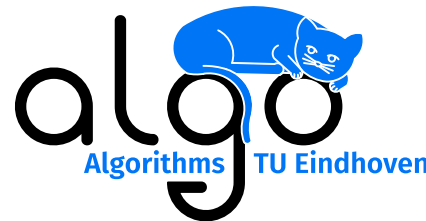


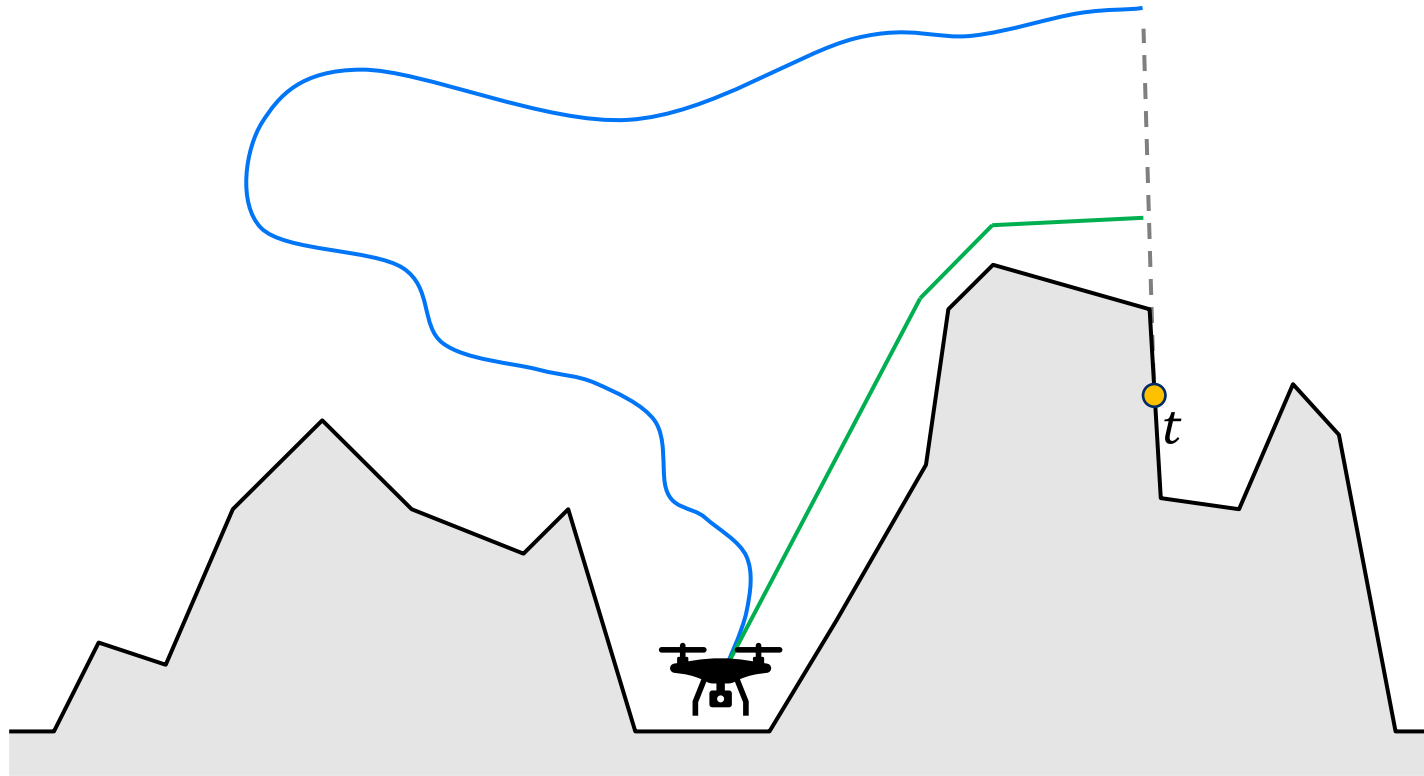
Competitive Searching over Terrains

Sarita de Berg
Utrecht University

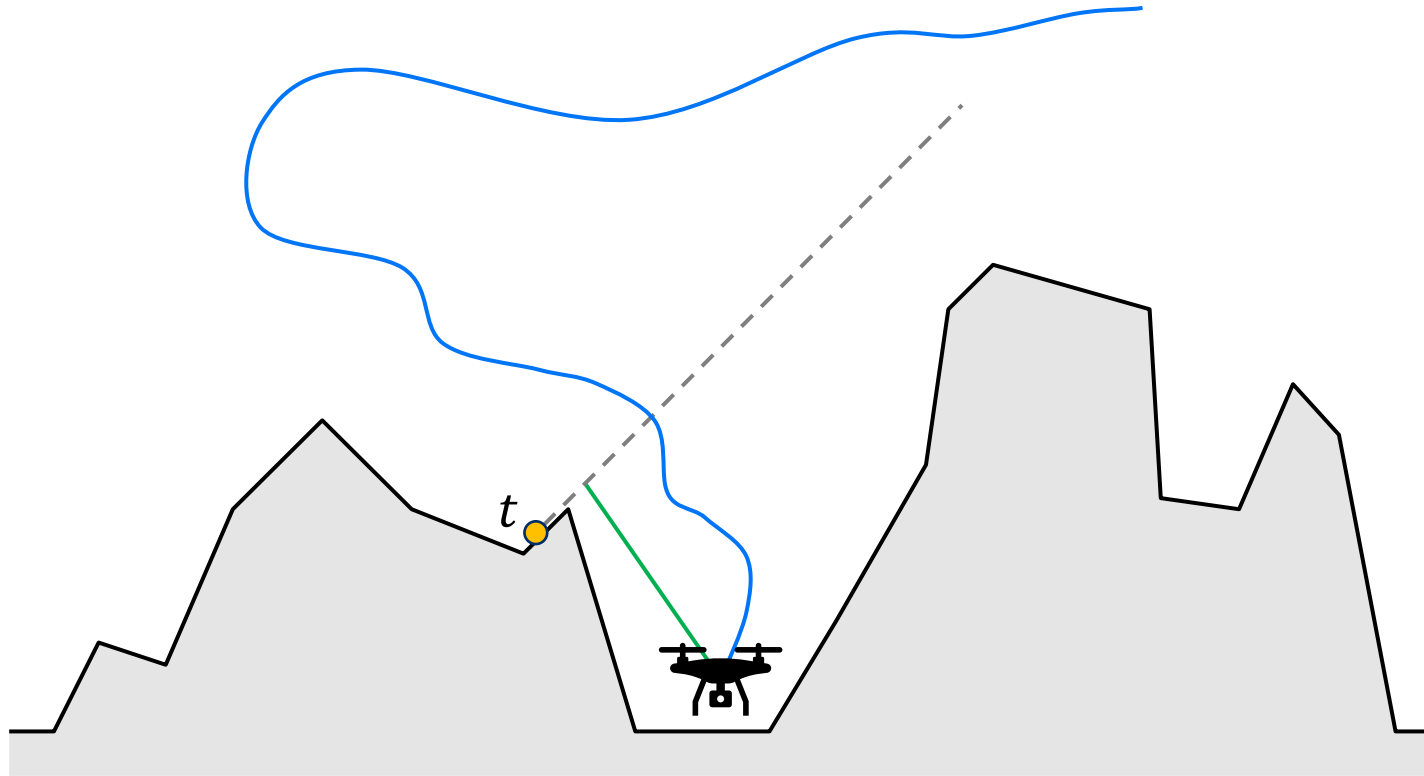
Nathan van Beusekom, Max van Mulken,
Kevin Verbeek, Jules Wulms
TU Eindhoven



Searching over Terrain



Searching over Terrain



Competitive Ratio c

Considers worst-case instance

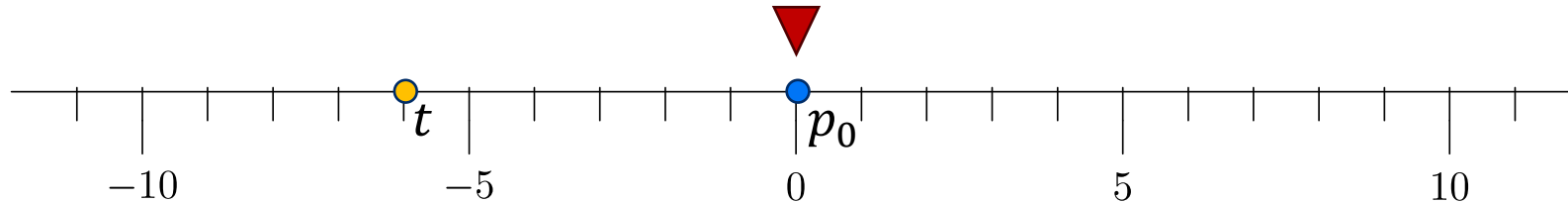
$$c = \max \frac{|\text{Strategy Path}|}{|\text{Shortest Path}|}$$

Goal: minimize c

Competitive Searching on a Line

Line with unit distances

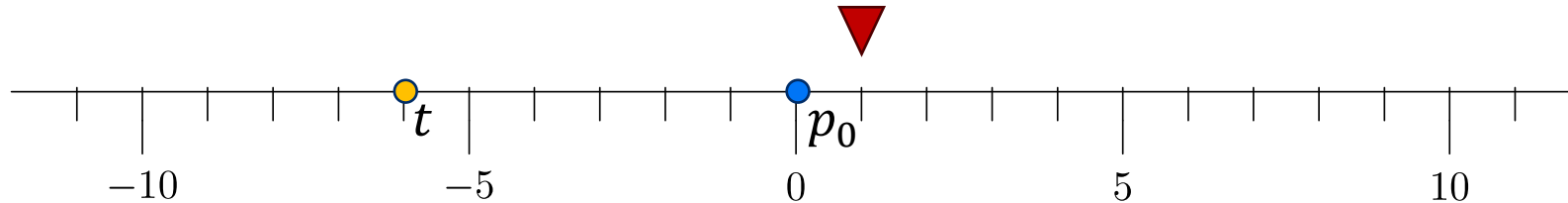
Doubling strategy (Beck & Newman, 1970)



Competitive Searching on a Line

Line with unit distances

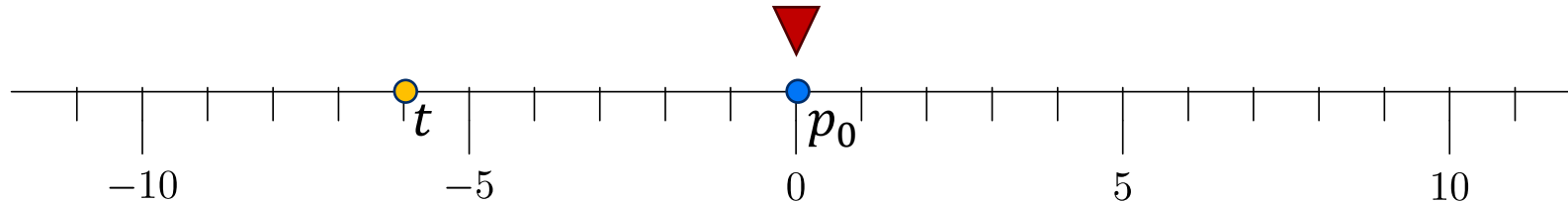
Doubling strategy (Beck & Newman, 1970)



