Image Denoising - Homework 7

Ex 2.1

$$E_\theta^{FoE}(u,v) = \frac{1}{2\sigma^2}\|u-v\|^2 + \sum_{x\in\Omega}\sum_{i=1}^{N}\phi_i(k_i * u(x)) + c$$

where $\phi_i(z) = \alpha_i \log\left(1+\frac{z^2}{2}\right)$

$$\nabla E_\theta^{FoE} = \frac{\cancel{2}}{\cancel{2}\sigma^2}(u-v) + \sum_{i=1}^{N}\phi_i'(k_i * u(x)) * \bar{k}_i$$

where $\bar{k}_i$ is a π-rotation of k_i since when deriving a convolution the kernels are shifted

Escaneado con CamScanner

Ex 2.2 | Let $L_\infty(\theta) = \mathbb{E}\{\log p_\theta(x)\} = \int \log(p_\theta(x))p(x)\,dx$

$$= \lim_{m\to\infty} \frac{1}{m}\sum_{j=1}^{m} \log p_\theta(x_j)$$

Let $KL(p(x)\|p_\theta(x)) = \int \log\left(\frac{p(x)}{p_\theta(x)}\right) p(x)\,dx$

$$KL(p(x)\|p_\theta(x)) = \int \log(p(x))\,p(x)\,dx - \int \log(p_\theta(x))\,p(x)\,dx$$

$$= \underbrace{\int \log(p(x))\,p(x)\,dx}_{\text{doesn't depend on } \theta} - L_\infty(\theta)$$

$\Rightarrow$ maximizing $L_\infty(\theta)$ minimizes $KL(p(x)\|p_\theta(x))$