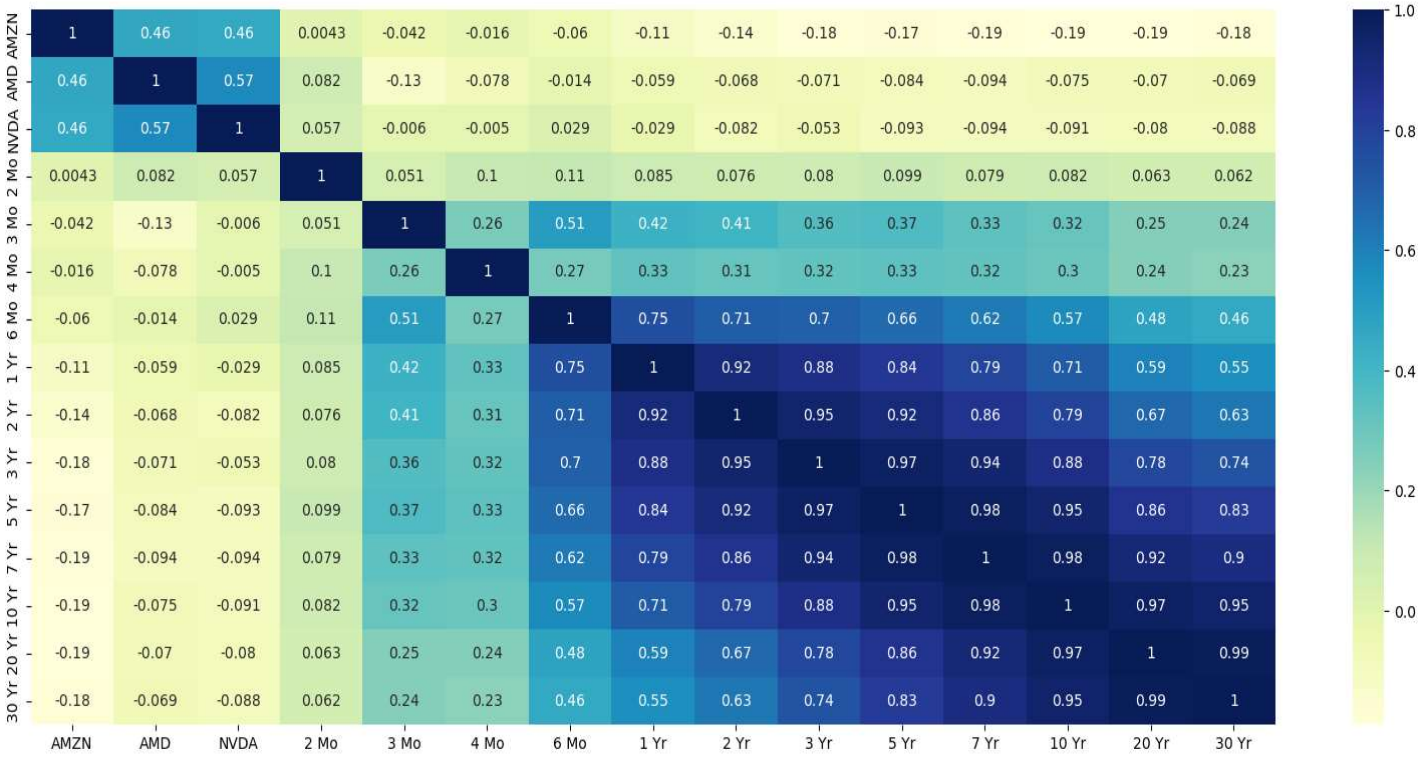
Equities & Interest Rate Modeling

Nancy Verma (2024)

