Physics of Small Brane Black Holes

work done with

Karasik Sahabandu Suranyi

INTRODUCTION

We have looked for static black hole solutions in low scale gravity models(ADD, RS).

BH's are attached to the brane.

Used an expansion scheme to find analytic solutions to Einstein's eqns.

Introduction

Frequently, properties of low mass BH's like cross sections , decay rates are estimated by assuming that they are described by the higher dimensional Schwarzschild solution.

Want to check if this is o.k. for ADD and RS.

We solved the Einstein's eqns for a point like matter distribution confined to the brane, in ADD as well as RS models.

Introduction

BH size << L where L is the size of the extra dimensions (for ADD), or, the ADS
ADS curvature length of the bulk(RS).
Solved the Einstein eqns with appropriate Junction conditions.
Solution was perturbative in the parameter BH size/L.

Introduction

- Are There Microscopic black objects stable w.r.t. Hawking Radiation
- Critical Black Branes in Einstein Gauss Bonnet Gravity

Conclusions ADD

- IN ADD expansion works out smoothly. Higher dimensional Schwarzschild approximation is quite good.
- Thermodynamic properties the horizon size could be worked out in this expansion.
- When its mass increases the BH grows into the bulk (oblate).

Conclusions RS

- The horizon becomes singular in the bulk when the first order correction is taken into account.
- Horizon is still regular on the brane.
- More studies are needed to clarify this behavior.

References

ADD solution to first order was also obtained by Harmark, and also by Gorbanos and Kol P.R.D.69,10405(04), JHEP 06,0539(2004).

K,S,S,W PRD 69, 064022(04) and PRD70,064007(04), P.R.D.024024(05)

Outline

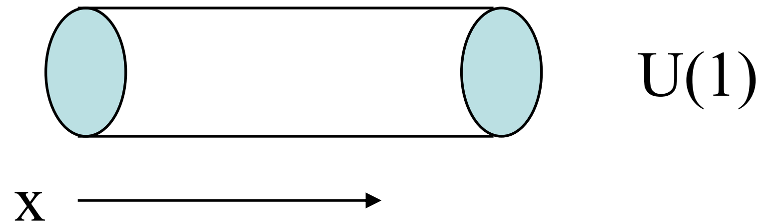
- Introduction (D>4 worlds - Compact dimensions)
- Brane Theories
- **Black holes in higher dimensional space**, Schwarzschild-Tangherlini Myers Perry solution
- Uniform and non-uniform **black strings**, Gregory-Laflamme instability
- Physical **quantities characterizing black holes** in compactified spaces, black hole thermodynamics, Smarr's formula, Phase diagram.

Outline

- **Numerical approach** to black holes
- **Analytic approximation** scheme: expansion in ratio of Schwarzschild radius to the AdS or compactification length (in AdS space and flat space)
- **Phase diagram**. Can a black hole turn into a black string?
- $D \to 4$ compactification, Rotating black holes

More than 4 dimensions

• Early attempts: Kaluza and Klein: 5 dimension includes electromagnetic field, 5th dimension compactified, U(1). Aim: Unification



• Strings are consistent only in 26 (bosonic) or 10 (supersymmetric) dimensions. 6 dimensions compacified, providing internal symmetries

• More than one compactified dimensions lead to group structure

• ADD, RS: aim is to solve the hierarchy problem

Size of dimensions

Einstein action ($G=G_4$, 4 dimensional Newton's constant):

$$S_g = \frac{1}{16\pi G_4} \int d^4x\sqrt{-g}R$$

In more than 4 dimensions (k flat directions compactified):

$$S^{(D)} = \frac{1}{16\pi G_D} \int d^D x\sqrt{-g}R \Rightarrow \frac{V^{(k)}}{16\pi G_D} \int d^{D-k}x\sqrt{-g}R$$

The relationship between Planck masses is,

$$M_{\mathrm{Pl(D\text{-}k)}}{}^{D-k-2} \sim r_k{}^k M_{\mathrm{Pl(D)}}{}^{D-2}$$

If D-k=4 then with appropriate $r_{D\text{-}4}$ we can choose the fundamental (D dim)
Planck mass at 1 TeV while the 4 dim Planck mass stays at physical value. Solve hierarchy problem (?).

