

## Chapter 3

### Binary Addition:

Four Basic Rules of Binary Addition:

$$0 + 0 = 0 \quad \text{sum} = 0, \text{carry} = 0$$

$$0 + 1 = 1 \quad \text{sum} = 1, \text{carry} = 0$$

$$1 + 0 = 1 \quad \text{sum} = 1, \text{carry} = 0$$

$$1 + 1 = 10 \quad \text{sum} = 0, \text{carry} = 1$$

$$\begin{array}{r}
 1 + 1 + 1 = ? \\
 \begin{array}{r}
 1 \\
 + 1 \\
 \hline
 10 \\
 + 1 \\
 \hline
 11 \\
 \begin{array}{cc}
 \nearrow & \nwarrow \\
 \text{carry} & \text{sum}
 \end{array}
 \end{array}
 \end{array}$$

Example 1:  $110 + 100 = ?$

$$\begin{array}{r}
 \overset{2}{1} \overset{0}{1} \overset{0}{0} \quad (6) \\
 + 100 \quad (4) \\
 \hline
 1010
 \end{array}$$

Example 2:  $111 + 11 = ?$

$$\begin{array}{r}
 \overset{2}{1} \overset{1}{1} \quad (7) \\
 + 11 \quad (3) \\
 \hline
 1010 \quad (10)
 \end{array}$$

### Binary Subtraction:

$$0 - 0 = 0$$

$$1 - 0 = 1$$

$$1 - 1 = 0$$

$$0 - 1 =$$

$$\begin{array}{r} 10 \text{ (2)} \\ \cancel{0} \\ - 1 \\ \hline 1 \end{array}$$

Example 1:  $101 - 011 = ?$

$$10 = 2$$

$$\begin{array}{r} \phantom{0} 10 \\ + \cancel{0} \phantom{0} 1 \\ - \phantom{0} 0 \phantom{0} 1 \phantom{0} 1 \\ \hline \phantom{0} 0 \phantom{0} 1 \phantom{0} 0 \end{array}$$

Example 2:  $110 - 101 = ?$

$$\begin{array}{r} \phantom{0} 0 \phantom{0} 10 \\ + \phantom{0} 1 \phantom{0} + \cancel{0} \phantom{0} 0 \\ - \phantom{0} 1 \phantom{0} 0 \phantom{0} 1 \\ \hline \phantom{0} 0 \phantom{0} 0 \phantom{0} 1 \end{array}$$

## Binary Multiplication:

Four Basic Rules of Binary Multiplication:

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

Example 2:  $111 \times 101 = ?$

The image shows a handwritten binary multiplication problem:  $111 \times 101$ . The numbers are aligned vertically, with the multiplier 101 on the right and the multiplicand 111 on the left. A horizontal line separates the two numbers. The multiplication is performed using the four basic rules of binary multiplication. The result is shown below the line, with the final sum of 110011.

$$\begin{array}{r} 111 \\ \times 101 \\ \hline 111 \\ 000 \\ 111 \\ \hline 110011 \end{array}$$

## Floating Point Numbers

→ **Scientific notation:** A notation that renders numbers with a single digit to the left of the decimal point.

**Normalized Number:** A number in scientific notation that has no leading 0s is called a normalized number.

→ 0.001

$1.0 \times 10^{-3}$

 → scientific Notation.

$10.0 \times 10^{-4}$

$0.1 \times 10^{-2}$

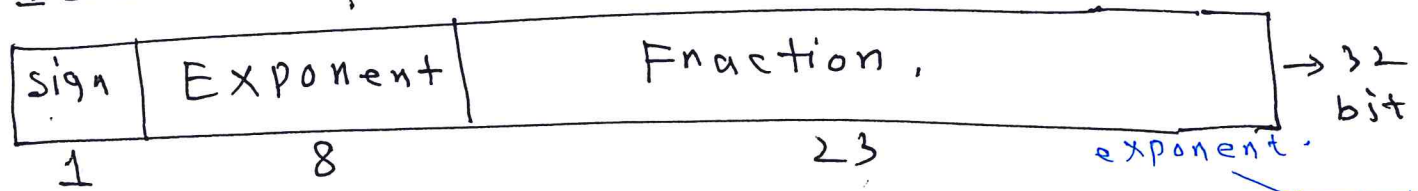
\* sign  
\* Fraction / Mantissa  
\* Exponent

- 
- Floating-point number (also known as a real number) consists of two parts plus a sign.
  - The mantissa is the part of a floating-point number that represents the magnitude of the number and is between 0 and 1.
  - The exponent is the part of a floating-point number that represents the number of places that the decimal point (or binary point) is to be moved.
  - For binary floating-point numbers, the format is defined by ANSI/IEEE Standard 754-1985 in three forms:
    - single-precision
    - double-precision

Example: Convert the decimal value 2.75 to IEEE-754 single precision format.  
Write your converted result in hexadecimal format.

