Geometric Precipices in String Cosmology

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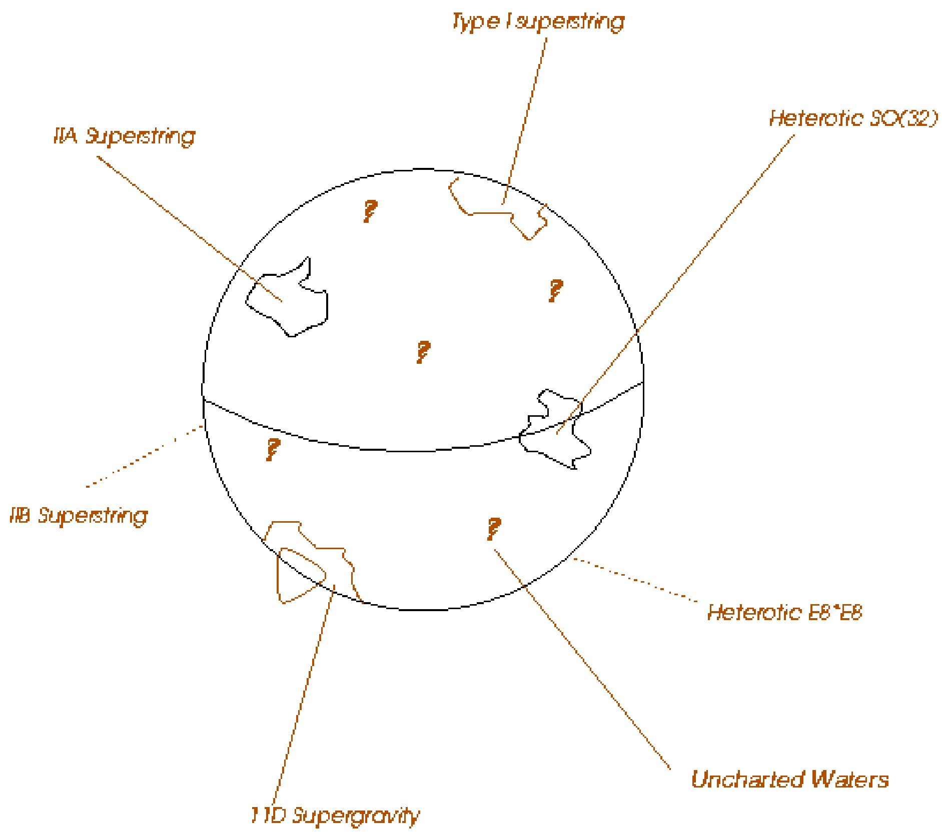
Based on work with G.S. Watson

Overview

- Stringy gas cosmology: a (very!) brief review
- Dilaton sector: a conserved discrete charge
- Singular precipices

Strings & Dualities

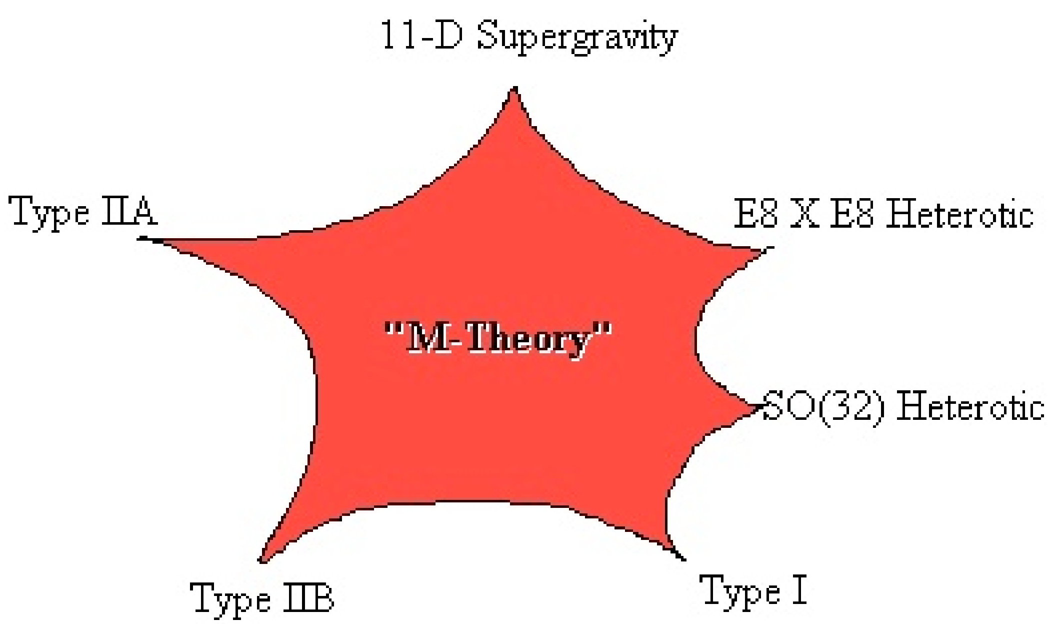
The M-theory planet



- Pre-95 revolution: The "islands" were not even known to be in the same universe, let alone "planet"...

