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DeepLearning.AI

Math for Machine Learning

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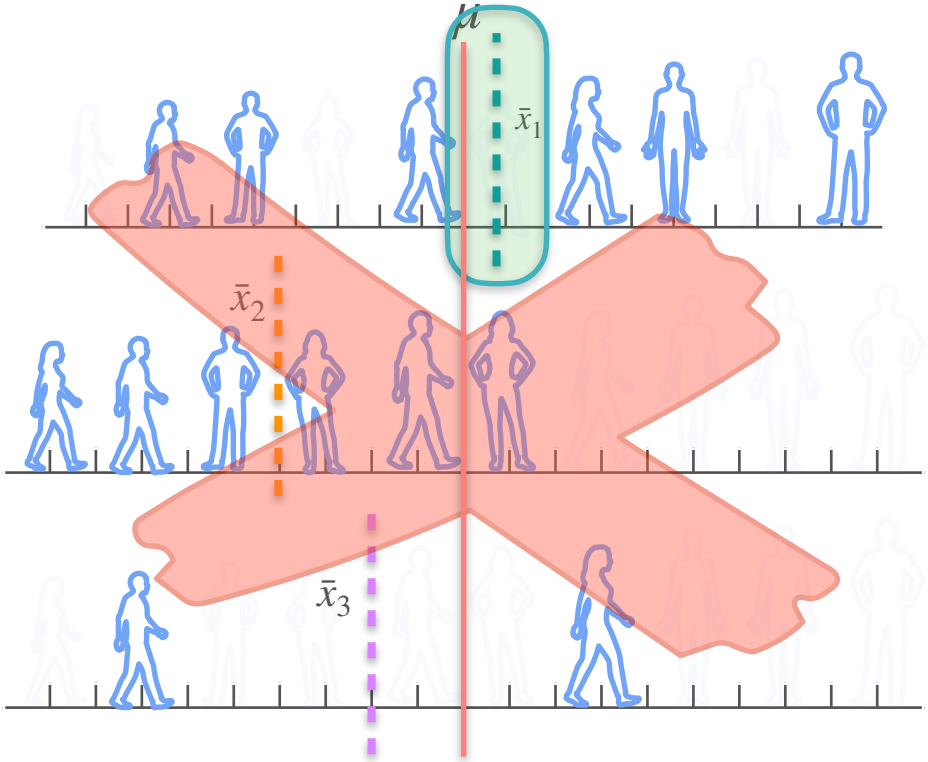
Probability and Statistics - Week 4

W4 Lesson 1

DeepLearning.AI

Confidence Interval (Known Standard Deviation)

Confidence Interval - Intuition



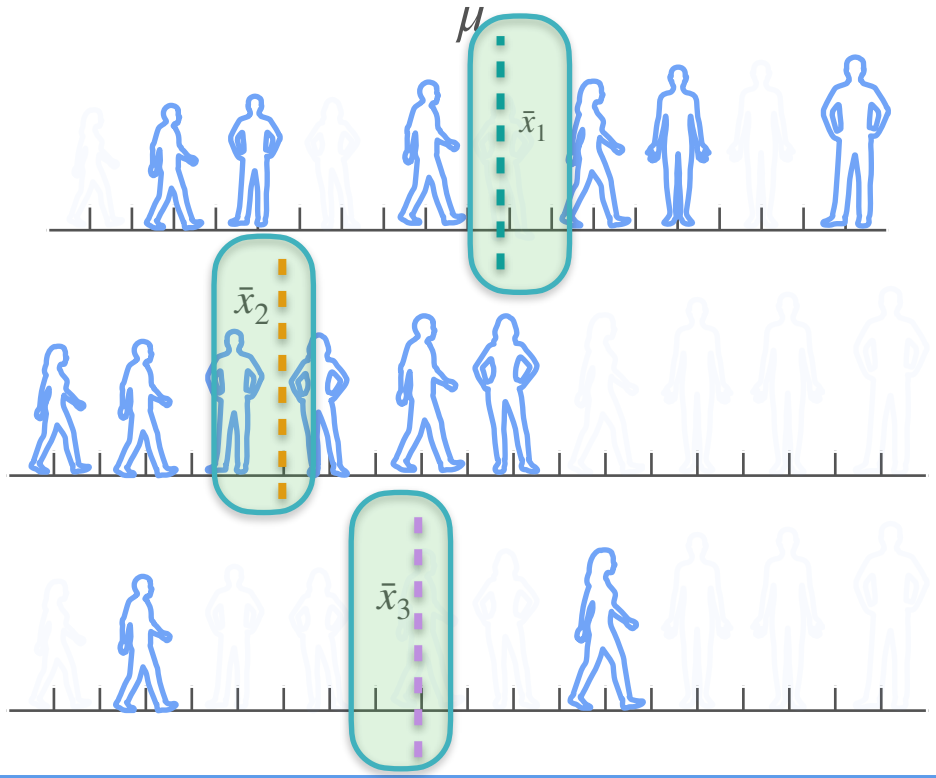
Statistopia

10,000 people

μ
(mean height of the population)

Can you use these sample means with some degree of certainty?

Confidence Interval - Intuition



Can you use these sample means with some degree of certainty?