Small black hole on a cylinder

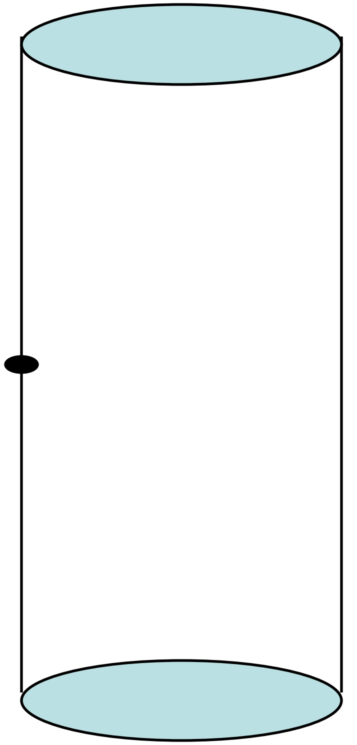
$\rho_0 = \text{Schwarzschild radius}$

$\rho < L$ Schwarzschild - Tangherlini black hole

$\rho > \rho_0$ Linearized approximation

$\rho_0^{d-3} = G_N M$

$\text{Expansion parameter} = \dfrac{\rho_0}{\mathrm{L}} \approx \mu^{1/(d-3)}$



Black hole $\longrightarrow$ Black string

- Start with a black object smaller than the extra dimension-it resembles a 5-d black hole with horizon topology S^3
- Increase its mass
- Eventually it will no longer fit the compact dimension
- A black string whose horizon has topology S^2 x S^1 could form

Black holes in more than 4 dim

If Gravity is strong at 1 TeV then black holes will be produced copiously at LHC, provided BH solutions exist {Giddings(Thomas+Katz+Eardley), Banks+Fishler, Dimopoulos+Landsburgh}. Production in cosmic rays (Anchrodoqui, Feng , Goldberg,Shapere)

In infinite D dim space, Tangherlini's generalized the Schwarzschild metric:(see also Myers Perry)

$$ds^2 = -(1 - \frac{\mu^{D-3}}{\rho^{D-3}})dt^2 + \frac{d\rho^2}{1 - \frac{\mu^{D-3}}{\rho^{D-3}}} + \rho^2 d\Omega^{(D-2)}$$

For D = 5

$$d\Omega^{(3)} = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)$$

If ADD or brane black holes are of "infinitesimal" size these solutions should become exact.

More on Black Holes

* Primordial 1TeV BH's immersed in hot plasma may affect cosmology.
* Define small: Schwarzschild radius less than L

- *Note also that,*

Newton Constant: $$G_n = M_n^{\;2-n}$$

$$r_H = \left(G_n M\right)^{\frac{1}{n-3}}$$

$$\left(M_4 = 10^{16}\,TeV\right)$$

$$r_H \cdot M = \left(\frac{M}{M_n}\right)^{\frac{n-2}{n-3}} \geq 1$$

Black strings

Only exact solution known in compactified space is the

O(3)xU(1)$^{D-4}$ symmetric **black string**:

$$ds^2 = -(1-\tfrac{2MG}{r})dt^2 + \frac{dr^2}{1-\frac{2MG}{r}} + \sum_{i=1}^{D-4} dw_i^{\,2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

• Gregory and Laflamme showed that below a **critical mass** (proportional to L, the compactification circumference) the black string is **unstable** against large wave-number s-wave perturbations

• Gregory and Laflamme proposed-Gubser (analytically, near critical point) and Wieseman (numerically) investigated the **non-uniform black string**.

• Non-uniform black string could pinch off at a point on its horizon and decay into one or many black holes, but not classically in finite affine time (Horowitz & Maeda). Structural transition (Kol)

Quantities characterizing black objects

Leading asymptotic terms of metric tensor (D dim black object):

$$g_{tt} = -1 + \frac{c_t}{r^{D-k-3}}, \quad g_{w_i w_i} = 1 + \frac{c_w}{r^{D-k-3}}$$

$$T_{00} = \rho(x), \;\; T_{w_i w_i} = \eta(x)$$

If k=D-4 long range force (Newtonian potential)

Define (ADM) mass and relative binding energy (relative tension) (Harmark and Obers):

$$M = \int d^{D-k-1} r \prod_{i} dw_i \, \rho(x), \quad n = \frac{1}{M} \int d^{D-k-1} r \prod_{i} dw_i \; \eta(x)$$

$$M = \frac{L^k \Omega_{D-k-2}}{16\pi G} \left[(D-k-2) c_t - k c_w \right]$$

$$n = \frac{c_t - (D-3) c_w}{(D-k-2) c_t - k c_w}$$

Examples for black objects

Phase diagrams

Black string:

$c_w = 0 \rightarrow$

$$n = \frac{c_t - (D-3)c_w}{(D-k-2)c_t - kc_w} = \frac{1}{D-k-2}$$

independent of M.

Black hole in non-compact space:

$T_{ww} = 0 \quad \rightarrow \quad \boldsymbol{n=0}$

Phase diagram of black objects: n vs. M.

Black hole in compactified space n has non trivial dependence on M.