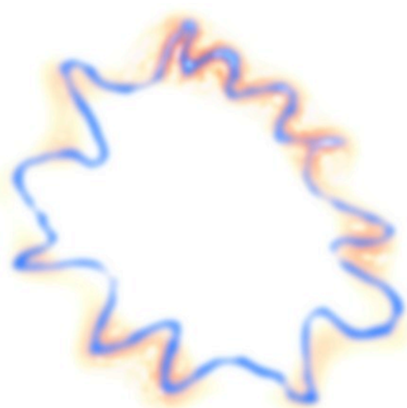
The Power of Dualities

- The "islands" are not only on the same "planet", but they are related by secret "tunnels": DUALITIES
- Different-looking perturbative theories are but aspects of the same underlying dynamical structure in different limits; the limits are related to each other by maps involving couplings and intechange of elementary and solitonic degrees of freedom



T-Duality

- An example: consider a string on a circle; its excitations are given by momentum and winding modes.
- T-duality: string theory on circle of radius R = string theory on a circle of radius l^2/R **iff**



$\longleftrightarrow$

R>l: LIGHT HEAVY

Enter Cosmology

- In cosmology, the universe expands monotonically at the largest scales, if filled by normal matter; in the far past therefore it was very small and dense. So it should have sampled the conditions where one may need string theory to describe it.
- *The state of the matter may thus well have been controlled by stringy dynamics. So from our low energy perspective, our universe may have been shaped by very exotic matter.*
- *An example: in a small, hot, dense universe inhabited by winding and momentum modes, they `recombine' into a Hagedorn-type gas stage, characterized by a limiting temperature, which cannot be exceeded.*
- *This is an attractive idea, because it introduces a physical cutoff in the momentum space. If it can be realized, perhaps it can yield a minimal size of the universe. This, if true, can help with cosmological singularities, and maybe also other things...*

Kripfganz, Perlt, 1990; Brandenberger, Vafa, 1991.

Dynamics of Stringy Gas Cosmology

- An ensemble of stringy excitations propagating in the background FRW geometry, governed by the light string modes

$$S_e = \frac{1}{2\kappa_N}\int d^{N+1}x\sqrt{-g_s}e^{-\phi_s}\left(R_s + (\partial\phi_s)^2 - \mathcal{L}_m\right)$$

- Truncate it on $ds^2 = -n(t)^2dt^2 + e^{2\lambda(t)}d\vec{x}^2$
- The effective 1D action

$$S_{1D} = \int dt\, n\left\{\frac{e^{-\varphi_s}}{M_s}\left(N\frac{\dot{\lambda}^2}{n} - \frac{\dot{\varphi}_s^2}{n}\right) + F(\lambda, n\beta)\right\}$$

- Claim: on homogeneus and isotropic backgrounds this may be a good description even for very small universe thanks to T-duality!
- *"... not expected on inhomogeneous/anisotropic backgrounds, but maybe those could be ignored ..."*
- Note that after duality revolution the isotropic string cosmologies may be viewed as anisotropic worlds in M-theory: warning flag!

Brandenberger & Vafa; *Tseytlin & Vafa, `91*; ...

Field Equations and Phase Space

- Matter sector yields the sources

$$E = -(F + \beta\partial_\beta F)\,, \quad P = -(\partial_\lambda E)|_{\beta=const}\,,$$

- Direct variation yields, for single fluid $P = \gamma E$ in the time gauge $E = \frac{dx}{dt}$.