Confidence Interval

lower limit $< \mu <$ upper limit

Confidence Interval - Intuition

$n = 1$

$\bar{x}$

Confidence level

%

$1 - \alpha$

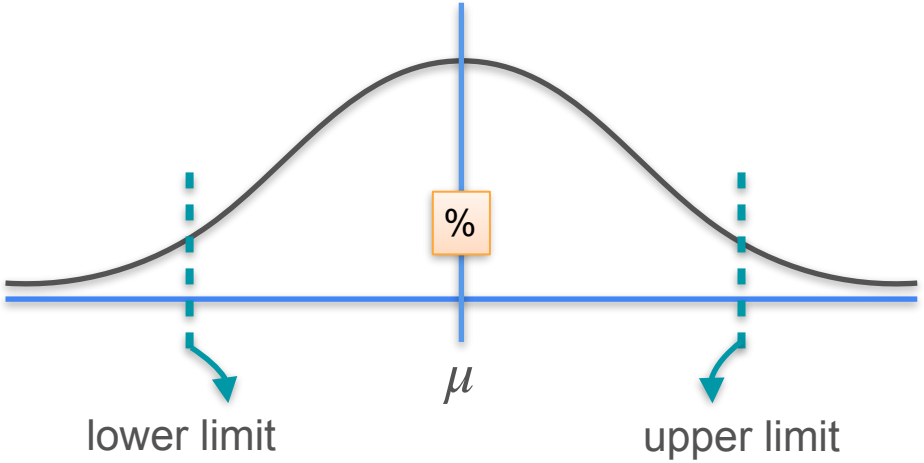
α

significance level

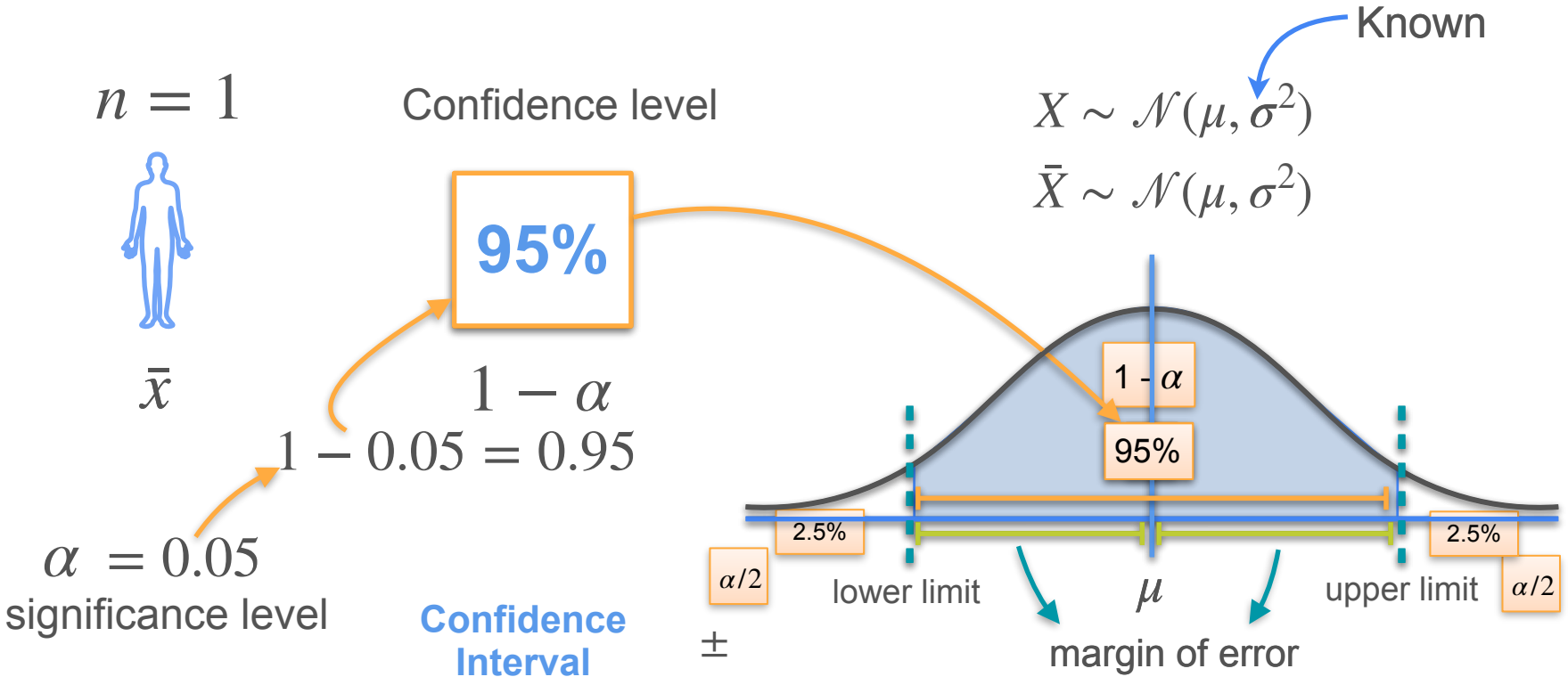
Known

$X \sim \mathcal{N}(\mu, \sigma^2)$

$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$



Confidence Interval - Intuition



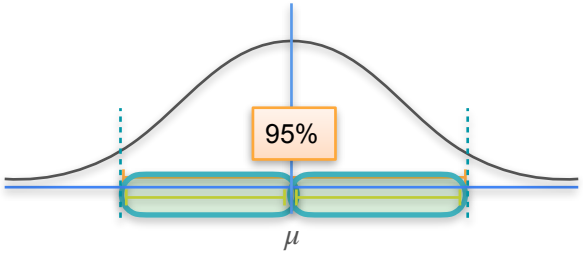
Confidence Interval - Intuition

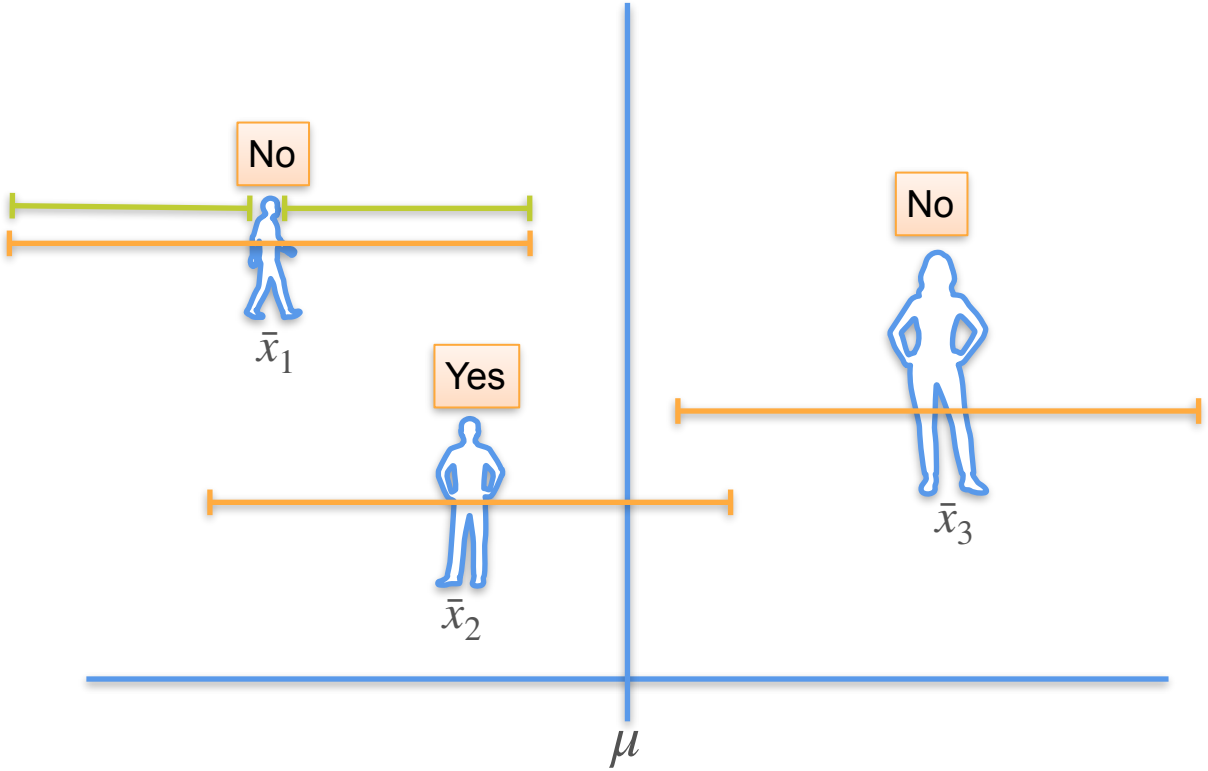
$n = 1$

Known σ

95%

Margin of error





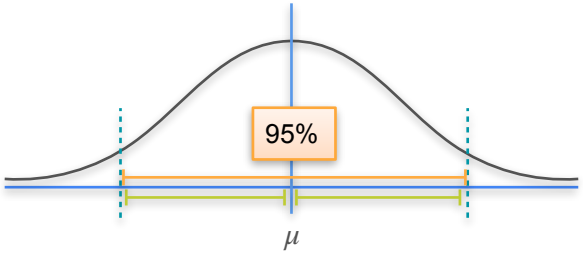
Confidence Interval - Intuition

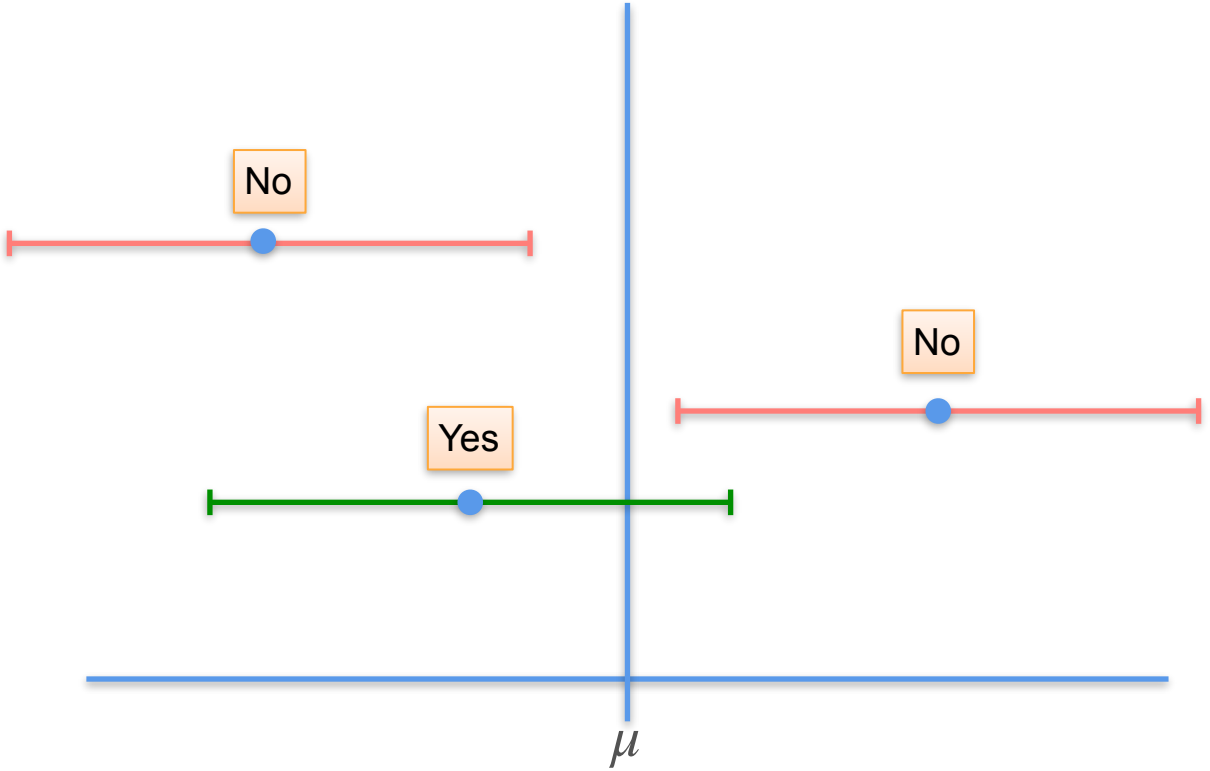
$n = 1$

Known σ

95%

Margin of error





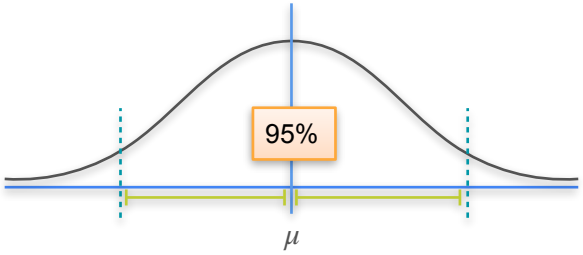
Confidence Interval - Intuition

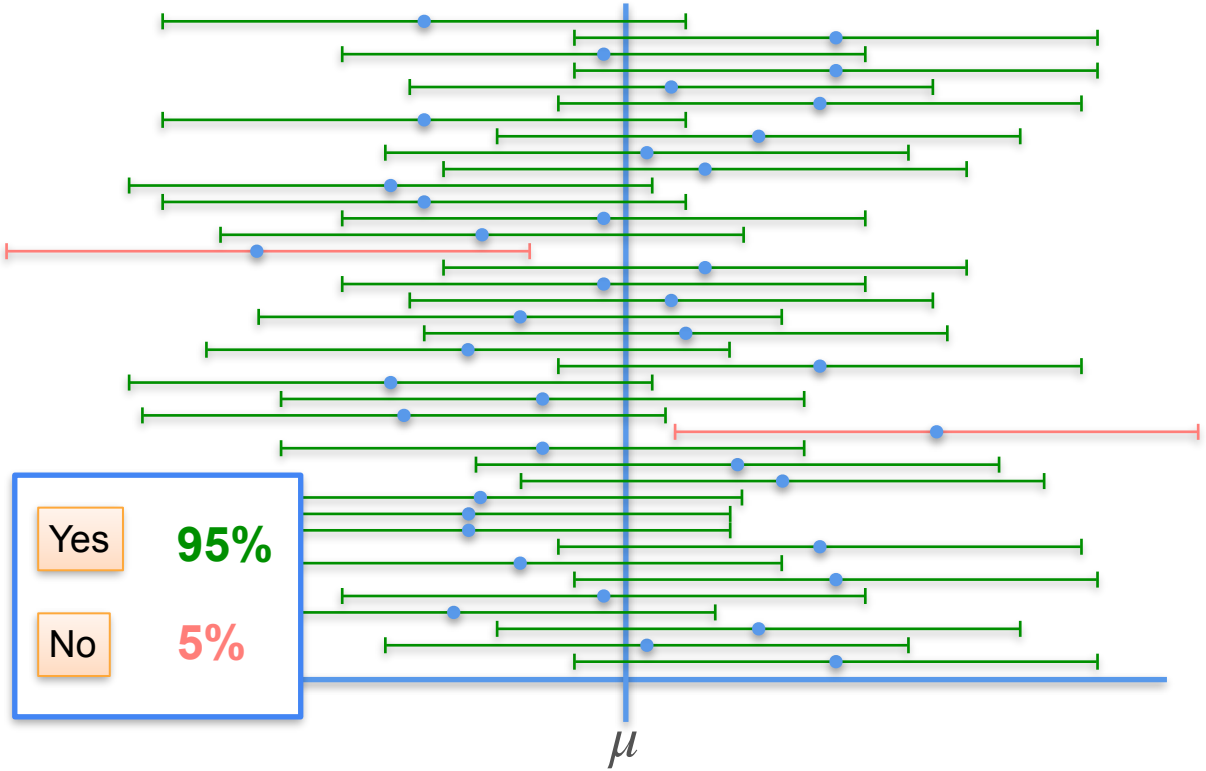
$n = 1$

Known σ

95%

Margin of error





Confidence Interval - Intuition

$n = 1$

$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$

$\bar{x}$

mean of the sample means

$\mu_{\bar{x}} = \mu$

population mean

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{1}} = \sigma$

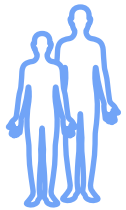
standard deviation of the sample means

Confidence Interval - Intuition

$n = 2$

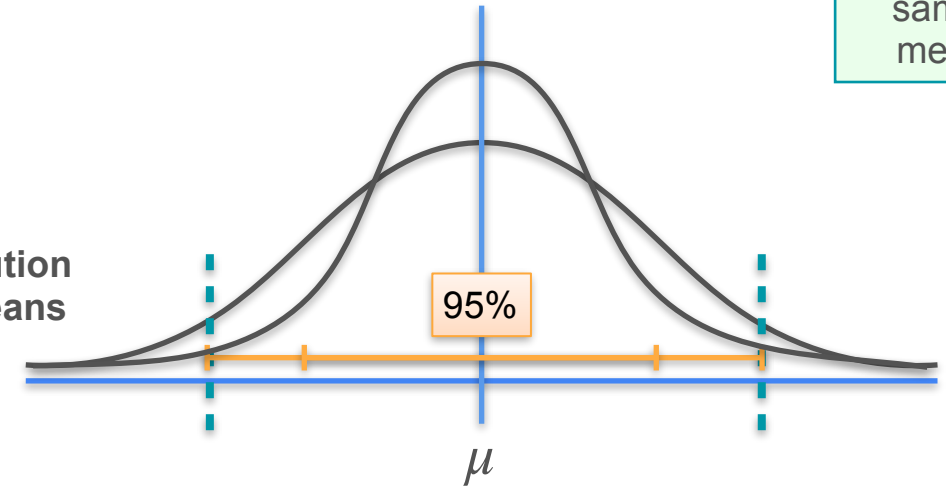
$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$

Known population standard deviation (σ)



