ME 226 - Mechanical Measurements (S2)

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1 Introduction and Definitions

1.1 Why experiments?

- Test hypothesis, theories and models - We need to test the theory or model before it can be used to predict things. We test these by doing experiments.

- Exploratory reasearch - Experiments are also used to explore the domains which we are not very well familiar.

- Measurement of properties - For example, if we are working with new material, we will need to use the material properties which can be found out by experiments.

- Field scale measurements - Non destructive testing. We want to know measure some properties without destroying the sample.

- Design of control systems - To design any thing, we need to measure somethings to behaviour of the system at those conditions.

1.2 Complexities in designing experiments

- Sound knowledge of engineering principles is required

- Should have a knowledge of governing principles

- Should have keen insight into the physical principles of the processes being investigated

1.3 Intrusive and Non intrusive based measurement techiques

Measurement techniques can be classified into intrusive and non intrusive.

- **Thermal inertia**:
  - Intrusive techniques have thermal inertia, i.e, the probe takes some time to give the actual reading of the measurements.
  - For example, a thermocouple used to measure the room temperature takes some time to give values near real temperature.
  - Non-intrusive techniques are inertia free, i.e, they give near real value readings within negligible time.
  - For example, light based/radiation based techniques take negligible time to give reading. It still takes finite time since speed of light is finite but this can be neglected.

- **Point measurement / Field measurement**:
  - Intrusive techniques give point measurements, i.e, they take the measurement from a small region.
  - For example, a thermocouple inserted in a water bath only measures temperature at which it is inserted. To measure temperatures at different points, we either need to change the position of the thermocouple or insert more number of thermocouples.
  - Non intrusive based techniques give field measurement, i.e, they give the measurement of whole region in a single shot.

- **Representative Elementary Volume (REV)**:
  - Intrusive based techniques give the volume averaged reading of small finite region. This region is generally called representative elementary volume (REV).
  - Non instrusive based techniques' REV is dependent on the resolution of the detector and lesser compared to intrusive based techniques.

- **Temporal and Frequency Response**:
  - As intrusive based techniques have thermal inertia, they have a poor temporal and frequency response, i.e, the rate at which they record data is low.
  - In the case of non intrusive based techniques, temporal and frequency response is only limited by the detector or camera.

- **Cost**:
  - Non intrusive based technique apparatus are generally costlier than intrusive based techniques.
  - Also non intrusive based techniques' data processing is also a bit difficult.

1.4 Transducers

- Transducers are devices that converts one physical effect to another by transforming one type of energy to another.

- Most used transducers are electrical transducers, i.e, devices which convert non-eletrical effects (quantities to be measured) into electrical effects. These types of transducers are prefered because the electrical signals can be easily measured and also that we can extract useful information from these signals using computers.

- Requirements of transuders:

  - **Linearity**: It is desired that the input signal and the output signal maintain linearity.
  - **Ruggedness**: Transducers are required to have high electrical and mechanical strength, so that when it gets overloaded it should not deterioate.
  - **Repeatability**: If we perform same experiment under same conditions multiple times, the results should be identical.
  - **High Signal to Noise Ratio**: The transducer should possess high SNR.
  - **Reliability**: It should be reliable.
  - **No hysteresis**: There should be no changes in output due to hysteresis, i.e, if we move from point A to point B or the other way around, there should be no change in the output. Hysteresis can occurr due to thermal effects, magnetic effects etc.

- These can be classified into -
  Active transducers:

  - These do not require any external power source.
  - They generate analog voltage or current when stimulated by some form of energy.
  - For instance, thermocouples outputs an voltage that is related to temperature being measured without any external power source.
  - Further amplification is required for active transducers as the output is usually low.

  Passive transducers:

  - These are extenally powered.
  - They provide output in the form of some variation in resistance, capacitance or any other electrical parameter which has to being converted into equivalent electrical current or voltage using external energy.
  - Passive transducers are complex than active transducers for getting required parameters.

1.5 Definitions in measurement

- **Validity**: It is the degree to which a measuring strategy (instrument, machine, or test) measures what is to be measured. A measurement is valid if it measures the required quantity accurately.

- **Reliability**: A measuring instrument is reliable if it is consistently gives same result for same experiment performed under identical conditions.

- **Readability**: Readability is the smallest difference between values that can be read from the instrument.

- **Least Count**: It is the smallest difference between indications that can be detected on the instrument scale.

- **Sensitivity**: It is the ratio of the linear movement of the pointer on an analog instrument to the change in the measured variable causing this motion

- **Hysteresis**: The system is said to exhibit hysteresis if there is a difference in reading depending on whether it is approached from above or below.

- **Accuracy**: Indicates the deviation of the reading from a known input. Accuracy is frequently expressed as a percentage of fullscale reading.

- **Precision**: Indicates its ability to reproduce a certain reading with a given accuracy.

1.6 Errors

- **Error in measurement**: It refers to difference between the measurement we obtain and true value of the variable.

- Sources of Error:
    - **Spacial resolution**: Probe is never a point, so the value we get is the volume averaged over some volume.
    - **Temporal error**: The reading is not measured instantaneously by the instrument, it requires some finite some time. This can lead to error.
    - **Dynamic errors**: Working with high frequency lead to this type of error. (//todo: complete this)
    - **Systematic and operational errors**: It is the difference between actual reading and the reading that instrument is showing.
    - **Hardware errors**
    - **Software errors**: Errors during data analysis using computer.