Thermodynamics of black objects

1. Zeroth law of black object thermodynamics (Racz and Wald)

κ is the constant surface gravity

$$T = \frac{\kappa}{2\pi}$$

2. First law of black object thermodynamics generalized to compactified 5 dim space:

$$\delta M = T\,\mathrm{d}S + knM\,\frac{dL}{L}$$

3. Smarr's formula (integrated first law) Smarr, Kol, Sorkin and Piran, hep-th/0309190, Harmark and Obers JHEP05 (2004) 043):

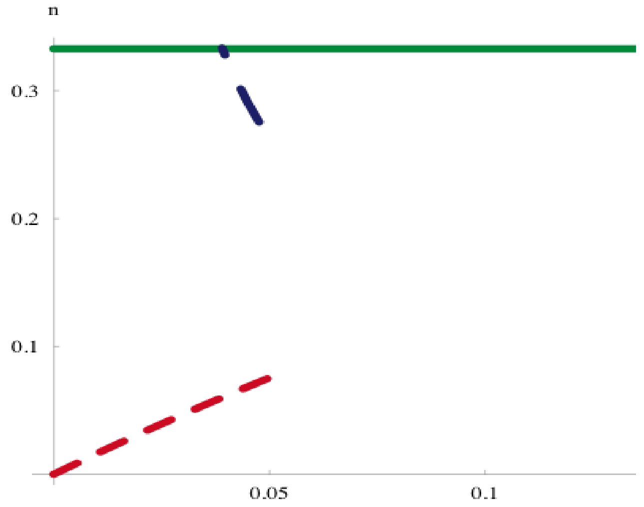
$$M = \frac{D-2}{D-3}TS + \frac{k}{D-3}nM \qquad S = \frac{A}{2G_D}$$

Entropy is a function of the ADM mass and tension

Numerical approach to black objects in $R^4 \times S^1$

• Non-uniform black strings investigated near the Gregory-Laflamme point by Gubser (hep-th/0110193). Away from the GL point, numerically, by Wiseman (hep-th/0211028). Wiseman and Kudoh, All D→D-1 (mostly D=6)

Phase diagram (blue dash-dot):

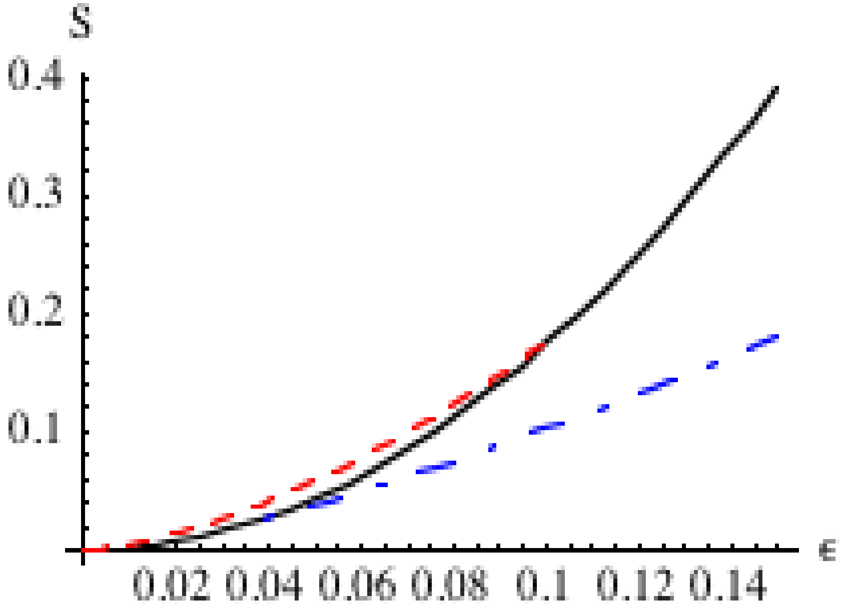


Disappointing: non-uniform black string exists in region of stability of black string and its entropy is lower. Is there another non-uniform black string branch that does not start from the black string?

$$\varepsilon \sim M$$

Entropies in numerical approach

Entropy of non-uniform black string (blue dash-dot) is always smaller than that of the uniform (solid black) at the same mass value. The dashed red line depicts the entropy of the black hole



Expansion scheme

• Expansion in $\varepsilon=\mu/L$, the ratio of the radius of the horizon μ, to the compactification radius, L. Such an approximation scheme was proposed in Karasik, Sahabandu, Suranyi, Wijewardhana (PR 69 064022 (2004) Applied to Randall-Sundrum compactification KSSW PR 70 064007 (2004), gr-qc/0309076

• Result: No solution in D=5 with regular horizon in iterative expansion. Singular solution (depending on half integer powers of ADM mass)

• In Arkani-Hamed, Dimoupoulos, and Dvali scenario the solutions are even function of these parameters, consequently the expansion parameter is,

$$\varepsilon=\left(\frac{\mu}{L}\right)^2$$

• Kol, Sorkin, and Piran (PR 69 064032 (2004)) proposed a similar scheme simultaneously, but calculated only to $O(\varepsilon)$. KSSW completed the calculations to $O(\varepsilon^2)$ calculating the phase diagram and thermodynamic quantities.