Competitive Searching on a Line

Line with unit distances

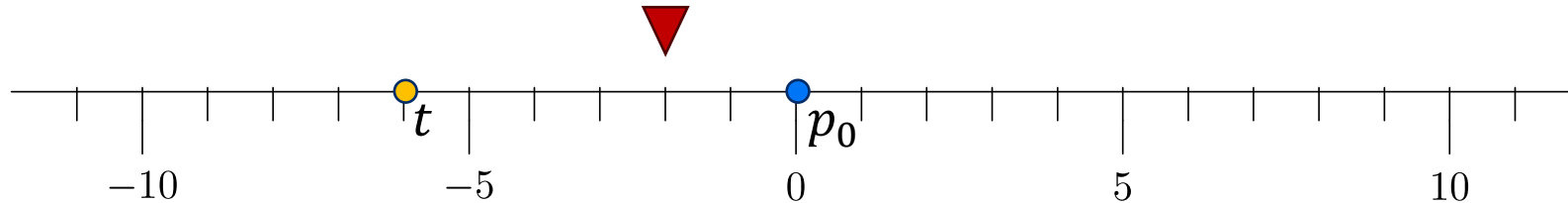
Doubling strategy (Beck & Newman, 1970)



Competitive Searching on a Line

Line with unit distances

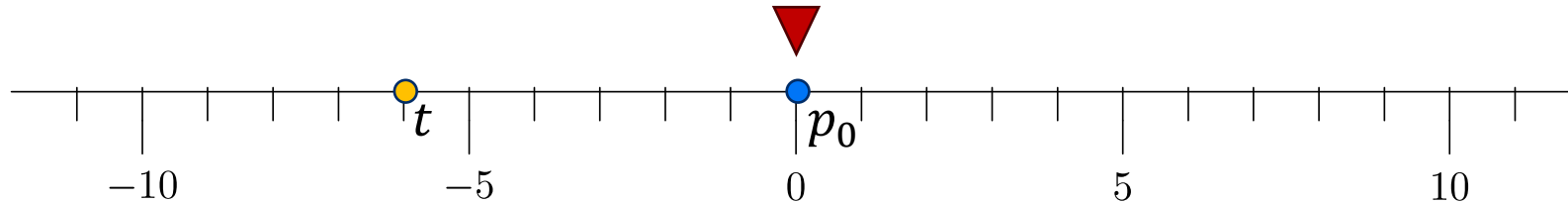
Doubling strategy (Beck & Newman, 1970)



Competitive Searching on a Line

Line with unit distances

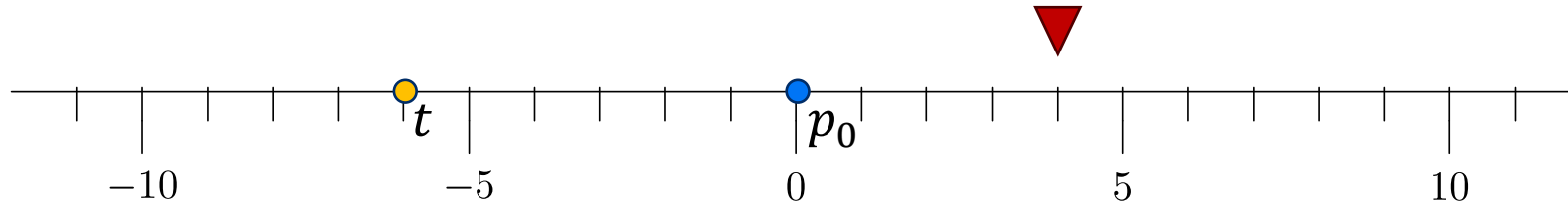
Doubling strategy (Beck & Newman, 1970)



Competitive Searching on a Line

Line with unit distances

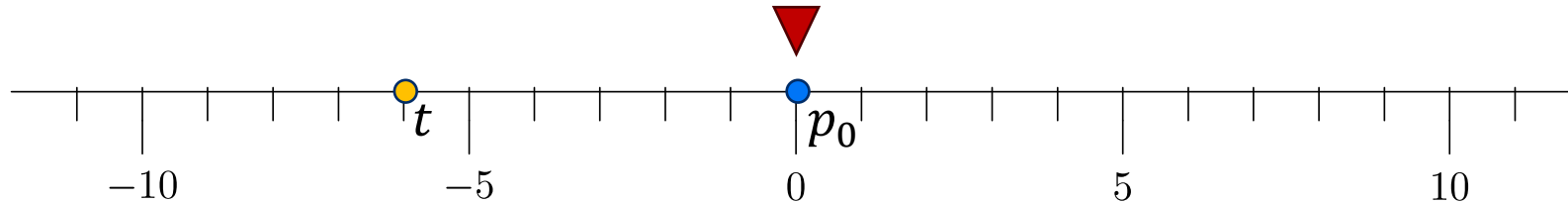
Doubling strategy (Beck & Newman, 1970)



Competitive Searching on a Line

Line with unit distances

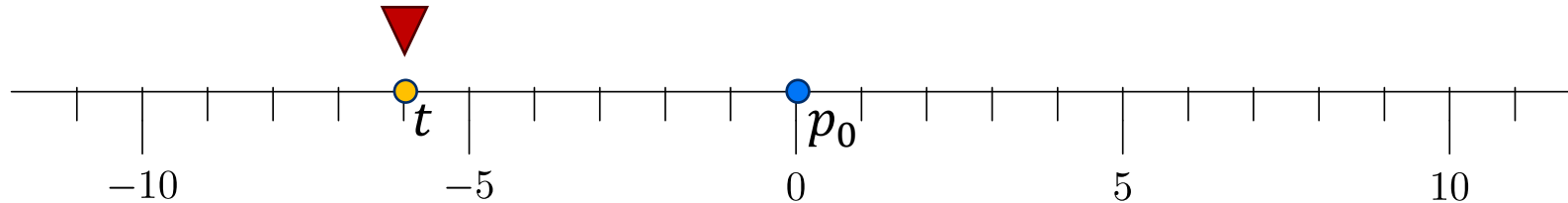
Doubling strategy (Beck & Newman, 1970)



Competitive Searching on a Line

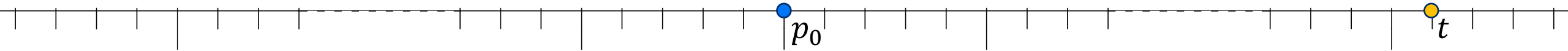
Line with unit distances

Doubling strategy (Beck & Newman, 1970)



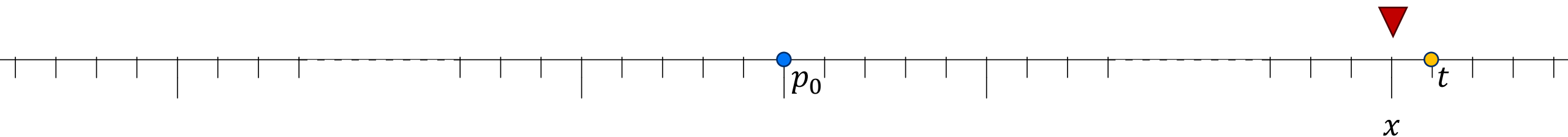
Competitive Searching on a Line

Competitive ratio?



Competitive Searching on a Line

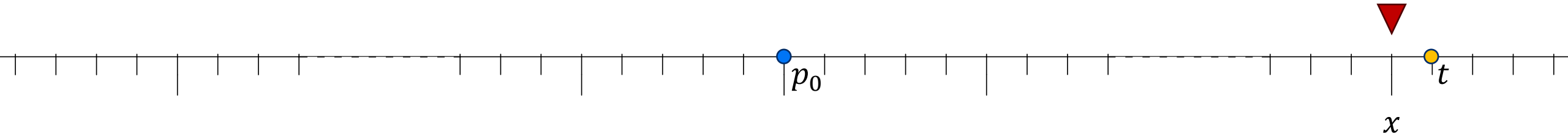
Competitive ratio?



Competitive Searching on a Line

Competitive ratio?

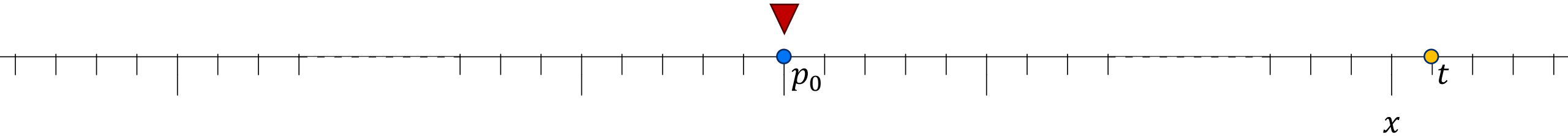
Distance traveled: $3x$



Competitive Searching on a Line

Competitive ratio?

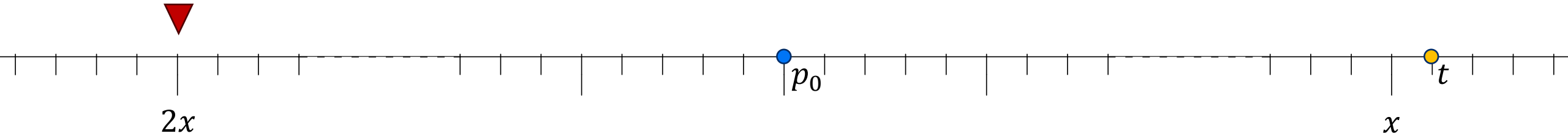
Distance traveled: $4x$



Competitive Searching on a Line

Competitive ratio?

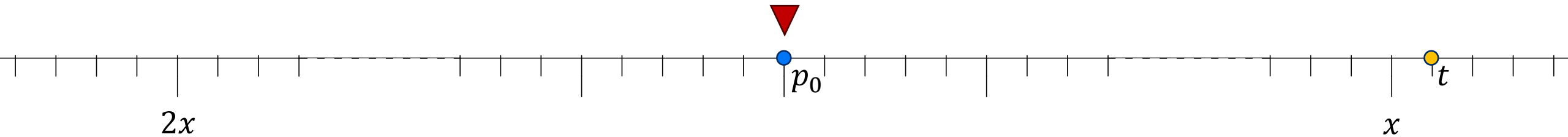
Distance traveled: $6x$



Competitive Searching on a Line

Competitive ratio?

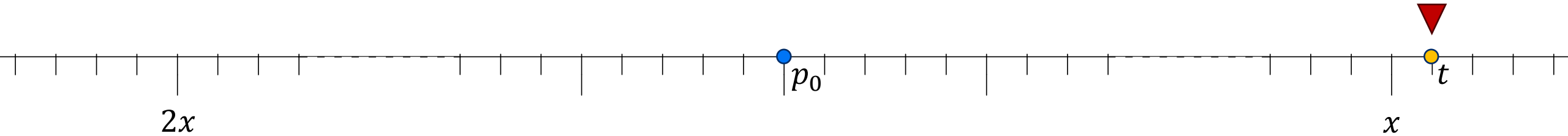
Distance traveled: $8x$



Competitive Searching on a Line

Competitive ratio?