Executive Summary

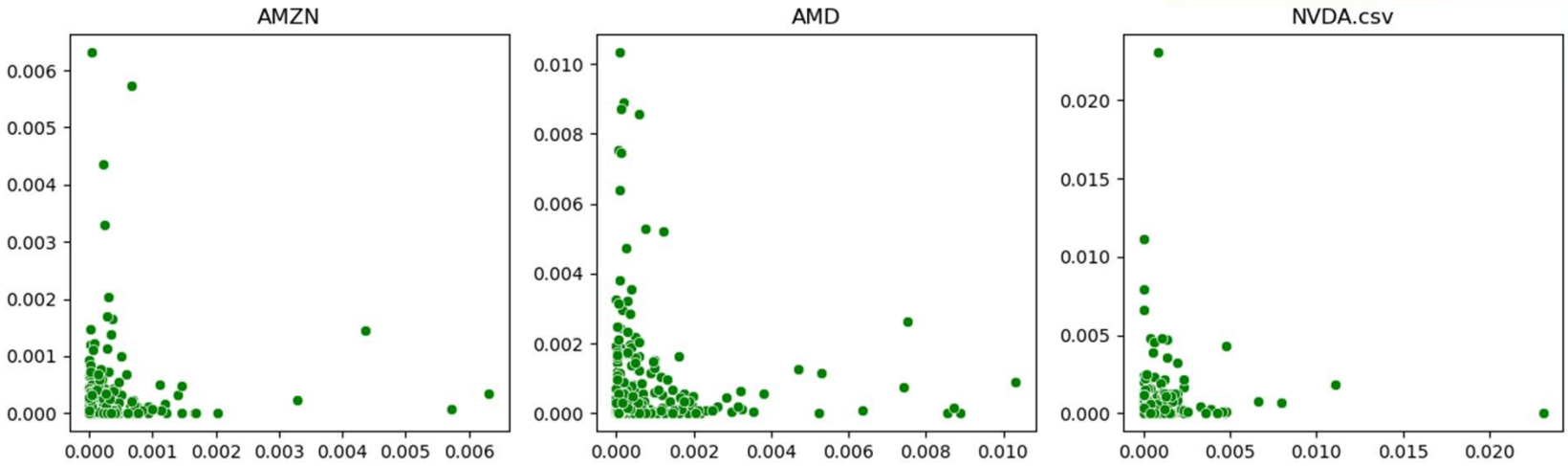
- This python project models equity (3 tech stocks) and treasury yield curve using econometric models
- Equity log-returns are modeled using GARCH(1,1) since log-returns exhibit heteroskedasticity
- Principal component analysis is performed on treasury yield curve for dimensionality reduction
- Finally, the joint returns of equity and yield curve are simulated using t-4 copula
- Risk measure of Value-at-Risk (Var) is illustrated using a dummy portfolio in stocks and 2-yr treasury bond.

- The analysis is performed on 3 tech stocks and treasury yield curve with maturities from 2 months to 30 Yrs using data from June'23 to June'24 (1-year)
- The heatmap shows correlation across 3 tech stocks log-returns: AMZN, AMD, NVDA and simple returns in treasury yield curve
- Tech stocks exhibit correlation of ~46% to ~57% among themselves, but equities have very low and sometimes negative correlation with yield returns



Exploratory Data Analysis

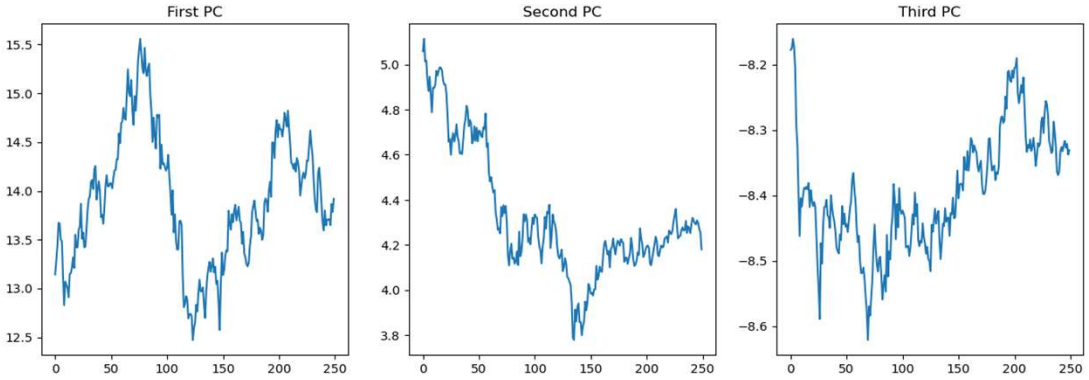
Below graph shows scatter plot between Lag-1 log-returns. The cone-like structure indicates heteroskedasticity in the returns. Hence, to model equity returns, GARCH (1,1) will be used.

