B)_Z = \{c \mid c = b + z, \text{for } b \in B\}$$

Example: Reflection and Translation



a b c

FIGURE 9.1 (a) A set, (b) its reflection, and (c) its translation by z.

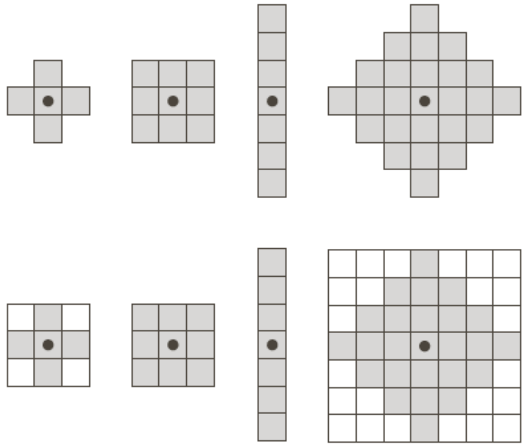
Preliminaries

- **Structure elements (SE)**

  Small sets or sub-images used to probe an image under study for properties of interest

  Similar to kernels in neighborhood operations

Examples: Structuring Elements



Erosion

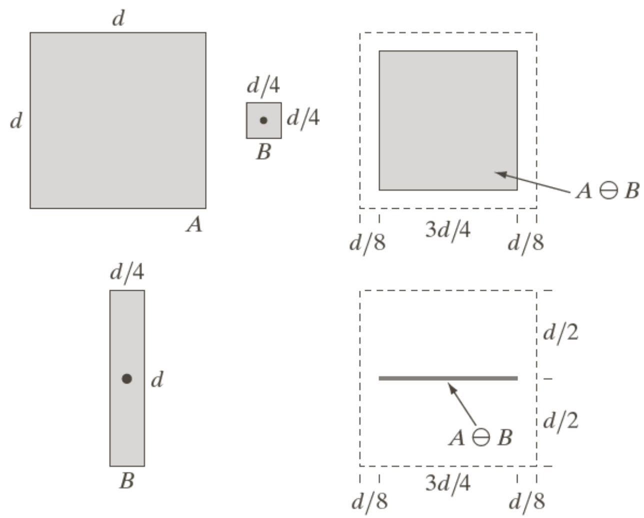
With A and B as sets in Z^2, the erosion of A by B, denoted $A \ominus B$, defined

$$A \ominus B = \{z \mid (B)_Z \subseteq A\}$$

The set of all points z such that B, translated by z, is contained by A.

$$A \ominus B = \{z \mid (B)_Z \cap A^c = \varnothing\}$$

Erosion Example



a b c
d e

FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Erosion Example

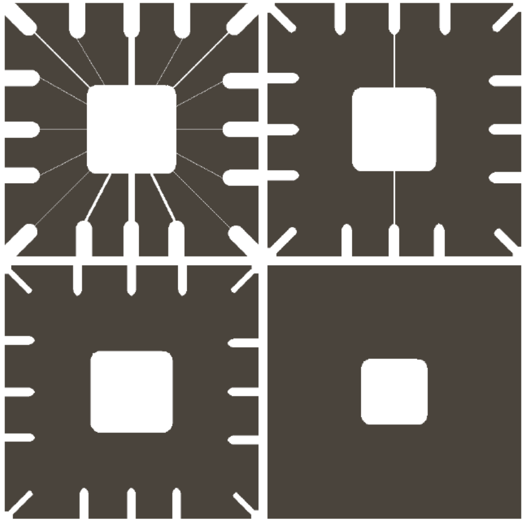


FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Dilation

With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

$$A \oplus B = \left\{ z \mid \left(\hat{B}\right)_z \cap A \neq \varnothing \right\}$$

The set of all displacements z, the translated $\hat{B}$ and A overlap by at least one element.

$$A \oplus B = \left\{ z \mid \left[\left(\hat{B}\right)_z \cap A \right] \subseteq A \right\}$$

