Basics of Number System

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EET 340

Recap: Basics of Number System

Binary Numbers

- Binary to Decimal Conversion
- Decimal to Binary Conversion

Hexadecimal Numbers

- Binary to Hexadecimal Conversion
- Hexadecimal to Binary Conversion

Binary Numbers

- The binary number system has base 2.
- The value of digit is determined by its position in the number.
- The two binary digits are: 1 and 0.
- The weights in a binary number are based on powers of two.

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Example of a binary full number: 101

Example of a binary fractional number: 101.01
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Whole number

Fractional part

TABLE 2–1

Decimal Number	Binary Number			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

TABLE 2–2

Binary weights.

Positive Powers of Two (Whole Numbers)									Negative Powers of Two (Fractional Number)					
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
256	128	64	32	16	8	4	2	1	1/2 0.5	1/4 0.25	1/8 0.125	1/16 0.625	1/32 0.03125	1/64 0.015625

The Weighting Structure of Binary Number

- A binary number is a weighted number. The right most bit is the LSB (least significant bit) in a binary whole number and has a weight of $2^0=1$. For whole numbers, the weights are positive powers of two that increases from right to left.
- Fractional decimal digits are placed to the right of decimal point. The left most bit is MSB in a binary fractional number and a weight of 2^{-1}. For fractional numbers, the weights are negative powers of two that decreases from left to right.
- The weight structure of a binary number is:

$$\underbrace{2^{n-1}\ \ldots\ldots\ 2^{3}\ 2^{2}\ 2^{1}\ 2^{0}}_{\textbf{Whole Number part}}\ \underset{\textbf{Binary Point}}{\underset{\downarrow}{.}}\ \underbrace{2^{-1}\ 2^{-2}\ 2^{-3}\ \ldots\ldots\ 2^{-n}}_{\textbf{Fractional part}}$$

Example 1: Convert 1101 to decimal.

Weights:	2^3	2^2	2^1	2^0
Binary Digit:	1	1	0	1

$$1101 = 2^3 + 2^2 + 2^0$$
$$= 8 + 4 + 1 = 13$$

Example 2: Convert 10.111 to decimal number.

Weights:	2^1	2^0	·	2^{-1}	2^{-2}	2^{-3}
Binary Digit:	1	0	.	1	1	1

$$10.111 = 2^1 + 2^{-1} + 2^{-2} + 2^{-3}$$
$$= 2 + 0.5 + 0.25 + 0.125 = 2.875$$

Converting whole decimal numbers to binary (Sum-of-weights Method)

- Determine the set of binary weights whose sum is equal to the decimal number.
- Place 1's and 0's on the appropriate weight positions determines the binary number for that decimal number.

- An easy way to remember binary weights is that the lowest is 1, which is 2^0, and that doubling any weight, you get the next higher weight. A list of seven binary weights :

Weights: 64 32 16 8 4 2 1

Weights: 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Converting 9 to binary number

- Step 1: The set of binary weights 8 and 1, whose sum is equal to the decimal number.

Weights: 64 32 16 $\boxed{8}$ 4 2 $\boxed{1}$

- Step 2: Place 1's and 0's on the appropriate weight positions determines the binary number for that decimal number.

Weights:	~~64~~	~~32~~	~~16~~	8	4	2	1
				1	0	0	1

Desired binary number

Example 1: Convert decimal number 25 to binary using Sum-of-Weights Method.

```
25 = 16 + 8+ 4 + 1

Weights:      16      8       4      2      1
Weights:      2^4     2^3     2^2    2^1    2^0

               1      1       0      0      1
Binary value = 11001
```

Example 2: Convert decimal number 58 to binary using Sum-of-Weights Method.

```
58 = 32 + 16 + 8 + 2

Weights:      32     16      8       4      2      1
Weights:      2^5    2^4     2^3     2^2    2^1    2^0

               1      1      1       0      1      0
Binary value = 111010
```