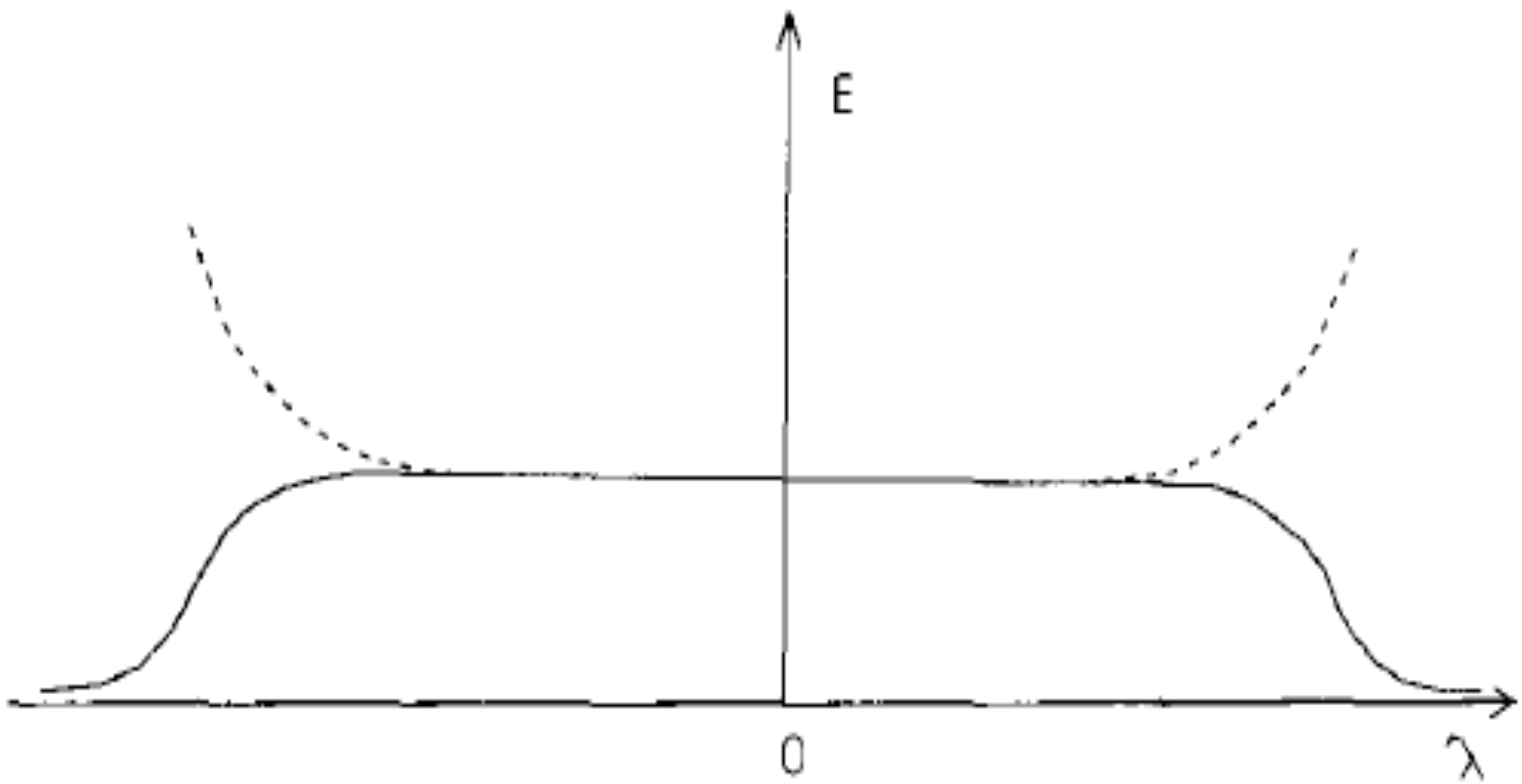
$$\begin{aligned}
\varphi_s'^{\,2} - N\lambda_s'^{\,2} &= E_0^{-1} e^{\varphi_s + \gamma N \lambda_s}\,, \\
\lambda_s'' - \gamma N \lambda_s'^{\,2} - \varphi_s' \lambda_s' &= \frac{1}{2}\gamma E_0^{-1} e^{\varphi_s + \gamma N \lambda_s}\,, \\
\varphi_s'' - \gamma N \varphi_s' \lambda_s' - N\lambda_s'^{\,2} &= \frac{1}{2} E_0^{-1} e^{\varphi_s + \gamma N \lambda_s}\,,
\end{aligned}$$

where the reduced dilaton and string coupling are

$$\varphi_s = \phi_s - N\lambda_s\,, \quad g_s^2 = \exp(\langle \phi_s \rangle)\,,$$

- These equations are in fact exactly integrable. But instead of writing solutions, let us understand them…