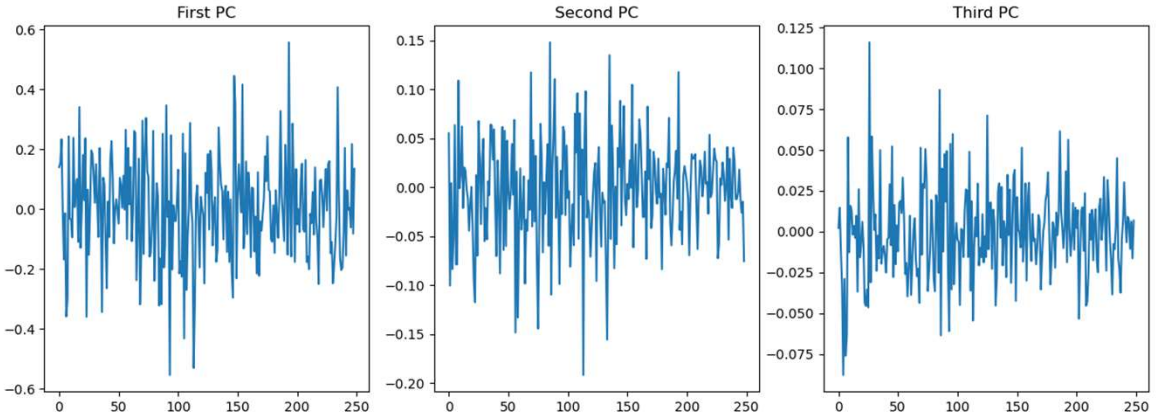


Exploratory Data Analysis

Treasury Yield Curve PCA

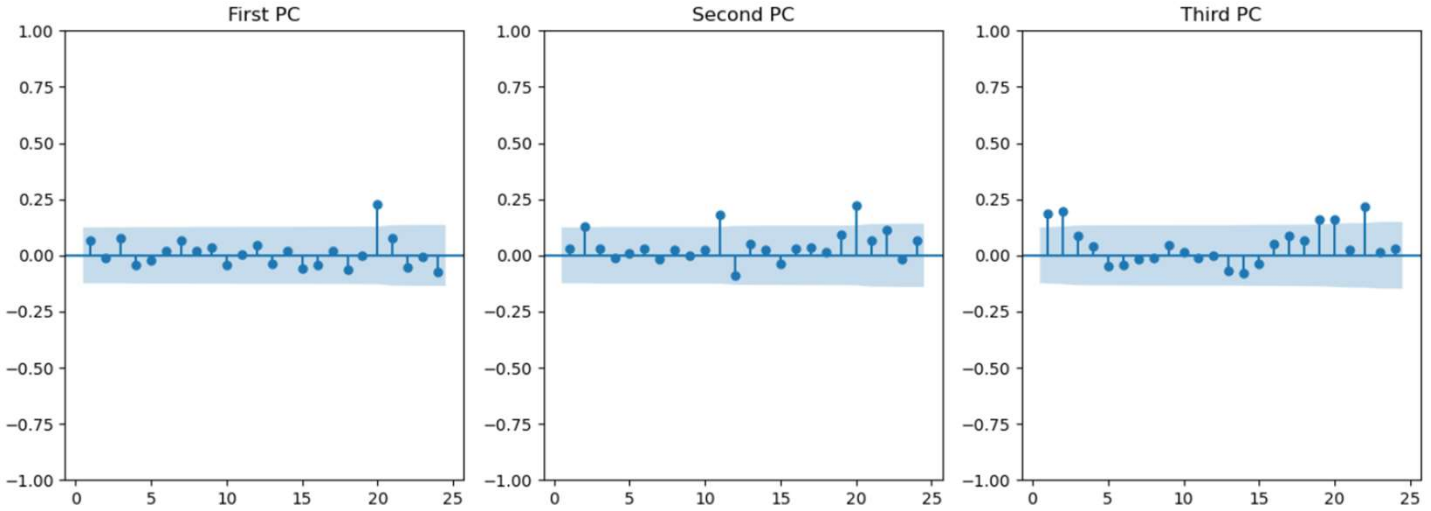
- First 3 principal components (PCs) explain 99% of the variance in treasury yield curve.
- Plots below show time-series for the values and simple returns of 3 principal components





