Detection Theory
Gestaltic Grouping of Line Segments

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MVA 2021-2022

1 Introduction

This project is an analysis of *Gestaltic Grouping of Line Segments* [**?**]. The paper takes inspiration from the Gestalt theory of visual perception, where already existent structures are recursively grouped to form more complex structures. At a high level, the algorithm proposed detect lines and then merge them into three groups: non-local alignments, good continuations, and parallel segments.

In this project we focus only on good continuations. We will show that the approach proposed in the paper does not follow the standard NFA definition and have some performance limitations in practice. Finally we propose a new theoretical formulation that solves most of these problems.

2 Original formulation

Definition: A sequence of k line segments $\ell_1, \ell_2, \ldots, \ell_k$ form a good continuation $g^{k,\rho,\theta}$ if and only if the maximum distance between two consecutive lines is $D_{max} < \rho$ and the absolute value of the maximum angle between two consecutive lines is $|\alpha_{max}| < \theta$.

The paper then defines a valid search region $\mathcal{S}^{\rho,\theta}$ for a good continuation of a given segment, as represented in Figure 1.

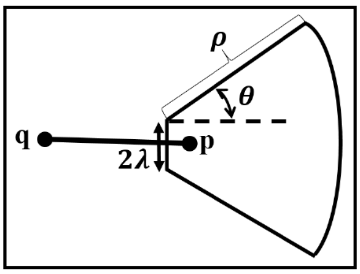


Figure 1: Search region $\mathcal{S}^{\rho,\theta}$ for a good continuation of the segment $q - p$. A good-continuation segment must have its tip inside $\mathcal{S}^{\rho,\theta}$ and orientation in the interval $(-\theta, \theta)$.

2.1 Good continuation event under background model

For the background model $\mathcal{H}_0$ we assume that the tips of the segments are uniform i.i.d. sampled on the image domain, of size $m \times n$. We consider first how to compute the probability of a good continuation between two segments ℓ_1 and ℓ_2. Under the background model, the probability of the tip of the segment ℓ_2 landing in the search region $\mathcal{S}^{\rho,\theta}$ of segment ℓ_1 is $\frac{\rho^2\theta}{mn} + 2\lambda\rho \simeq \frac{\rho^2\theta}{mn}$. We also need the angle between ℓ_1 and ℓ_2 to be in the interval $(-\theta, \theta)$. Then, the probability of segment ℓ_2 being a good continuation of ℓ_1 is $\frac{\rho^2\theta}{mn}\frac{\theta}{\pi}$. Finally, the probability under the background model of a good continuation of exactly k segments with maximum distance ρ and maximum angle θ, is given by

$$\mathbb{P}(g^{k,\rho,\theta}|\mathcal{H}_0) = \left(\frac{\rho^2\theta^2}{mn\pi}\right)^{k-1} \tag{1}$$

At this point we stress that this formulation does not follow the standard definition of a NFA. Usually, we observe a quantity of interest and then compute its probability under the background model. Instead, here we define the event $g^{k,\rho,\theta}$ with fixed parameters k, ρ, and θ and then compute its probability under the background model.

2.2 Number of tests

Given N line segments there are $\frac{N!}{(N-k)!}$ ways of forming a sequence with k segments. Then, the number of tests for event $g^{k,\rho,\theta}$ is

$$N_{tests} = \frac{N!}{(N-k)!} \tag{2}$$

The paper propose to use several fixed values of ρ and θ. In order to account for the number of pairs (ρ, θ) tried, called c_1, we redefine the number of tests to be

$$N_{tests} = c_1 \frac{N!}{(N-k)!} \tag{3}$$

In practice the algorithm tests chains of size $k = \{1, \ldots, k_{max}\}$ with $k_{max} = 5$ by default. However, if this is not incorporated in the NFA function the real bound on the expected number of false alarms would be $k_{max}\epsilon$ instead of ϵ. We account for this by setting the number of tests to be

$$N_{tests} = k_{max} c_1 \frac{N!}{(N-k)!} \tag{4}$$

Multiplying the number of tests N_{tests} and the probability of a good continuation under the background model $\mathbb{P}(g^{k,\rho,\theta}|\mathcal{H}_0)$ we arrive at our definition of the $\text{NFA}(k, \rho, \theta) = N_{tests}\mathbb{P}(g^{k,\rho,\theta}|\mathcal{H}_0)$

$$\text{NFA}(k, \rho, \theta) = k_{max} c_1 \frac{N!}{(N-k)!} \left(\frac{\rho^2\theta^2}{mn\pi}\right)^{k-1} \tag{5}$$