Hexadecimal Numbers

- The hexadecimal number system has sixteen characters; it is used primarily as a compact way of displaying or writing binary numbers because it is very easy to convert between binary and hexadecimal.
- As you are probably aware, long binary numbers are difficult to read and write because it is easy to drop or transpose a bit.
- The hexadecimal number system has a base of sixteen; that is, it is composed of 16 numeric and alphabetic characters.
- Most digital systems process binary data in groups that are multiples of four bits, making the hexadecimal number very convenient because each hexadecimal digit represents a 4-bit binary number

Hexadecimal Numbers

TABLE 2–3

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Binary-to-Hexadecimal Conversion

- Simply break the binary number into 4-bit groups, starting at the right-most bit and replace each 4-bit group with the equivalent hexadecimal symbol.

Convert the following binary numbers to hexadecimal:
(a) 1100101001010111 (b) 111111000101101001

Solution

(a) $\underbrace{1100}_{\downarrow \atop C}\underbrace{1010}_{\downarrow \atop A}\underbrace{0101}_{\downarrow \atop 5}\underbrace{0111}_{\downarrow \atop 7} = \mathbf{CA57}_{16}$

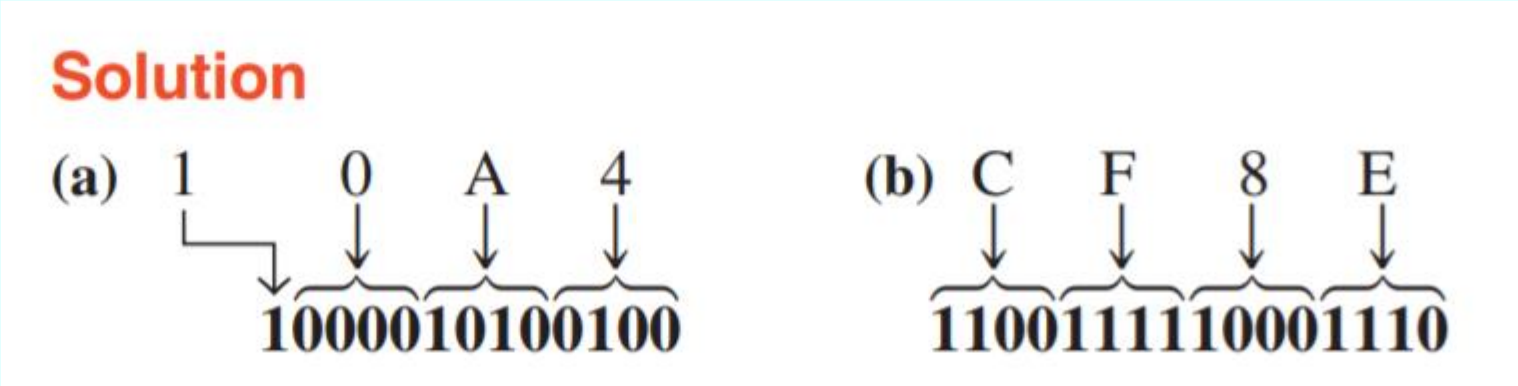
(b) $\underbrace{0011}_{\downarrow \atop 3}\underbrace{1111}_{\downarrow \atop F}\underbrace{0001}_{\downarrow \atop 1}\underbrace{0110}_{\downarrow \atop 6}\underbrace{1001}_{\downarrow \atop 9} = \mathbf{3F169}_{16}$

Two zeros have been added in part (b) to complete a 4-bit group at the left.

Hexadecimal-to-Binary Conversion

- To convert from a hexadecimal number to a binary number, reverse the process and replace each hexadecimal symbol with the appropriate four bits.

Determine the binary numbers for the following hexadecimal numbers: (a) $10A4_{16}$ (b) $CF8E_{16}$



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Source:

1.Digital Fundamentals (11^{th} Edition) by Floyd