Treasury Yield Curve PCA

- The autocorrelation plots below for the simple returns in 3 principal components show that there is significant correlation between lagged returns beyond Lag-1. However, for the simplicity of this project, simple returns in 3 PCs have been modeled using GARCH (1,1) along with equity log-returns



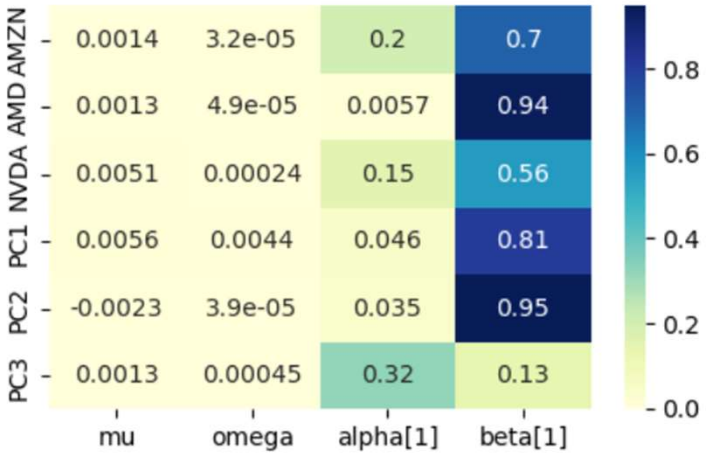
GARCH (1,1) Fitting

$$r_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t = \sigma_t z_t$$

$${\sigma_t}^2 = \omega + \alpha {\varepsilon_t}^2 + \beta {\sigma_{t-1}}^2$$

Calibrated parameters for tech stocks (log-returns) and 3 yield principal components (simple returns) are shown in the table below



Distribution fitting & t-copula

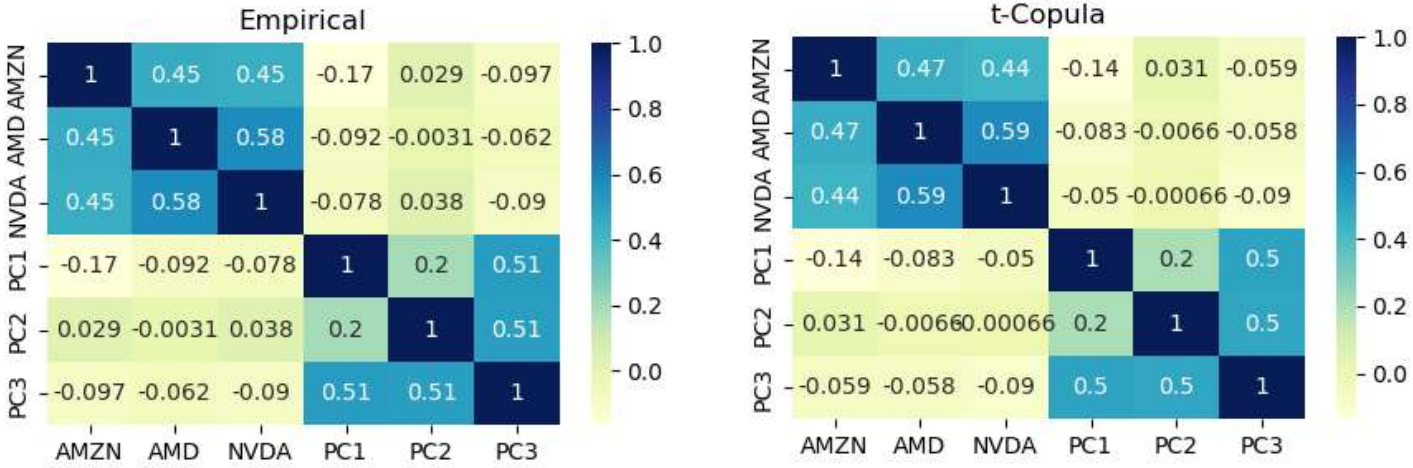
- i.i.d. residuals (z_t) from the GARCH(1,1) model are extracted from the historical time-series and t-distribution is fitted on the residuals. The table below shows degrees of freedom for each of them:

i.i.d Residuals	t-dist fitted dof
AMZN	8
AMD	5
NVDA	5
PC1	21
PC2	13
PC3	14

- A high degrees of freedom for the principal component residuals indicate it can be modeled using normal distribution
- Uniform variates are generated for the i.i.d residuals of 3 equities and 3 principal components using the respective cumulative distribution function (cdf)
- Following approach of **t-copula**, uniform variates are then mapped to t-distribution with degrees of freedom 4 using inverse cdf

Empirical vs. t-copula correlation

- A monte-carlo simulation using t-copula generated correlation matrix is performed to generate 10,000 random variates of the residuals
- Below is a comparison of empirical correlation and simulated correlation of t-distributed variates with dof =4
- As can be seen here, historical correlation is maintained between the residuals to a reasonable degree

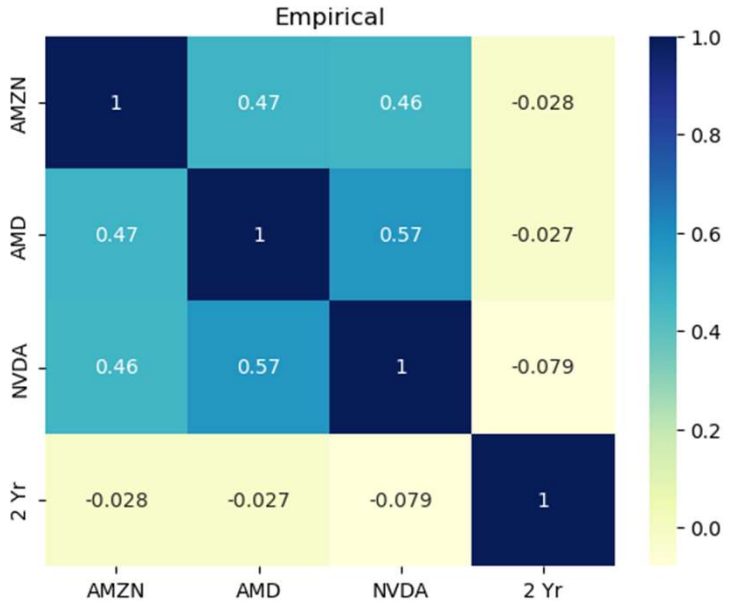


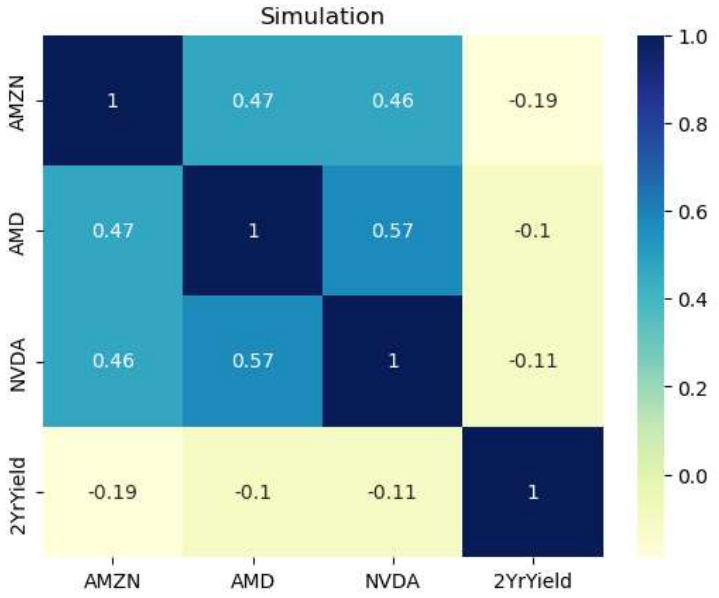
Equity & Yield Correlation

-Equity return scenarios and Principal components scenarios are reversed from the GARCH model, using i.i.d residual scenarios.
-Yield curve scenarios are generated by doing inverse transformation from principal components scenarios and principal component loadings
-Below, correlation across equity log-returns and yield simple returns is presented from the simulation. It is observed that simulated correlation across yield curve is higher than the empirical correlation observed in initial data analysis

