The Lee Wick Standard Model.

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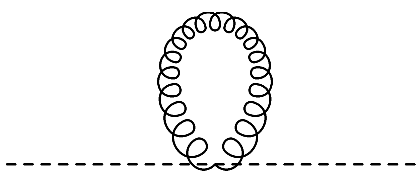
Outline

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	**Lorentz**
$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	3	2	1/6	(1/2,0)
u_R^i	3	1	2/3	(0,1/2)
d_R^i	3	1	-1/3	(0, 1/2)
$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$	1	2	-1/2	(1/2,0)
e_R^i	1	1	-1	(0,1/2)
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	1	2	1/2	(0,0)

The Hierarchy Problem

- Motivation for new physics beyond weak scale, Hierarchy Problem. SM Higgs potential: $V(H) = \lambda(H^\dagger H - v^2/2)^2$ implies vacuum expectation value $H^0 = v/\sqrt{2}$ physical Higgs scalar mass $m_h \sim \sqrt{\lambda}v$. $v/M_{PL} \sim 10^{-17}$.

- One loop radiative corrections from coupling to W boson induce ; $\delta m_h^2 \sim g_2^2 \Lambda_c^2 / 16\pi^2$.



- If Λ_c is of order the Planck scale then $\delta m_h^2 \sim (10^{18}\text{GeV})^2$. To keep v small requires a delicate fine tuning between bare parameters and radiative corrections.

- Some of solutions proposed: Dynamical Symmetry Breaking (Technicolor), Low Energy Supersymmetry, Large Extra Dimensions, Warped Extra Dimensions, Anthropic Principle A.

- Lee-Wick Standard Model presented in a paper with Donal O'Connell and Benjamin Grinstein Phys. Rev. D**77**:025012 (2008). It builds on ideas first presented in two papers, T. D. Lee and G. C. Wick, Nucl Phys B**9**, 209 (1969) and T.D. Lee and G. C. Wick, Phys. Rev. D**2**, 1033 (1970). Paradigm shift not causal.

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A Toy Model

- Lagrange density

$$\mathcal{L}_{\text{hd}} = \frac{1}{2}\partial_\mu\widehat{\phi}\partial^\mu\widehat{\phi} - \frac{1}{2M^2}(\partial^2\widehat{\phi})^2 - \frac{1}{2}m^2\widehat{\phi}^2 - \frac{1}{3!}g\widehat{\phi}^3,$$

propagator of $\widehat{\phi}$

$$\widehat{D}(p) = \frac{i}{p^2 - p^4/M^2 - m^2}.$$

$M \gg m$, poles at $p^2 \simeq m^2$ and also at $p^2 \simeq M^2$. More than one degree of freedom. Residue of $p^2 \simeq M^2$ wrong sign. Unstable.

- Auxiliary Lee Wick scalar field $\tilde{\phi}$. Write the theory as

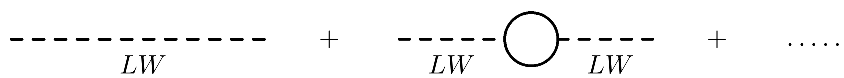
$$\mathcal{L} = \frac{1}{2}\partial_\mu \hat{\phi}\partial^\mu \hat{\phi} - \frac{1}{2}m^2\hat{\phi}^2 - \tilde{\phi}\partial^2\hat{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 - \frac{1}{3!}g\hat{\phi}^3.$$

  Removing $\tilde{\phi}$ from $\mathcal{L}$ with their equations of motion reproduces $\mathcal{L}_{\mathrm{hd}}$.

- Define $\phi = \hat{\phi} + \tilde{\phi}$ to remove $\tilde{\phi}$-$\hat{\phi}$ kinetic mixing.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 - \frac{1}{2}m^2(\phi-\tilde{\phi})^2 - \frac{1}{3!}g(\phi-\tilde{\phi})^3.$$

  Two kinds of scalar field: a normal scalar field ϕ and a new LW-field $\tilde{\phi}$. $\tilde{\phi}$ can decay to two ϕ's. Lee Wick resonance, calculate its decay width from two-point function of $\tilde{\phi}$.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- Bubble sum,

$$\begin{aligned}\tilde{D}(p) &= \frac{-i}{p^2 - M^2} + \frac{-i}{p^2 - M^2}(-i\Sigma(p^2))\frac{-i}{p^2 - M^2} + \cdots \\ &= \frac{-i}{p^2 - M^2 + \Sigma(p^2)}.\end{aligned}$$