$\bar{x}$

sampling distribution of the sample means



mean of the sample means

$\mu_{\bar{x}} = \mu$

population mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

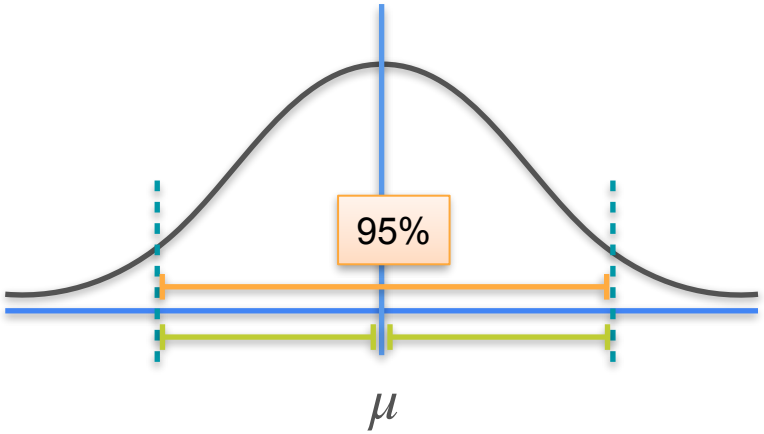
standard deviation of the sample means

Confidence Interval - Intuition

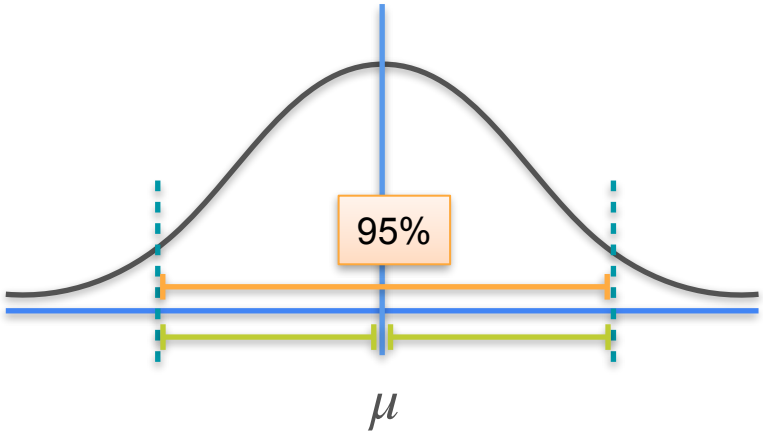
95%

$n = 1$

$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$

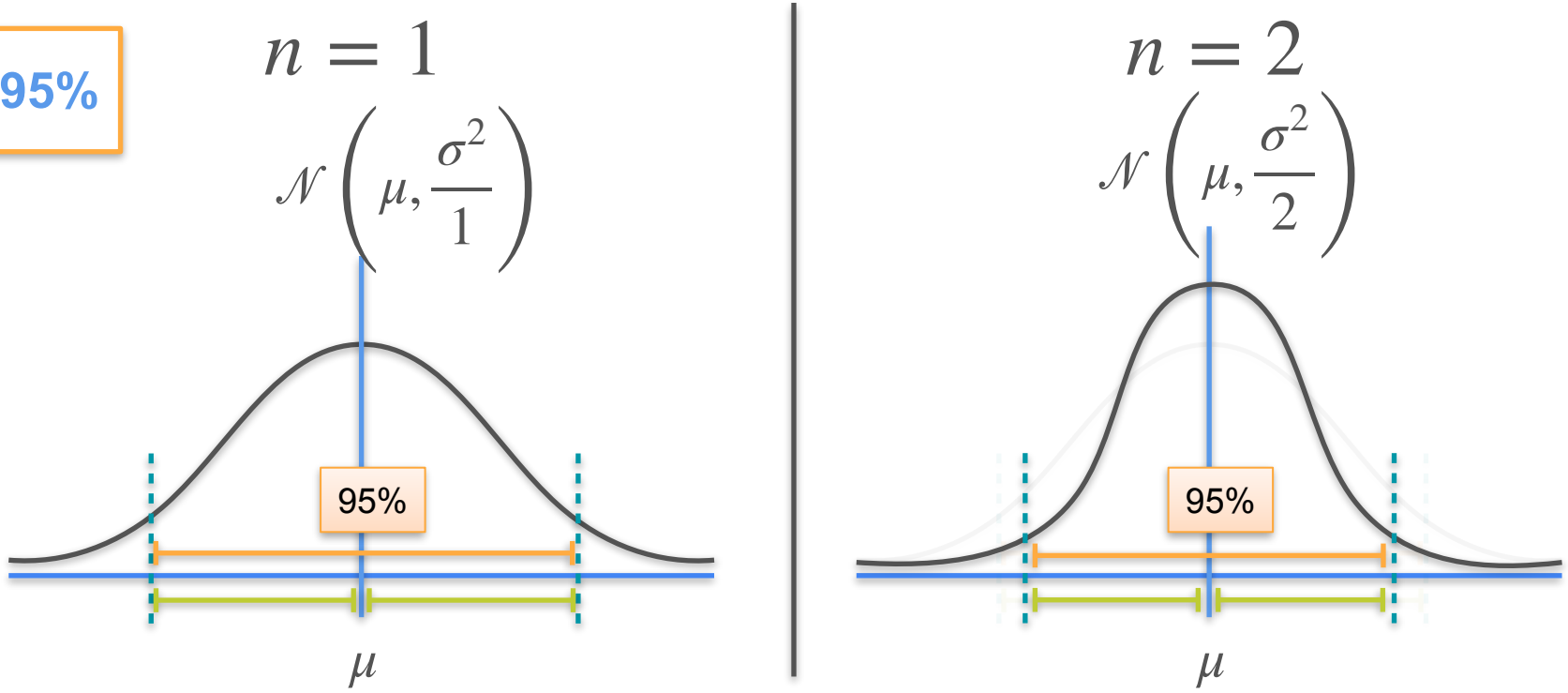


$n = 2$



Confidence Interval - Intuition

95%

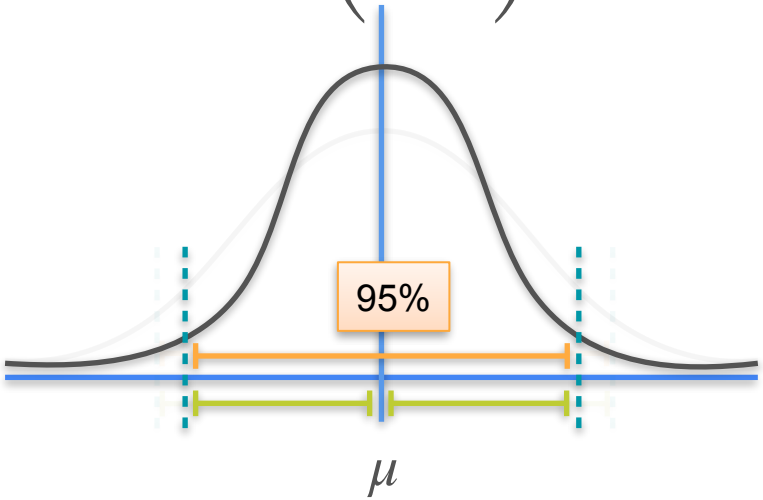


Confidence Interval - Intuition

95%

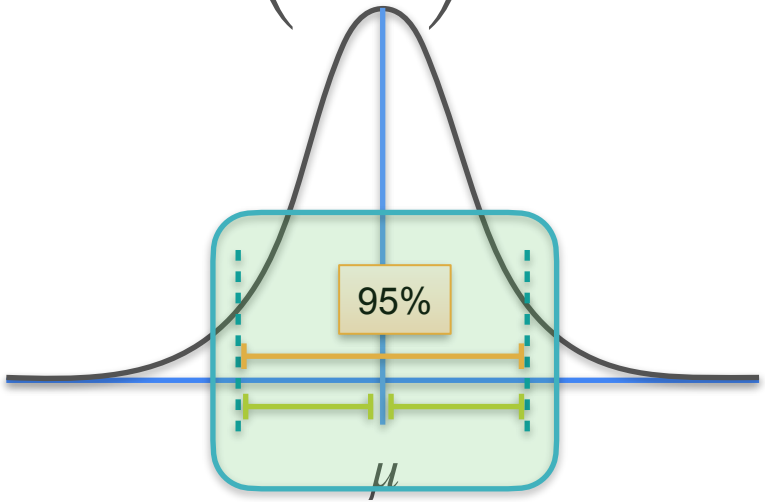
$n = 2$

$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$

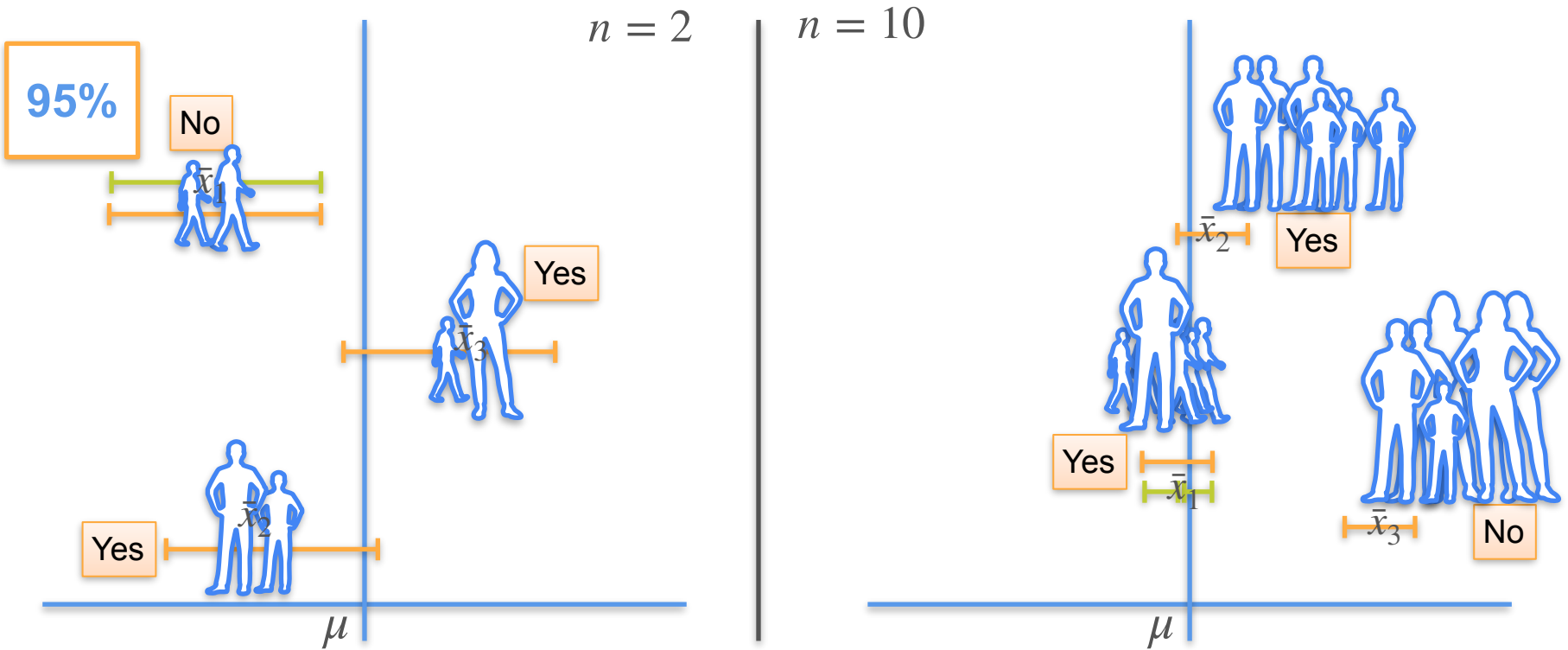


$n = 10$

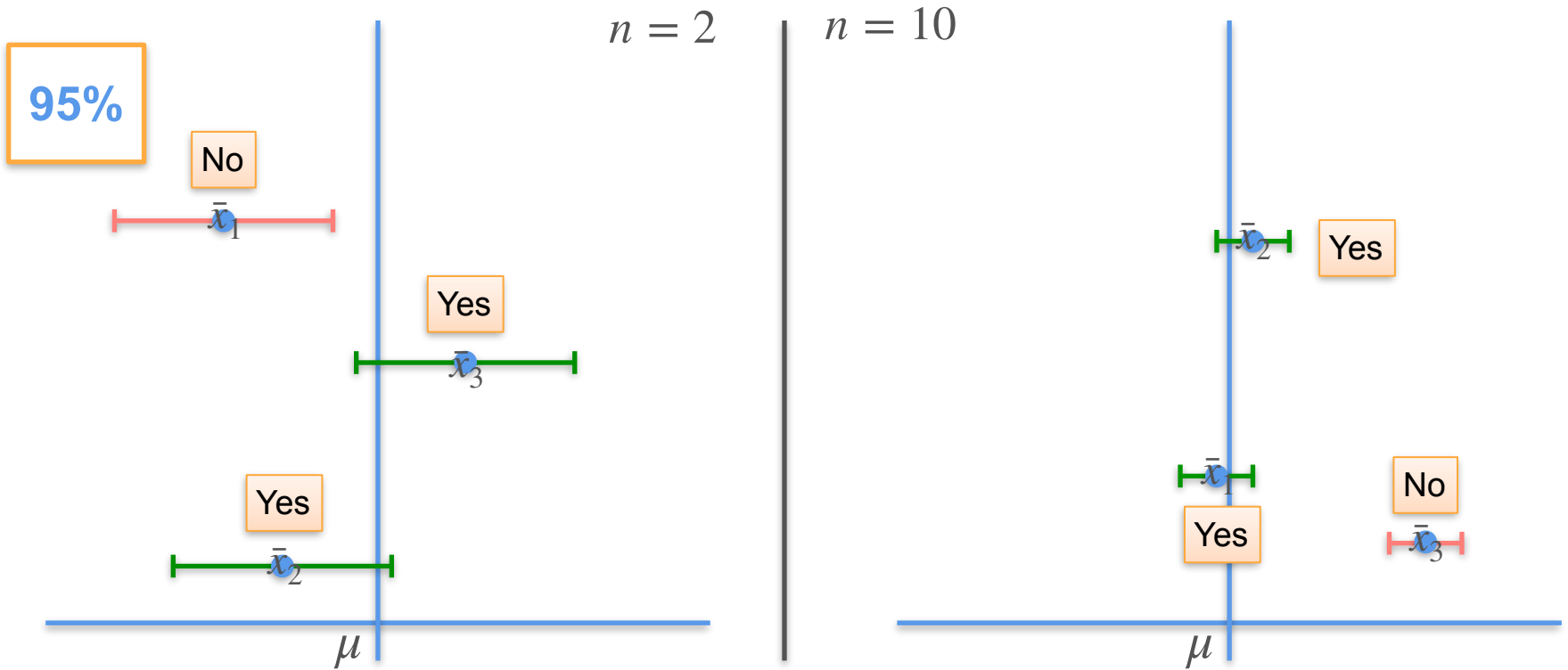
$\mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$