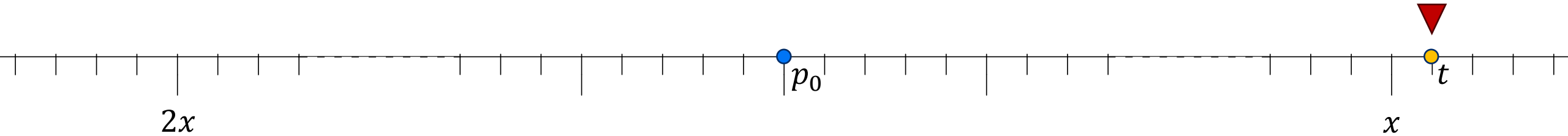
Distance traveled: $9x + 1$



Competitive Searching on a Line

Competitive ratio?

Competitive ratio: $\frac{9x+1}{x+1}$

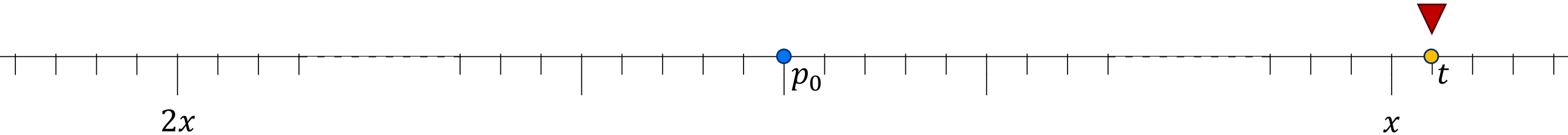


Competitive Searching on a Line

Competitive ratio?

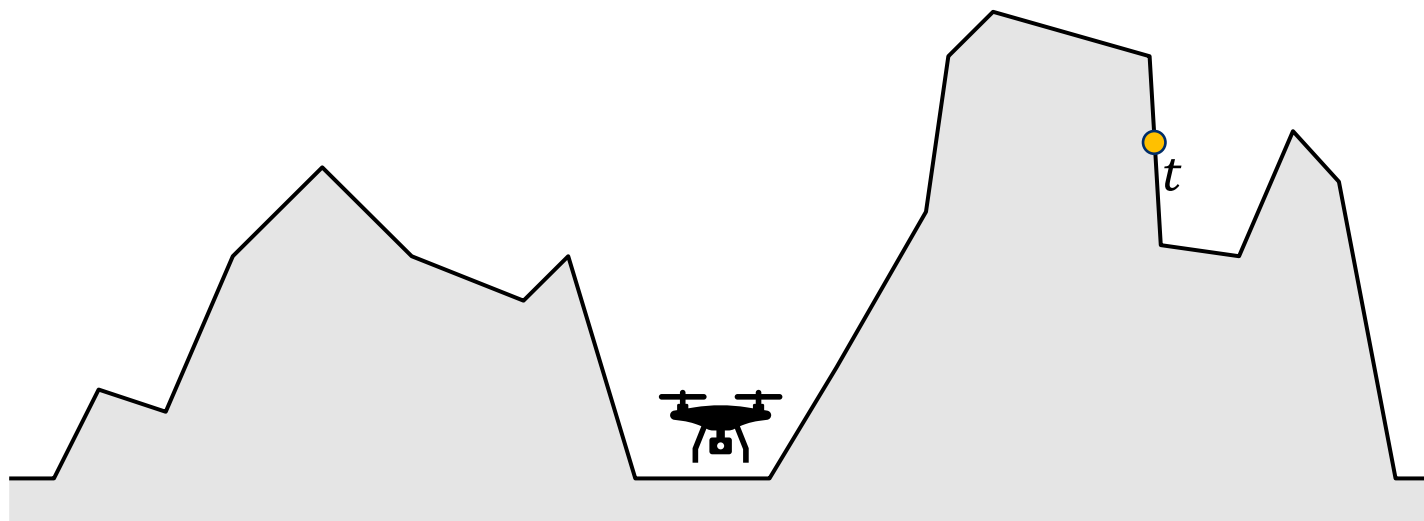
Competitive ratio: $\lim_{x \rightarrow \infty} \frac{9x+1}{x+1} = 9$