Equity & Yield Correlation

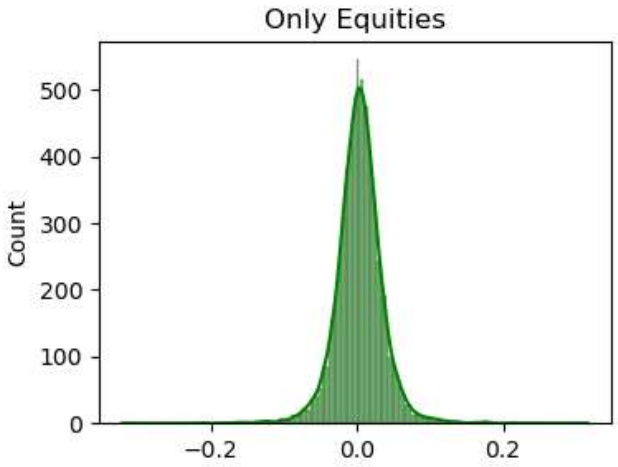
- Below is a comparison of empirical vs. simulated correlation between equities and 2-year treasury yield
- It is observed that simulation increases negative correlation between tech stocks and 2-year yield compared to historical correlation

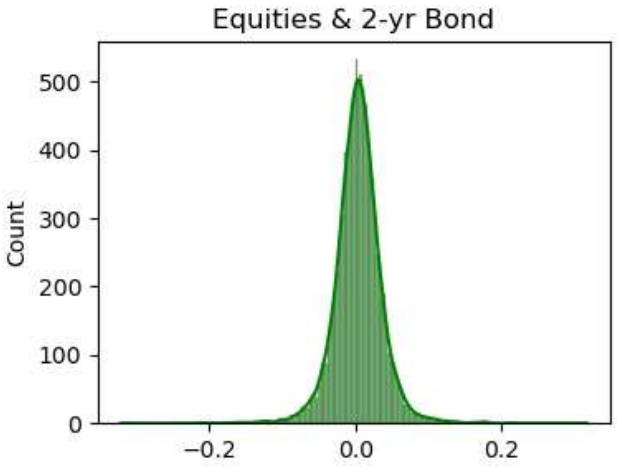




Simple Portfolio Construction

- An equities-only portfolio is compared to an equities + bond portfolio and VaR is calculated using 95% confidence level
- Equities-Only: 20% in AMZN, 30% in AMD, 50% in NVDA. Sharpe Ratio = 9.59%, VaR = 4.54%
- Equities + Bond: 20% in AMZN, 20% in AMD, 50% in NVDA, 10% 2-yr Bond with 3.75% quarterly coupon rate. Sharpe Ratio=10%, VaR = 4.10%





Conclusions

- Equity log-returns exhibit heteroskedasticity and can be modeled using GARCH (1,1)
- 99% of the variance in the yield curve can be explained by first 3 principal components
- Negative correlation between equities and treasury yields from 2023-2024 is representative of post-covid market movements where stock-bond correlation deviated from its historical pattern of offsetting each other.
- t-distribution is appropriate to model invariants/residuals in the equity log-returns, while normal distribution is better suited for invariants/residuals in principal components
- Equities and principal components residuals can be simulated using t-4 copula and it was observed that historical correlation is preserved in the simulated correlation between equities and principal components, however correlation is over-estimated between equities and yield curve post-simulation
- Even with slight negative correlation between yields & equities, it is low enough for bonds to still provide some offset to equities-only portfolio, resulting in a higher sharpe ratio and lower VaR

Future enhancements:

- Invariants in equity returns exhibit heteroskedasticity even after applying GARCH (1,1) model which means other variations of GARCH(p,q) model needs to be explored
- Simulated correlation across yield curve is overestimated post inverse-transformation of principal components, compared to the historical correlation. Data quality of yield curve needs further investigation as well a different econometrics model for principal components needs to be explored