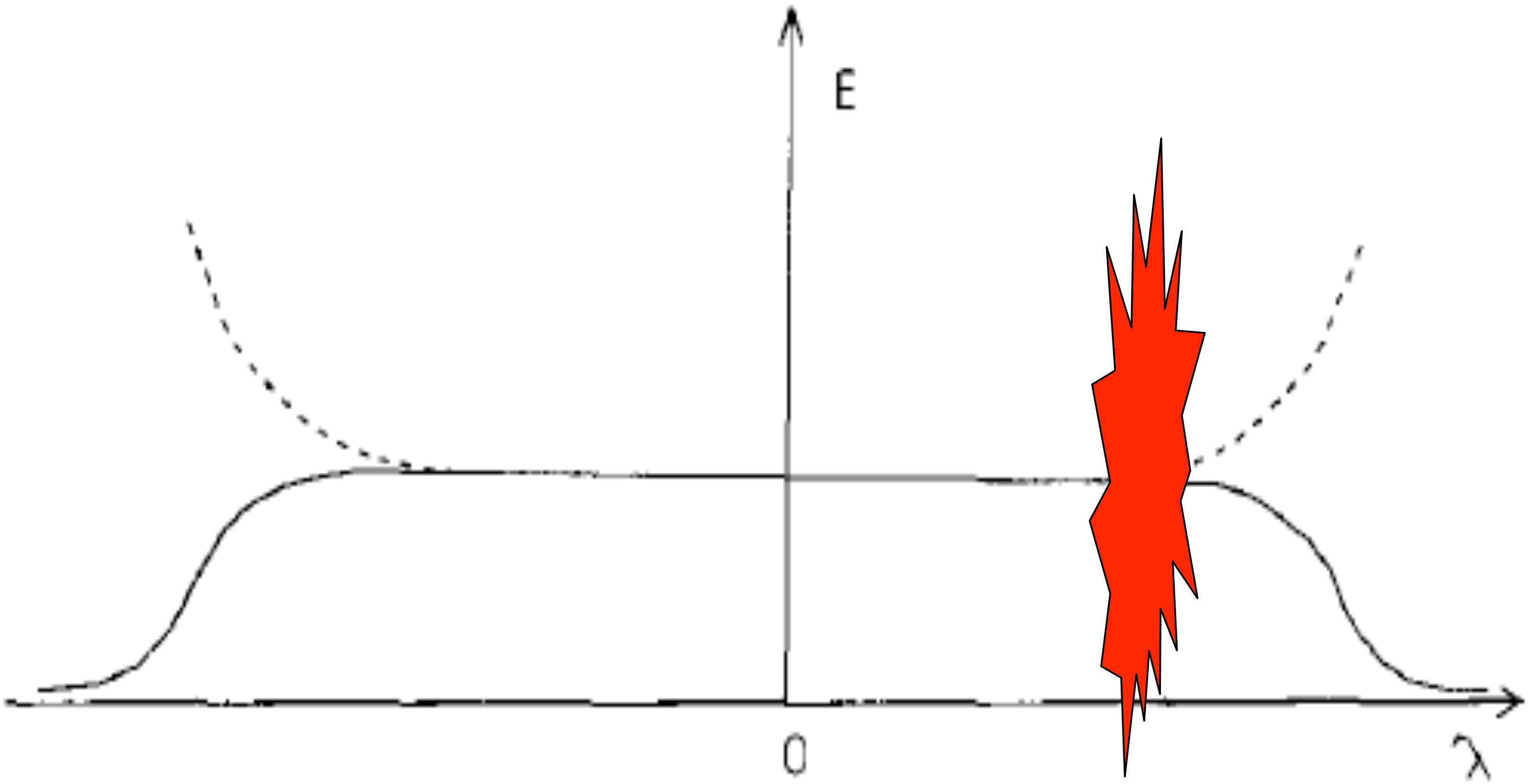
Tseytlin-Vafa Proposal



Spinoffs

- This has triggered a lot of activity in the attempt to develop phenomenologically interesting scenarios.
- Main goal: assuming the validity of the T-V proposal, explore the phase space of the theory for the corners where evolution yields cosmologically attractive predictions, specifically nearly scale-invariant perturbations.
- Works by Brandenberger, Vafa & others.
- It was also shown that phenomenologically interesting scenarios are difficult, at best, to realize within the standard stringy dynamics (Greene, Kabat & collaborators).
- Here, we will show that in fact such dynamics require violation of the NULL ENERGY CONDITION - ghosts???

Tseytlin-Vafa Proposal



What's Really Going On?

- Dilaton crucial: at high energies $\sim M_S$, not stabilized - theory is T duality invariant to start with.
- The sign of $\dot{\varphi}_s = \pm\sqrt{NH_s^2 + e^{\phi_s}\rho_s}$ is constant if energy is positive - a conserved discrete charge!
- Single fluid cosmologies, with the required asymptotics, classified by the arrow of time and the dilaton sign.
- It is easy to get exact solutions.
- Four classes of solutions, flowing to/fro the special solutions - evolutionary saddle points. They are (+) and (-) branches (signs of $\dot{\varphi}_s$), just like in PBB.

Solutions

$$\lambda_s = \lambda_{s0} + \frac{\gamma}{\alpha}\ln\left[x(x-x_*)\right] + \frac{1}{\alpha\sqrt{N}}\ln\left(1-\frac{x_*}{x}\right) ,$$

$$\varphi_s = \varphi_{s0} - \frac{1}{\alpha}\ln\left[x(x-x_*)\right] - \frac{\gamma\sqrt{N}}{\alpha}\ln\left(1-\frac{x_*}{x}\right) ,$$

$$\phi_s = \phi_{s0} - \frac{1-N\gamma}{\alpha}\ln\left[x(x-x_*)\right] - \frac{(\gamma-1)\sqrt{N}}{\alpha}\ln\left(1-\frac{x_*}{x}\right) ,$$

- The 4 classes of solutions are parameterized by the signs of x and x_* for fixed external parameters γ and N (note that $\alpha = 1 - N\gamma^2$)

However, instead of staring at equations, one can use pictures! Phase space analysis is most revealing.