Optimal



Competitive Searching over 1.5D Terrain

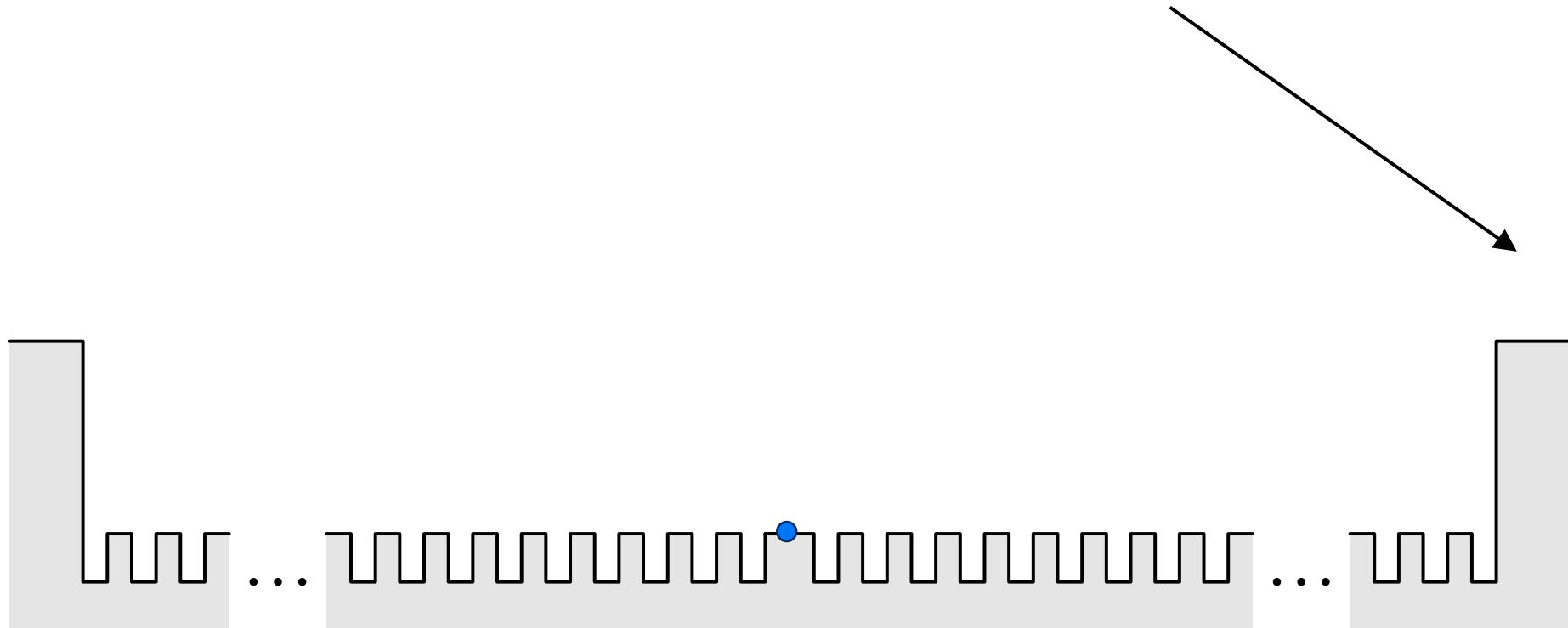
1-dimensional height function



1.5D Terrains – Lower Bound



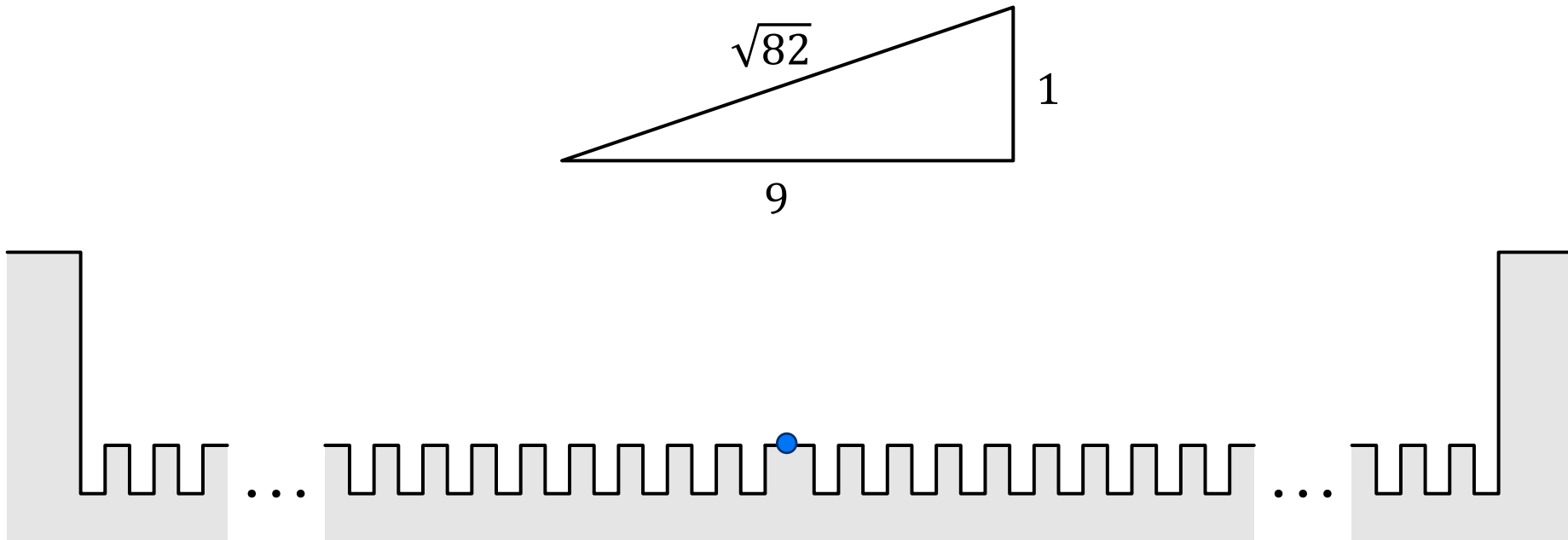
1.5D Terrains – Lower Bound



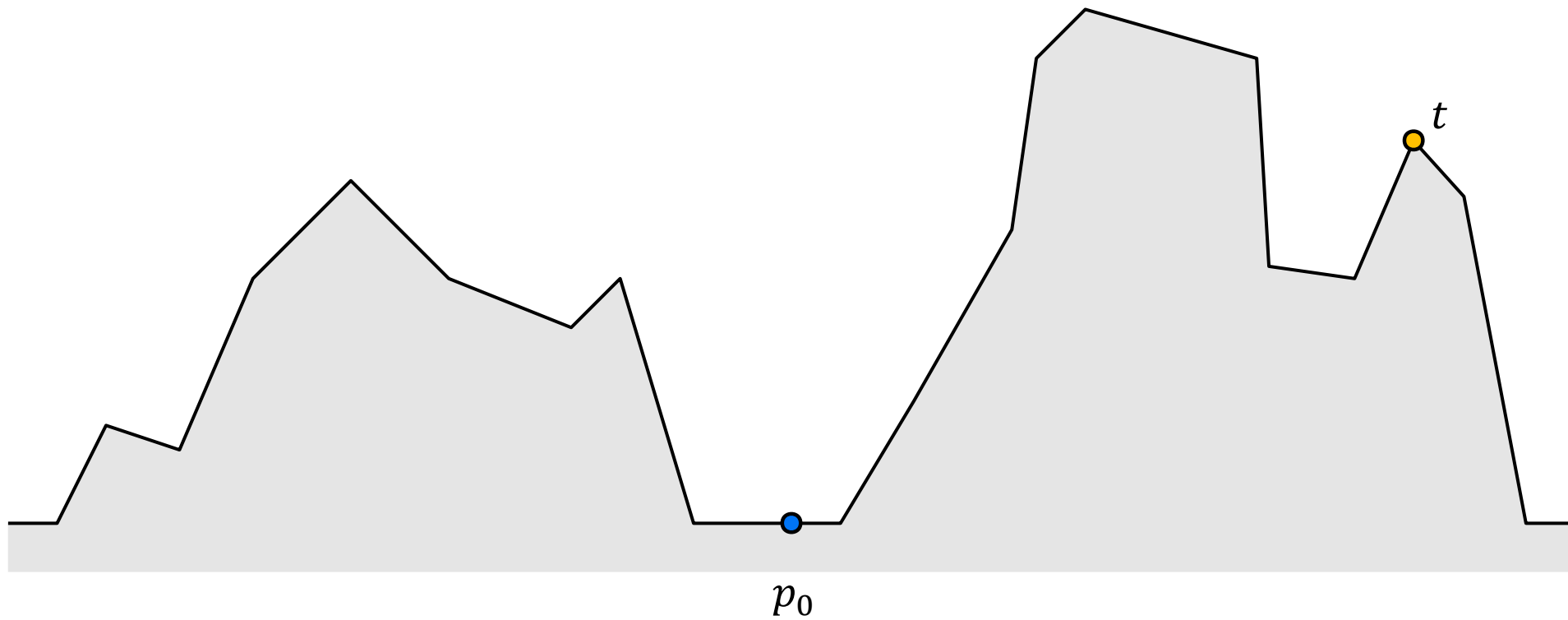
1.5D Terrains – Lower Bound

Lower bound for any searching strategy:

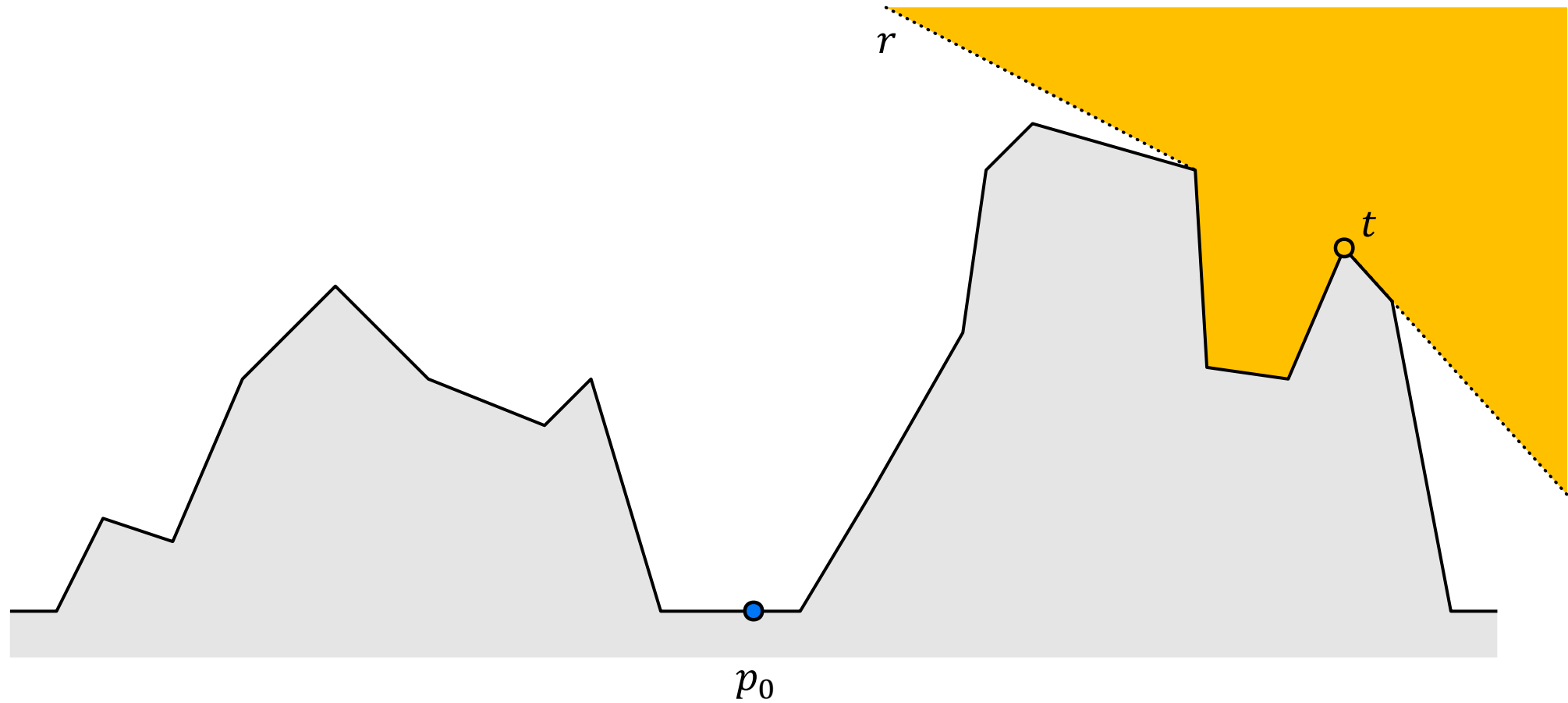
$$c \geq \sqrt{82}$$



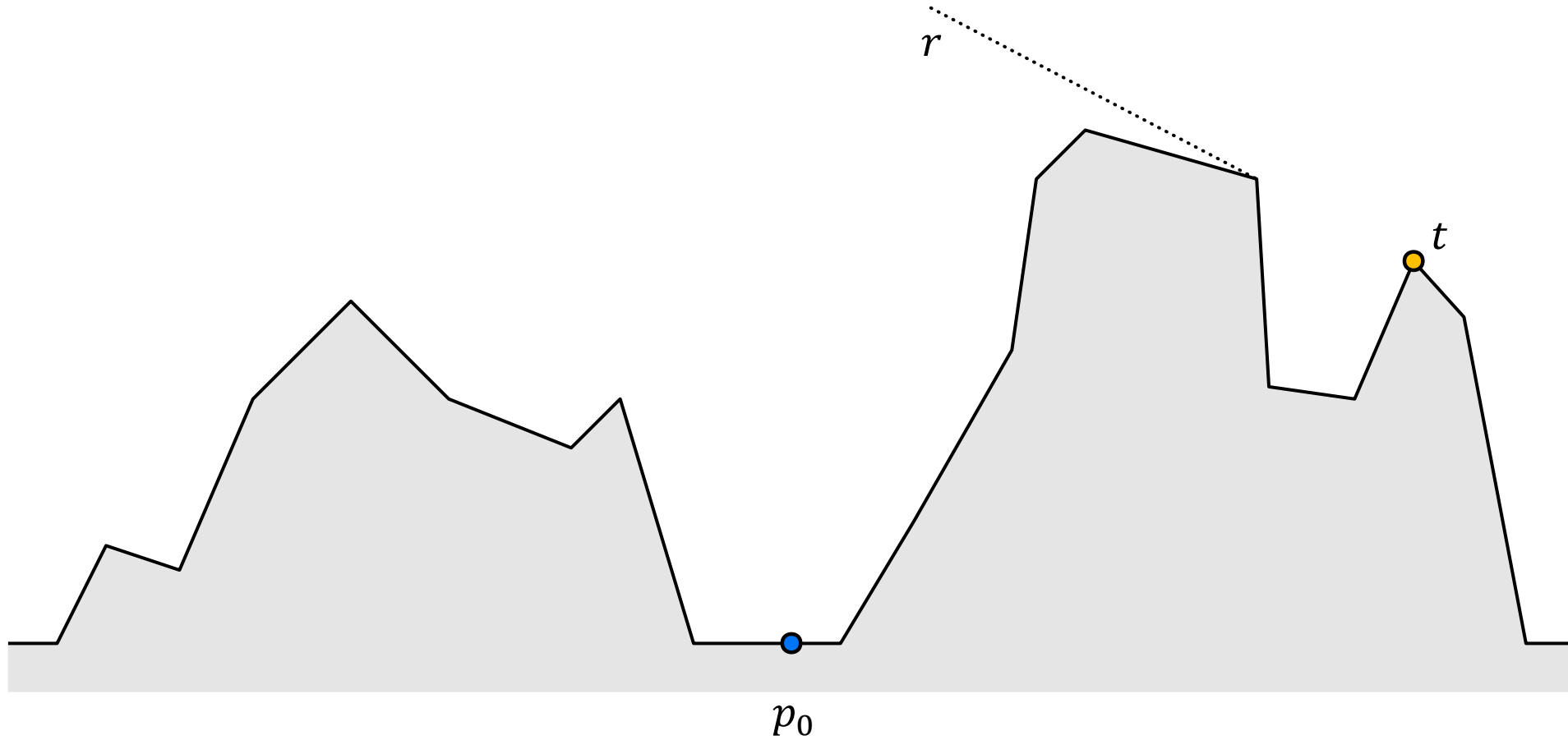
1.5D Terrains – Strategy



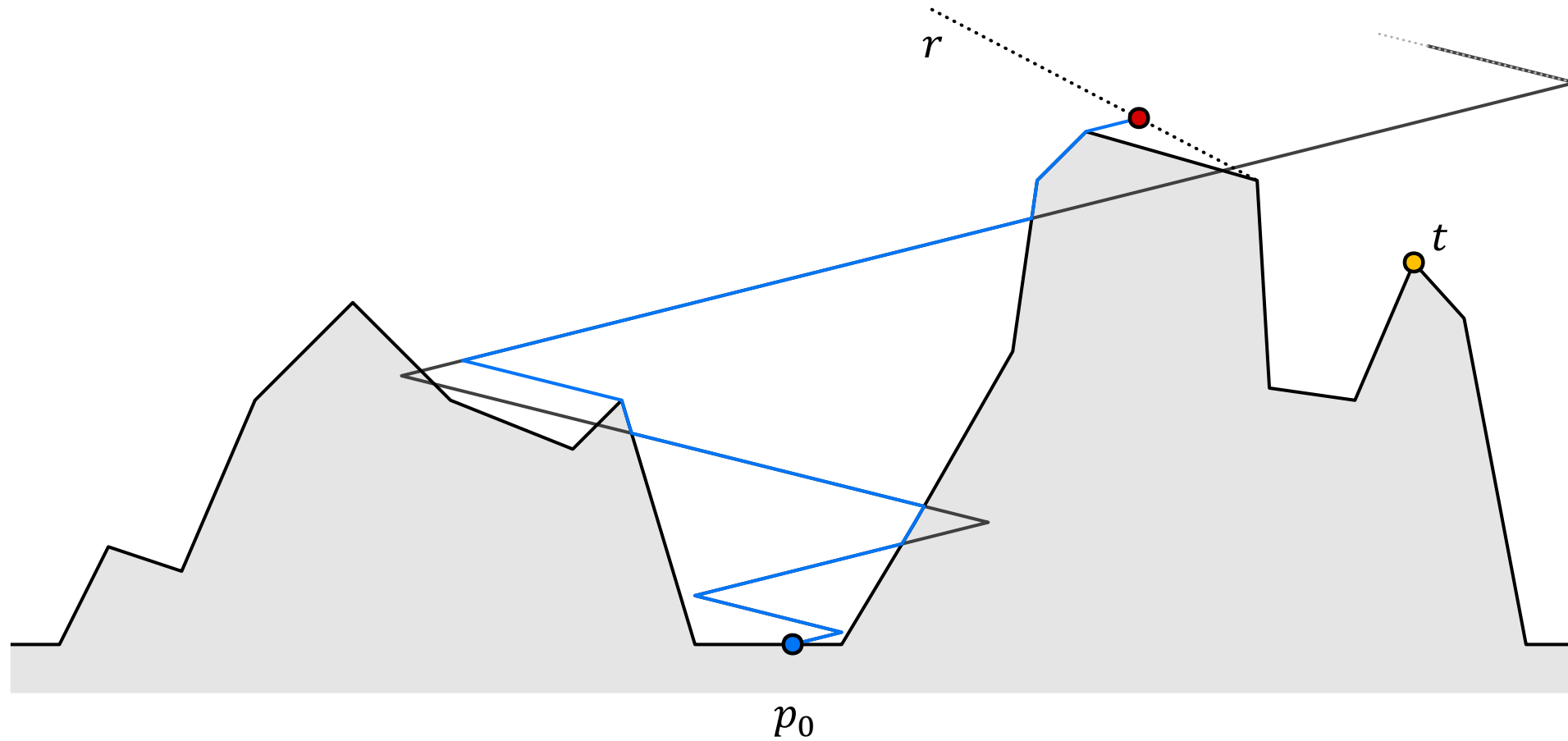
1.5D Terrains – Strategy



1.5D Terrains – Strategy



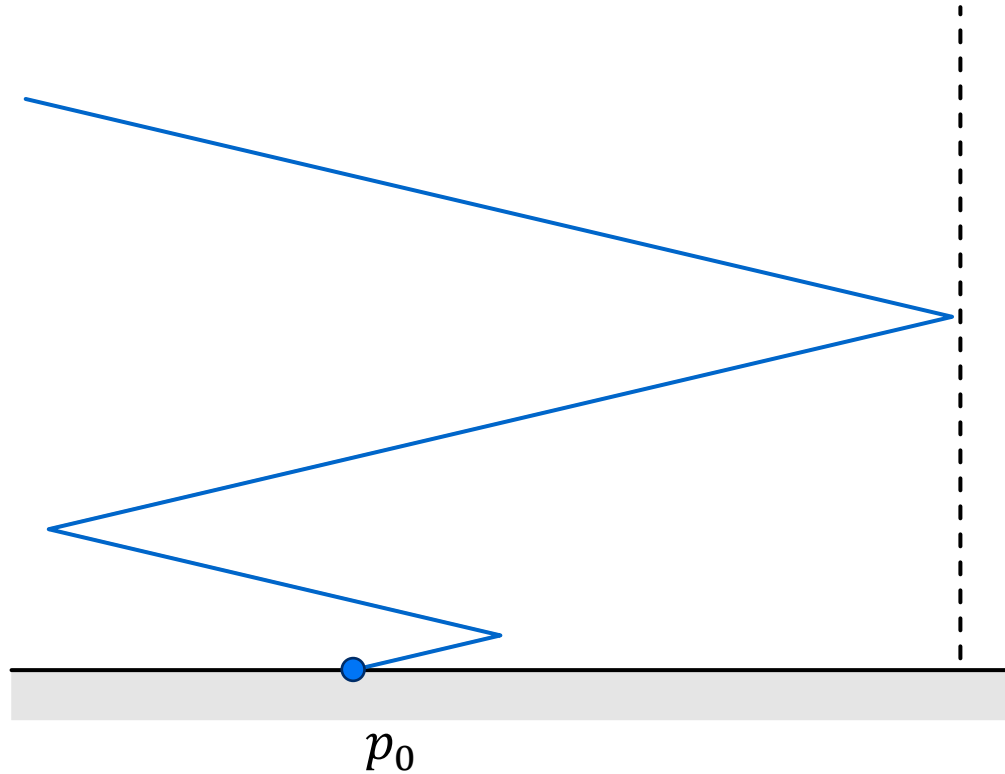
1.5D Terrains – Strategy



1.5D Terrains – Upper Bound

Worst-case instance

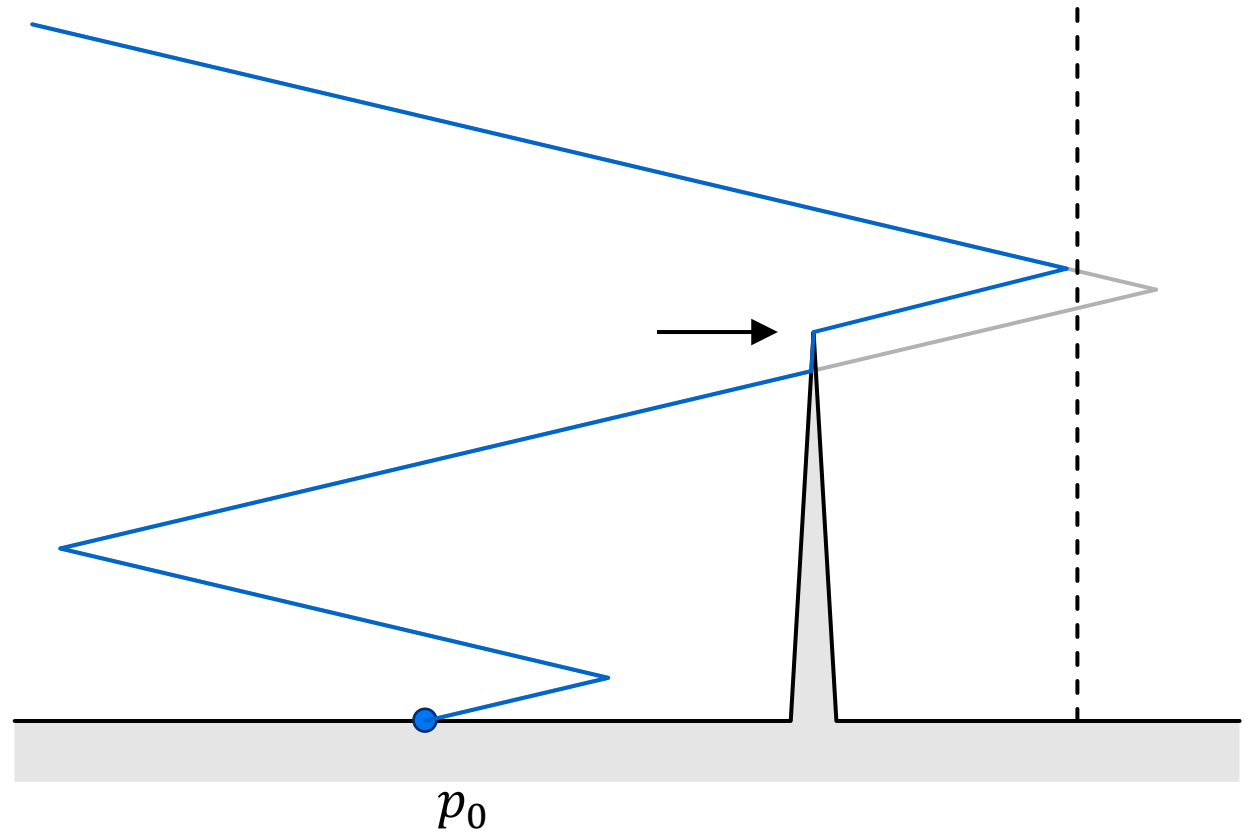
Vertical ray r



1.5D Terrains – Upper Bound

Worst-case instance

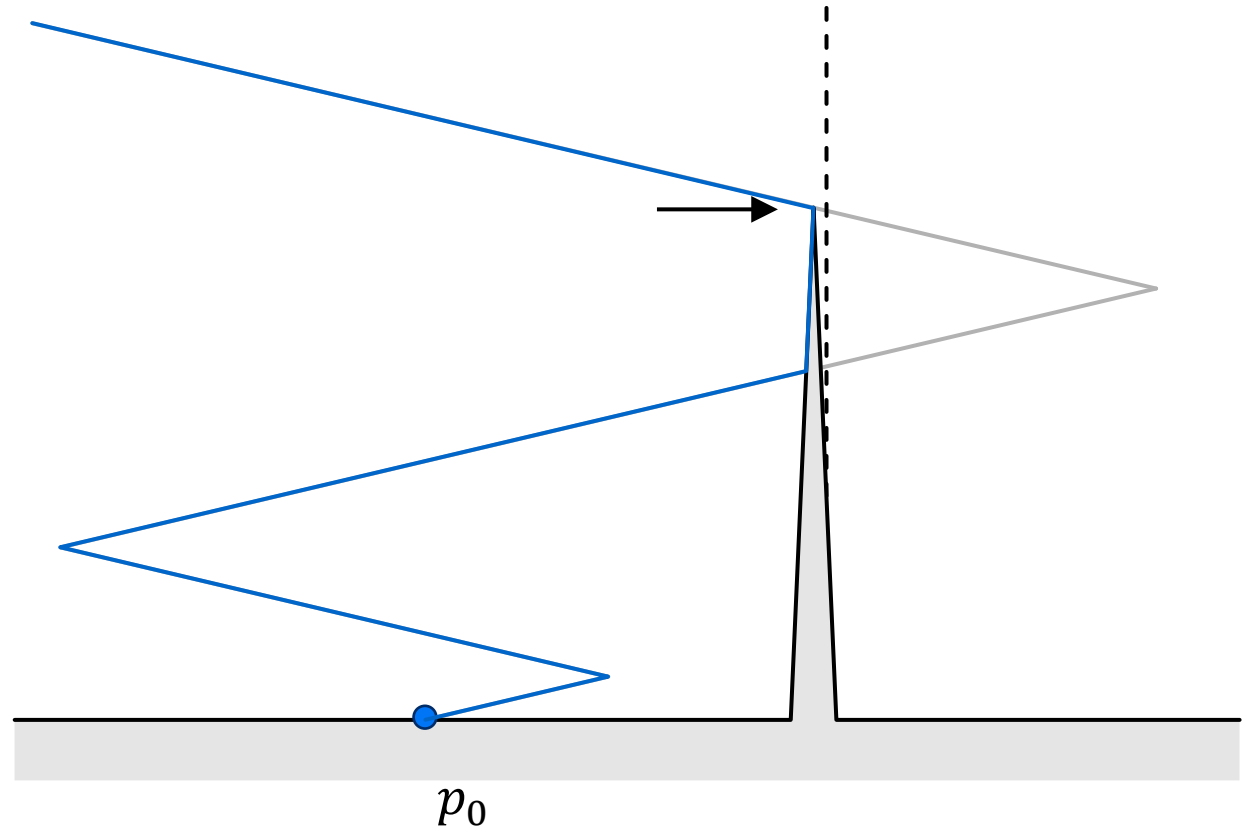