Examples of Dilation

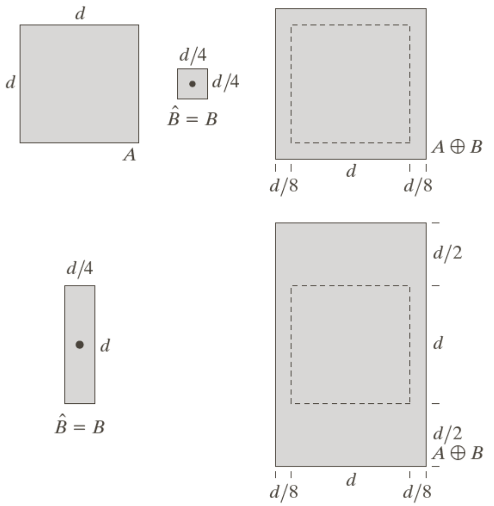
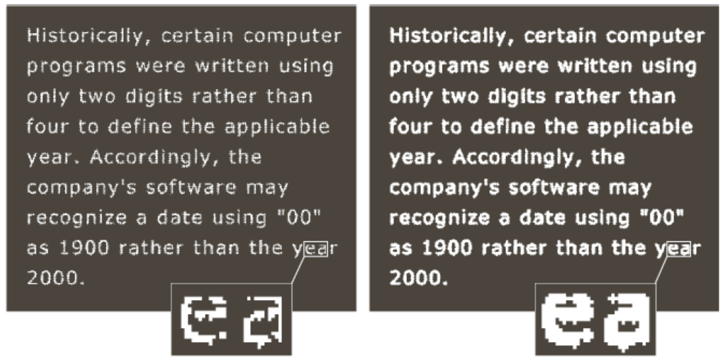


FIGURE 9.6 (a) Set A. (b) Square structuring element (the dot denotes the origin). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Examples of Dilation



0	1	0
1	1	1
0	1	0

FIGURE 9.7 (a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

and

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$\begin{aligned}(A \ominus B)^c &= \left\{z \mid (B)_Z \subseteq A\right\}^c \\ &= \left\{z \mid (B)_Z \cap A^c = \varnothing\right\}^c \\ &= \left\{z \mid (B)_Z \cap A^c \neq \varnothing\right\} \\ &= A^c \oplus \hat{B}\end{aligned}$$

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$\begin{aligned}(A \oplus B)^c &= \left\{z \mid (\hat{B})_Z \cap A \neq \varnothing\right\}^c \\ &= \left\{z \mid (\hat{B})_Z \cap A^c = \varnothing\right\} \\ &= A^c \ominus \hat{B}\end{aligned}$$

Opening and Closing

- Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions
- Closing tends to smooth sections of contours but it generates fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour

Opening and Closing

The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = (A \ominus B) \oplus B$$

The closing of set A by structuring element B, denoted $A \bullet B$, is defined as

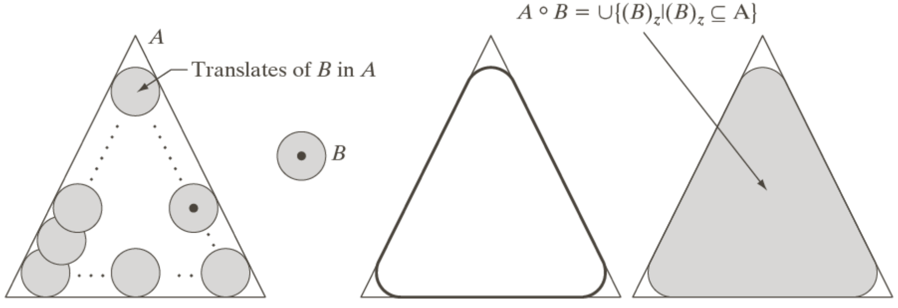
$$A \bullet B = (A \oplus B) \ominus B$$

Opening

The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = \bigcup \{ (B)_Z \mid (B)_Z \subseteq A \}$$

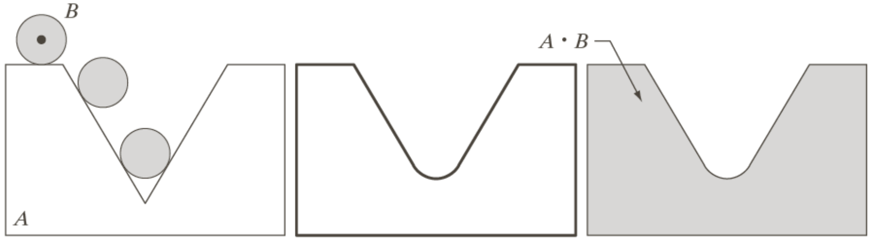
Example: Opening



a b c d

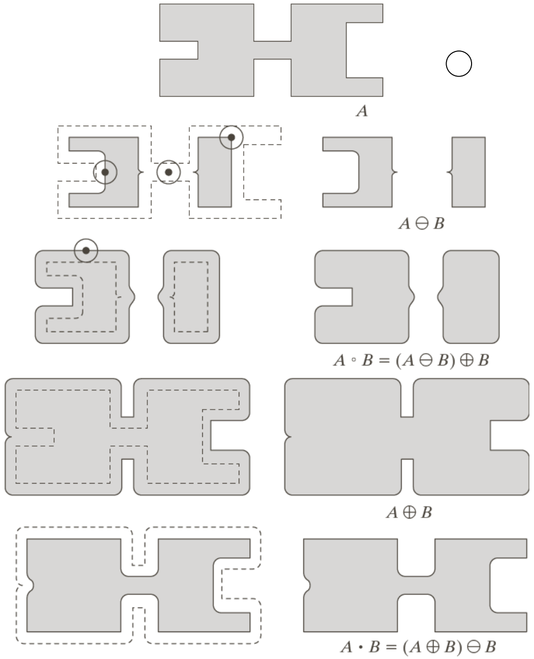
FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Example: Closing



