Chapter 3

Binary Addition:

Four Basic Rules of Binary Addition:

$0 + 0 = 0$ sum = 0, carry = 0

$0 + 1 = 1$ sum = 1, carry = 0

$1 + 0 = 1$ sum = 1, carry = 0

$1 + 1 = 10$ sum = 0, carry = 1

carry sum

Example 1: $110 + 100 = ?$

$$\begin{array}{r} 110 \\ 100 \\ \hline 1010 \end{array}$$

$1 + 1 + 1 = 11$

$$\begin{array}{r} 1 \\ +1 \\ \hline 10 \\ +1 \\ \hline 11 \end{array}$$

Example 2: $111 + 11 = ?$

$$\begin{array}{r} 111 \\ 111 \\ 11 \\ \hline 1010 \end{array}$$

Binary Subtraction:

$1 - 0 = 1$

$0 - 0 = 0$

$1 - 1 = 0$

$0 - 1 = ?$

$\overset{2}{10} - 1 = 1$

$10 = 2$

Example 1: 101 - 011 = ?

$$\begin{array}{r} \overset{0}{\cancel{1}}\ \overset{10}{\cancel{0}}\ 1 \\ 0\ 1\ 1 \\ \hline 0\ 1\ 0 \end{array}$$

$10_{(2)}$

Example 2: 110 - 101 = ?

$$\begin{array}{r} 1\ \ \cancel{1}0\ \ \overset{10}{\cancel{0}} \\ 1\ \ 0\ \ 1 \\ \hline 0\ \ 0\ \ 1 \end{array}$$

Binary Multiplication:

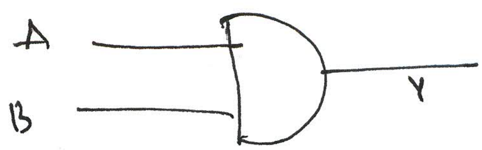
Four Basic Rules of Binary Multiplication:

0 x 0 = 0

0 x 1 = 0

1 x 0 = 0

1 x 1 = 1



Boolean Multiplication

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Example 2: 111 X 101 = ?

```
      1 1 1
      1 0 1
  -----------
      1 1 1
 1  1
    0 0 0
  1 1 1
  -----------
 1 0 0 0 1 1
```

$\boxed{3} \rightarrow \underbrace{0011}$

$\pi = 3.1416$

$\pi = 3.14159265358 9 \ldots$

Floating Point Numbers

Scientific notation: A notation that renders numbers with a single digit to the left of the decimal point.

Normalized Number: A number in scientific notation that has no leading 0s is called a normalized number.

0.001

Not Normalized.

1.0×2^{-3} S = 0

Normalized exponent = −3

Scientific Number. Fraction = 0

~~1.0 × 2~~

$1.0.0 \times 2^{-4}$.

* sign
* Mantissa. (Fraction)
* Exponent.

- Floating-point number (also known as a real number) consists of two parts plus a sign.
- The mantissa is the part of a floating-point number that represents the magnitude of the number and is between 0 and 1.
- The exponent is the part of a floating-point number that represents the number of places that the decimal point (or binary point) is to be moved.
- For binary floating-point numbers, the format is defined by ANSI/IEEE Standard 754-1985 in three forms:
    - single-precision
    - double-precision

8	4	2	1	.	0.5	0.25	0.125
		1	0	.	1	1	

Example: Convert the decimal value 2.75 to IEEE-754 single precision format. Write your converted result in hexadecimal format.

IEEE-754 Single precision Format

S	Exponent	Fraction
1	8	23

S : Sign bit (1 → Negative 0 → Non-negative)

Exponent = Actual exponent + bias

For single precision, bias = 127

Fraction = 23-bit fractions.

2.75

10 . 11

$0.75 \times 2 = 1.5$

$0.5 \times 2 = 1.0$

Repeated Multiplication by 2

$$\boxed{2.75_{10} = 10.11_2}$$

$$= 1.011 \times 2^1$$ (Normalized)

S = 0

Exponent = 1 + 127 = 128_{10}

10000000_2

Fraction = 011

011000 0

Sign	Exponent	Fraction
0	1000 0000 0	011 0000 0000 0000 0000 0000

4 0 3 0 0 0 0 0

Hex value = 0X 4030 0000

↑ Hex

Whole fractional

Example: Convert the decimal value -4.25 to IEEE-754 single precision format. Write your converted result in hexadecimal format.

	Whole					Fractional		
Weight	8	4	2	1	.	0.5	0.25	0.125
		1	0	0	.	0	1	

$$-4.25_{10} = -100.01_2 = \underbrace{-1.0001 \times 2^2}_{\text{Normalized number}}$$

$S = 1$

Exponent $= 2 + 127 = 129_{10}$

10000001_2

Fraction $= 0001$.

Sign	Exponent	Fraction
1	1000 0001	0001000 0000 0000 0000 0000

1100 | 0000 | 1000 | 1000 | 0000 | 0000 | 0000 | 0000

C 0 8 8 0 0 0 0

Hex Value = 0X C0880000