Obstructing terrain



1.5D Terrains – Upper Bound

Worst-case instance

Obstructing terrain



1.5D Terrains – Upper Bound

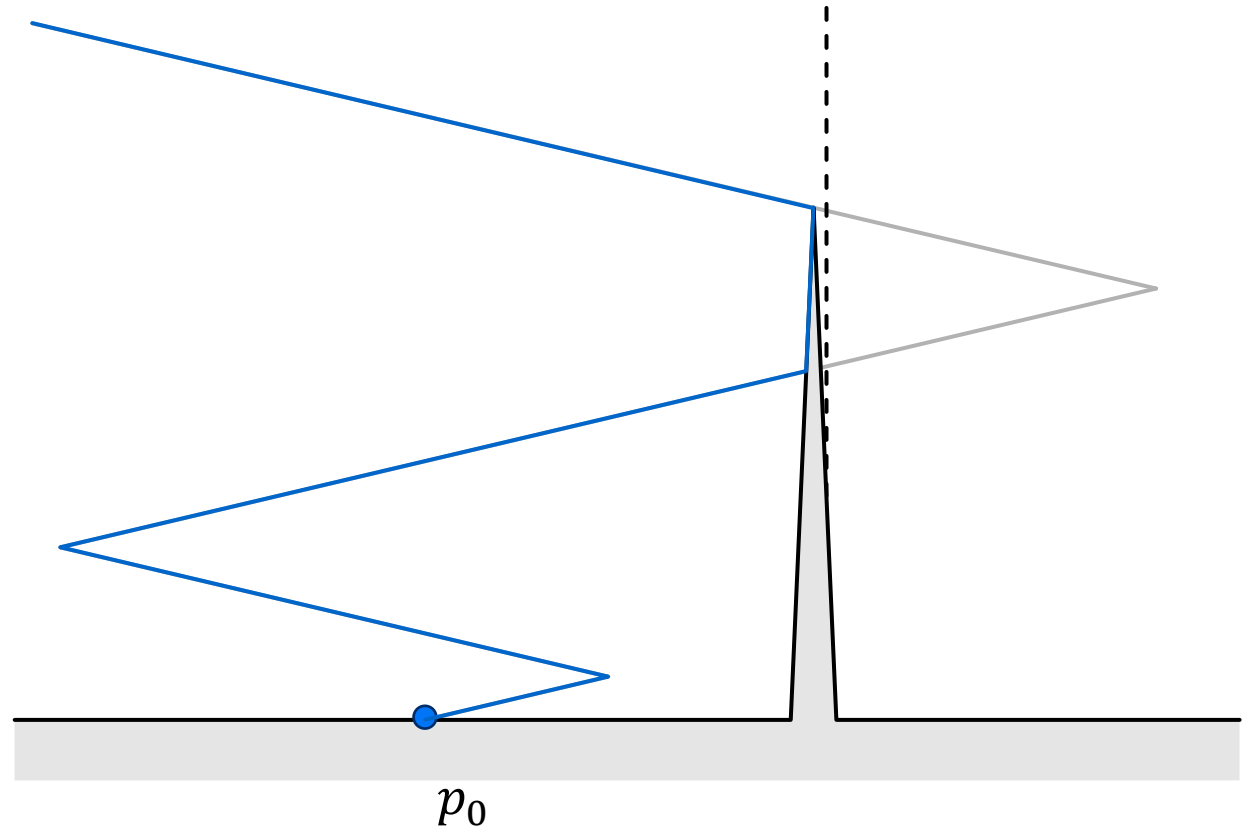
Worst-case instance

Obstructing terrain

Choose suitable slope: $s = \frac{\sqrt{2}}{6}$

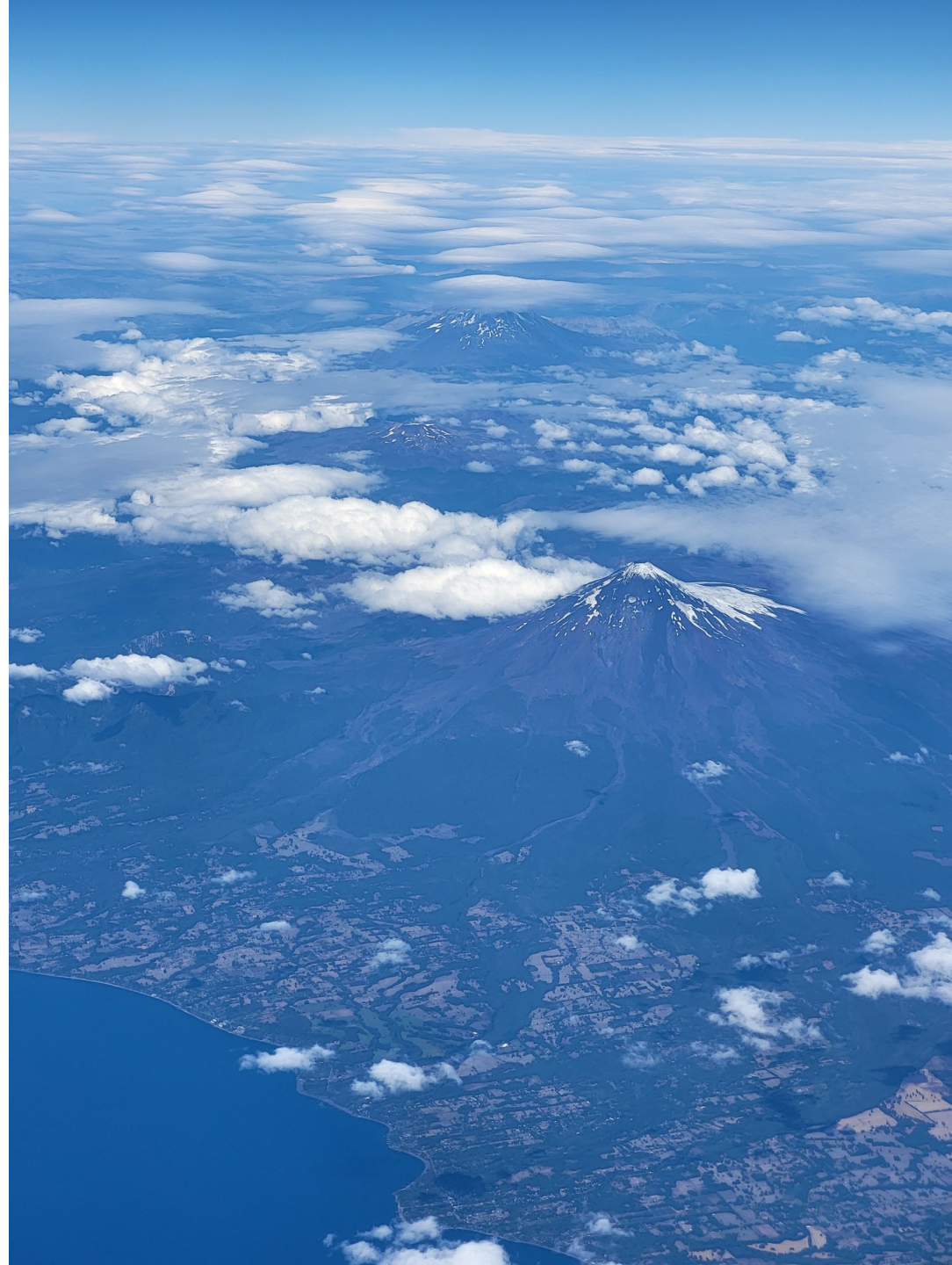
$$c \leq 3\sqrt{19/2}$$

$$9.055 \leq c \leq 9.247$$

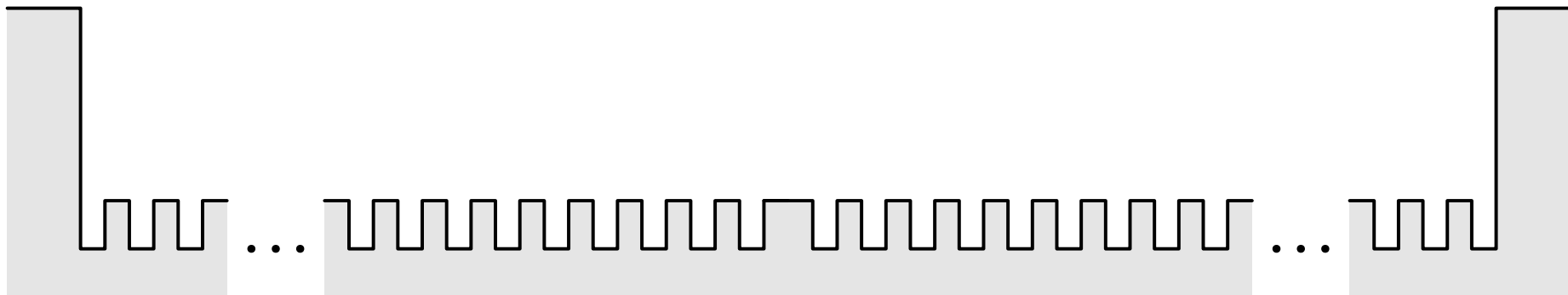


2.5D Terrain

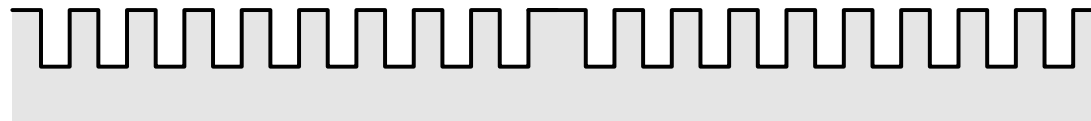
Can we do anything?