- $(-i\Sigma(p^2))$ same as usual but extra minus sign with each $(-i\Sigma(p^2))$. Width,

$$\Gamma = -\frac{g^2}{32\pi M}\sqrt{1 - \frac{4m^2}{M^2}}.$$

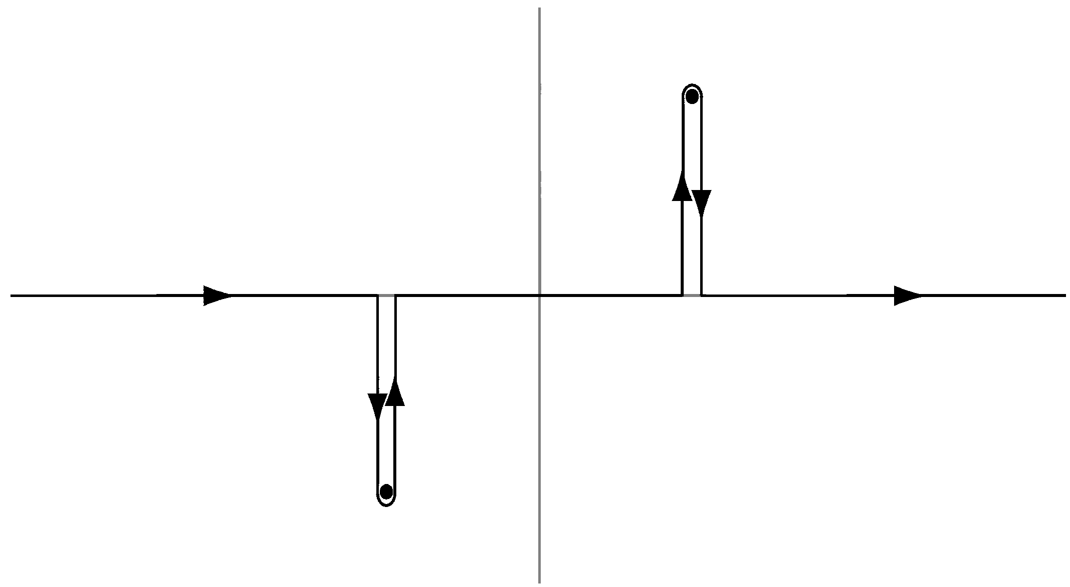
- Not narrow width approximation normal resonance

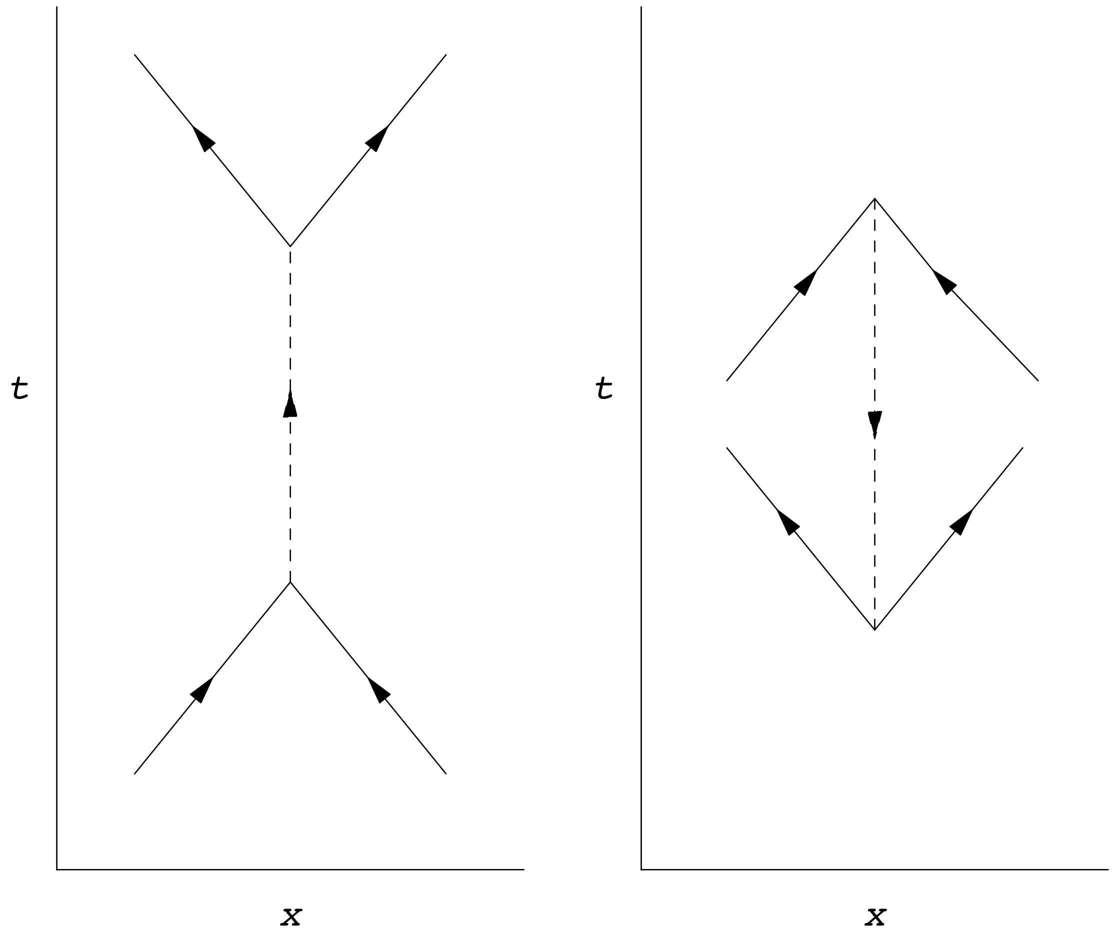
$$D_\chi(p^2) = \frac{i}{\pi}\int_{4m^2}^{\infty} ds \frac{\rho(s)}{p^2 - s + i\epsilon} \simeq \frac{i}{p^2 - M^2 + iM\Gamma}.$$