Coordinates used

ρ and *r* are the 4 and 3 dimensional radii

$$\rho \sin\psi = r,$$

$$\rho \cos\psi = \tfrac{L}{\pi} \sin \tfrac{\pi w}{L}, \quad w \in \left[-\tfrac{L}{\pi}, \tfrac{L}{\pi}\right],$$

$$\rho = \sqrt{r^2 + \tfrac{L^2}{\pi^2} \sin^2 \tfrac{\pi w}{L}},$$

$$\psi = \tan^{-1} \frac{\pi r}{L \sin(\pi w / L)}.$$

Conditions to impose on solution

1. Zeroth law: Surface gravity is constant
2. Killing horizon: The metric is static, therefore the normal vector to the horizon, $g_{tt,\mu}$ should be null (on the horizon)
3. Solution periodic: $g_{\mu\nu}(w+L) = g_{\mu\nu}(w)$
4. Solution is Z_2 symmetric: $g_{\mu\nu}(-w) = g_{\mu\nu}(w)$, except for $g_{iw}(-w) = -g_{iw}(w)$, $i \neq w$
5. Small ε limit at $\rho \ll L$ Myers-Perry solution
6. Small ε , large ρ limit: linearized (Newton) gravity
7. No black string: the Kretschmann scalar is finite on the $r = 0$ axis.

Need for ‘near’ and ‘asymptotic’ solutions

We need a ‘near’ solution (N) that is valid near the horizon to calculate thermodynamic quantities. All the conditions can be imposed on the near solution, except for the asymptotic condition. Therefore N is not completely fixed. To fix N one also needs an asymptotic solution (A). After imposing the asymptotic condition on A we match the two solutions in a region where both are valid.

How to find the two solutions?

If $\varepsilon=(\mu/L)^2 \ll 1$ then either μ (horizon radius) is a small parameter, or $1/L$, or both.

$$g^{\mathrm{N}}{}_{\mu\nu}(x) = \sum_{n=0} (\mu/L)^{2n} g^{\mathrm{N},n}{}_{\mu\nu}(x/\mu), \quad \text{expansion in } 1/L \text{ with } x, \mu < L$$

$$g^{\mathrm{A}}{}_{\mu\nu}(x) = \sum_{n=0} (\mu/L)^{2n} g^{\mathrm{A},n}{}_{\mu\nu}(x/L), \quad \text{expansion in } \mu < L \text{ and } x > \mu$$

Matching the two solutions

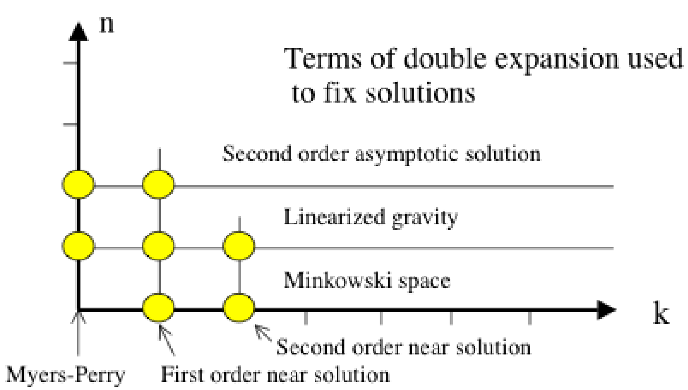
$$g^{\mathrm{N}}{}_{\mu\nu}(x) = \sum_{n=0} (\mu/L)^{2n} g^{\mathrm{N},n}{}_{\mu\nu}(x/\mu), \quad \text{expansion in } 1/L, \quad x/\mu \text{ arbitrary}$$

$$g^{\mathrm{A}}{}_{\mu\nu}(x) = \sum_{n=0} (\mu/L)^{2n} g^{\mathrm{A},n}{}_{\mu\nu}(x/L), \quad \text{expansion in } \mu, \quad x/L \text{ arbitrary}$$

To match these in a region where both are valid, $\mu \ll \rho \ll L$ we expand them in μ/ρ and ρ/L respectively and compare order by order powers of

$$\underbrace{\left(\frac{\mu}{\rho}\right)^{2n}\left(\frac{\rho}{L}\right)^{2k}}_{\text{term of double expansion}} = \underbrace{\varepsilon^{n}\left(\frac{\rho}{L}\right)^{2(k-n)}}_{\text{term of } g^{\mathrm{A}}} = \underbrace{\varepsilon^{k}\left(\frac{\mu}{\rho}\right)^{2(n-k)}}_{\text{term of } g^{\mathrm{N}}}$$

$$\varepsilon = \left(\frac{\mu}{L}\right)^{2}$$



Order of $\varepsilon=\mu^2/L^2$	1	μ^2/L^2	μ^4/L^4
Asympt. soln.	$1,\ \rho^2/L^2,\ \rho^4/L^4, \dots$	$L^2/\rho^2,\ 1,\ \rho^2/L^2, \dots$	$L^4/\rho^4,\ L^2/\rho^2,\ 1, \dots$
Near solution	$1,\ \mu^2/\rho^2,\ \mu^4/\rho^4, \dots$	$\rho^2/\mu^2,\ 1,\ \mu^2/\rho^2, \dots$	$\rho^4/\mu^4,\ \rho^2/\mu^2,\ 1, \dots$