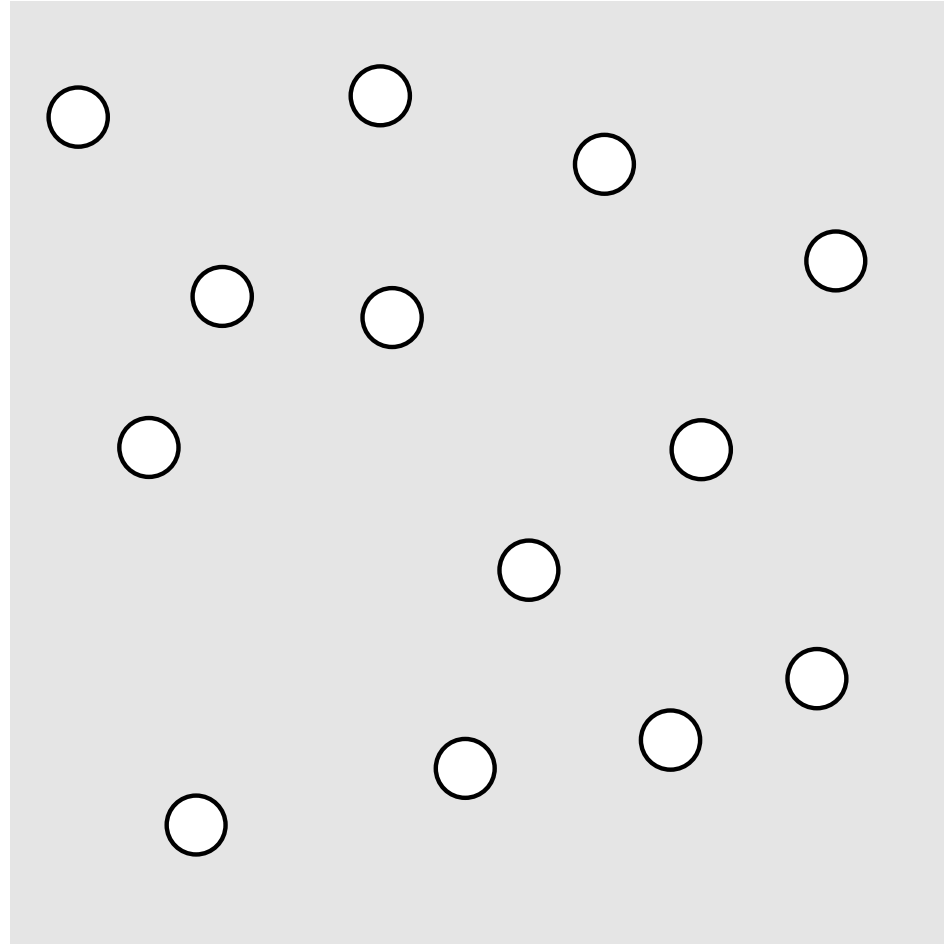
2.5D Terrain



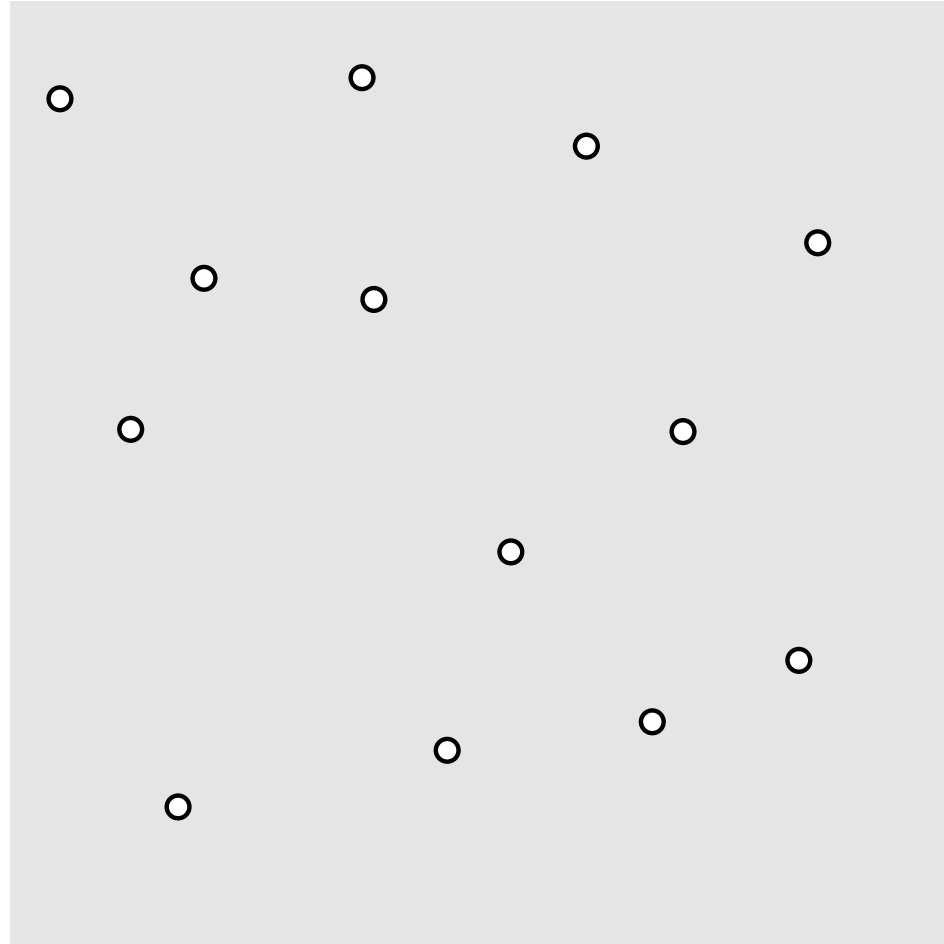
2.5D Terrain



2.5D Terrain



2.5D Terrain

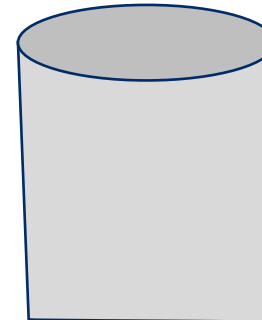
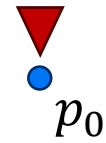


2.5D Terrain

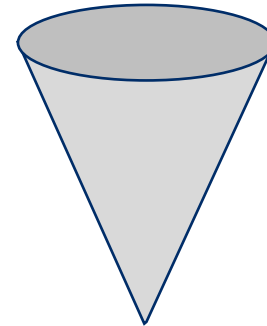
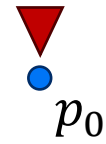
Cannot do anything without knowledge of terrain