- Not narrow width approximation Lee Wick resonance

$$D_{\tilde{\phi}}(p^2) = \frac{-i}{p^2 - M_c^2} + \frac{-i}{p^2 - M_c^{*2}} + \frac{i}{\pi}\int_{4m^2}^{\infty} ds \frac{\rho(s)}{p^2 - s + i\epsilon}$$

where $M_c^2 = M^2 + iM\Gamma$.

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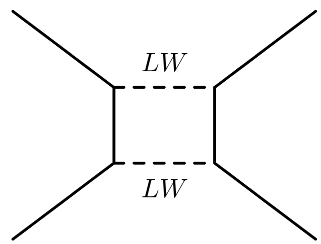
LW
LW

- For loop diagrams we must understand how to define expressions such as

$$I = \int \frac{d^4p}{(2\pi)^4} \frac{-i}{(p+q)^2 - M_1^2} \frac{-i}{p^2 - M_2^2}$$

where M_1 and M_2 may be complex masses, either in the upper or lower half plane of the Feynman integration.

- Let us consider the p^0 integral. The integrand has four poles. For time-like q we can go to a frame where $\vec{q} = 0$ and the poles are located at,

$$p^0 = \pm\sqrt{\vec{p}^2 + M_2^2}, \qquad p^0 = -q^0 \pm \sqrt{\vec{p}^2 + M_1^2}$$

- At $g = 0$ the widths vanish, so M_1 and M_2 are real masses. Then contour is defined to be the usual Feynman contour. As g increases, we define the contour so that the poles do not cross the contour; a pole which was initially below the contour remains below the contour, for example. Leads to a well-defined contour that can be Wick rotated unless poles pinch the contour. Pinching can occur because we have seen that a pole at M_1^2 is accompanied by another pole at M_1^{*2} - that is, we could have $M_2^2 = M_1^{*2}$ and for some q^0 two poles overlap. In this case, Cutkowsky et al., define integral by taking the masses M_1^2 and M_1^{*2} to be unrelated complex mass parameters so that the poles don't overlap. At the end of the calculation the condition that M_2^2 is the complex conjugate of M_1^2 is imposed. Leads to Unitary scattering amplitudes.

LW-Gauge Bosons and the Hierarchy Problem

- Higher derivative Lagrange density

$$\mathcal{L}_{\text{hd}} = -\frac{1}{2}\text{tr}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \frac{1}{M_A^2}\text{tr}\left(\hat{D}^{\mu}\hat{F}_{\mu\nu}\right)\left(\hat{D}^{\lambda}\hat{F}_{\lambda}{}^{\nu}\right),$$

where $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - ig[\hat{A}_\mu, \hat{A}_\nu]$, and $\hat{A}_\mu = \hat{A}_\mu^A T^A$ with T^A the generators of the gauge group G in the fundamental representation. Choose this higher derivative term because can remove it using LW-field $\tilde{A}_\mu$.

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$$\mathcal{L} = -\frac{1}{2}\text{tr}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - M_A^2\text{tr}\tilde{A}_\mu\tilde{A}^\mu + 2\text{tr}\hat{F}_{\mu\nu}\hat{D}^\mu\tilde{A}^\nu,$$

where $\hat{D}_\mu\tilde{A}_\nu = \partial_\mu\tilde{A}_\nu - ig[\hat{A}_\mu, \tilde{A}_\nu]$.