Hagedorn gas

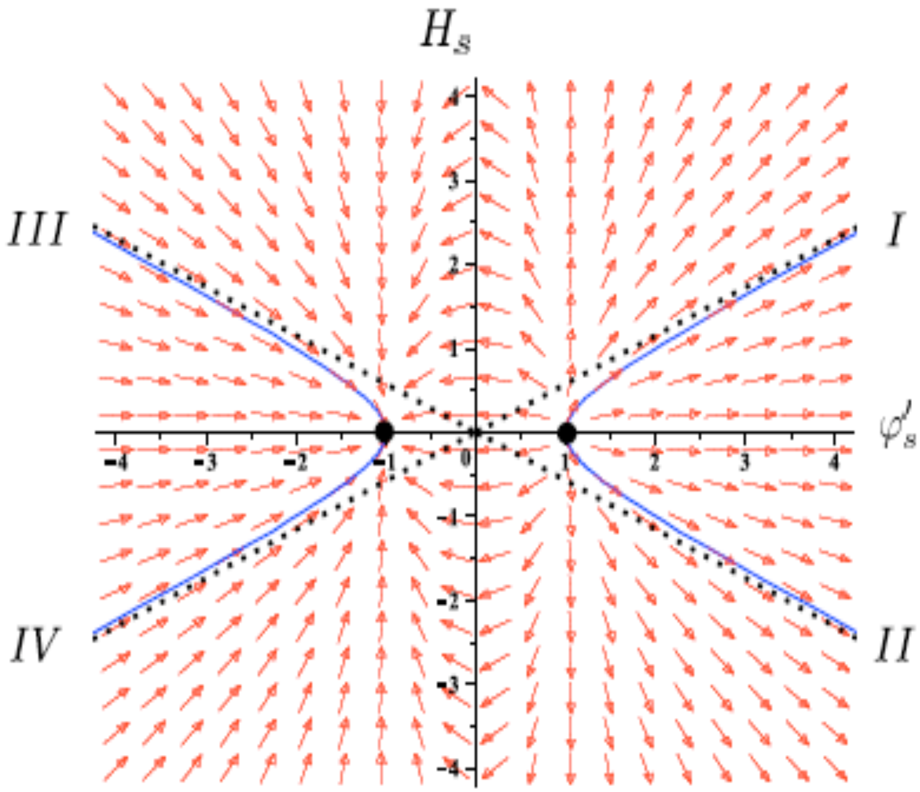


Figure 4: The (φ_s', H_s) phase diagram of Hagedorn cosmologies, with phase space flow and the limiting envelopes (dotted lines) describing the case $E_0 = 0$.

Momentum mode gas

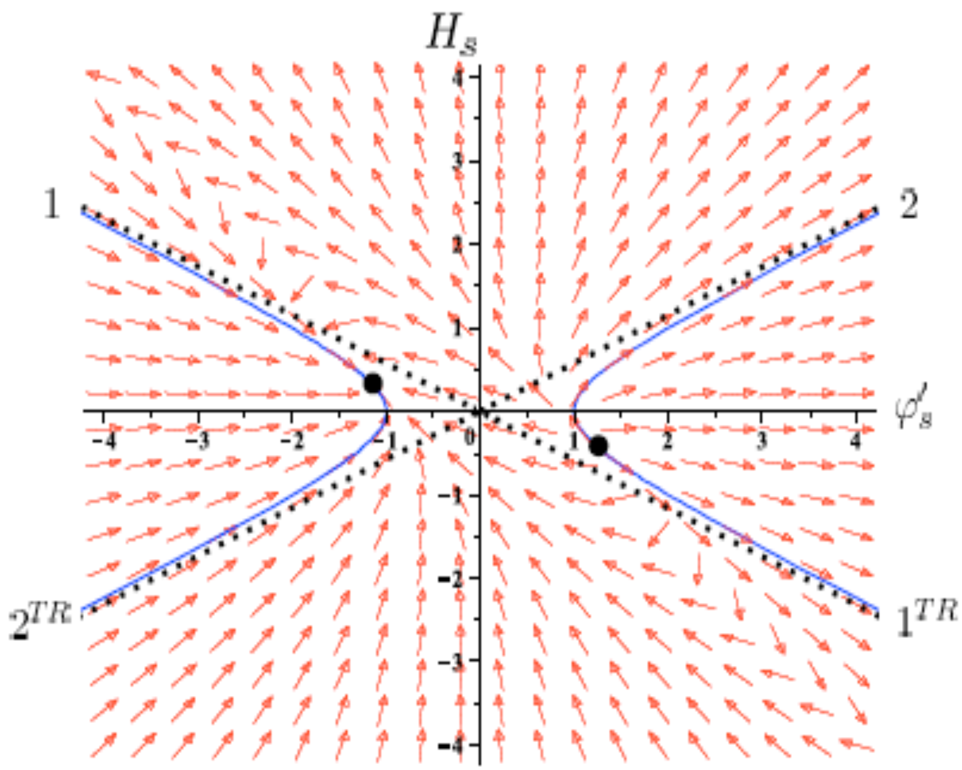


Figure 5: The (φ'_s, H_s) phase diagram of momentum mode cosmologies, with phase space flow and the limiting envelopes (dotted lines) describing the case $E_0 = 0$.