Asymptotic solution in first order of ε

Use zeroth order near solution (MP) to fix first order asymptotic solution

$$g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon\, h_{\mu\nu}$$

Einstein equation:

$$h_{tt,rr} + \tfrac{2}{r} h_{tt,r} + h_{tt,ww} = 0$$

Solution of Z_2 symmetry in w

$$h_{tt}(r,w) = \sum_{n=0} c_n e^{-2\pi n r/L} \cos(2\pi n w/L)$$

c_n is found from matching with zeroth order near solution

$$h_{tt}(r,w) = \frac{\pi L}{r} \frac{\sinh(2\pi n r/L)}{\cosh(2\pi n r/L) - \cos(2\pi n w/L)}$$

satisfies $$\varepsilon h_{tt}(r,w) \to \frac{2G_4 M}{r} \quad \text{if } r \to \infty$$

Near solution in first order of ε and higher orders

Ansatz:

$$ds^2 = -[1 - \tfrac{\mu^2}{\rho^2} - \varepsilon B_1(\rho,\psi)]dt^2 + \frac{[1+\varepsilon A_1(\rho,\psi)]d\rho^2}{1 - \frac{\mu^2}{\rho^2} - \varepsilon B_1(\rho,\psi)}$$

$$+\rho^2[1+\varepsilon U_1(\rho,\psi)]d\psi^2 + \varepsilon V_1(\rho,\psi) + \rho^2 \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)]$$

Einstein equation is solved in terms of a single function F, satisfying a wave equation and a gauge function. The wave equation can be solved and its constants fitted using the near conditions and matching to the first order asymptotic solution.

The gauge is fixed by requiring that g is independent of ψ on the horizon.

The same ansätze can be used in the higher order asymptotic and near solutions and the same wave equations must be solved with an inhomogeneity, determined by lower orders.

Results

$$S_{BH} = \frac{\pi^2 L^3}{2G_5}\varepsilon^{3/2}\left(1 + \frac{\pi^2}{8}\varepsilon + \frac{\pi^4}{384}\varepsilon^2 + O(\varepsilon^3)\right)$$

$$n_{BH} = \frac{\pi^2}{6}\varepsilon - \frac{\pi^4}{36}\varepsilon^2 + O(\varepsilon^3)$$

radius of horizon:

$$\rho_H = \mu\left(1 + \frac{\pi^2}{24}\varepsilon - \frac{23\pi^4}{1920}\varepsilon^2 + O(\varepsilon^3)\right)$$

Black string

$$S_{BS} = \frac{9\pi^3 L^3}{16G_5}\varepsilon^2,$$

$$n = \frac{1}{2}$$

Non-uniform Black string (numerical)

$$S_{NBS} = 0.0265\,\frac{L^3}{G_5}\left[1 + 1.12\left(\frac{\varepsilon}{\varepsilon_{GL}} - 1\right)\right]$$

$$n_{NBS} = \frac{0.0125}{\varepsilon} + 0.013,$$

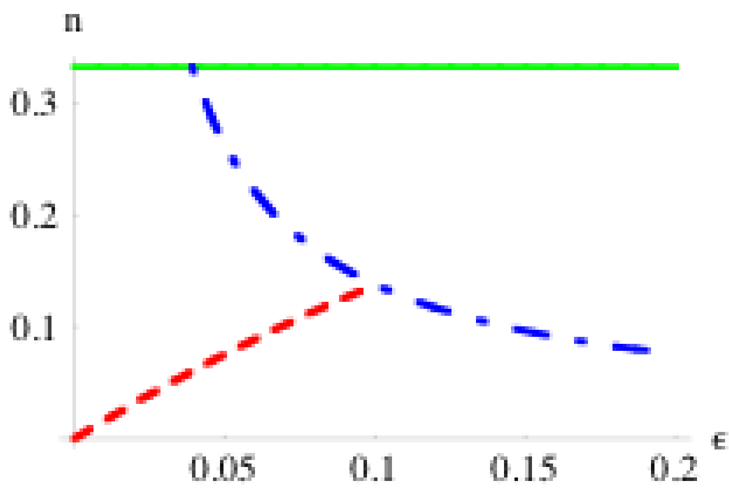
$$\varepsilon_{GL} = 0.039$$

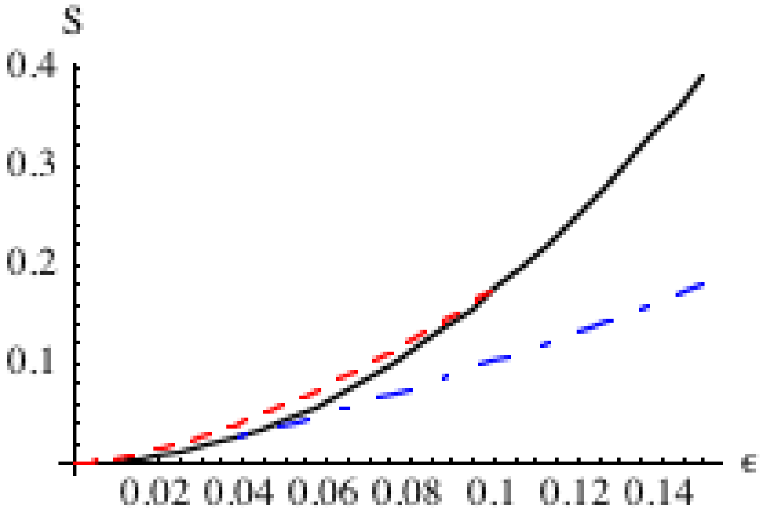
Analysis of n and S

Radius of horizon reaches *L*: $\varepsilon = 0.096$ (in 1st order 0.094)

n coincides for NBS and BH: $\varepsilon = 0.100$ (in 1st order 0.092)

