$$\text{performance} = \frac{1}{\text{Execution Time}}.$$

Problem: If computer A runs a program in 10 seconds and computer B runs the same program in 15 seconds, how much faster is A than B?

$$\text{performance}_A = \frac{1}{\text{Execution Time}_A}.$$

$$\text{performance}_B = \frac{1}{\text{Execution Time}_B}.$$

$$\frac{\text{performance}_A}{\text{performance}_B} = \frac{\text{Execution Time}_B}{\text{Execution Time}_A}$$

$$= \frac{15}{10}$$

$$= 1.5.$$

$$\boxed{\therefore \text{performance}_A = 1.5 \times \text{performance}_B}$$

$$\text{CPU Time} = \frac{\text{CPU clock cycles}}{\text{Clock Rate}}.$$

Problem: Our favorite program runs in 10 seconds on computer A, which has a 2GHz clock. We are trying to help a computer designer build a computer, B, which will run this program in 6 seconds. The designer has determined that a substantial increase in the clock rate is possible, but this increase will affect the rest of the CPU design, causing computer B to require 1.2 times as many clock cycles as computer A for this program. What clock rate should we tell the designer to target?

Computer A

$\text{CPU Time}_A = 10 \text{ sec}$

$\text{clock Rate}_A = 2 \text{ GHz}$
$= 2 \times 10^9 \text{ Hz}$

Computer B

$\text{CPU Time}_B = 6 \text{ sec.}$

$\text{clock cycle}_B = 1.2 \times \text{clock-cycle}_A$

$\text{Clock Rate}_B = ?$

$$\boxed{\text{CPU Time}_A = \frac{\text{CPU clock cycles}_A}{\text{Clock Rate}_A}}$$

$\therefore \text{CPU clock cycles}_A = \text{CPU Time}_A \times \text{Clock Rate}_A$
$= 10 \times 2 \times 10^9$
$= 20 \times 10^9 \text{ cycles}.$

$$\boxed{\text{CPU Time}_B = \frac{\text{CPU clock cycles}_B}{\text{Clock Rate}_B}}$$

$\therefore \text{Clock Rate}_B = \cancel{\text{CPU Time}_B} \times \frac{\text{CPU clock cycles}_B}{\text{CPU Time}_B}$
$= \cancel{6 \text{ sec} \times} \frac{1.2 \times \text{clock-cycles}_A}{\text{CPU Time}_B}$
$= \frac{1.2 \times 20 \times 10^9}{6}$
$= 4 \times 10^9$
$= 4 \text{ GHz}$

$$\boxed{\begin{aligned} &\rightarrow a = 2 \\ &\rightarrow b = a + 3 \\ &\rightarrow c = a + b \end{aligned}}$$

Instruction Count and CPI

Instruction Count: The number of instructions executed by the program.

CPI (Clock cycles per instruction): Average number of clock cycles per instruction for a program.

The number of clock cycles required for a program can be written as

$$\boxed{\text{CPU Clock cycles} = \text{Instruction count} \times \text{CPI}}$$

$$* \quad \boxed{\text{CPU Time} = \text{CPU clock cycles} \times \text{Clock cycle Time}}$$

$$* \quad \boxed{\text{CPU Time} = \frac{\text{CPU clock cycles}}{\text{Clock Rate}}}.$$

$$* \quad \boxed{\text{CPU Time} = \frac{\text{Instruction Count} \times \text{CPI}}{\text{Clock Rate}}}$$

$$* \quad \boxed{\text{CPU Time} = \text{Instruction count} \times \text{CPI} \times \text{clock cycle Time}}.$$

Number of instructions = I.

Problem: Suppose we have two implementations of the same instruction set architecture. Computer A has a clock cycle time of 250ps and a CPI of 2.0 for some program, and computer B has a clock cycle time of 500ps and a CPI of 1.2 for the same program. Which computer is faster for this program and by how much?

Computer A	Computer B
Clock cycle $\text{Time}_A = 250\text{ ps}$	Clock cycle $\text{Time}_B = 500\text{ ps}$
$CPI_A = 2$	$CPI_B = 1.2$
$IC_A = I$	$IC_B = I$.

$$* \ \text{CPU Time}_A = IC_A \times CPI_A \times \text{Clock cycle Time}_A$$

$$\text{CPU Time}_B = IC_B \times CPI_B \times \text{clock cycle Time}_B$$

$$\frac{\text{CPU Time}_B}{\text{CPU Time}_A} = \frac{IC_B \times CPI_B \times \text{clock cycle Time}_B}{IC_A \times CPI_A \times \text{clock cycle Time}_A}$$

$$= \frac{I \times 1.2 \times 500}{I \times 2 \times 250}$$

$$\frac{\text{CPU Time}_B}{\text{CPU Time}_A.} = 1.2$$

$$\frac{\frac{1}{\text{performance}\ \cancel{\text{Execution}}\ \cancel{\text{Time}}\ B}}{\frac{1}{\text{performance}\ \cancel{\text{Execution}}\ \text{Time}\ A}} = 1.2$$

$$\cancel{\text{Execution Time}_A = 1.2 \times \text{Executi}}$$

$$\therefore \boxed{\text{performance}_A = 1.2 \times \text{performance}_B}$$

$$\text{CPU Time}_B = \cancel{\text{Execution Time}_B}\ \text{performance}_B$$

$a = 2$
$b = a + 3$
$c = a + b.$

CPI in More Detail

If different instruction class take different numbers of cycles

$$\text{CPU clock cycles} = \sum_{i=1}^{n} \left(CPI_i \times C_i \right)$$

Instruction Count

$$\boxed{\text{CPU clock cycles} = CPI_1 \times C_1 + CPI_2 \times C_2 + \ldots + CPI_n \times C_n}$$

* Weighted Average CPI

$$\boxed{CPI = \frac{\text{CPU clock cycles}}{\text{Instruction Count}}}$$

Problem: A compiler designer is trying to decide between two code sequences for a computer. For a particular high-level language, the compiler writer is considering two code sequences that requires the following instruction counts. Which code sequence executes the most instructions? Which will be faster? What is the CPI for each sequence?

	Class	A	B	C	
	CPI	1	2	3	
program1 *	IC in code sequence 1	2	1	2	2+1+2=5
program2 *	IC in code sequence 2	4	1	1	4+1+1=6

sequence 2 executes the most instruction

$$\text{CPU clock cycles}_1 = 2 \times 1 + 1 \times 2 + 2 \times 3$$
$$= 2 + 2 + 6 = 10$$

$$\text{CPU clock cycles}_2 = 4 \times 1 + 1 \times 2 + 1 \times 3$$
$$= 4 + 2 + 3 = 9.$$

Code Sequence 2 Will be faster.

$$CPI_1 = \frac{\text{CPU clock cycles}_1}{\text{Instruction count}_1}$$

$$= \frac{10}{5} = 2$$

$$CPI_2 = \frac{\text{CPU clock cycles}_2}{\text{Instruction count}_2}$$

$$= \frac{9}{6} = 1.5$$