n chose k

Number of ways to chose k elements from a set of n elements when order does not matter and

i there is no replacement:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

ii there is replacement (multichose):

$$\left(\!\!\binom{n}{k}\!\!\right) = \binom{n+k-1}{k}$$

Pascal's identity:

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

Similarly:

$$\left(\!\!\binom{n}{k}\!\!\right) = \left(\!\!\binom{n-1}{k}\!\!\right) + \left(\!\!\binom{n}{k-1}\!\!\right)$$

Hockey stick identity:

$$\sum_{x=k}^{n} \binom{x}{k} = \binom{n+1}{k+1}$$

Binomial identity:

$$(x+y)^n = \sum \binom{n}{k} x^k y^{n-k}$$

Other properties:

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\sum \binom{n}{k} = 2^n$$

Fibonacci Numbers

Recursive definition:

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-1}$$

Binet's formula:

$$F_n = \frac{1}{5}(\phi^n - \bar{\phi}^n)$$

$$\phi = \frac{1+\sqrt{5}}{2} \text{ and } \bar{\phi} = \frac{1-\sqrt{5}}{2}$$

$$\text{Note that } \phi \text{ and } \bar{\phi} \text{ are solutions to } x^2 = x + 1$$

First 13 numbers of Fibonacci series:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144$$

All the ways (f_n) to tile a board of length $1 \times n$ with squares (1×1) and dominoes (1×2):

$$f_n = F_{n+1}$$

Sets

The union of two sets, $A \cup B$, is the set of all elements contained in both A or B.

The intersection of two sets, $A \cap B$ or simply AB, is the set of all elements in A and B.

Two sets are disjoint if $A \cap B$ is the null set $\{\}$

The size of a set A is denoted by $|A|$

Rule of sum: given disjoint sets $A_1, A_2, A_3, ..., A_n$, the size of there union is

$|A_1 \cup A_2 \cup A_3 \cup ... \cup A_n| = \sum |A_i| - \sum \sum |A_i A_j| + \sum \sum \sum |A_i A_j A_k| - ... - (-1)^n |A_i A_j A_k ... A_n|$