S coincides for BS and BH: $\varepsilon = 0.102$ (in 1st order 0.101)





Similarity of ε values is tantalizing but not well understood.

Shape of horizon

prolate or oblate?

Measure maximal area of cross section of horizon parallel perpendicular to the w direction

$$e = \frac{A_{\parallel}}{A_{\perp}} - 1 = \frac{4\pi^4}{135}\varepsilon^2$$

prolate (football shaped)

Does proper distance between two poles of the horizon vanish? (Near that point the near solution can be used)

$$L_{\text{poles}} = 2\int_{\rho_H}^{z} \sqrt{g^N{}_{\rho\rho}}\,d\rho + 2\int_{\frac{L}{\pi}\sin^{-1}(\pi z/L)}^{L/2} \sqrt{g^A{}_{ww}}\,dw$$

$$\approx 2\int_{\rho_H}^{z} \sqrt{g^N{}_{\rho\rho}}\,d\rho = L\left(\frac{149}{60\pi} - \frac{17\pi}{24}\varepsilon - \frac{1003\pi^3}{1440}\varepsilon^2\right)$$

L_{poles} vanishes at: $\varepsilon = 0.146$ (in 1st order 0.355)

What happens when we throw matter into black hole?

Nonuniform black string of Gubser and Wiseman is poor candidate:

1. It has too low entropy
2. It does not constrict at L/2 completely at critical ε
3. It does not have a string singularity attached to it

Turning into non-uniform black string can happen as follows:

Range is $-L/2<w<L/2$

At very small mass O(4) symmetry

(Tangherlini-Schwartzschild)

Motion as function of increasing mass

QuickTime™ and a
Animation decompressor
are needed to see this picture.

Possibly new branch of NBS starts at critical point, Not reachable by numerical methods starting from the GL point

Investigate its existence at large ε perturbing uniform BS?

D→4 Compactification
(work in progress)

Compactification on $\prod_{1}^{D-4} \otimes S_1$

Problems:

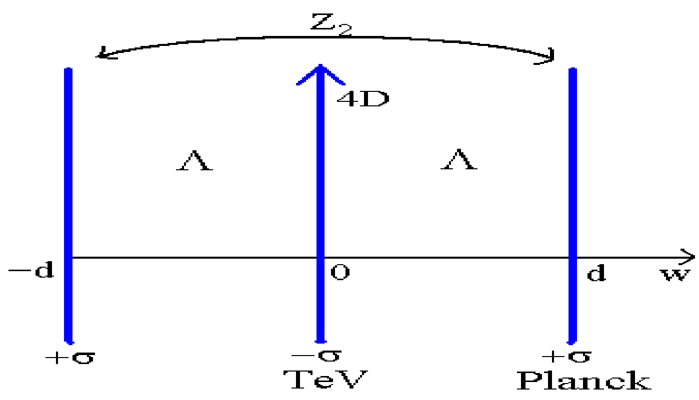
1. Dependence on D-3 variables.
2. No closed form for leading order near solution

Both problems are solvable to order 1 in ε.

Other possibility: **rotating black hole**:

Again three variables. Expand in angular momentum?

Randall-Sundrum 1 Scenario



$$ds^2 = \left(\frac{\ell}{\ell - |w|}\right)^2 \eta_{AB} dy^A dy^B$$

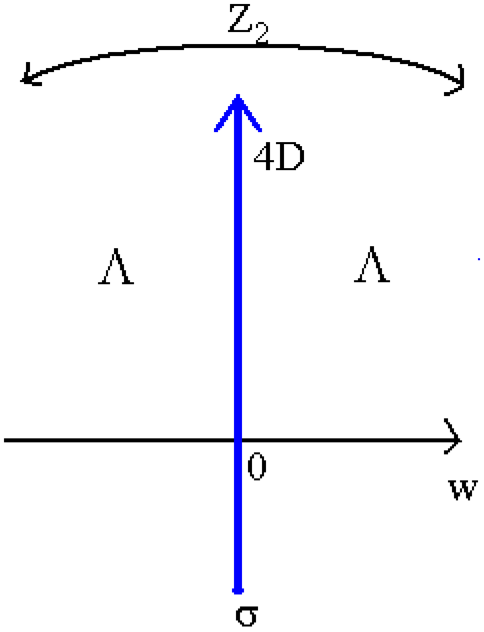
$$8\pi G_5 \Lambda = -\frac{6}{\ell^2}$$