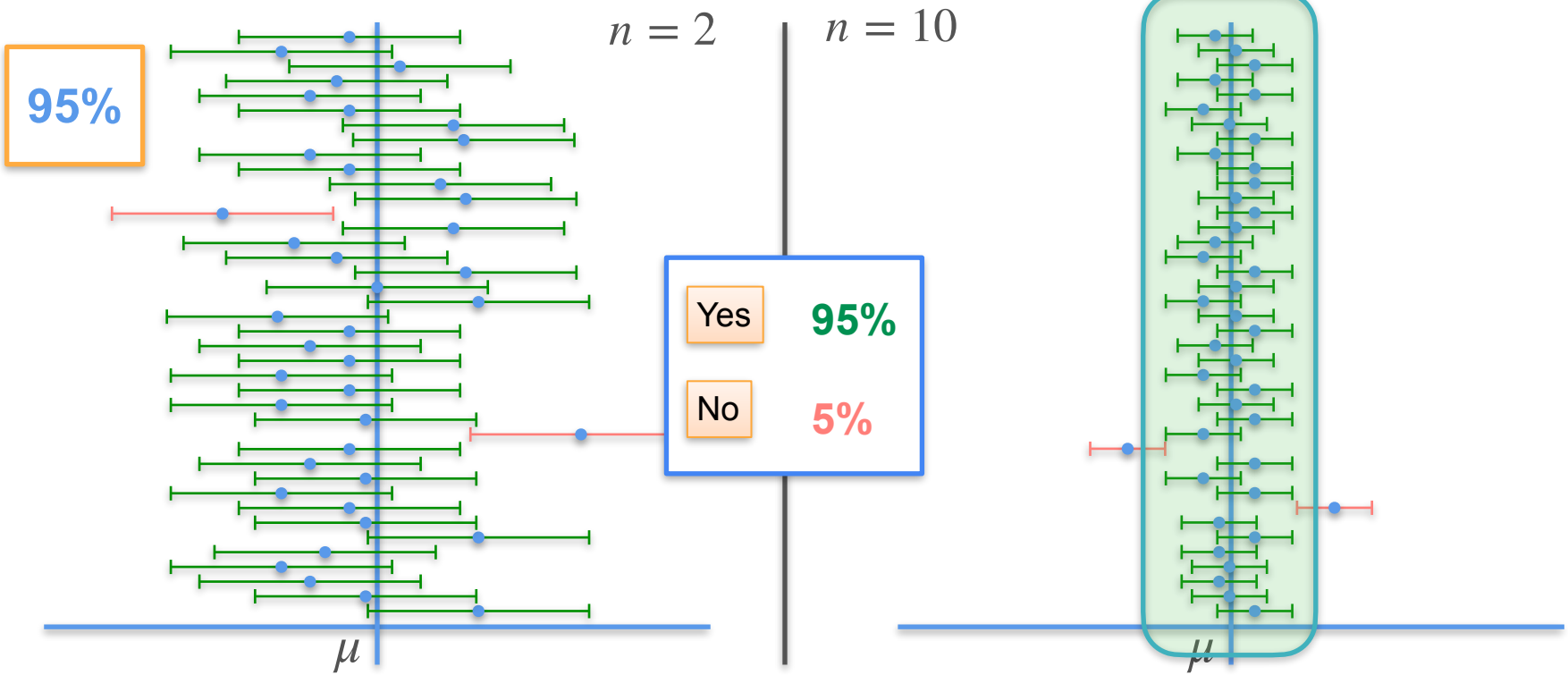
Confidence Interval - Intuition



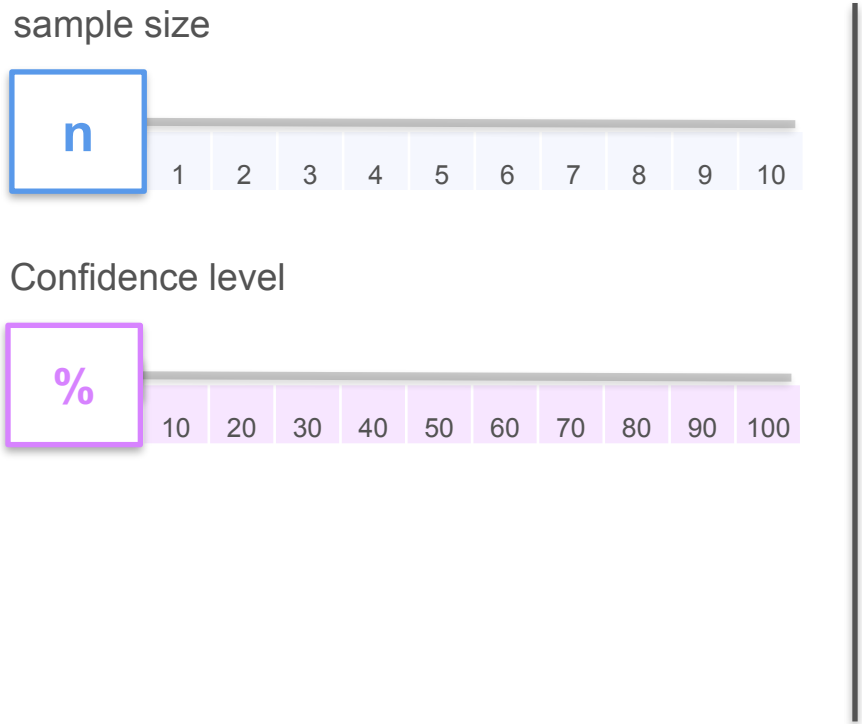
Confidence Interval - Intuition



Confidence Interval - Intuition

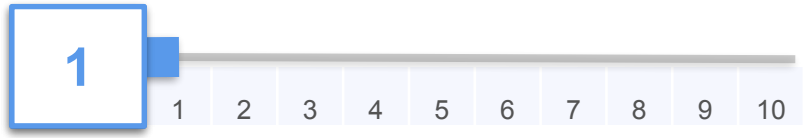


Effect of the Sample Size

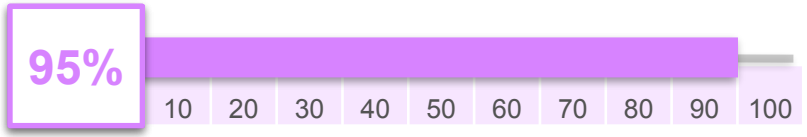


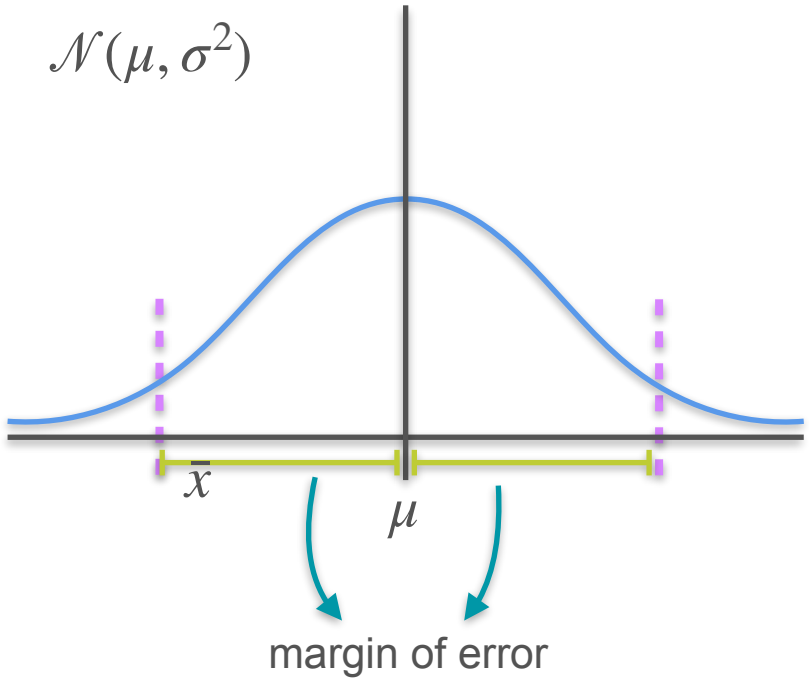
Effect of the Sample Size

sample size



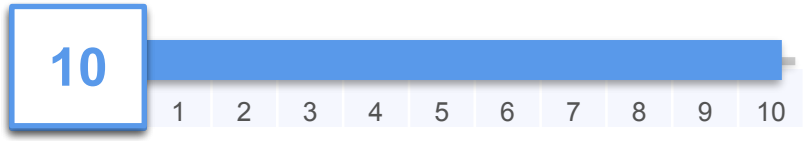
Confidence level



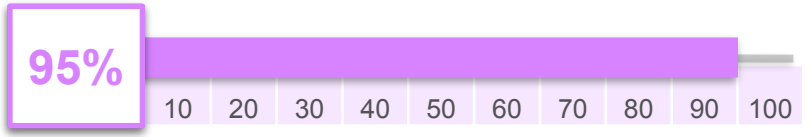


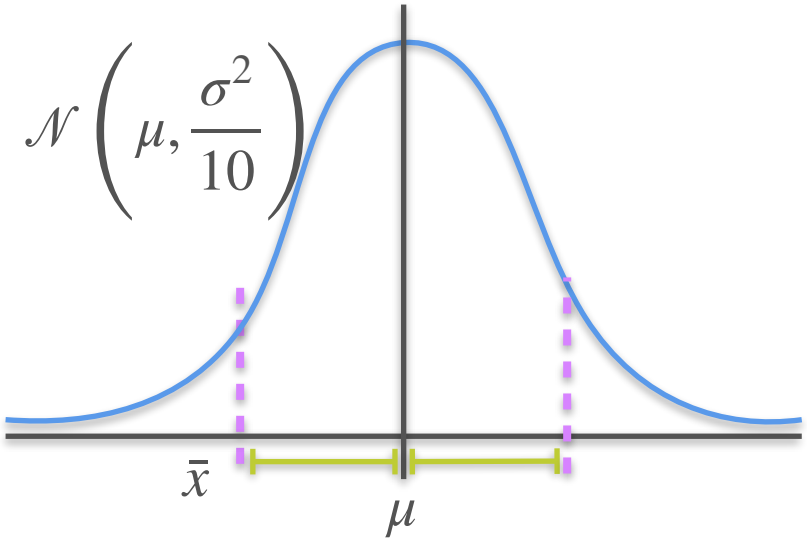
Effect of the Sample Size

sample size

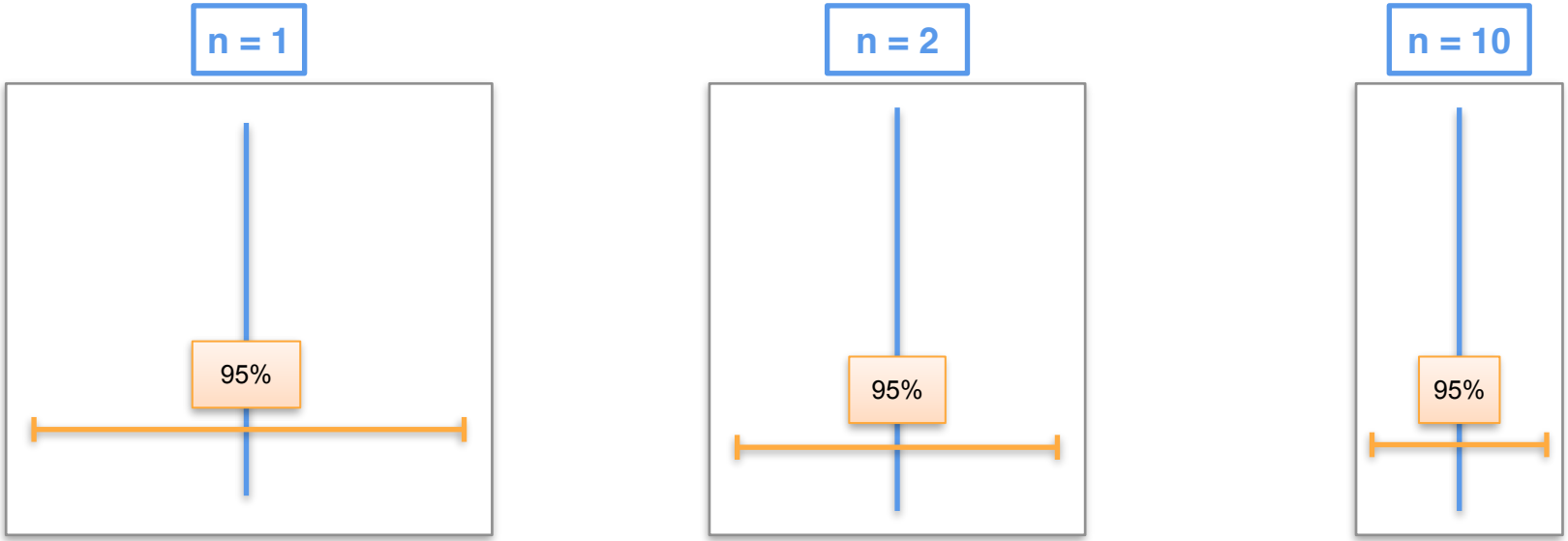


Confidence level





Effect of the Sample Size



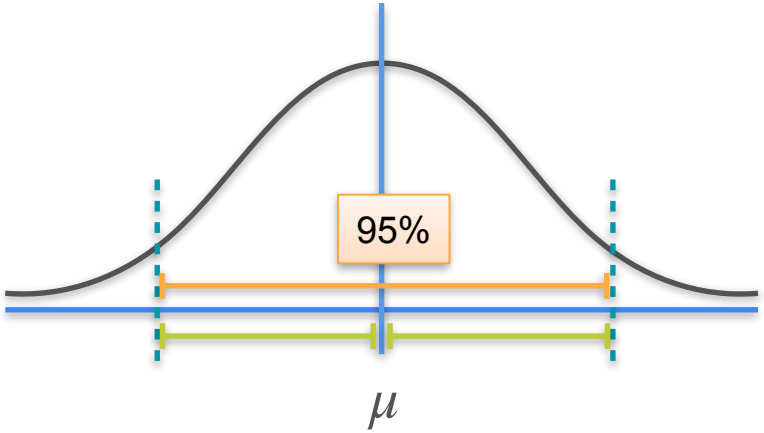
As n increases, the confidence interval shrinks