Winding mode gas

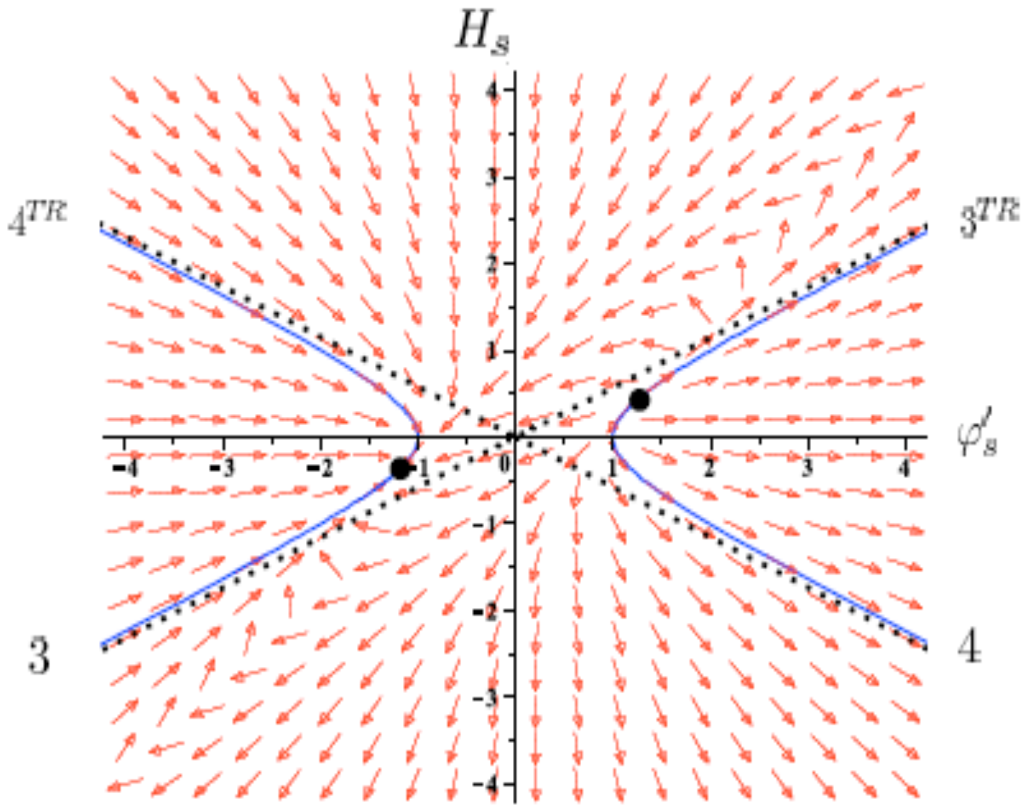
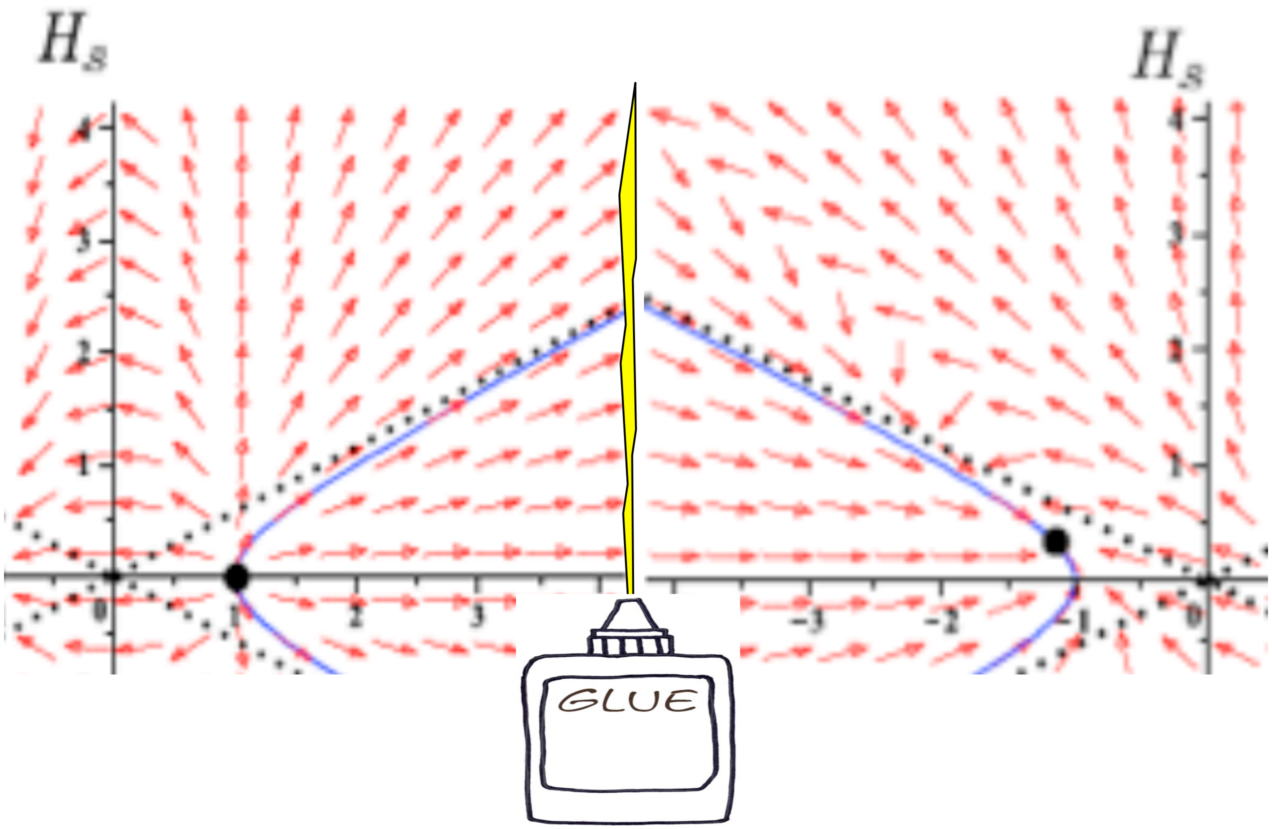