Bound maximum slope λ of the terrain

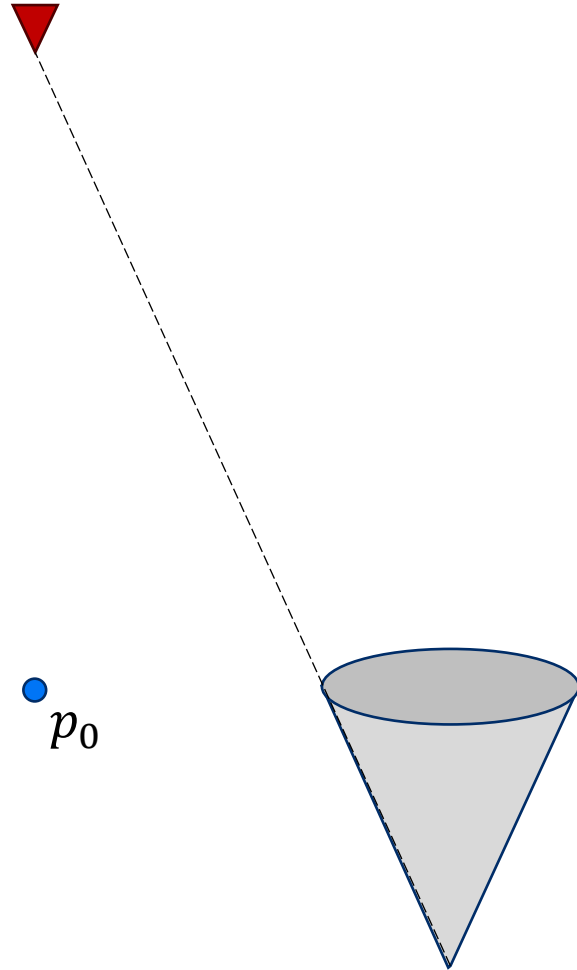
2.5D Terrain – Bounds



2.5D Terrain – Bounds

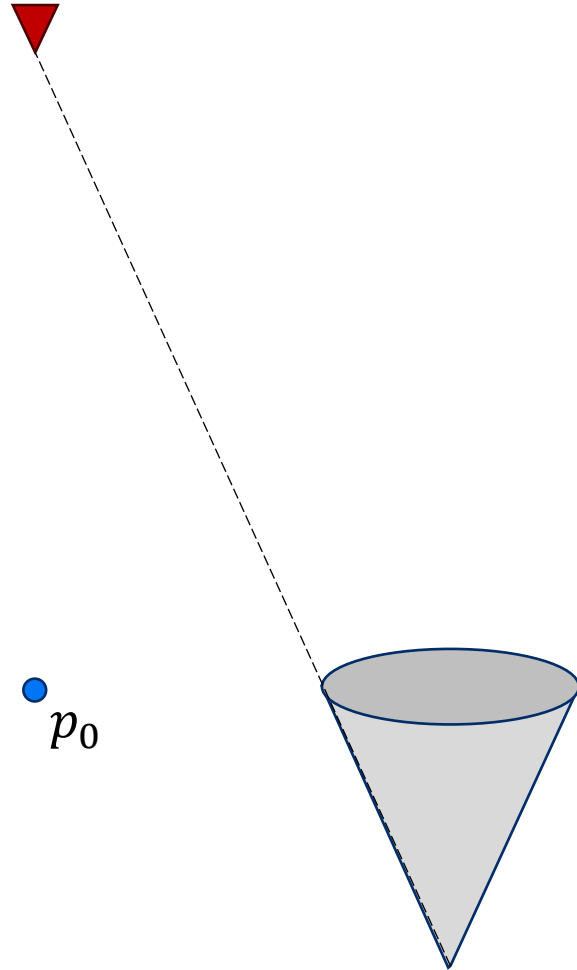


2.5D Terrain – Bounds



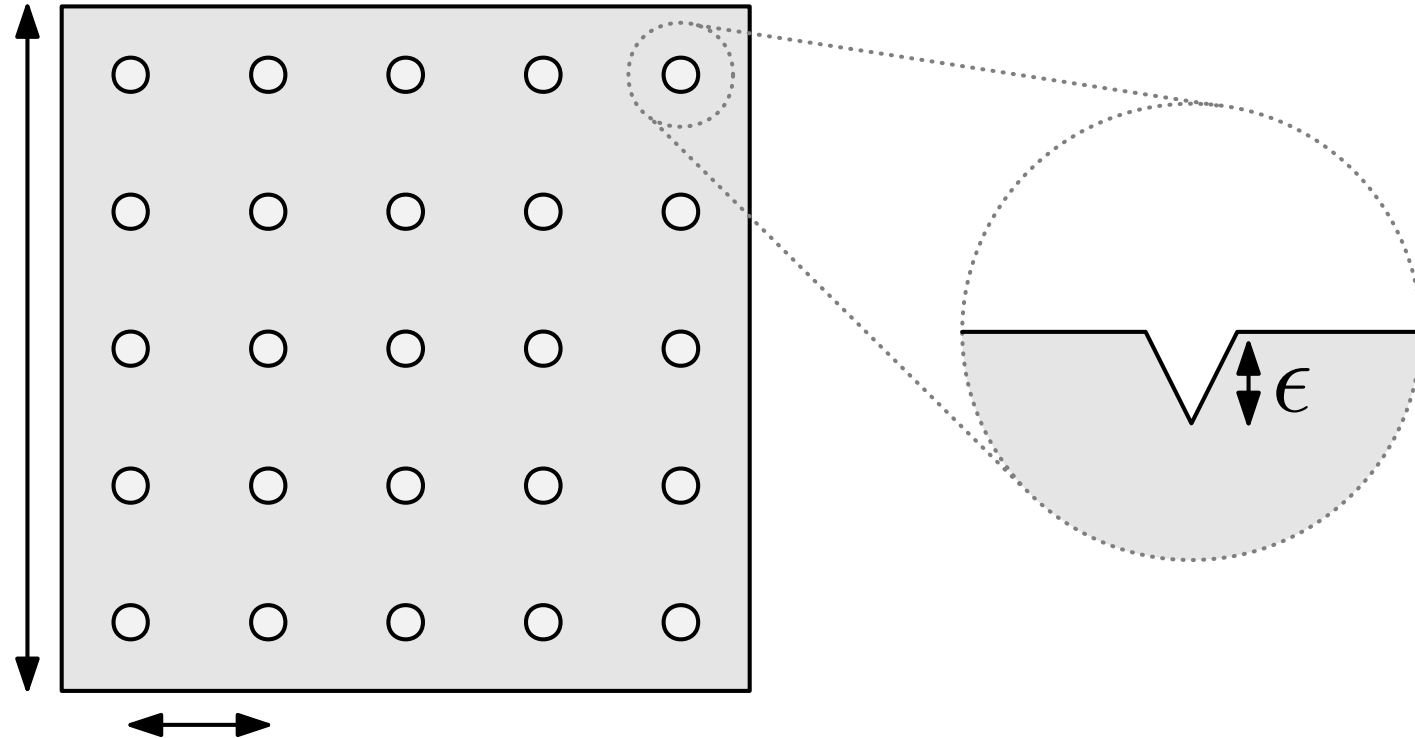
2.5D Terrain – Bounds

Moving upwards: $O(\lambda)$

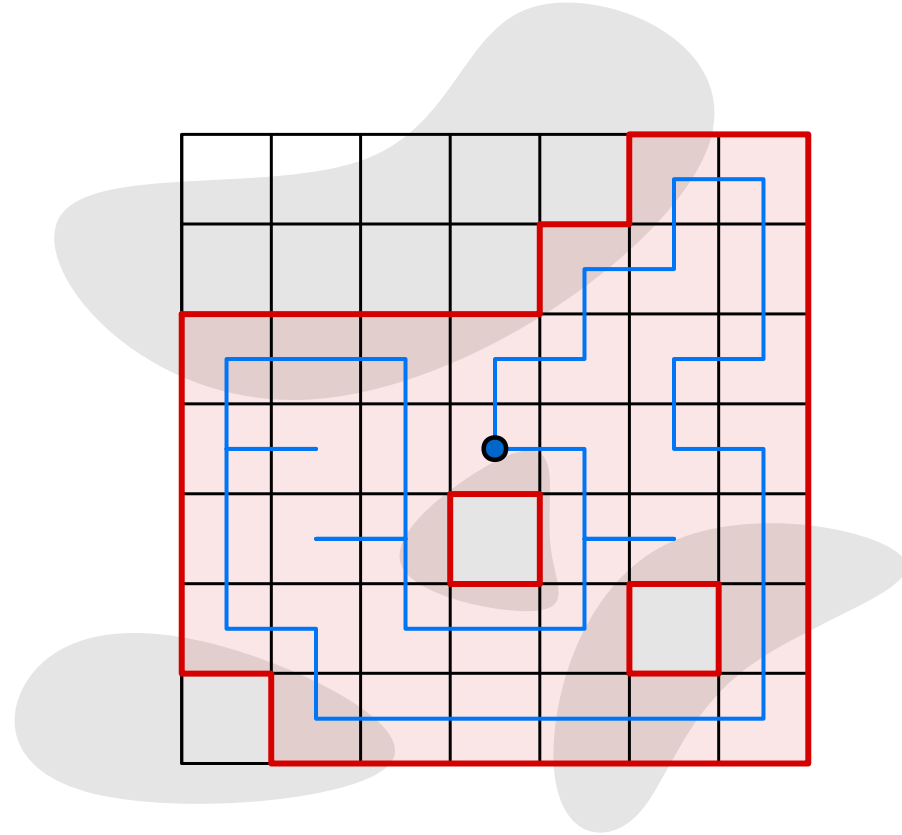
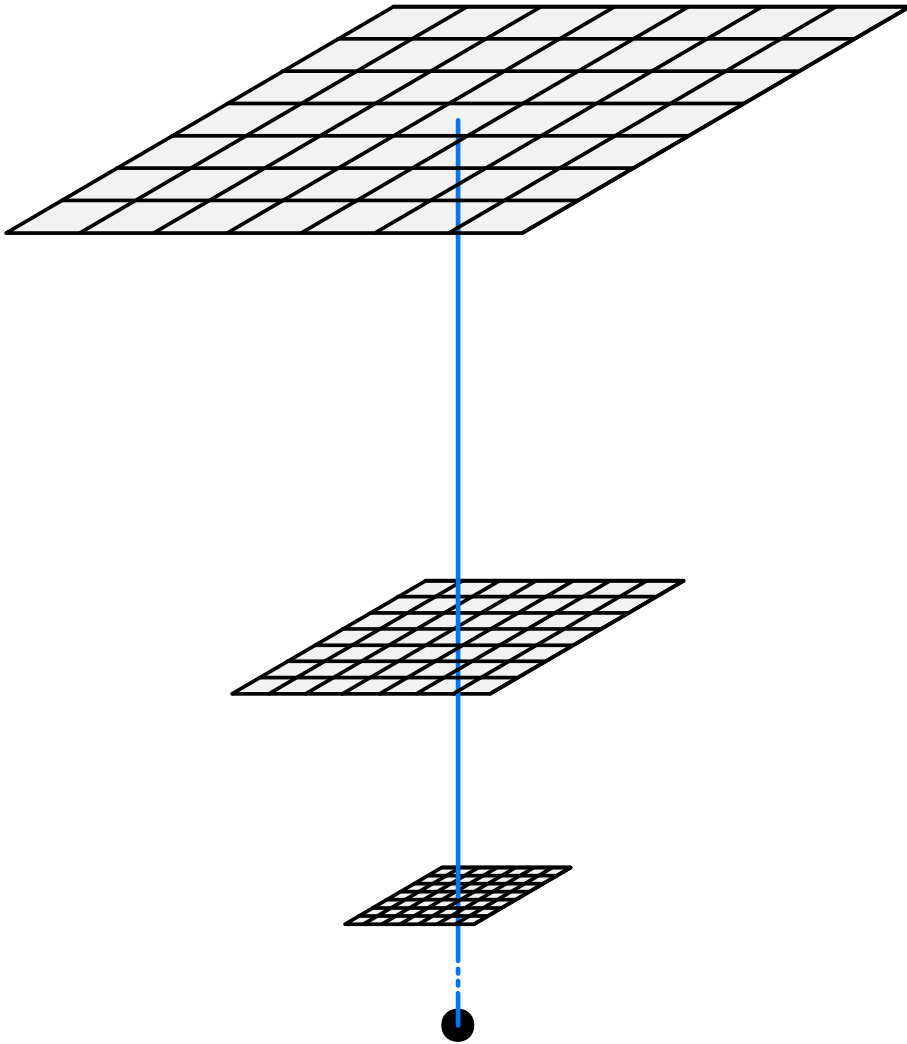


2.5D Terrain – Lower Bound

$$c \in \Omega(\sqrt{\lambda})$$

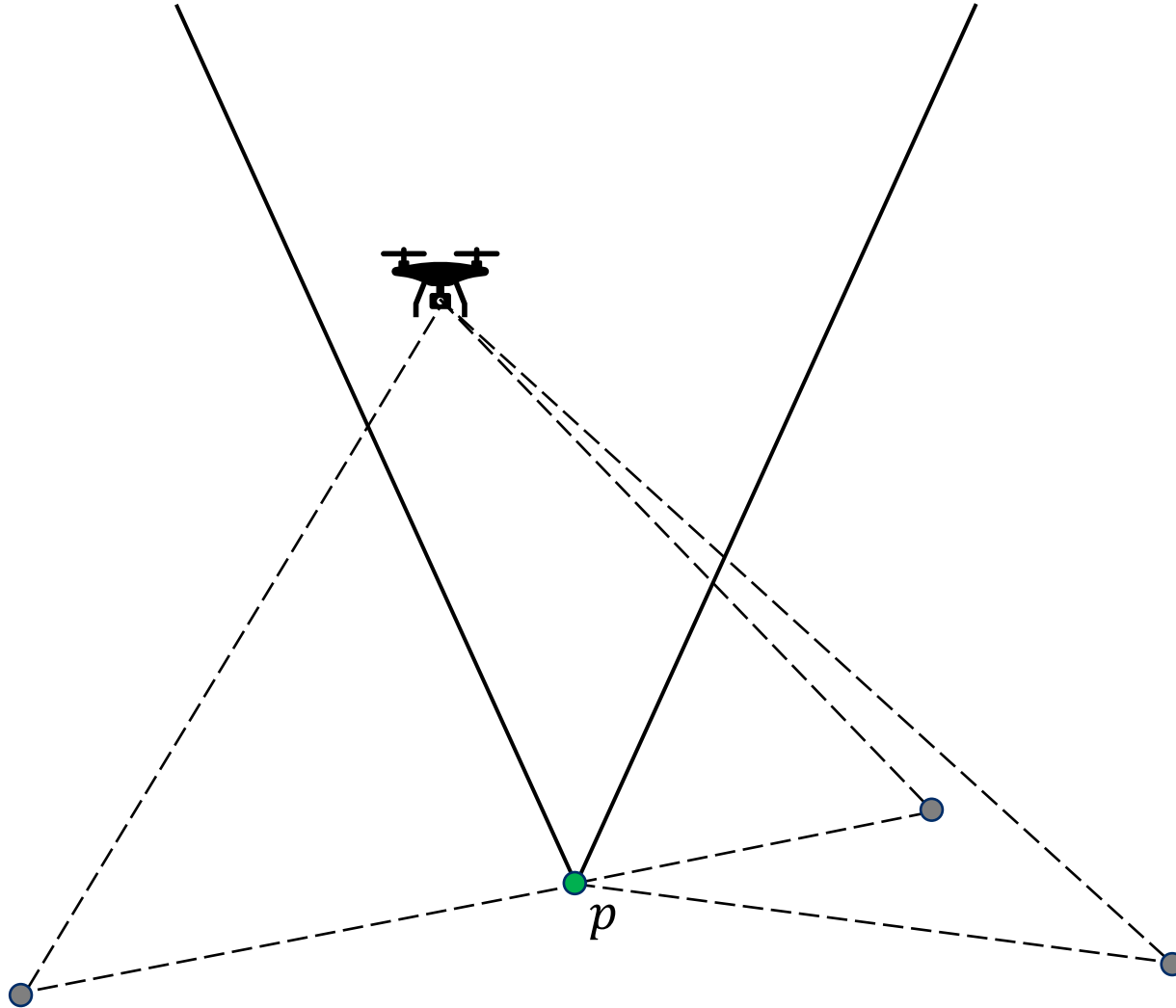


2.5D Terrain – Strategy



2.5D Terrain – Upper Bound

$$c \in O(\sqrt{\lambda})$$



Conclusion

1.5D Terrain

$$c \geq \sqrt{82}$$

$$c \leq 3\sqrt{19/2}$$

2.5D Terrain

$$c \in \Omega(\sqrt{\lambda})$$

$$c \in O(\sqrt{\lambda})$$