2.3 Checking the NFA property

The fundamental property of the *a contrario* framework is that we can control the expected number of false alarms in our detection task by only accepting detections with a score $\text{NFA}(k, \rho, \theta) \leq \epsilon$. Let's try to verify this for our NFA function.

$$\begin{aligned}
\mathbb{E}\left[\sum_{i=1}^{N_{tests}} \mathbb{1}_{\{g^{k,\rho,\theta}|\mathcal{H}_0\}}\right] &= \sum_{i=1}^{N_{tests}} \mathbb{E}[\mathbb{1}_{\{g^{k,\rho,\theta}|\mathcal{H}_0\}}] \\
&= \sum_{i=1}^{N_{tests}} \mathbb{P}(g^{k,\rho,\theta}|\mathcal{H}_0) \\
&= N_{tests}\mathbb{P}(g^{k,\rho,\theta}|\mathcal{H}_0) \\
&= \text{NFA}(k, \rho, \theta) \\
&\leq \epsilon
\end{aligned}$$

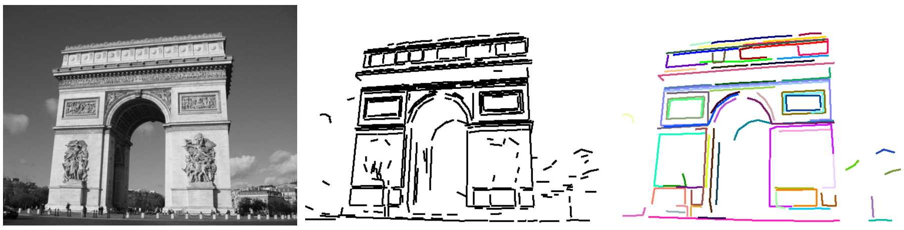
For the last inequality to hold we just need to set $\text{NFA}(k, \rho, \theta) \leq \epsilon$. This proves that we can control the average number of false alarms by only accepting individual detections where $\text{NFA}(k, \rho, \theta) \leq \epsilon$.

2.4 Candidate proposal

The line segments are detected using LSD [?]. To speed up the computations, for each tip of a segment only the three closest tips are considered. The maximum number of segments in a chains is limited to $k_{max} = 5$ by default.

2.5 Non-maximum suppression

In order to eliminate overlapping detections, we only keep sequences that contain no smaller sub-sequence with a smaller NFA.



(a) Input image (b) LSD detections (c) Good continuation detections

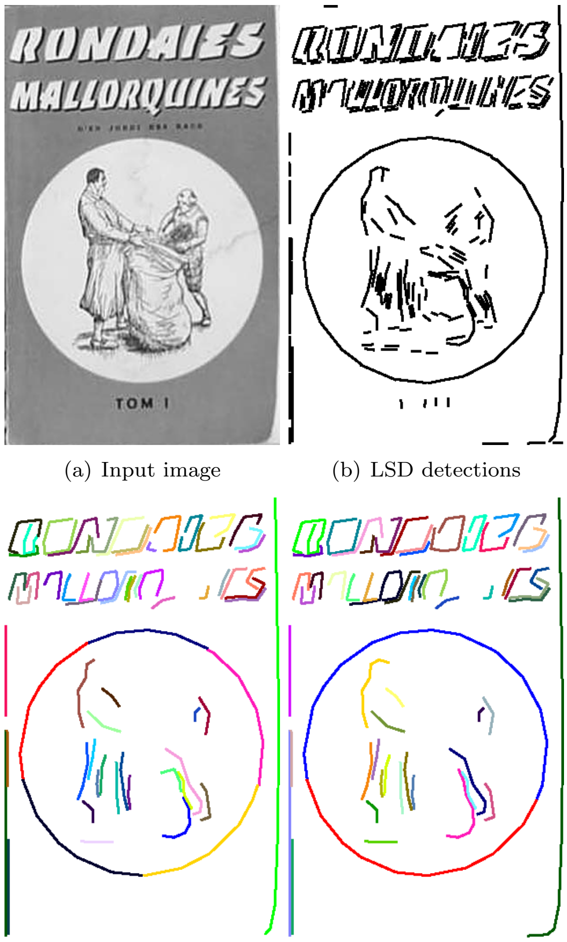
Figure 2: Detection example.

3 Limitations

The original formulation in the paper does not follow the standard definition of NFA. Usually, we observe a quantity of interest and then compute its probability under the background model. This automatically sets the detection thresholds on the quantity of interest. Instead, since we have three quantities of interest, the paper propose to define the event $g^{k,\rho,\theta}$ with fixed parameters k, ρ, and θ and then compute its probability under the background model. This effectively controls the expected number of false alarms but it is not strictly a NFA because the thresholds are not set automatically.