Figure 6: The (φ_s', H_s) phase diagram of winding mode cosmologies, with phase space flow and the limiting envelopes (dotted lines) describing the case $E_0 = 0$.

Cosmological Transitions?

- The stringy gas cosmology wants to link different solutions and avoid singularity; eg. of Tseytlin-Vafa:

Hagedorn I → Momentum 1.



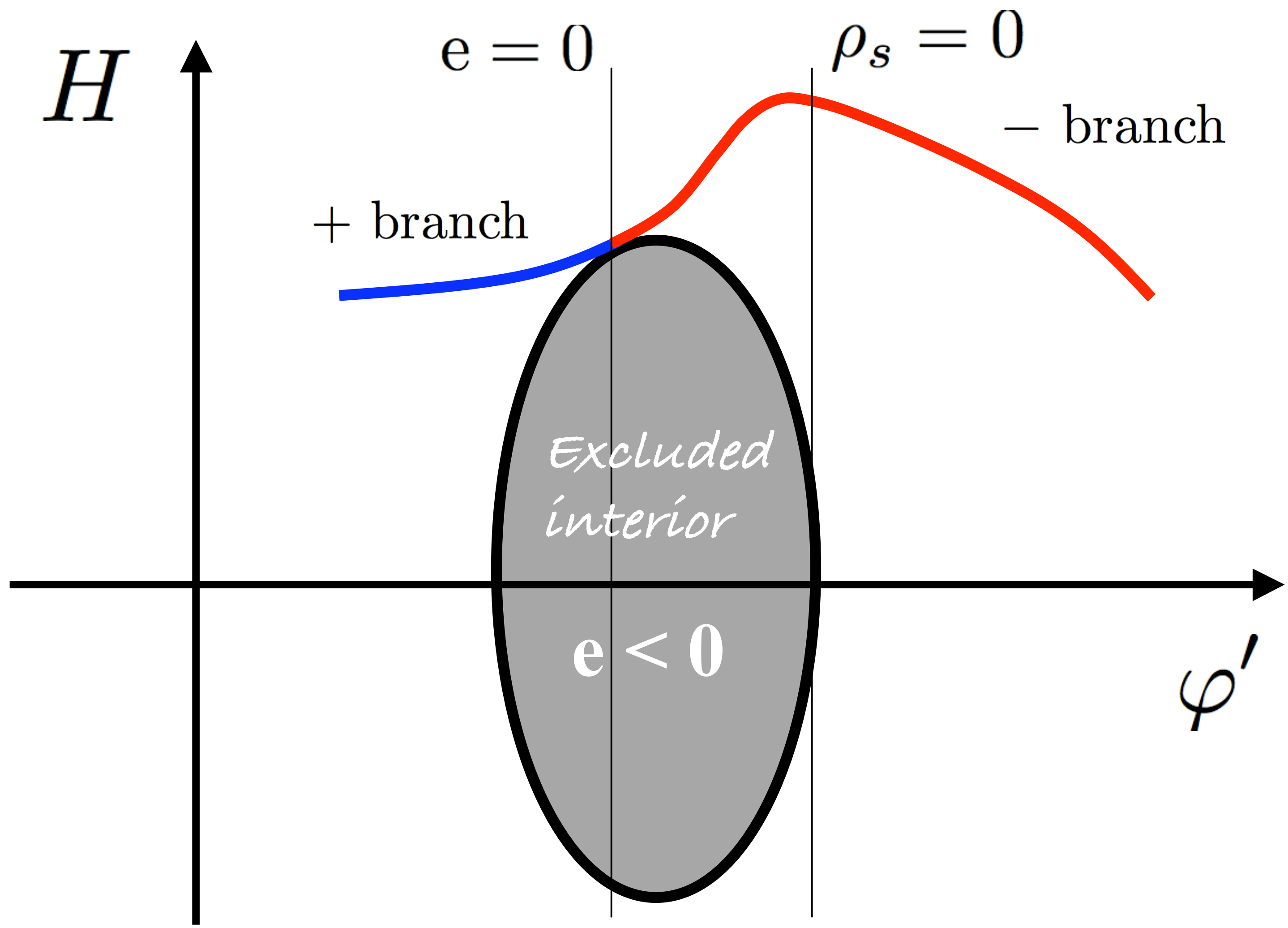
No-go & Null Energy

- These solutions are on different branches: (+) and (-)!
- For matching to occur, dilaton velocity must reverse: there must be region where the `egg function' vanishes!

$$\mathsf{e} = N H_s^2 + e^{\phi_s} \rho_s$$

- For this to happen we must ensure:
  - Negative energy density somewhere
  - Collision between the trajectory and this region
  - Subsequent escape without further collision

Negative energy density somewhere: put in a potential which dips below zero for some values of the dilaton. But that in itself is NOT ENOUGH! What is required is something like:



- Consider evolution of e along a trajectory:

$$\pm \frac{d}{dt}\left(\sqrt{\mathsf{e}}\right) = -\dot{H}_s + H_s\dot{\phi}_s + \frac{1}{2}e^{\phi_s}\left(\rho_s + p_s\right)$$

- (+/-) refers to the branch on which this evolves.
- Assume that a trajectory hits the egg, and escapes and integrate between the hit and the escape time:

$$\int_{t_h}^{t_e} dt \left[\frac{d}{dt}\left(\pm\sqrt{\mathsf{e}} + H_s\right) - H_s\,\dot{\phi}_s\right] = \frac{1}{2}\int_{t_h}^{t_e} dt\; e^{\phi_s}\left(\rho_s + p_s\right)$$

- The 3rd term on the LHS is the negative area between the curve and the horizontal axis. Rewrite as

$$\left(\mp\sqrt{N} - 1\right)H_s(t_e) + H_s(t_h) + \mathcal{A} = -\frac{1}{2}\int_{t_h}^{t_e} dt\, e^{\phi_s}\left(\rho_s + p_s\right)$$

Same idea as in NK, Madden, Olive, 1995.

- If you start on (+) branch:

$$\left(\sqrt{N}-1\right)H_s(t_e)+H_s(t_h)+\mathcal{A}=-\frac{1}{2}\int_{t_h}^{t_e}dt\;e^{\phi_s}\left(\rho_s+p_s\right)$$

- LHS positive definite: to have the transition from + to - must get RHS to be negative.
- If you start on (-) branch:

$$\left(\sqrt{N}+1\right)H_s(t_e)=H_s(t_h)+\mathcal{A}+\frac{1}{2}\int_{t_h}^{t_e}dt\;e^{\phi_s}\left(\rho_s+p_s\right)$$

- Branch change from - to + can occur with normal matter.