Effect of the Confidence Level

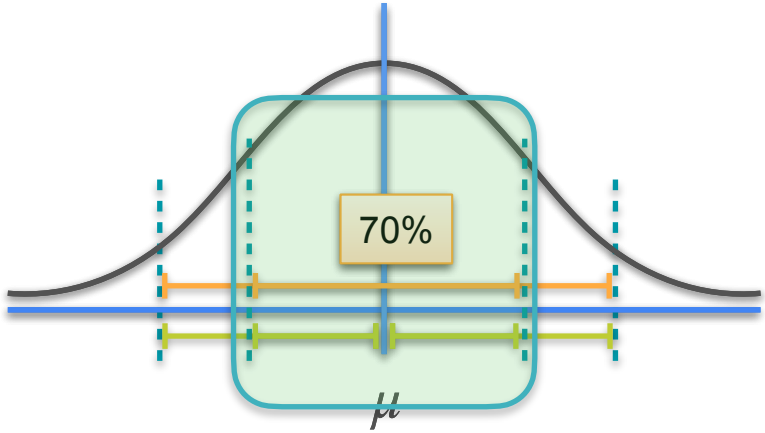
$n = 1$

$\mathcal{N}(\mu, \sigma^2)$

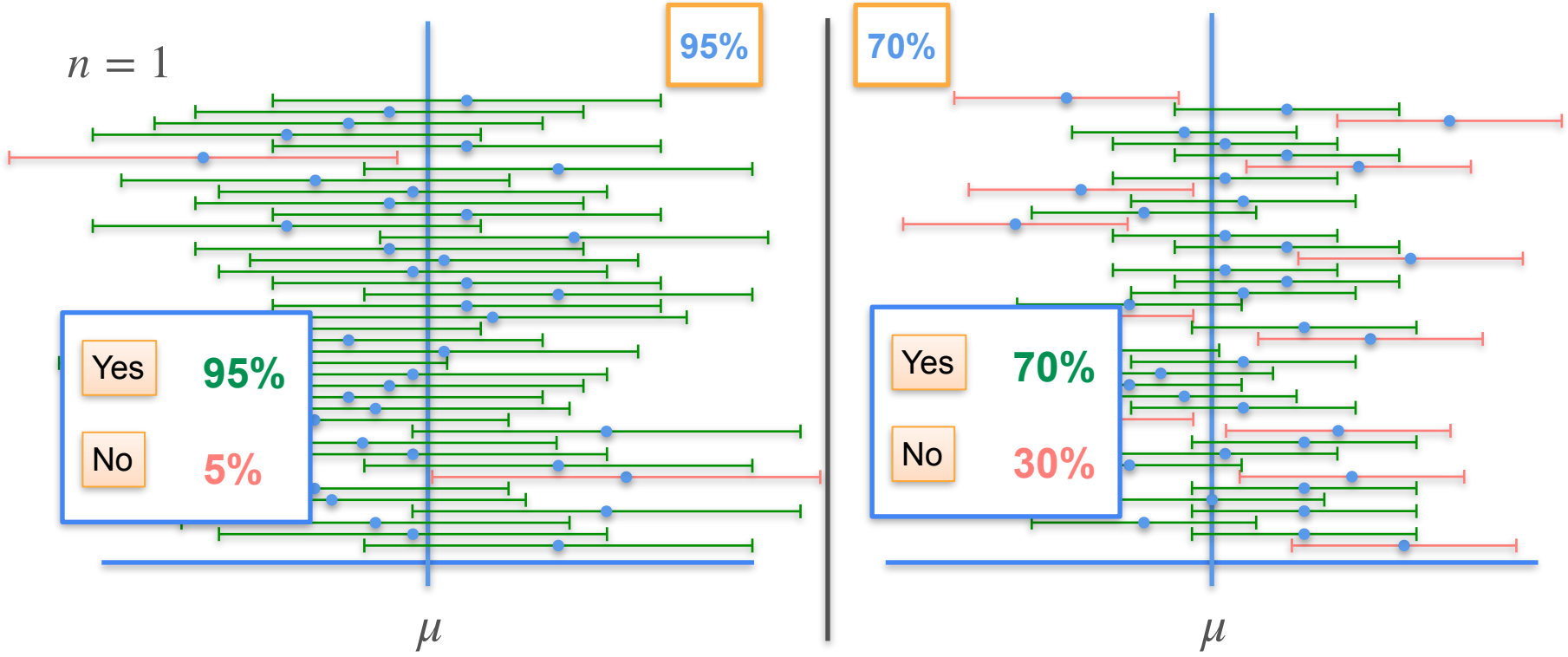
95%



70%



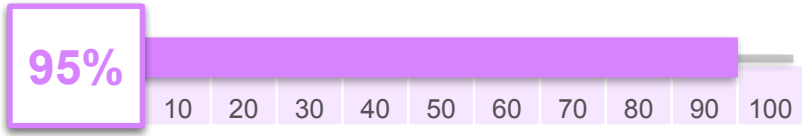
Effect of the Confidence Level

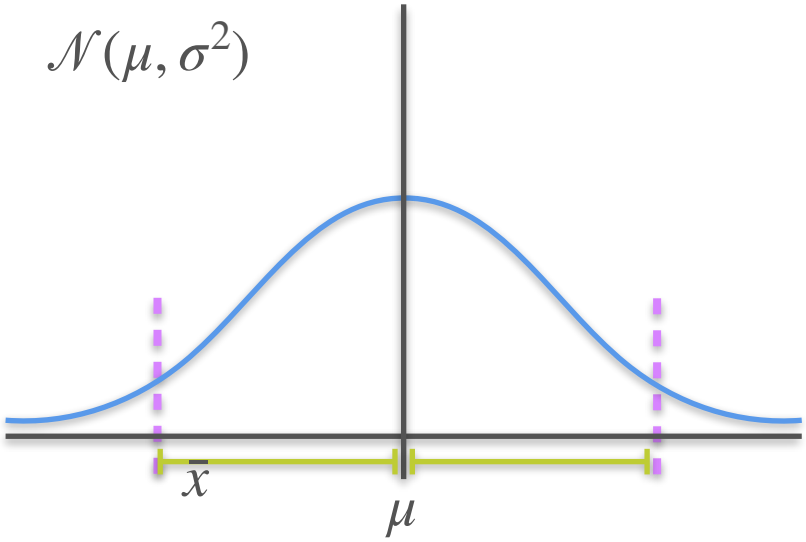


Effect of the Confidence Level

sample size

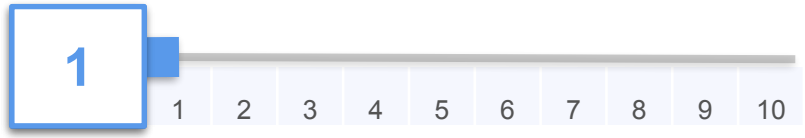
Confidence level



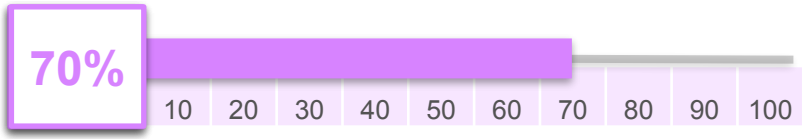


