Image Denoising - Homework 4

9.1| MAP formulation

$$P(\tilde{P}|P) = \frac{1}{(2\pi)^{d/2}\det(\sigma^2)^{1/2}} e^{-\frac{1}{2}\frac{\|P-\tilde{P}\|^2}{\sigma^2}}$$

Using Bayes Theorem:

$$P(P|\tilde{P}) = \frac{P(\tilde{P}|P)\,P(P)}{P(\tilde{P})}$$

Since we want to maximize, $P(\tilde{P})$ is just a constant
So we can maximize $\log P(\tilde{P}|P) + \log P(P)$

Note that we can simplify the term $\log P(\tilde{P}|P)$ since $\log \frac{1}{(2\pi)^{d/2}\det(\sigma^2)^{1/2}}$ is just a constant for the maximization

Therefore is sufficient to maximize $-\frac{1}{2}\frac{\|P-\tilde{P}\|^2}{\sigma^2} + \log P(P)$

Note that this is equivalent to minimize

$$\frac{1}{2}\frac{\|P-\tilde{P}\|^2}{\sigma^2} - \log P(P)$$

Finally, this is equivalent to maximize

$$E(P) = \frac{\|P-\tilde{P}\|^2}{\sigma^2} - \log P(P)$$

9.2| Technically, EPLL would be the log-likelihood if the patches were independent. This is not true, since patches may overlap.
However, the proportion of overlapping patches is small, so it's a fair approximation