

$$\text{CPU Time} = \text{CPU Clock Cycles} \times \text{Clock Cycle Time}$$

$$\text{CPU Time} = \frac{\text{CPU Clock Cycles}}{\text{Clock Rate}}$$

$$\text{Clock cycle Time} = \frac{1}{\text{Clock Rate}}$$

Problem: Our favorite program runs in 10 seconds on computer A, which has a 2GHz clock. We are trying to help a computer designer build a computer, B, which will run this program in 6 seconds. The designer has determined that a substantial increase in the clock rate is possible, but this increase will affect the rest of the CPU design, causing computer B to require 1.2 times as many clock cycles as computer A for this program. What clock rate should we tell the designer to target?

computer A

$$\text{CPU Time}_A = 10 \text{ s}$$

$$\begin{aligned} \text{Clock Rate}_A &= 2 \text{ GHz} \\ &= 2 \times 10^9 \text{ Hz} \end{aligned}$$

computer B

$$\text{CPU Time}_B = 6 \text{ s}$$

$$\text{Clock-cycle}_B = 1.2 \times \text{Clock-cycles}_A$$

$$\text{Clock Rate}_B = ?$$

A

$$\text{Clock-cycle}_A = \text{CPU Time}_A * \text{Clock Rate}_A$$

$$= 10 * 2 * 10^9$$

$$= 20 * 10^9 \text{ Hz}$$

$$\text{Clock-cycle}_B = 1.2 * 20 * 10^9$$

$$= 24 * 10^9$$

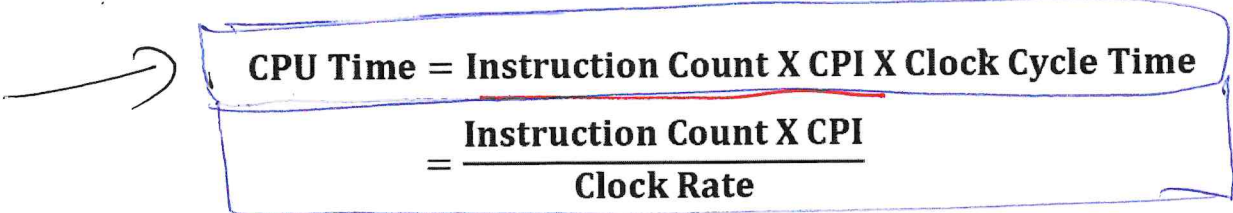
$$\begin{aligned}
 \text{Clock-Rate}_B &= \frac{\text{CPU clock cycles}}{\text{CPU Time}} \\
 &= \frac{24 \times 10^9}{6} \\
 &= 4 \times 10^9 \text{ Hz.}
 \end{aligned}$$

Instruction Count and CPI

(IC) Instruction Count: The number of instructions executed by the program.

(CPI) CPI (Clock cycles per instruction): Average number of clock cycles per instruction for a program.

$$\text{CPU clock-cycles} = \text{IC} \times \text{CPI}.$$



$$\begin{aligned}
 \text{CPU Time} &= \text{Instruction Count} \times \text{CPI} \times \text{Clock Cycle Time} \\
 &= \frac{\text{Instruction Count} \times \text{CPI}}{\text{Clock Rate}}
 \end{aligned}$$

$$\frac{\text{performance}_A}{\text{performance}_B} = \frac{\text{CPU Time}_B}{\text{CPU Time}_A} \quad IC = I$$

Problem: Suppose we have two implementations of the same instruction set architecture. Computer A has a clock cycle time of 250ps and a CPI of 2.0 for some program, and computer B has a clock cycle time of 500ps and a CPI of 1.2 for the same program. Which computer is faster for this program and by how much?

$$\begin{aligned} &\underline{A} \\ &\text{clock-cycle Time}_A = 250 \text{ ps} \\ &\text{CPI}_A = 2.0 \\ &IC_A = I \end{aligned}$$

$$\begin{aligned} &\underline{B} \\ &\text{clock-cycle Time}_B = 500 \text{ ps} \\ &\text{CPI}_B = 1.2 \\ &IC_B = I \end{aligned}$$

$$\begin{aligned} \text{CPU Time}_B &= I * 1.2 * 500 \text{ ps} \quad \text{--- (1)} \\ \text{CPU Time}_A &= I * 2.0 * 250 \text{ ps} \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} \frac{\text{CPU Time}_B}{\text{CPU Time}_A} &= \frac{\cancel{I} * 1.2 * 500 \text{ ps}}{\cancel{I} * 2.0 * 250 \text{ ps}} \\ \frac{\text{performance}_A}{\text{performance}_B} &= 1.2 \end{aligned}$$

$$\text{performance}_A = 1.2 \times \text{performance}_B$$

- ① $a = 2$
- ② $a = a - 3$
- ③ $b = a * 5$

CPI in More Detail

If different instruction class take different number of cycles

$$\begin{aligned}
 \text{CPU clock cycles} &= \sum_{i=1}^n \text{CPI}_i \times C_i \\
 &= \text{CPI}_1 C_1 + \text{CPI}_2 C_2 + \dots + \text{CPI}_n C_n
 \end{aligned}$$

Instruction count

Weighted Average CPI

$$\text{CPI} = \frac{\text{CPU clock cycles}}{\text{Instruction count}}$$

Problem: A compiler designer is trying to decide between two code sequences for a computer. For a particular high-level language, the compiler writer is considering two code sequences that requires the following instruction counts. Which code sequence executes the most instructions? Which will be faster? What is the CPI for each sequence?

Class	A	B	C
CPI	1	2	3
IC in code sequence 1	2	1	2
IC in code sequence 2	4	1	1

$$2 + 1 + 2 = 5$$

$$4 + 1 + 1 = 6$$

Code sequence 2 executes most instruction

$$\begin{aligned}
 \text{CPU clock-cycles}_A &= \text{CPI}_A C_A + \text{CPI}_B C_B + \text{CPI}_C C_C \\
 &= 1 \cdot 2 + 2 \cdot 1 + 3 \cdot 2 \\
 &= 2 + 2 + 6 \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 \text{CPU clock-cycles}_B &= \cancel{\text{CPI}_B C_B} + \dots \\
 &= 4 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 \\
 &= 9
 \end{aligned}$$

Code sequence 2 is faster.

$$\begin{aligned}
 \text{CPI}_1 &= \frac{\text{CPU clock cycles}_1}{\text{Instruction Count}_1} \\
 &= \frac{10}{5} = 2
 \end{aligned}$$

$$\text{CPI}_2 = \frac{9}{6} = 1.5$$