IST 772 Midterm Exam

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Instructions: Compose brief answers to each of the following questions, typing your response in *italics* below each question. Please also include a .R file with IF RELEVANT.

1. Describe the conceptual connection between $\boldsymbol{\mu}$ ("mu," the population mean), $\{\bar{x}\}$ ("x-bar," a sample mean) and the sampling distribution. How are they connected to each other?

Population means and sample means are connected because sample means are essentially point estimates of the population mean. To get more specific, by the law of large numbers, the mean of many sample means approaches the population mean. We can also infer the population mean by looking at the distribution of the sample means. By the central limit theorem, we know that a large number of sample means will approach a normal distribution. Given all of this, and the fact that normal distribution is symmetrical (meaning mode = mean), we can look at the modal value of the sample mean distribution and know that over many samples, that value will approach the population mean.

2. Your boss at the New York Times asks you to conduct an A/B test on two different headlines about the same story. Each headline is displayed on n=140 high traffic social media pages:

Headline 1 gets an average of 2400 clicks per hour.
Headline 2 gets an average of 2200 clicks per hour.

The 95% confidence interval is as follows:

100 < (mean difference between Headline 1 and 2) < 300.

Answer the following questions about that confidence interval:

a. On the basis of this confidence interval conduct a hypothesis test at the 0.05 level under the alternative hypothesis that average clicks per hour are not equal. State the null and alternative hypotheses.

Let mu1 denote the population mean for clicks per hour of Headline 1.
Let mu2 denote the population mean for clicks per hour of Headline 2.

Technical:
H0: mu1 = mu2
H1: mu1 ≠ mu2

Conceptual:
H0: There is no difference in the average clicks per hour between Headline 1 and Headline 2
H1: There is a difference in the average clicks per hour between Headline 1 and Headline 2

b. Would you reject or not reject the null hypothesis. Why or why not?

I would reject the null hypothesis. This is because we have an alpha of 0.05 and a 95% confidence interval for the difference between the two means that does not include 0. This means that we have significant evidence to say that a result of "mu1 - mu2 = 0", or "mu1 = mu2", is not not an expected result at the 0.05 level.

c. Based on your answer in b. What is your conclusion about the difference between the headlines? Is headline 1 or 2 better and why?

Based on the hypothesis test conducted in b, we only have statistically significant evidence to say that the average clicks per hour between Headline 1 and Headline 2 are different. However, we still can make a decision between the two. The key in telling which to choose is in how we actually decided to reject the null hypothesis in part b, which was the 95% confidence interval. Not only are we confident that mu1 - mu2 is not 0, but because the 95% confidence interval includes only positive numbers, we have statistically significant evidence to say that a result of "mu1 - mu2 < 0", or "mu1 < mu2", is not expected. That is, at the 0.05 level, we have statistically significant evidence to say that Headline 1 is better than Headline 2.

d. Your friend calculates the p-value for the hypothesis mentioned in questions a-c and finds that p = 0.25. Does this sound plausible to you? Why or why not?

This is not plausible because we concluded that with an alpha of 0.05, we can reject the null hypothesis, which would require a p-value less than 0.05. Obviously 0.25 is not less than 0.05, so given this p-value, we would actually fail to reject the null hypothesis, which is not plausible given our findings in parts a-c.

e. Your boss tells you to run the same experiment 999 more times, calculating a new confidence interval each time. Now you have a collection of 1000 confidence intervals, each of which was constructed in the same way, but from new data samples: What can you say about this collection of confidence intervals?

This collection of confidence intervals is expected to contain the actual mean difference between clicks per hour of Headline 1 and Headline 2 in about 95% of the confidence intervals. More specifically, we would expect about 950 of the 1000 confidence intervals to contain the actual mean difference between clicks per hour of Headline 1 and Headline 2.

f. Which command in R would you use to produce the confidence interval for each of the 1000 that you constructed?

t.test(mu1,mu2), given our definitions of mu1 and mu2 given in part a.

3. Tests for detecting diseases such as HIV are not 100% accurate and one can use Bayes' theorem to assess the probability that someone is actually infected HIV given a positive test. Please use the following facts to calculate the probability that someone has HIV after having received a positive test:

- For someone with HIV, the probability of a positive test is 99%
- The probability that someone has HIV is 3%.
- The probability of getting a positive test is 4%.

$$P(H|D) = \frac{P(D|H)*P(H)}{P(D)}$$

$$P(HIV|+) = \frac{P(+|HIV)*P(HIV)}{P(+)} = \frac{0.99*0.03}{0.04} = \frac{0.0297}{0.04} = \frac{297}{400} = 0.7425 = 74.25\%$$

Thus, per Bayes' Theorem and our likelihood (99%), prior (3%), and evidence (4%), we can conclude that the probability that someone has HIV after having received a positive test is 74.25%.

4. : The Null Hypothesis Significance test (NHST) is the classic inferential test used throughout the 20th century. The NHST comprises a set of logical steps that lead to a consideration of the viability of a stated null hypothesis. Following the material presented on page 77 of *Reasoning with Data*, here is an unordered list of the steps:

Calculate the test statistic **4**
Assert a null hypothesis **1**
Collect data **3**
Find the p-value associate with the test statistic **5**
Choose an alpha level **2**
Reject the null hypothesis **7**
Fail to reject the null hypothesis **7**
Evaluate the p-value with respect to alpha **6**

Place these steps in the correct order and add a brief one or two sentence explanation that describes the purpose and importance of each step.

1. ***Assert a null hypothesis test: This will essentially be the hypothesis that nothing of significance results from the NHST.***

2. ***Choose an alpha level: This is the threshold for acceptable p-values, or the upper bound of p-values that we will allow when rejecting the null hypothesis.***

3. ***Collect data: Collecting data will allow us to perform a statistical test, specifically a NHST, on those data.***

4. ***Calculate the test statistic: Calculating the test statistic is mostly important for finding the p-value.***

5. ***Find the p-value associated with the test statistic: This p-value is important because it gives us the probability of falsely rejecting the null hypothesis. We will use this to compare with our chosen alpha level.***

6. ***Evaluate the p-value with respect to alpha: If the p-value is at or below the alpha level, then we have significant evidence to reject the null hypothesis. If the p-value is above the alpha level, we do not have significant evidence to reject the null hypothesis.***

7. ***Reject the null hypothesis OR Fail to reject the null hypothesis: Using the evaluation discussed in step 6, this is where we truly determine the results of our NHST.***