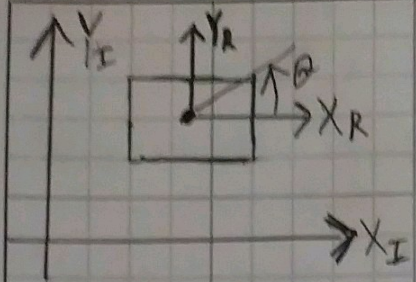


$\xi = \begin{bmatrix} X \\ Y \\ \theta \end{bmatrix}_I$ = Pose in Global Frame $\{X_I, Y_I\}$

*X_R is forward heading of robot

P = center of Robot chassis
A = position of wheel in $\{X_R, Y_R\}$ in polar coordinates (l, α)

l = dist. from robots origin P to center of wheel A
α = angle of $\overline{PA}$ w.r.t. $\{X_R, Y_R\}$

β = angle of wheel plane relative to the robots chassis

r = radius of wheel
$\varphi(t)$ = angular position of the wheel w.r.t. time

γ = angle between main wheel plane and axis of circumferential rollers

> *Refer to [1] Sec. 3.2.1 for how to represent the robot position w.r.t. the global frame

2 types of Constraints for Wheels

Rolling Constraint = the wheel must roll when motion takes place in the appropriate direction

Sliding Constraint = the wheel must not slide orthogonal to the wheel plane

Swedish Wheel

*Note: Swedish 45-degree wheel has $\gamma = 45^\circ$ or -45°, and would be considered fixed

Rolling Constraint: ☆☆☆☆☆

$$[\sin(\alpha+\beta+\gamma), -\cos(\alpha+\beta+\gamma), (-l)\cos(\beta+\gamma)]\, R(\theta)\dot{\xi}_I - r\dot{\varphi}\cos\gamma = 0$$

"Sliding Constraint":

$$[\cos(\alpha+\beta+\gamma), \sin(\alpha+\beta+\gamma), l\sin(\beta+\gamma)]\, R(\theta)\dot{\xi}_I - r\dot{\varphi}\sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$

*Note: r_{sw} = radius of circumferential rollers
$\dot{\varphi}_{sw}$ = angular velocity of circumferential rollers

$R(\theta)$ = orthogonal rotation matrix for mapping motion in Global frame $\{X_I, Y_I\}$ to motion in terms of Local frame $\{X_R, Y_R\}$

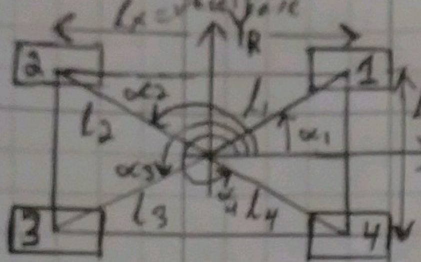
$\dot{\xi}$ = Velocity of the robot in Global frame $\{X_I, Y_I\}$ as $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$

[1] Introduction to Autonomous Mobile Robots, Siegwart/Nourbakhsh/SCARAMUZZA

Given a robot of M wheels (M = # of wheels): 4 swedish wheels

*Note: Swedish wheels impose no kinematic constraints on robot chassis since $\dot{\xi}$ can range freely in all cases due to the internal dofs

*So for 4 swedish wheels: l, α_i, β_i, γ_i ($=45°$ or $-45°$ → depending on left/right wheel config.), r (consistent between wheels), $\dot{\varphi}_i$



Combining all wheels rolling constraints into a matrix equation:
(and since no steerable wheels exist $[J_1(\beta_s) \rightarrow J_{1f}]$)

$$J_{1f} R(\theta) \dot{\xi}_I - J_2 \dot{\varphi} = 0$$

where:

$$R(\theta) = \begin{bmatrix} \cos\theta, & \sin\theta, & 0 \\ -\sin\theta; & \cos\theta; & 0 \\ 0; & 0, & 1 \end{bmatrix}$$

$$J_{1f} = \begin{bmatrix} \sin(\alpha_1+\beta_1+\gamma_1), & -\cos(\alpha_1+\beta_1+\gamma_1), & -l\cos(\beta_1+\gamma_1) \\ \cdots 2 & \cdots 2 & \cdots 2 \\ \cdots 3 & \cdots 3 & \cdots 3 \\ \cdots 4 & \cdots 4 & \cdots 4 \end{bmatrix}$$

$\dot{\xi}$ = forward kinematic model (velocity)

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_I = f(l, \alpha_{1-4}, \beta_{1-4}, \theta, \dot{\varphi}_{1-4})$$

$$J_2 = \begin{bmatrix} r_1\cos\gamma, & 0, & 0, & 0 \\ 0; & r_2\cos\gamma; & 0; & 0 \\ 0; & 0; & r_3\cos\gamma, & 0 \\ 0; & 0; & 0, & r_4\cos\gamma \end{bmatrix}$$

$$\dot{\varphi} = \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \\ \dot{\varphi}_4 \end{bmatrix}$$

*Rearrange Rolling Constraint Matrix gives

$$\underbrace{\dot{\xi}_I = R(\theta)^{-1} J_{1f}^{-1} J_2 \dot{\varphi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_I}_{\text{Forward Kinematic Model for a 4 Swedish Wheeled Robot Chassis}}$$

*Note: Using the combination of sliding constraints ($C_{1f} R(\theta) \dot{\xi}_I = 0$) the mobility ($\delta_m$) of this robot can be shown to be $\delta_m = 3$ (and is therefore, omnidirectional)

Additionally, the degree of steerability (δ_s) is $\delta_s = 0$, because wheel velocities are used to control motion.

And therefore, the degree of maneuverability (δ_M) is $\delta_M = 3$

*Refer to Section 3.3 from Ref [1]

Need to verify!!!

4 wheeled Mecanum Robot → Forward Kinematics

*Note: $\gamma = 45°$ means top roller of Mecanum Wheel has its axis of rotation directed inward towards robot chassis so

$\gamma_1 = \gamma_3 = 45°$ and $\gamma_2 = \gamma_4 = -45°$

Also $r_1 = r_2 = r_3 = r_4 = r$ is the radius of wheel

* Also ignoring $R(\theta)^{-1}$ for the moment to find $\dot{\xi}_R$ w.r.t. robot chassis

Also $\alpha_2 = 180° - \alpha_1$, $\alpha_3 = 180° + \alpha_1$, $\alpha_4 = -\alpha_1$

Also $\beta_1 = 90° - \alpha_1$, $\beta_2 = -\beta_1$, $\beta_3 = \beta_1$, $\beta_4 = -\beta_1$

Also $l_1 = l_2 = l_3 = l_4 = l$ is length from center of robot chassis to Wheel

*derived in [2] in Eq. 14, 15, and 16

[Define Robot Parameters as defined in Table 1 [2]] where l and r are constant

so Forward Kinematics is... and Inverse Kinematics is

$$\boxed{\dot{\xi}_R = J_f^{-1} J_2 \dot{\varphi}} \qquad \boxed{\dot{\varphi} = J_f J_2^{-1} \dot{\xi}}$$

Need to verify!!!!

[2] Kinematic Model of a Four Mecanum Wheeled Mobile Robot