Descriptive Statistics

Elements of Probability

Random Variables and Expectation

Random Variables

Définition. *Quantities determined by the result of an experiment. They have probabilityy distributions.*

Types of Random Variables

Discrete

Définition. *For a discrete random variable X, its probability mass function is*

$$p(a) = P\{X = a\}$$

Additionally,

$$p(x) \geq 0$$
$$\sum_{i=1}^{\infty} p(x_i) = 1$$

Définition. *The cumulative distribution function F is*

$$F(a) = \sum_{x \leq a} p(x)$$

Continuous

Définition. *For a continuous random variable X, its probability density function is*

$$f : \mathbb{R} \to [0, \infty]$$

$$f(x) = P\{X = x\}$$

Additionally,

$$f(x) \geq 0 \forall x$$
$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Définition. *The cumulative distribution function F is*

$$F(x) = P\{X \leq x\} = \int_{-\infty}^{\infty} f(x)dx$$

Jointly Distributed Random Variables

Let X, Y be random variables. The joint CDF of X and Y is

$$F(x, y) = P(X \leq x, Y \leq y), x, y \in \mathbb{R}$$

Note: X and Y can be both discrete or continuous.
If X, Y are both discrete, then the joint pmf is

$$p(x, y) = P(X = x, Y = y)$$

If X, Y are both continuous, then the joint pmf is

$$f(x, y) : \mathbb{R}^2 \to [0, \infty]$$
$$f(x, y) \geq 0 \forall x, y$$
$$\int \int f(x, y)dxdy = 1$$

We call the (cdf, pmf, pdf) of X and Y the marginal (cdf, pmf, pdf).

Independent R.V.s

Independence
Two random variables X,Y are said to be independent if

$$P(X \leq x, Y \leq y) = P(X = x) \times P(Y = y) \forall x, y$$

If X and Y are discrete, X and Y are independent if

$$P(X = x, Y = y) = P(X = x)P(Y = y)$$

If X and Y are continuous, X and Y are independent if

$$f(x, y) = f_X(x)f_Y(y)$$

Expectation

Expectation = E[X] = where does X take values? (location parameter)

X is discrete

$$E[X] = \sum_{\text{t possible values}} tP(X = t)$$

X is continuous

$$E[X] = \int_{-\infty}^{\infty} tf(t)dt$$

Properties of the Expected Value

Expected Value of Sums of Random Variables

Variance

Variance = Var(X) = how scattered are the values? (scale parameter)

$$Var(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

Covariance and Variance of Sums of R.V.s

Moment Generating Functions