Image Denoising - Homework 8

$\boxed{\text{Ex 3.1}}$ Blind-spot network $\mathcal{F}$

noise: $\cdot\ \mathbb{E}_v[v|u] = u \quad \forall u$

$\cdot\ v_x \perp v_y \,|\, u$ (for $x \neq y$)

1) $\mathbb{E}_{u,v}[\|\mathcal{F}^\lambda(v) - u\|^2] = \mathbb{E}_{u,v}[\|\lambda \mathcal{F}(v) + (1-\lambda)v - u\|^2]$

$= \mathbb{E}_{u,v}[\|\lambda \mathcal{F}(v) - \lambda u + (1-\lambda)v - (1-\lambda)u\|^2$

$= \mathbb{E}_{u,v}[\|\lambda(\mathcal{F}(v) - u) + (1-\lambda)(v-u)\|^2]$

$= \lambda^2 \mathbb{E}_{u,v}[\|\mathcal{F}(v) - u\|^2] + (1-\lambda)^2 \mathbb{E}_{u,v}[\|v-u\|^2]$

$+ 2\lambda(1-\lambda)\mathbb{E}_{u,v}[\langle \mathcal{F}(v) - u, u - v\rangle]$

2) $\mathbb{E}_{u,v}[\langle \mathcal{F}(v) - u, v - u\rangle] = \mathbb{E}_u \mathbb{E}_v[\langle \mathcal{F}(v) - u, v - u\rangle | u]$

$= \mathbb{E}_u \mathbb{E}_v[\sum_j (\mathcal{F}(v)_j - u_j)(v_j - u_j) | u]$

$\mathcal{F}$ is j-invariant

$= \sum_j \mathbb{E}_u[(\mathbb{E}_v[\mathcal{F}(v)_j - u_j | u])\underbrace{(\mathbb{E}_v[v_j - u_j | u])}_{=0}]$

0 because $\mathbb{E}_v[v|u] = u$

3) $\frac{\partial J}{\partial \lambda} = \cancel{2}\lambda \mathbb{E}_{u,v}\{\|\mathcal{F}(v) - u\|^2\} - \cancel{2}(1-\lambda)\mathbb{E}_{u,v}\{\|v-u\|^2\} = 0$

$\lambda(\mathbb{E}_{u,v}\{\|\mathcal{F}(v) - u\|^2\} + \mathbb{E}_{u,v}\{\|v-u\|^2\}) = \mathbb{E}_{u,v}\{\|v-u\|^2\}$

$$\Rightarrow \lambda^* = \frac{\mathbb{E}_{u,v}\{\|v-u\|^2\}}{\mathbb{E}_{u,v}[\|\mathcal{F}(v)-u\|^2] + \mathbb{E}_{u,v}\{\|v-u\|^2\}}$$

$\mathbb{E}_{u,v}\{\|v-u\|^2\} = \mathbb{E}_u \mathbb{E}_v\{\|v-u\|^2 | u\} =$

$= \underbrace{\mathbb{E}_u \mathbb{E}_v[\langle v, v\rangle | u]}_{\text{Var}\{v|u\}} + \underbrace{\mathbb{E}_u \mathbb{E}_v[\langle u, u\rangle | u]}_{=0} - \underbrace{2\mathbb{E}_u \mathbb{E}_v[\langle v, u\rangle | u]}_{=0}$

u is the clear image.

also because $\mathbb{E}_v[v|u] = u$

4) $\mathrm{Var}(v|\mu) = d\sigma^2 \quad \forall \mu \in \mathbb{R}^d$

$$\lambda^* = \frac{d\sigma^2}{\mathbb{E}_{\mu,v}[\|\mathcal{F}(v) - \mu\|^2] + d\sigma^2}$$

From Proposition 3.4 $\rightarrow$ $\underbrace{\mathbb{E}_v[\|\mathcal{F}(v) - v\|^2]}_{R_{N2S}} = \underbrace{\mathbb{E}_{\mu,v}[\|\mathcal{F}(v) - \mu\|^2]}_{R_{N2C}} + \underbrace{\mathbb{E}_\mu \mathrm{Var}(v|\mu)}_{d\sigma^2}$

$$\boxed{\lambda^*} = \frac{d\sigma^2}{R_{N2S}(\mathcal{F}) - \cancel{d\sigma^2} + \cancel{d\sigma^2}} = \frac{d\sigma^2}{R_{N2S}(\mathcal{F})}$$

$\boxed{\text{Ex } 3.2}$

$$\mu = \mathbb{E}_v[v|\mu]$$

$$\mathbb{E}_v[\|\hat{\mu}(v) - \mu\|^2|\mu] = \mathbb{E}_v\left[\|\mathbb{E}_v[\hat{\mu}(v)|\mu] - \mu + \hat{\mu}(v) - \mathbb{E}_v[\hat{\mu}(v)|\mu]\|^2 \middle| \mu\right]$$

$$= \mathbb{E}_v\left[\|\mathbb{E}_v[\hat{\mu}(v)|\mu] - \mu\|^2 \middle| \mu\right] + \mathbb{E}_v\left[\|\hat{\mu}(v) - \mathbb{E}_v[\hat{\mu}(v)|\mu]\|^2 \middle| \mu\right]$$

$$+ \underbrace{2\,\mathbb{E}_v\left[\langle \mathbb{E}_v[\hat{\mu}(v)|\mu] - \mu, \hat{\mu}(v) - \mathbb{E}_v[\hat{\mu}(v)|\mu]\rangle \middle| \mu\right]}$$

$$= \sum_j (\mathbb{E}_v[\hat{\mu}(v)|\mu]_j - \mu_j)\,\mathbb{E}_v\big(\underbrace{\hat{\mu}(v)_j - \mathbb{E}_v[\hat{\mu}(v)|\mu]_j}_{=0}\big) = 0$$

$$= \|\mathbb{E}_v[\hat{\mu}(v)|\mu] - \mu\|^2 + \mathbb{E}_v\left[\|\hat{\mu}(v) - \mathbb{E}_v[\hat{\mu}(v)|\mu]\|^2 \middle| \mu\right]$$

Escaneado con CamScanner