3.1 Fixed chain length

Since we defined our NFA test to deal with a finite number k_{max} of possible chain lengths, we sometimes detect a concatenation of small good continuations when there should be a single one good continuation containing them all. This is illustrated in Figure 3, where the algorithm cannot obtain a detection for the whole circle. Instead, it detects arcs.



(a) Input image

(b) LSD detections

(c) Good continuation detections with $k_{max} = 5$

(d) Good continuation detections with $k_{max} = 15$

Figure 3: Limitation of the good continuation detection. The algorithm detects arcs instead of the full circle. Allowing for larger continuations helps at the cost of more tests, but the problem still persists.

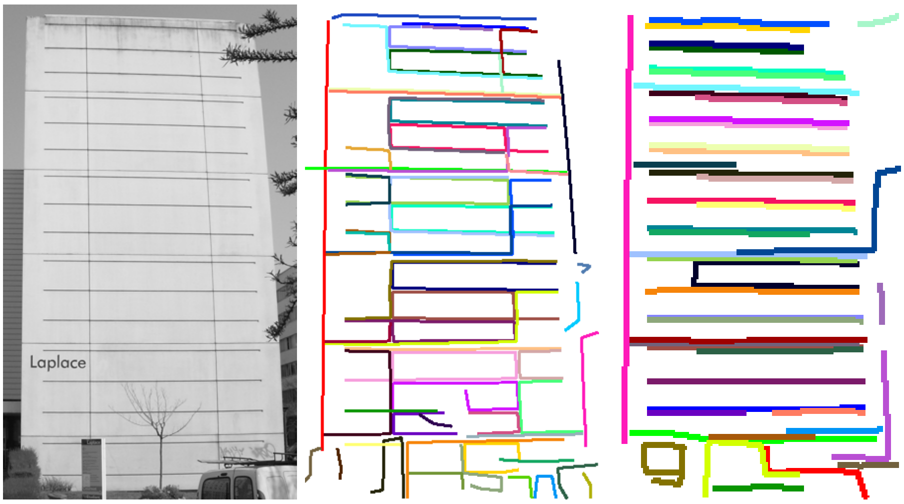
We can modify the formulation to allow for any chain length by considering

$$N_{tests} = 2^k c_1 \frac{N!}{(N-k)!} \tag{6}$$

This is equivalent to limit the detection of a chain of length k by $\frac{\epsilon}{2^k}$

3.2 Scale variant

Since the parameter ρ must be specified beforehand in pixel units the algorithm is not invariant to scale. The paper tris to overcome this by setting $\rho = \min(10, 0.1\max(m,n))$, but in practice this still leads to very different detections at different scales for the same image, as shown in Figure 4.



(a) Input image (b) Good continuation detections at full scale (c) Good continuation detections at half scale

Figure 4: Limitation of the good continuation detection due to the fixed ρ steps. The algorithm is not scale invariant.

4 Alternative formulation

We want to formulate the problem more similar to the standard form seen during the lectures. For this purpose, instead of fixing both parameters ρ and θ we just fix θ and measure ρ.

Consider two segments ℓ_1, ℓ_2. We observe that the tip of ℓ_2 lands at a distance r of the tip of ℓ_1. Under a uniform background model for the distance between the tips, denoted by ρ, the probability of $\rho \le r$ is $\mathbb{P}(\rho \le r | \mathcal{H}_0) = \frac{\pi r^2}{mn}$ If we consider chains of every possible length k and n_θ possible values of θ to test we get the following NFA test function

$$\mathrm{NFA}(\theta) = 2^k n_\theta \frac{N!}{(N-k)!} \left(\frac{\pi r^2}{mn}\right)^{k-1} \tag{7}$$

We can think of the detection test as consisting of two stages

1. Hard test on θ: if the measured angle α is $|\alpha| \geq \theta$, decide no detection.
2. Soft test on r: if $|\alpha| \leq \theta$, use the NFA formula to decide if the test is ϵ-meaningful

Note that is not still a fully standard NFA formulation because the NFA test is only valid under a fixed value of θ. However, this formulation frees us from specifying a fixed value of ρ beforehand and makes the algorithm naturally scale-invariant (fixed angles doesn't affect the scale).

5 Conclusions

In the project we analyzed an *a contrario* approach for detection of good continuations of line segments. This approach effectively controls the expected number of false alarms but within a non-standard formulation that forces use to fix all the important parameters (chain length, distance tolerance ρ, and angle tolerance θ) beforehand, leading to a non-robust algorithm in practice. We propose an alternative formulation that only need to fix θ and automatically sets the detection threshold on the distance ρ, and also works for any possible length of the chain.