

$$\begin{aligned} & \vdash \Box \Diamond F \Leftrightarrow \Box \Diamond \neg \Box F \\ & \equiv \Box \models (\Diamond \Box \Diamond F \Leftrightarrow \Box \Diamond F) \\ & \equiv (\Box \models \Diamond \Box \Diamond F) \Leftrightarrow (\Box \models \Box \Diamond F) \end{aligned}$$

$$\begin{aligned} & \equiv \dots \exists m \in \mathbb{N} : \forall n \in \mathbb{N} : \exists k \in \mathbb{N} : \sigma^{(n+n+k)} \models F \\ & \Leftrightarrow \forall n \in \mathbb{N} : \exists k \in \mathbb{N} : \sigma^{(n+k)} \models F \\ & \equiv \text{TRUE} \end{aligned}$$

$$\begin{aligned} \text{c) } & \Box [A \wedge B]_{\forall} \Leftrightarrow \Box [A]_{\forall} \wedge \Box [B]_{\forall} \\ & \equiv \sigma \models (\Box [A \wedge B]_{\forall}) \\ & \equiv \forall n \in \mathbb{N} : \sigma^{+n} \models A \wedge B \\ & \equiv \forall n \in \mathbb{N} : (\sigma^{+n} \models A) \wedge (\sigma^{+n} \models B) \\ & \equiv \forall n \in \mathbb{N} : \Box [A]_{\forall} \wedge \Box [B]_{\forall} \\ & \Rightarrow \text{It is tautology.} \end{aligned}$$

$$\begin{aligned}
 b) & \neg \Box (F \vee G) \Rightarrow \Diamond \neg F \wedge \Diamond \neg G \\
 & \equiv \neg \Box (F \vee G) \Rightarrow \Diamond \neg (F \vee G) \Rightarrow \Diamond \neg F \wedge \Diamond \neg G \\
 & \equiv \Box \vdash (\neg \Box (F \vee G)) \\
 & \equiv \neg (\forall n \in \mathbb{N} : \sigma^{+n} \models F \vee G) \\
 & \equiv \exists n \in \mathbb{N} : \neg (\sigma^{+n} \models F \vee G) \\
 & \Rightarrow \text{It is not tautology}
 \end{aligned}$$

Example: If $\Box F$ is False and $\Box G$ is False. \neg
 The $\Box \vdash \neg \Box (F \vee G)$ is TRUE but
 $\Box \vdash \Diamond \neg F \wedge \Diamond \neg G$ is False and
 ~~$\Box \vdash \neg \Box (F \vee G)$~~

$$d) \Box \Diamond (A \wedge B) \vee \Box \Diamond A \vee \Box \Diamond B \quad (\text{Note})$$

$$\begin{aligned}
 & \Box \vdash (\Box \Diamond (A \wedge B)) \\
 & \equiv \forall n \in \mathbb{N} : \sigma^{+n} \models \Box \Diamond (A \wedge B) \\
 & \equiv \forall n \in \mathbb{N} : \exists m \in \mathbb{N} : \sigma^{+(n+m)} \models (A \wedge B) \\
 & \equiv \Box \vdash \Box \Diamond A \vee \Box \Diamond B \\
 & \equiv \Box [A \wedge B] \\
 & \equiv \Box A \vee \Box B \\
 & \Rightarrow \text{It is tautology}
 \end{aligned}$$