- To diagonalize the kinetic terms, we introduce shifted fields defined by

$$\widehat{A}_\mu = A_\mu + \tilde{A}_\mu.$$

The Lagrangian becomes

$$\mathcal{L} = -\frac{1}{2}\mathrm{tr} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\mathrm{tr}\left(D_\mu \tilde{A}_\nu - D_\nu \tilde{A}_\mu\right)\left(D^\mu \tilde{A}^\nu - D^\nu \tilde{A}^\mu\right)$$
$$-ig\,\mathrm{tr}\left(\left[\tilde{A}_\mu, \tilde{A}_\nu\right] F^{\mu\nu}\right) - \frac{3}{2}g^2\mathrm{tr}\left(\left[\tilde{A}_\mu, \tilde{A}_\nu\right]\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\right)$$
$$-4ig\,\mathrm{tr}\left(\left[\tilde{A}_\mu, \tilde{A}_\nu\right] D^\mu \tilde{A}^\nu\right) - M_A^2\mathrm{tr}\left(\tilde{A}_\mu \tilde{A}^\mu\right).$$

- Only dimension four operators but longitudinal massive vector bosons can cause bad high energy behavior. Standard model with no Higgs but mass terms for W-bosons $\mathcal{A}(LL \to LL) \sim g^2E^2/M^2$. Arises from $\epsilon_L(p) = (|\mathbf{p}|, E(p)\hat{\mathbf{p}})/M$.

- For Lee-Wick massive gauge bosons no bad high energy behavior of scattering amplitudes at tree level $\mathcal{A}(LL \to LL) \sim g^2 M^2/E^2$, $\mathcal{A}(LL \to LT) \sim g^2 M/E$, $\mathcal{A}(LL \to TT) \sim g^2$.

- To discuss hierarchy problem add scalar multiplet in fundamental representation

$$\mathcal{L}_{\mathrm{hd}} = \left(\hat{D}_\mu \hat{\phi}\right)^\dagger \left(\hat{D}^\mu \hat{\phi}\right) - \frac{1}{M_\phi^2} \left(\hat{D}_\mu \hat{D}^\mu \hat{\phi}\right)^\dagger \left(\hat{D}_\nu \hat{D}^\nu \hat{\phi}\right).$$

- $\hat{\phi} = \phi - \tilde{\phi}$

$$\begin{aligned}\mathcal{L} &= (D_\mu \phi)^\dagger D^\mu \phi - (D_\mu \tilde{\phi})^\dagger D^\mu \tilde{\phi} + M_\phi^2 \tilde{\phi}^\dagger \tilde{\phi} \\ &+ ig(D^\mu \phi)^\dagger \tilde{A}_\mu^A T^A \phi + g^2 \phi^\dagger \tilde{A}_\mu^A T^A \tilde{A}^{B\mu} T^B \phi - ig\phi^\dagger \tilde{A}_\mu^A T^A D^\mu \phi \\ &- ig(D^\mu \tilde{\phi})^\dagger \tilde{A}_\mu^A T^A \tilde{\phi} + ig\tilde{\phi}^\dagger \tilde{A}_\mu^A T^A D^\mu \tilde{\phi} - g^2 \tilde{\phi}^\dagger \tilde{A}_\mu^A T^A \tilde{A}^{B\mu} T^B \tilde{\phi}.\end{aligned}$$

- Work in higher derivative theory and fix gauge in usual way. Gauge boson propagator

$$\hat{D}^{AB}_{\mu\nu}(p) = \delta^{AB}\frac{-i}{p^2 - p^4/M_A^2}\left(\eta_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2} - \xi\frac{p_\mu p_\nu}{M_A^2}\right).$$

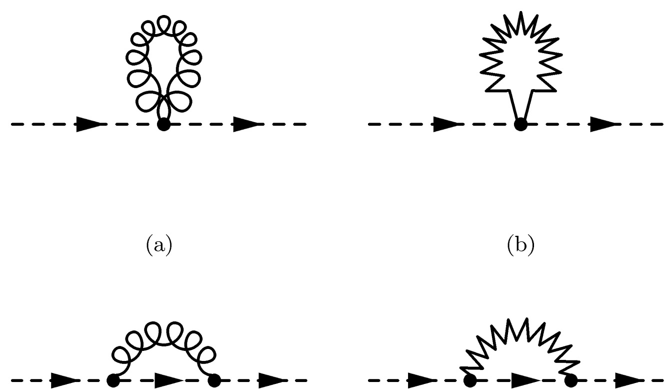
  No quadratic divergence in theory in Landau $\xi = 0$ gauge; $d = 6 - 2L - E - E' - 2E_g$.