- But then all the - branch solutions are past-singular and + ones are future singular.
- This excludes the transitions proposed by Tseytlin and Vafa, and their applications to phenomenology by Brandenberger et al. This also excludes the loitering universe idea of Brandenberger et al, for positive energy.

- If energy is strictly positive, it is impossible to smoothly connect different branches.
- Impossible to remove the singularity from the EFT picture UNLESS one violates the null energy condition. Only then + to - transitions, (ie. evolution between saddle points) occur - and perhaps might be pushed past them by T-duality.
- In the standard framework where null energy condition is maintained such evolution does not happen and singularity is present somewhere.
- A variant of the Hawking-Penrose singularity theorem, which one expects in the Einstein frame!

- Thus: the claims about getting interesting cosmological perturbations from stringy gases are not reliable - there is no calculational control at present.
- So: are these frameworks completely useless?
  - Transitions from - to + may occur if $\rho < 0$ were allowed. May be of some interest, to probe the perturbations in a small universe limit, using T-duality.
  - Perturbations will break T-duality, but presumably weakly. Is there a way to explore perturbations at the `trans-planckian' (well, really `trans-stringy') scales using the T-dual?

So what do the ***phenomenological applications*** of stringy gases in cosmology – to date – look like?

I still don't know what the idea is!

It's about nothing!

I think you may have something there...

Summary

- Early universe in string theory: very important; it may yield new insights into the nature of initial conditions.
- Many cosmological problems: counting of relevant DOFs? Can string cosmology (gassy or not) help here?
- But user beware: thermodynamic description is only valid at LARGE SCALES compared to the string scale!
- An example: can't ignore the dilaton.
- Dilaton driven singularities may divide stages of stringy gases. There, no calculational control.
- Before phenomenology: must secure the dynamics required for the underlying backgrounds, as in other models which strive to bridge the singularities (pre-Big Bang, ekpyrotics etc)…