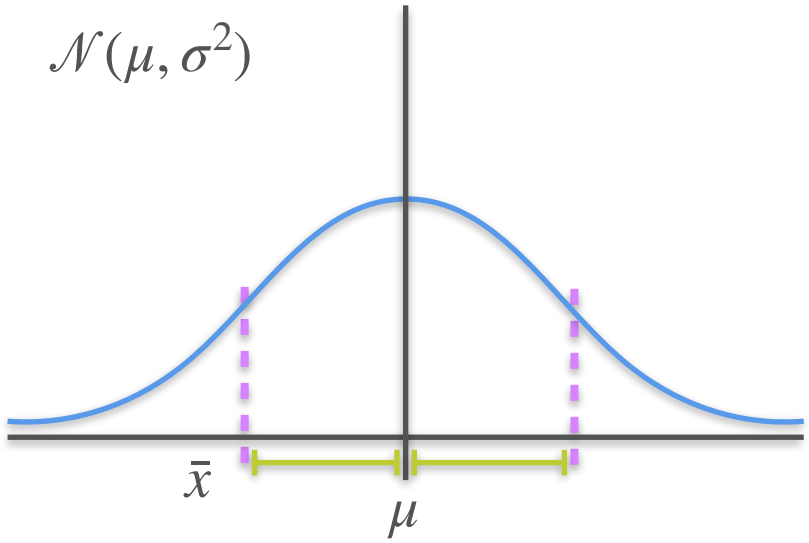
Effect of the Confidence Level

sample size



Confidence level

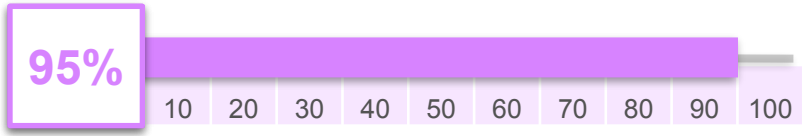


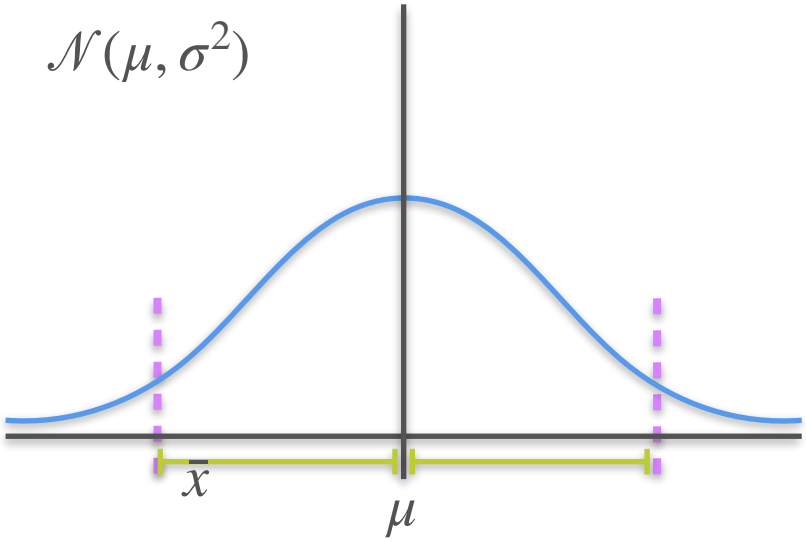


Effect of the Confidence Level

sample size

Confidence level





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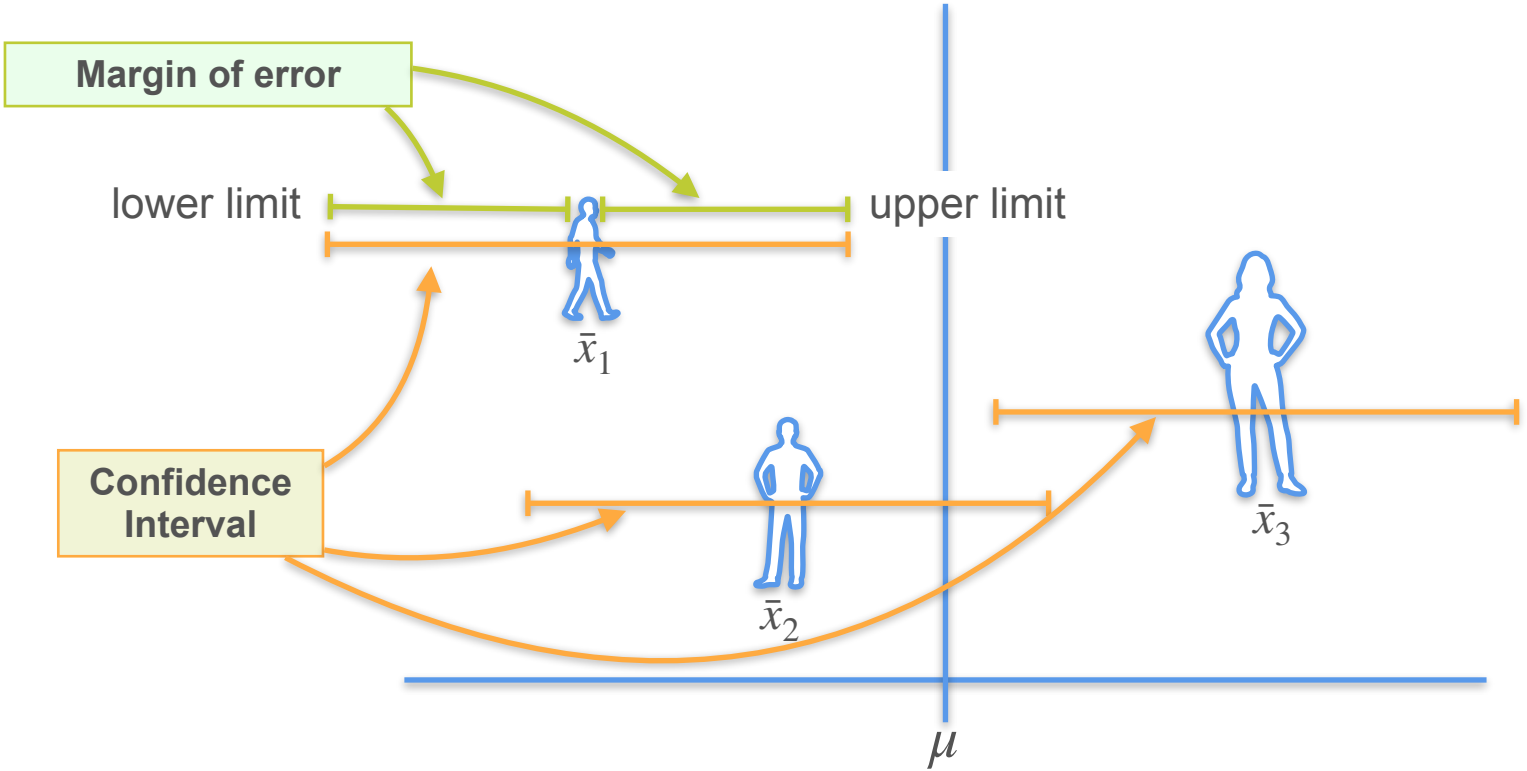
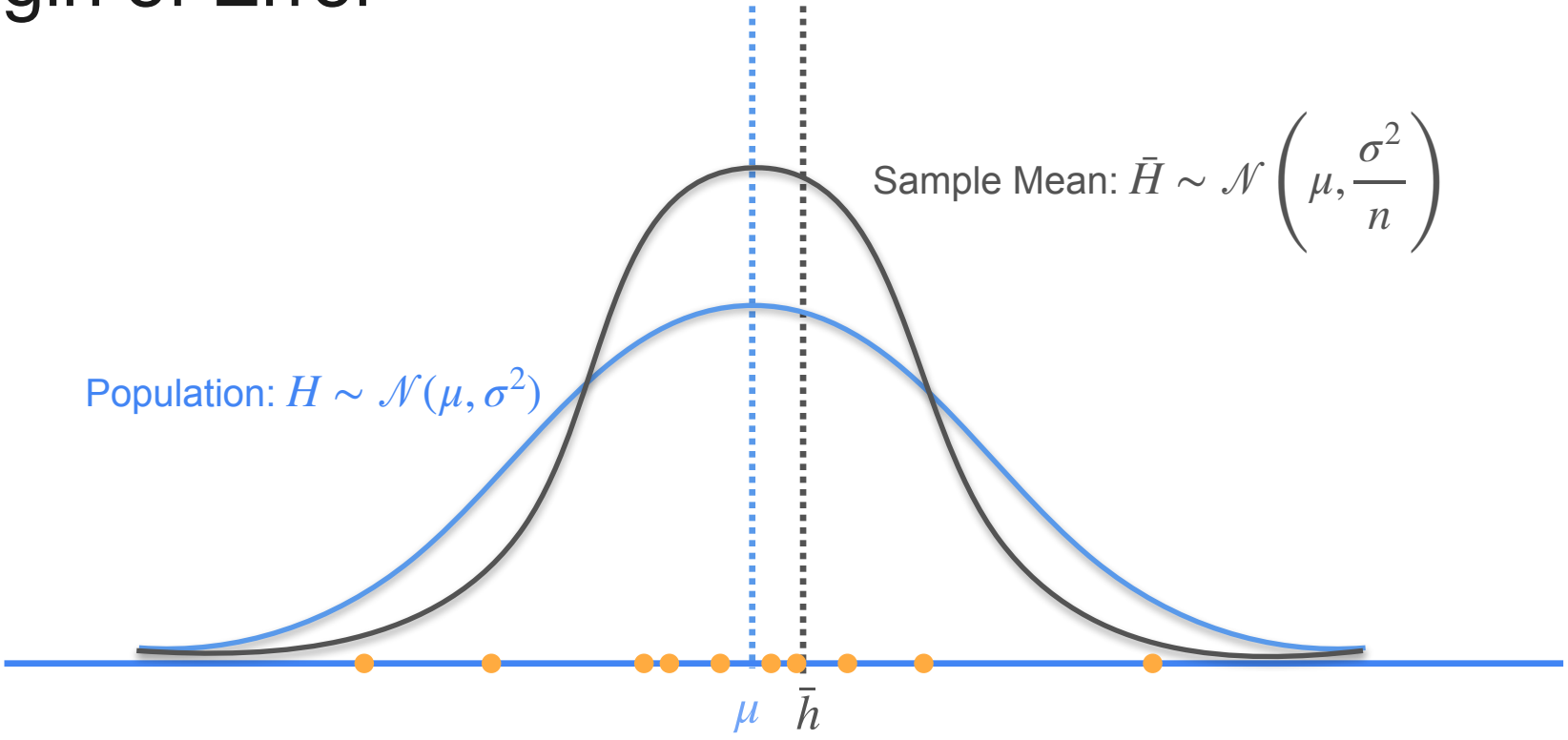
Confidence Interval