- Explicit one loop calculation in LW-version of theory with no higher derivative terms. Propagators for gauge bosons,

$$D^{AB}_{\mu\nu}(p) = -\delta^{AB}\frac{i}{p^2}\left(\eta_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right),$$

  while the propagator for the LW-vector field is

$$\tilde{D}^{AB}_{\mu\nu}(p) = \delta^{AB}\frac{i}{p^2 - M_A^2}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_A^2}\right).$$



(a) (b)

(c) (d)

- One loop scalar mass corrections

$$-i\Sigma_a(0) = g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \frac{n}{k^2}$$
$$-i\Sigma_b(0) = -g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \left(\frac{n-1}{k^2 - M_A^2} - \frac{1}{M_A^2} \right)$$
$$-i\Sigma_c(0) = -g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2}$$
$$-i\Sigma_d(0) = -g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \frac{1}{M_A^2}.$$

- $$\delta m_h^2 \sim \frac{g^2 M_{\mathrm{LW}}^2}{16\pi^2} \ln \left(\frac{\Lambda^2}{M_{\mathrm{LW}}^2} \right).$$

- Magnitude of LW masses for $S(2) \times U(1)$ constrained by precision electroweak physics. Self energies in higher derivative formulation. Neglect Higgs vev

$$\Pi(q^2) = q^2 - \frac{q^4}{M_{1,2}^2}$$

Parameters Y and W defined by

$$Y = \frac{M_W^2}{2}\Pi''_{BB}(0) \quad W = \frac{M_W^2}{2}\Pi''_{33}(0).$$

LW-Standard Model at tree level

$$Y = -\frac{M_W^2}{M_1^2}, \qquad W = -\frac{M_W^2}{M_2^2}$$

Precision electroweak data imply at 99% confidence level when $M_1 = M_2$ that $M_{1,2} > 3\mathrm{TeV}$. When one of masses much greater than other find $M_2(M_1) > 2(2.5)$ TeV .

Fermions

- $$\mathcal{L}_{\text{hd}} = \overline{\widehat{Q}}_L i\widehat{\not{D}}\widehat{Q}_L + \frac{1}{M_Q^2}\overline{\widehat{Q}}_L i\widehat{\not{D}}\widehat{\not{D}}\widehat{\not{D}}\widehat{Q}_L.$$

- Eliminate the higher derivative term by introducing LW-quark doublets $\tilde{Q}_L$, $\tilde{Q}'_R$ which form a real representation of the gauge groups. Lagrangian in this formulation becomes

$$\mathcal{L} = \overline{\widehat{Q}}_L i\widehat{\not{D}}\widehat{Q}_L + M_Q\left(\overline{\tilde{Q}}_L\tilde{Q}'_R + \overline{\tilde{Q}'}_R\tilde{Q}_L\right)$$
$$+\overline{\tilde{Q}}_L i\widehat{\not{D}}\widehat{Q}_L + \overline{\widehat{Q}}_L i\widehat{\not{D}}\tilde{Q}_L - \overline{\tilde{Q}'}_R i\widehat{\not{D}}\tilde{Q}'_R.$$

Remove LW-fermions with their equations of motion

$$\tilde{Q}'_R = -\frac{i\widehat{\not{D}}}{M_Q}\widehat{Q}_L, \quad \tilde{Q}_L = \frac{\widehat{\not{D}}\widehat{\not{D}}}{M_Q^2}\widehat{Q}_L,$$

- To diagonalize the kinetic terms, we introduce the shift $\widehat{Q}_L = Q_L - \tilde{Q}_L$, and the Lagrangian becomes

$$\mathcal{L} = \overline{Q}_L i \not{D} Q_L - \overline{\tilde{Q}}_L i \not{D} \tilde{Q}_L - \overline{\tilde{Q}'}_R i \not{D} \tilde{Q}'_R + M_Q \left(\overline{\tilde{Q}}_L \tilde{Q}'_R + \overline{\tilde{Q}'}_R \tilde{Q}_L \right)$$
$$- \overline{Q_L} \gamma_\mu \tilde{\mathbf{A}}^\mu Q_L + \overline{\tilde{Q}_L} \gamma_\mu \tilde{\mathbf{A}}^\mu \tilde{Q}_L + \overline{\tilde{Q}'_R} \gamma_\mu \tilde{\mathbf{A}}^\mu \tilde{Q}'_R .$$

- New FCNC but small.

Conclusion

PLEASE LET THE LHC WORK