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.



a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Duality of Opening and Closing

- Opening and closing are duals of each other with respect to set complementation and reflection

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

$$(A \circ B)^c = (A^c \bullet \hat{B})$$

The Properties of Opening and Closing

- Properties of Opening

(a) $A \circ B$ is a subset (subimage) of A

(b) if C is a subset of D, then $C \circ B$ is a subset of $D \circ B$

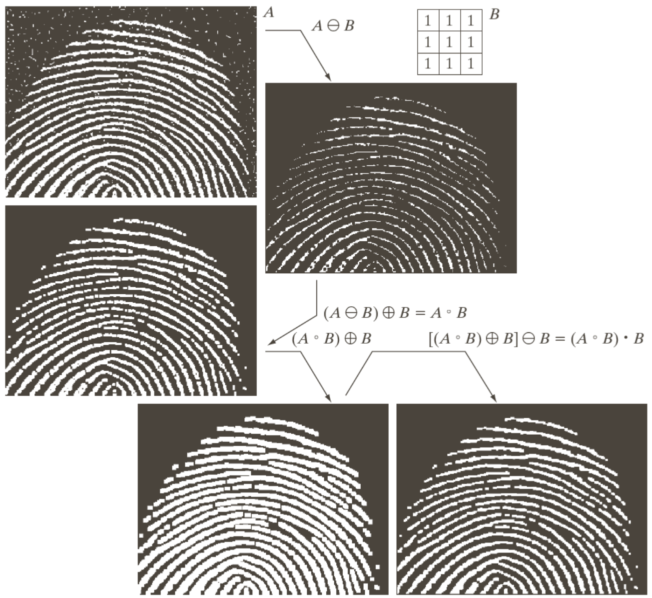
(c) $(A \circ B) \circ B = A \circ B$

- Properties of Closing

(a) A is subset (subimage) of $A \bullet B$

(b) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$

(c) $(A \bullet B) \bullet B = A \bullet B$



a b
d c
e f

FIGURE 9.11 (a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Opening of A. (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

The Hit-or-Miss Transformation

if B denotes the set composed of D and its background, the match (or set of matches) of B in A, denoted $A \circledast B$,

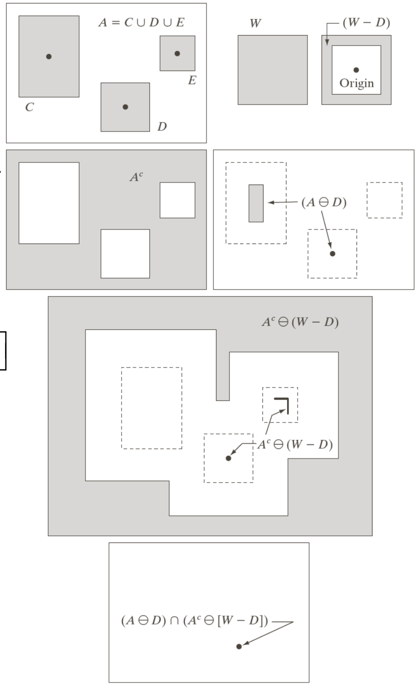
$$A \circledast B = (A \ominus D) \cap \left[A^c \ominus (W - D)\right]$$

$$B = (B_1, B_2)$$

B_1 : object

B_2 : background

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$



a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

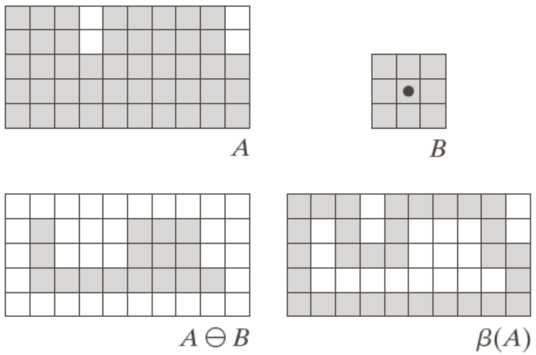
Some Basic Morphological Algorithms (1)

- **Boundary Extraction**

  The boundary of a set A, can be obtained by first eroding A by B and then performing the set difference between A and its erosion.

$$\beta(A) = A - (A \ominus B)$$

Example 1



a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Example 2

a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Some Basic Morphological Algorithms (2)

- **Hole Filling**

  A hole may be defined as a background region surrounded by a connected border of foreground pixels.

  Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e., a hole). Given a point in each hole, the objective is to fill all the holes with 1s.

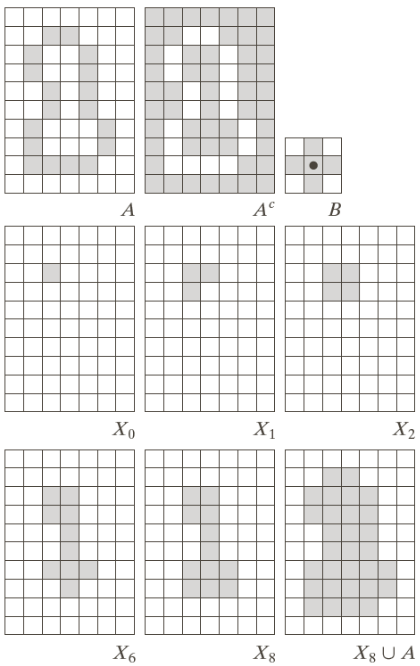
Some Basic Morphological Algorithms (2)

- **Hole Filling**

  1. Forming an array X_0 of 0s (the same size as the array containing A), except the locations in X_0 corresponding to the given point in each hole, which we set to 1.

  2. $X_k = (X_{k-1} \oplus B) \cap A^c \quad k=1,2,3,\ldots$

  Stop the iteration if $X_k = X_{k-1}$



a b c
d e f
g h i

FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

Some Basic Morphological Algorithms (3)

- **Extraction of Connected Components**

  Central to many automated image analysis applications.

  Let A be a set containing one or more connected components, and form an array X_0 (of the same size as the array containing A) whose elements are 0s, except at each location known to correspond to a point in each connected component in A, which is set to 1.

Some Basic Morphological Algorithms (3)

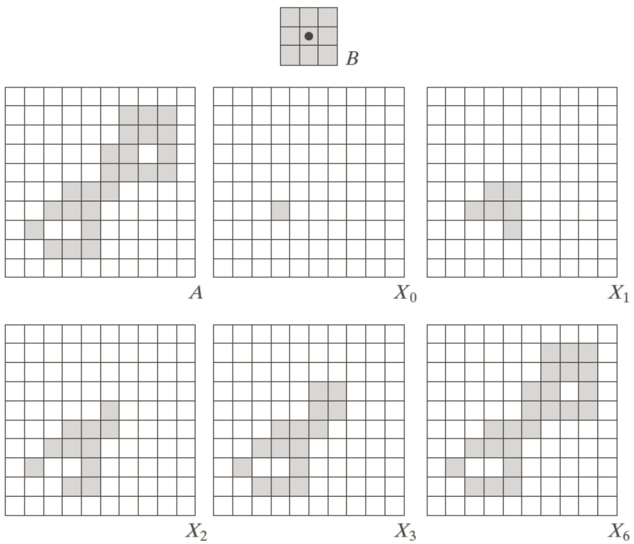
- **Extraction of Connected Components**

  Central to many automated image analysis applications.

$$X_k = (X_{k-1} \oplus B) \cap A$$

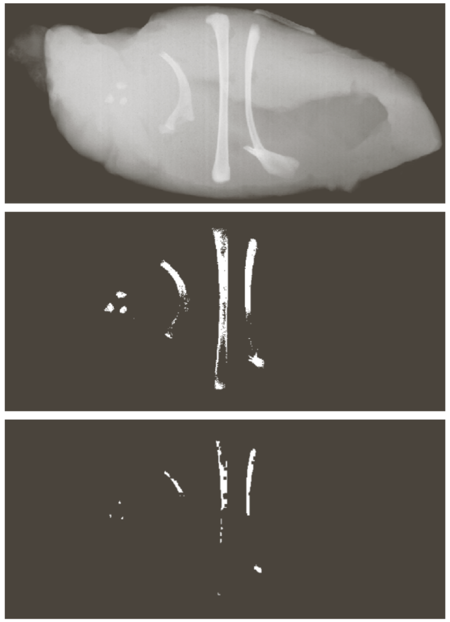
B : structuring element

$$\text{until } X_k = X_{k\text{-}1}$$



a
b c d
e f g

FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)