Margin of Error

Margin of Error - Introduction

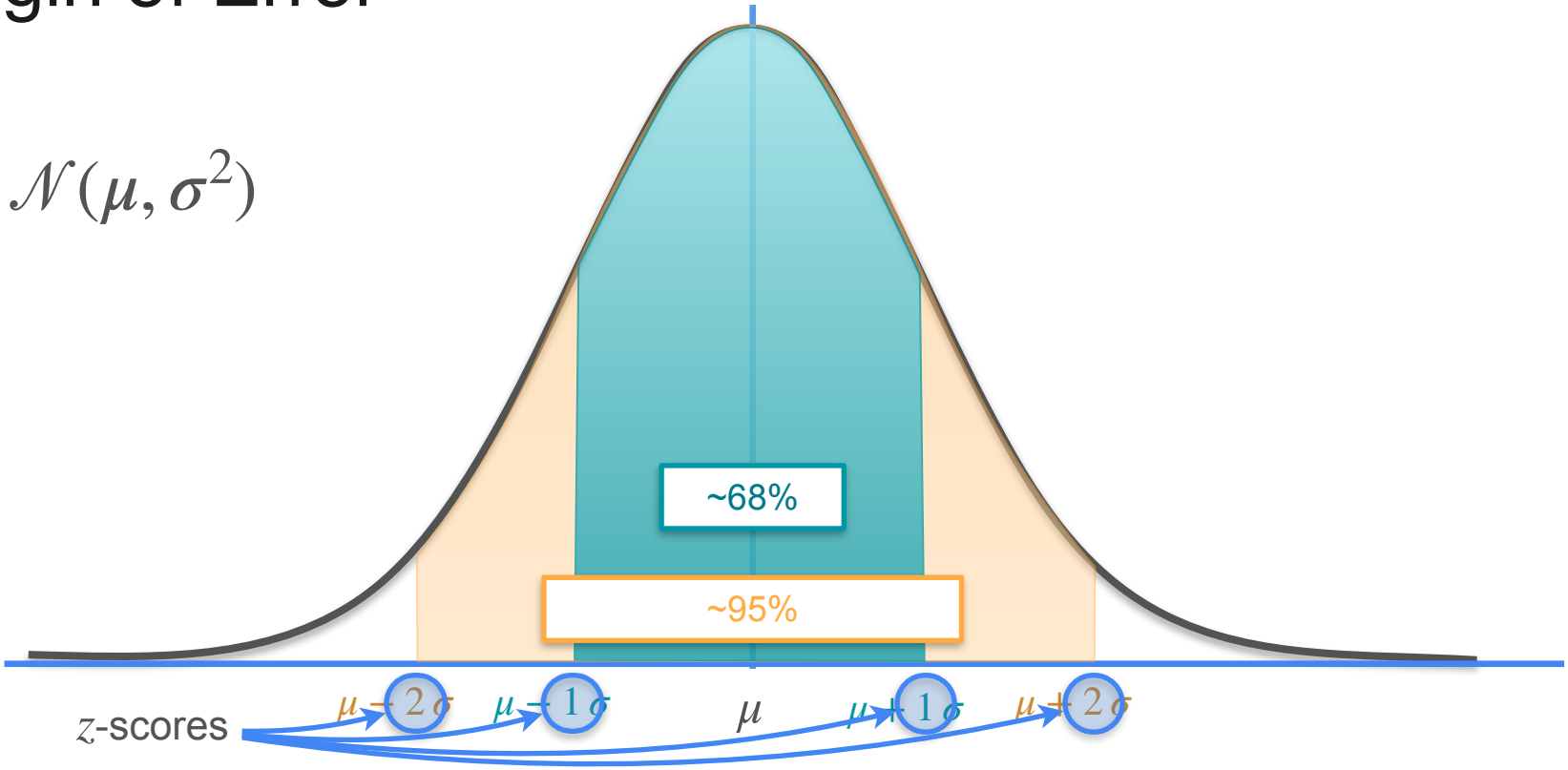


Margin of Error



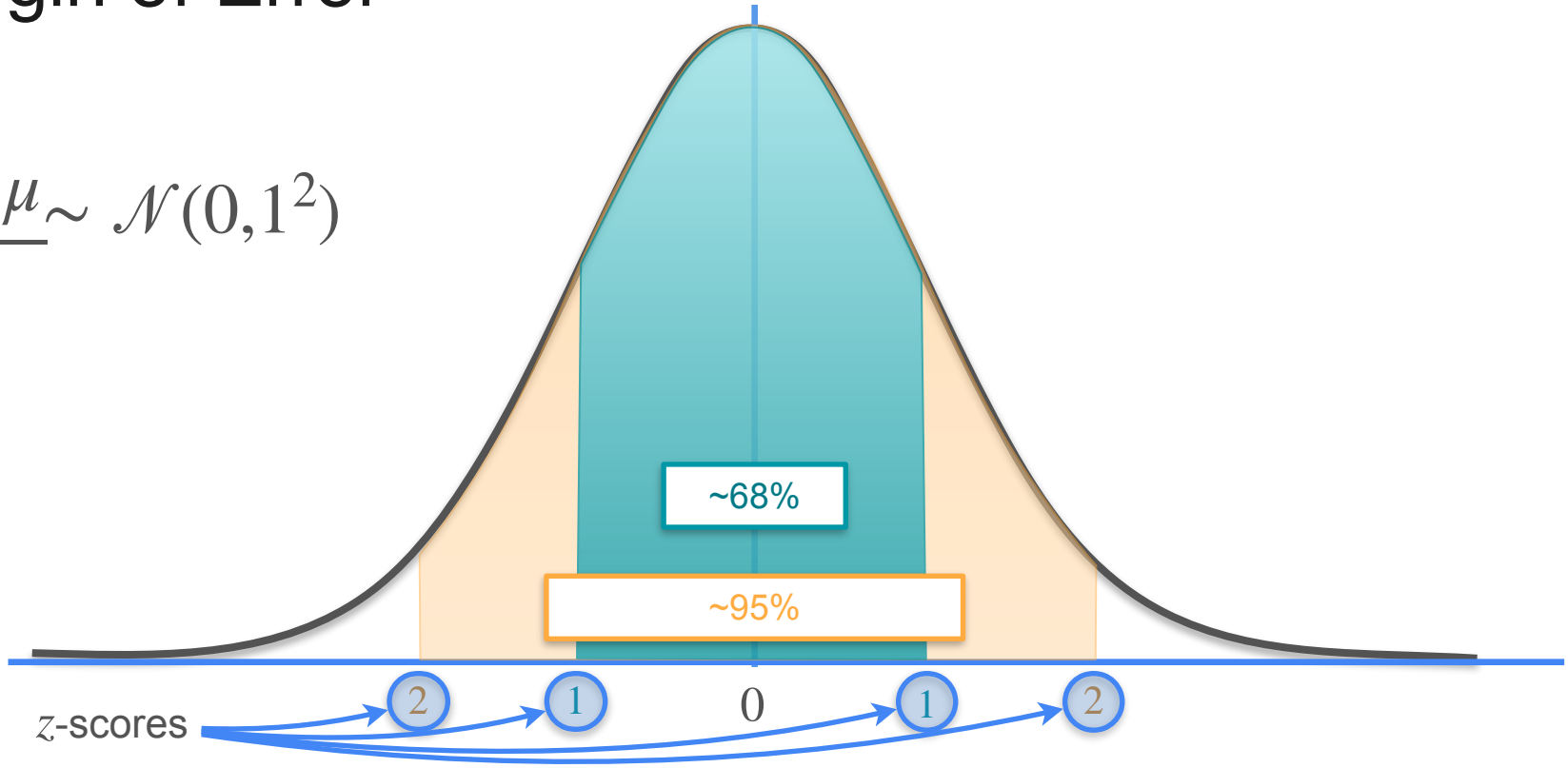
Margin of Error

$$X \sim \mathcal{N}(\mu, \sigma^2)$$



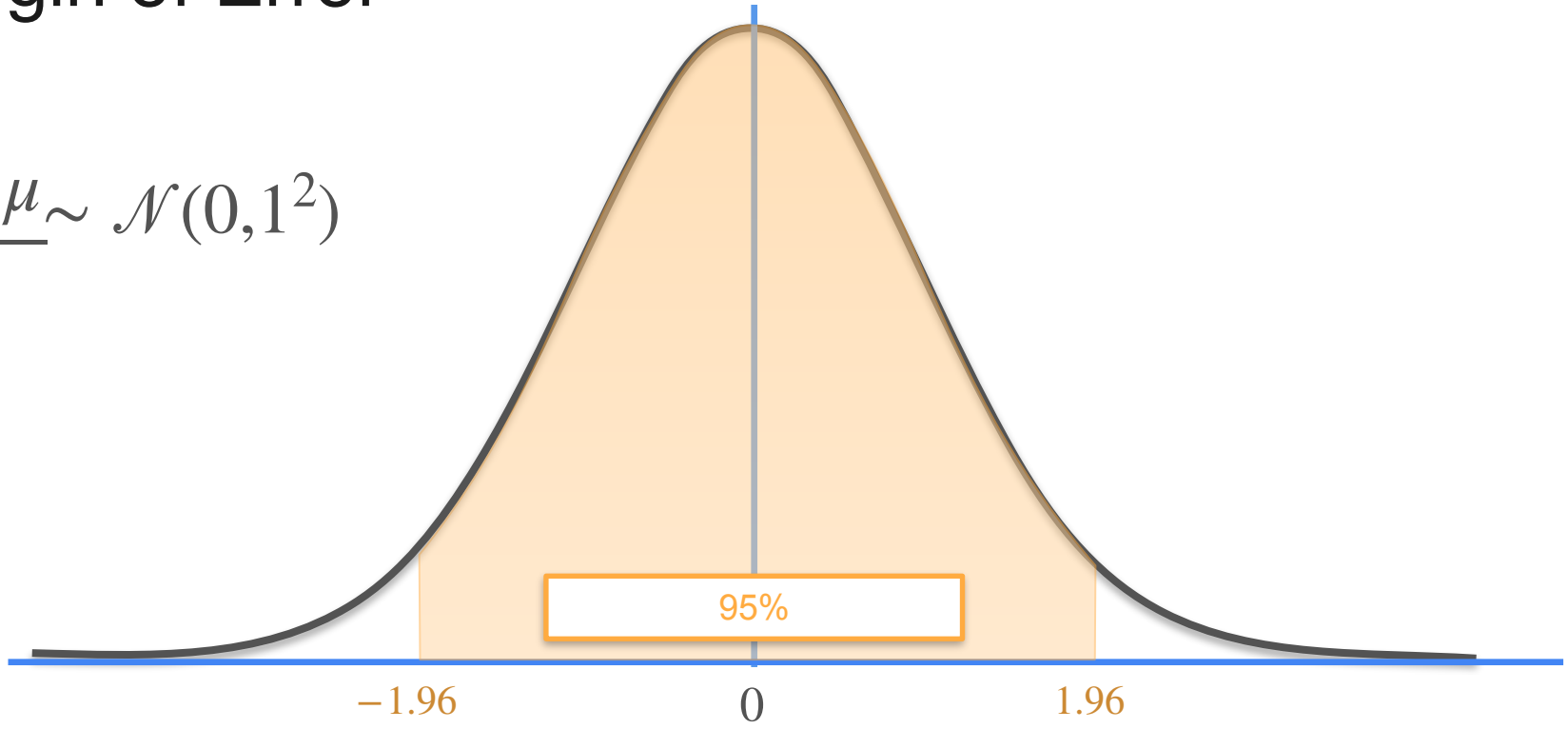
Margin of Error

$$\frac{X-\mu}{\sigma} \sim \mathcal{N}(0,1^2)$$



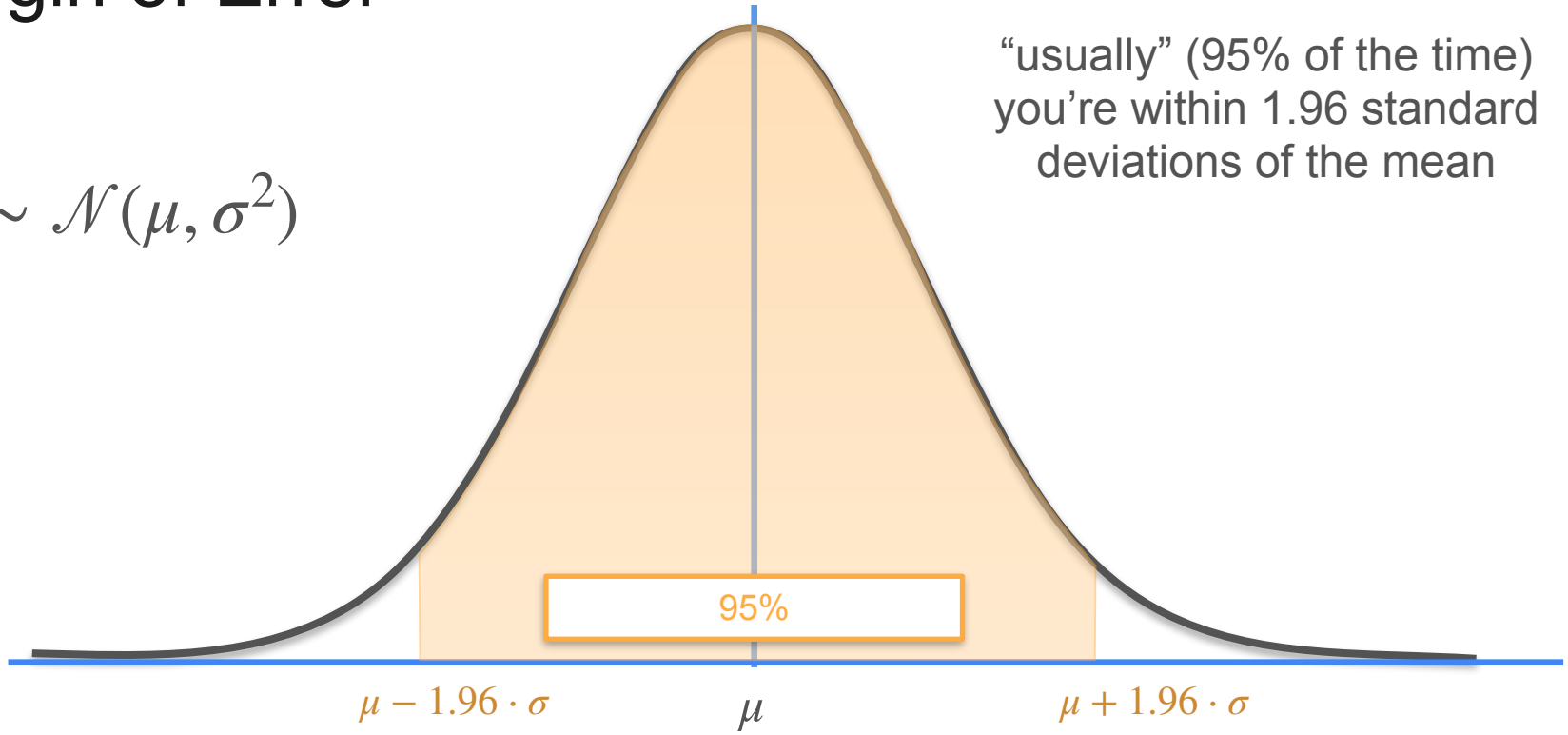
Margin of Error

$$\frac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1^2)$$



Margin of Error

$$X \sim \mathcal{N}(\mu, \sigma^2)$$



"usually" (95% of the time) you're within 1.96 standard deviations of the mean

Margin of Error

$$\frac{X - \mu}{\sigma} \sim \mathcal{N}(0,1^2)$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

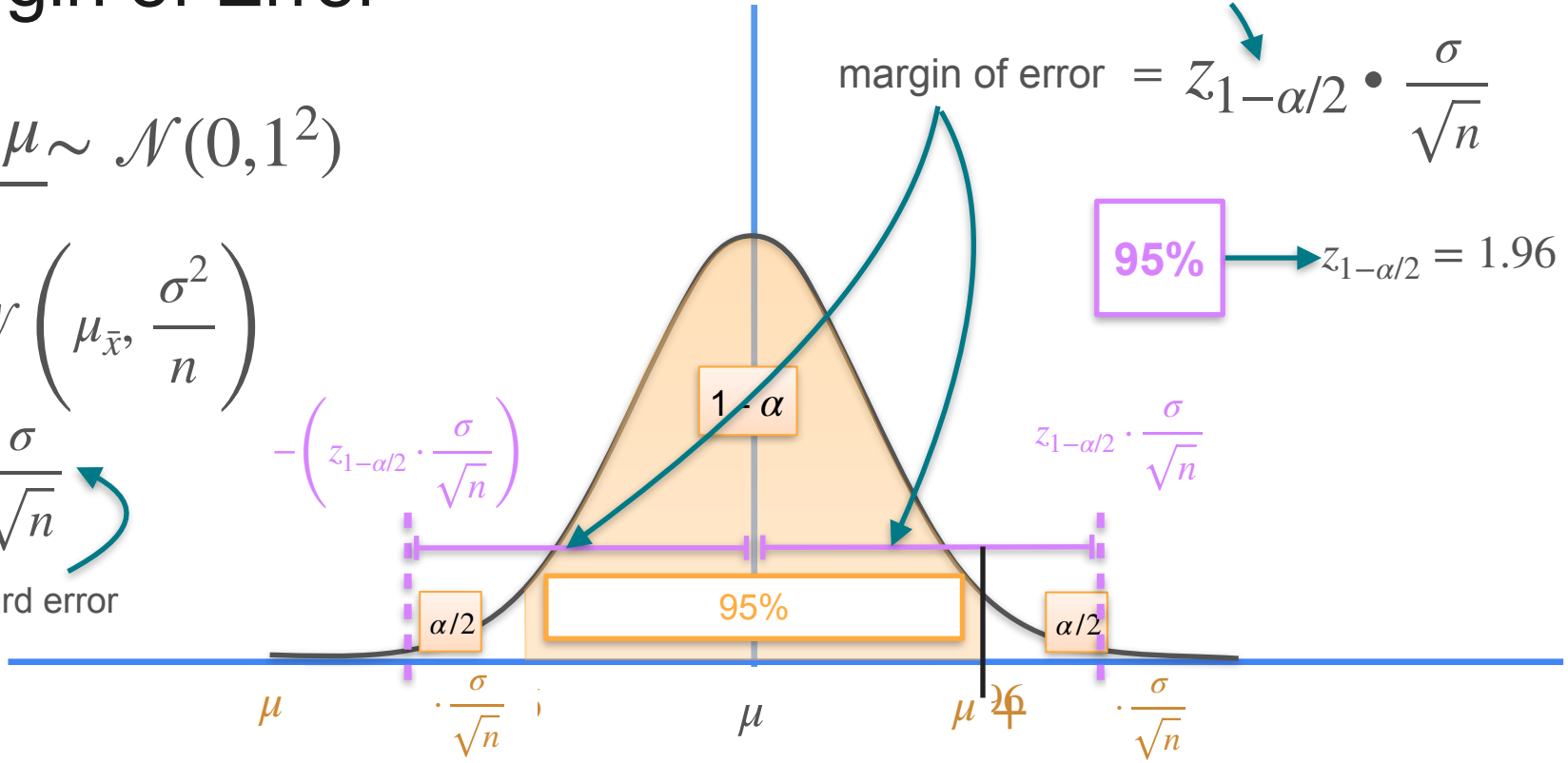
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error

Critical Value

$$\text{margin of error} = z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

95% → $z_{1-\alpha/2} = 1.96$



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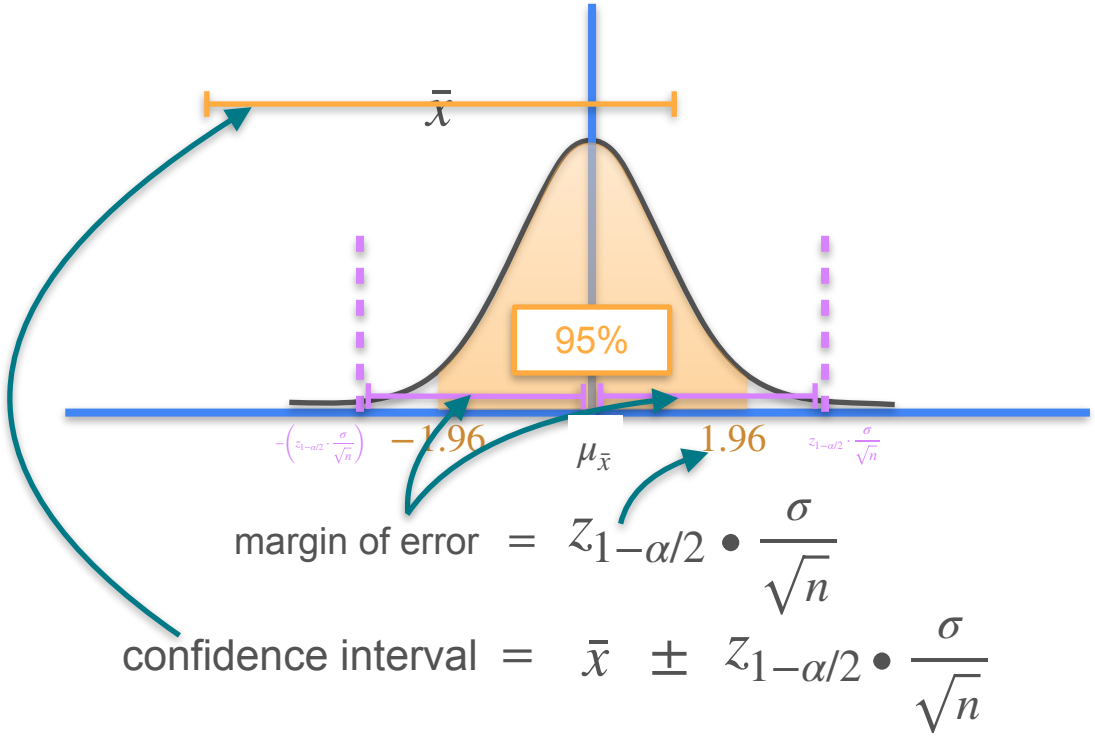
Confidence Interval

Confidence Interval - Calculation Steps

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
- Get the critical value ($z_{1-\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean



Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
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- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Assumptions

- Simple random sample
- Sample size > 30 or population is approximately normal

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Confidence Interval - Example

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49

$\bar{x} = 170cm$

$\sigma = 25cm$

95% → $z_{1-\alpha/2} = 1.96$

Calculate a 95% confidence interval for the average height of adults on Statistopia.

Confidence Interval - Example

Random Selection

49

$\sigma = 25cm$

95% → $z_{1-\alpha/2} = 1.96$

Confidence Interval

$170cm \pm$ margin of error

$$\text{margin of error} = z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \cdot \frac{25}{\sqrt{49}}$$

$$= 1.96 \cdot \frac{25}{7}$$

$$= 7$$

Confidence Interval - Example

Random Selection

2500

$\sigma = 10cm$

95% → $z_{1-\alpha/2} = 1.96$

Confidence Interval

$170cm \pm$ margin of error

margin of error $= 7$

Confidence Interval

$170cm - 7 = 163cm$

$170cm + 7 = 177cm$

$163cm < \mu < 177cm$