IEEE-754 single precision,

$$\text{bias} = 127.$$



$$2.75_{10} = 10.11_2 = 1.011 \times 2^1$$

Fraction

Weight: 2 1 0.5 0.25

Binary: 1 0 . 1 1

$$\text{Sign} = 0$$

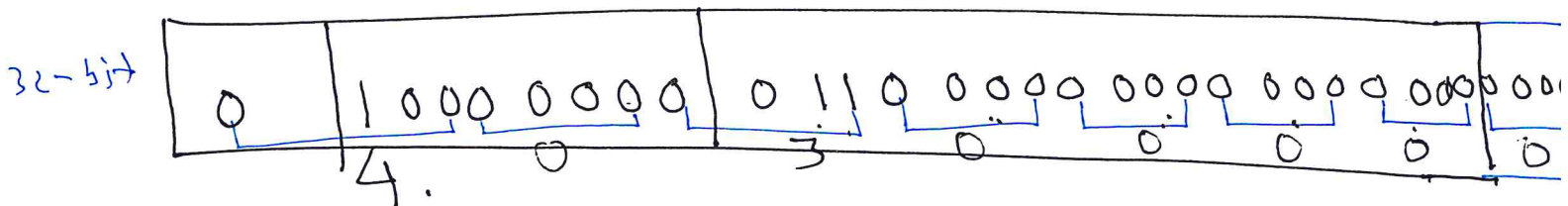
Actual Exponent = Exponent + bias

$$= 127 + 1$$

$$= 128, \quad \begin{array}{r} 1000 \ 0000 \\ \hline 8 \text{ bits} \end{array}$$

Fraction = 01100000000000000000000

23 bits



**Example: Convert the decimal value -4.25 to IEEE-754 single precision format. Write your converted result in hexadecimal format.**

Weight :  $2^2 \quad 2^1 \quad 2^0 \quad 2^{-1} \quad 2^{-2} \quad (0.25)$

Binary :  $1 \quad 0 \quad 0 \quad 0 \quad 1$

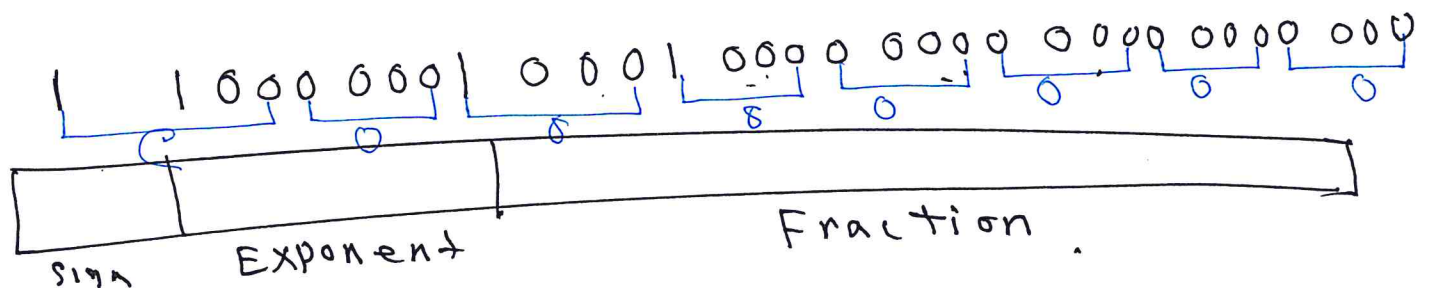
$$-4.25 = -100.01 = -1.0001 \times 2^2$$

sign = 1

Bias = 127

Exponent =  $127 + 2 = 129$

Fraction = 0001



0XC0880000