$$8\pi G_5 \sigma = \frac{6}{\ell}$$

$$d = \ell\left(1 - \lambda\right)$$

$$\lambda = 10^{-19}$$

RS2

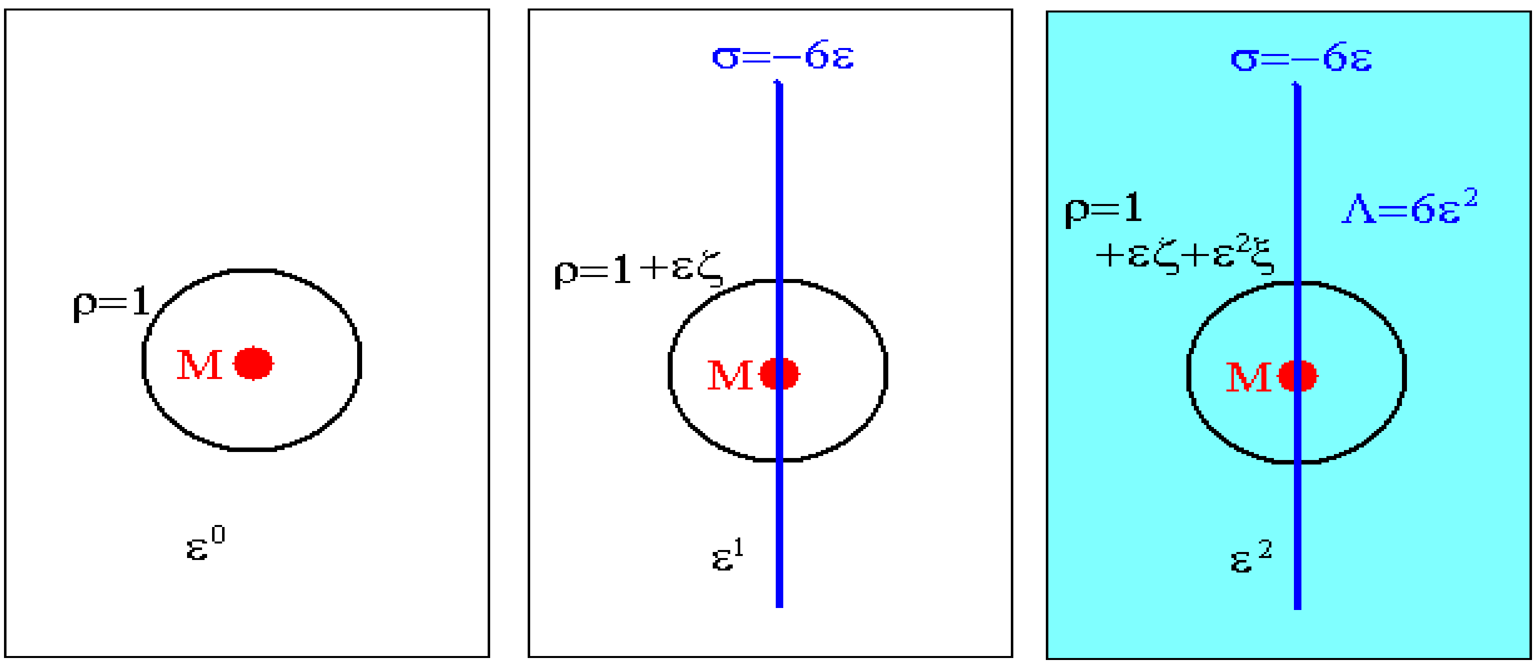


- Einstein equations:

$$R_{AB} - \frac{1}{2} R g_{AB} = 6\epsilon^2 g_{AB}$$

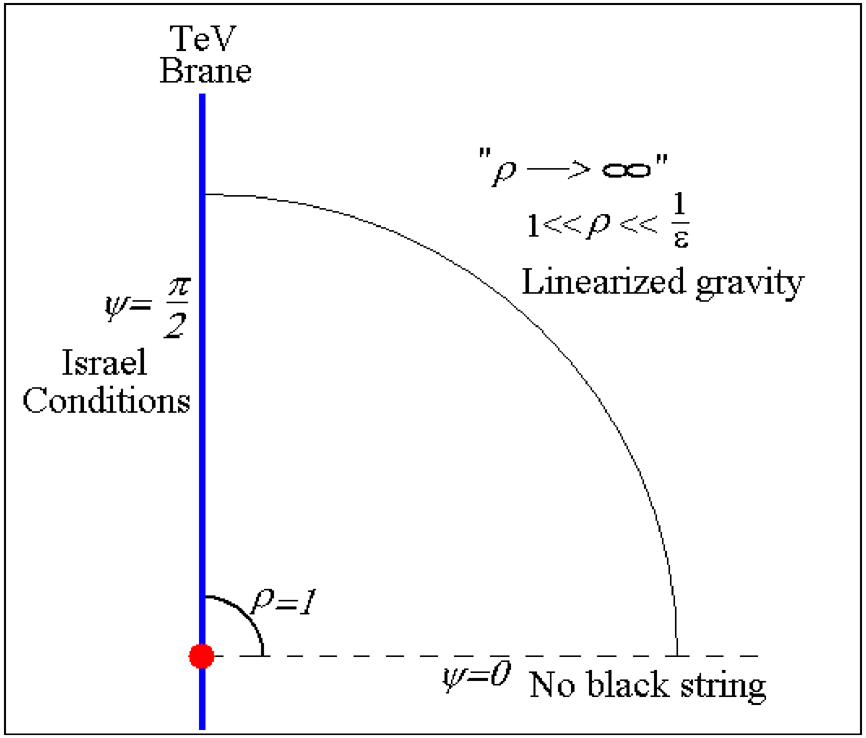
- Israel junction conditions:

$$2\left(K\gamma_{\mu\nu} - K_{\mu\nu}\right) = 6\varepsilon\gamma_{\mu\nu}$$



Solution

- Boundary conditions:



The solution up to the first order in ε

Satisfies Einstein eqns in the bulk

Israel junction conditions on the brane

Asymptotic form fixed by the weak field approximation

Investigating the horizon

- Solve $g_{tt} = 0$
- Check if the resulting surface is a killing horizon, i.e. a null surface with a killing field normal to the surface
- Whether the surface gravity is constant

Surface Gravity

The surface gravity is proportional to:

$1+2\ \mathrm{Sin}\ \psi$

It is not a Constant

According to a theorem by Racz, Wald the horizon is singular

Give up the post linearized assumption

Give up the assumption that the asymptotic solution can be expanded in integer powers of M, but still keep the linearized solution as the asymptotic boundary condition.
Horizon could be made regular.
This condition is unphysical.

Conclusions

- RS horizon is singular in the bulk but regular on the brane.
- Physical interpretation is not clear.
- Naked singularities are supposed to be unstable objects.
- Conclusion also holds for RS2 BH's.
- Use of Schwarzschild approximation is not justified.

EINSTEIN GAUSS-BONNET GRAVITY

$$S = -\frac{1}{16\pi G_D}\int d^D x \sqrt{-g}\left[R + \alpha L_{GB}\right]$$

$$L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$$

EQ^N $$G_{\mu\nu} = -\alpha \; L_{\mu\nu}$$

$$L_{\mu\nu} \equiv \text{LANCZOS TENSOR}$$

$$= -\frac{1}{2} g_{\mu\nu} L_{GB} + 2RR_{\mu\nu} - 4R_{\mu\gamma}R^{\gamma}{}_{\nu} - 4R_{\gamma\delta}R^{\gamma\delta}{}_{\mu\nu} + 2R_{\mu\gamma\delta\lambda}R_{\nu}{}^{\gamma\delta\lambda}$$

SPHERICALLY SYM SOL^N

(BOULWARE - DESER)
P.R.L 55

$$ds^2 = -f\,dt^2 + \frac{1}{f}\,dr^2 + r^2 d\Omega_{D-2}$$

$$f(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{4\alpha M}{r^{D-1}}}\right)$$

THERMODYNAMICS BY

MYERS, SIMON P.R.D 38
WILTSHIRE P.R.D 38

IN $D=5$ THERE IS A CRITICAL MASS BELOW WHICH NO B.H.

$$T \rightarrow (M - M_{cr})^{\frac{1}{2}}$$

STABLE B-H. REMNENTS IN A 5-D WORLD.

BLACK BRANES IN
$D \geq 5$ E.G.B. GRAVITY

NO EXACT SOLUTIONS

NUMERICAL SOLUTION BY
KOBAYASHI TANAKA
P.R.D 71, 084005 (2005).

α EXPANSION FOR $D \geq 5$
BY SAHABANDU, SURANYI, VAZ
AND W P.R.D 73, 044009 (2006)

RECENT IMPROVEMENT ON
THIS BY SURANYI, VAZ, W
HEP-TH/ 0810 0525.

B- STRING ~~BRAN~~ ANSATZ

$$ds^2 = -f(r)\,dt^2 + \frac{g(r)}{f(r)}\,dr^2 + r^2\,d\Omega^2 + h(r)\,dw^2$$

① EXPAND f, g, h in

$\rho = \left(\frac{r}{r_h} - 1\right)$ [HORIZON EXPANSION]

② α

③ $\frac{1}{r}$

THERE IS A CRITICAL MASS $M_c \propto \sqrt{\alpha}$

FOR $M > M_c$ THE BLACK BRANE HAS 2 HORIZONS.

$$T \propto (M - M_c)^{\frac{1}{4}}$$

$M = M_c$ HORIZONS COINCIDE

EXTREMAL B. BRANE

PRIMORDIAL BLACK BRANES REDUCED TO ITS CRITICAL MASS BY HAWKING RADIATION IS A STABLE NON-BARYONIC OBJECT.