Name:____________________
Math 40, Section: __
Homework #2

#15 Find $\mathrm{d}(\mathbf{u}, \mathbf{v})$ where $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

#26 Find the angle between $\mathbf{u}$ and $\mathbf{v}$ where $\mathbf{u} = [4, 3, -1]$ and $\mathbf{v} = [1, -1, 1]$

#46 There are to ways of computing the area of a triangle with vectors:

(a) A = $\frac{1}{2}\|\mathbf{u}\|\|\mathbf{v} - \text{proj}_{\mathbf{u}}(\mathbf{v})\|$

(b) A = $\frac{1}{2}\|\mathbf{u}\|\|\mathbf{v}\| \sin\theta$, and $\sin\theta = \sqrt{1-\cos^2\theta}$

Compute the area of the triangle with the given vertices using both methods:

$$A = (1, -1), B = (2, 2), C = (4, 0)$$

#60 Suppose we know that $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ Does is follow that $\mathbf{v} = \mathbf{w}$? If it does, give a proof that is valid in $\mathbb{R}^n$; otherwise, give a counter example (i.e., a specific set of vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ for which $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ but $\mathbf{v} \neq \mathbf{w}$).

#62

(a) Prove that $\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$ for all vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$.

(b) Draw a diagram showing $\mathbf{u}$, $\mathbf{v}$, $\mathbf{u}+\mathbf{v}$, and $\mathbf{u}-\mathbf{v}$ in $\mathbb{R}^2$ and use (a) to deduce a result about parallelograms.

#68b Prove that if $\mathbf{u}$ is orthogonal to both $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $s\mathbf{v} + t\mathbf{w}$ for all scalars s and t.

#6 Write the equation of the line passing through $\mathrm{P} = (3, 0, -2)$ with direction vector $\mathbf{d} = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ in (a) vector form and (b) parametric form.

#10 Write the equation of the plane passing through $P = (6, -4, -3)$ with direction vectors $\mathbf{u} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ in (a) vector form and (b) parametric form.

#16ace Consider the vector equation $\mathbf{x} = \mathbf{p} + t(\mathbf{q} - \mathbf{p})$, where $\mathbf{p}$ and $\mathbf{q}$ correspond to distinct points P and Q in $\mathbb{R}^2$ or $\mathbb{R}^3$.
(a) Show that this equation describes the line segment $\overline{PQ}$ as t varies from 0 to 1.
(c) Find the midpoint of $\overline{PQ}$ when $P = (2, -3)$ and $Q = (0, 1)$.
(e)Find the two points that divide $\overline{PQ}$ in part (c) into three equal parts.

#18 The line ℓ passes through the point $P = (1, -1, 1)$ and has direction vector $\mathbf{d} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$. For each of the following planes $\mathscr{P}$, determine whether ℓ and $\mathscr{P}$ are parallel, perpendicular, or neither:

(a) $2x + 3y - z = 1$

(b) $4x - y + 5z = 0$

(c) $x - y - z = 3$

(d) $4x + 6y - 2z = 0$

#46 Show that the plane given by $4x - y - z = 6$ and the line given by $x = t$, $y = 1 + 2t$, $2 + 3t$ intersect and the find the acute angle of intersection between them.

#4 Use the cross product to help find the normal form of the equation of the plane.

(a) The plane passing through $P = (1, 0, -2)$, parallel to $\mathbf{u} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

(b) The plane passing through P$= (0, -1, 1)$, $Q = (2, 0, 2)$, and $R = (1, 2, -1)$

#34 Solve the linear system:
$2x_1 + 3x_2 - x_3 = 1$
$x_1 + x_3 = 0$
$-x_1 + 2x_2 - 2x_3 = 0$

#42 Find substitutions (changes of variables) that convert each system into a linear system and use this linear system to help solve the given system:

$$x^2 + 2y^2 = 6$$
$$x^2 - y^2 = 3$$