

Problem 1:

We have $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$.

$$\Rightarrow P(x_1) - P(x_2) = \sum_{i=0}^n a_i (x_1^i - x_2^i)$$

We also have:

$$x_1^k - x_2^k = (x_1 - x_2) \sum_{j=0}^{k-1} x_1^j x_2^{k-j-1}$$

Therefore:

$$P(x_1) - P(x_2) = 0 \pmod{(x_1 - x_2)} \quad (1)$$

However, Bob function gives:

$$P(20) = 7 \text{ and } P(15) = 5$$

$$7 - 5 \neq 0 \pmod{(20 - 15)} \quad (\text{contradict to (1)})$$

\Rightarrow There was a mistake in Bob's implementation.