- If true value is possible to calculate, then error can be found using

$$Error = x_{measured} - x_{true}$$

- **Scatter** : If the values recorded from the a no of runs are taken, then the values deviate from the their mean, this is called scatter. The extent of scatter (standard deviation of the readings) is called uncertainity.

- **Confidence interval** is the percentage of the readings that lie in the range $mean \pm 2\sigma$ (or whatever we specify).

- Types of experimental errors
    - **Gross blunders** : The apparatus or instrument contruction invalidates the data, i.e, wrong construction.
    - **Fixed errors** : This will cause error to be shifted by a approximately same amount. This are also called as bias error or systematic error. We can model these types of errors and correct the measured values.
    - **Random errors** : Human errors, random electronic fluctuations in instruments. This *generally* a statistical distribution.

2 Uncertainity analysis

Let $x_1, x_2, \ldots, x_n$ be independent variables and they are used calculate $R(x_1, x_2, \ldots, x_n)$. Let the uncertainties in the variables be $w_1, w_2, \ldots, w_n$. Then the uncertainity in the calcuated result is given by

$$\boxed{w_R = \sqrt{\left(\frac{\partial R}{\partial x_1}w_1\right)^2 + \left(\frac{\partial R}{\partial x_2}w_2\right)^2 + \ldots + \left(\frac{\partial R}{\partial x_n}w_\nu\right)^2}}$$

If R is of the form $R = x_1^{a_1} x_2^{a_2} \ldots x_n^{a_n}$, then

$$\frac{\partial R}{\partial x_i} = x_1^{a_1} x_2^{a_2} \ldots (a_i x_i^{a_i - 1}) \ldots x_n^{a_n}$$

$$\frac{1}{R}\frac{\partial R}{\partial x_i} = \frac{a_i}{x_i}$$

Using this,

$$\frac{w_R}{R} = \sqrt{\sum_{i=1}^{n}\left(\frac{a_i w_i}{x_i}\right)^2}$$

3 Laplace Transform

$$\mathcal{L}(f(t)) = \int_{0^-}^{\infty} e^{-st} f(t) dt$$

$$\mathcal{L}(f^{(n)}(t)) = s^n \mathcal{L}(f(t)) - \sum_{k=0}^{n-1} s^{n-1-k} f^{(k)}(0^-)$$

4 Dynamic systems

- Static measurements - When the quantity is not changing with time.
- Dynamic measurements - When the quantity is changing with time
A system may be described in terms of variable $x(t)$ which can be written as

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1}x}{dt^{n-1}} + \ldots + a_1 \frac{dx}{dt} + a_0 x = F(t)$$

where $F(t)$ is the force imposed on the system.

The **order of system** is defined as the order of the above differential equation.

5 Zero Order systems

$$a_0 x = F(t)$$

Here, $x = \frac{1}{a_0} F(t)$ variable x instants tracks the change in $F(t)$, $\frac{1}{a_0}$ is called the static sensitivity.
The real measurements strictly do not follow the zeroth order system

6 First Order Systems

$$a_1 \frac{dx}{dt} + a_0 x = F(t)$$

$$\frac{a_1}{a_0} \frac{dx}{dt} + x = \frac{1}{a_0} F(t)$$

$\frac{a_1}{a_0}$ is called the time constant of the system and has dimensions of time.

Consider an example -

$$F(t) = \begin{cases} 0 & t < 0 \\ A & t \geq 0 \end{cases}$$

Initial condition: At $x = x_0$ at $t = 0$
On solving, we have

$$x(t) = \frac{A}{a_0} + \left(x_0 - \frac{A}{a_1} \right) e^{-\frac{t}{\tau}}$$

where, $\tau = \frac{a_1}{a_0}$ and is called the time constant
$\frac{A}{a_0}$ is the steady state value
Rearranging it and putting $\frac{A}{a_0} = x_\infty$, we have

$$\frac{x - x_\infty}{x_0 - x_\infty} = e^{-\frac{t}{\tau}}$$

- **Time constant** is the time taken by the system to achieve 63.3% response of the steady state response (steady state response $= x_\infty - x_0$)

- **Rise constant** is the time taken by the system to achieve 90% response of the steady state value

$$t_{rise} = ln(10)\tau$$

- A response is usually assumed to be completely after 5τ.

- First order systems exhibit storage and dissipation capabilities.

7 Thermocouples

- **Seebeck Effect**: When two disimilar wire are connected at two junctions to make a loop and one of the junctions is kept at a higher temperature than the other, then there is a emf (and hence, current) in produced in the loop.

- In general, the relation between emf produced and temperature is non linear. But for small changes in temperatures, this can be approximated as

$$\Delta e_{AB} = \alpha \Delta T$$

  where α is **seebeck coefficient** can be approximately treated as constant.

- Two wires made of same metal do not produce any emf even if their junctions have a temp difference.

- Thermocouples are designed based on Seebeck effect. There are different thermocouples available with different metals which produce different changes in emf for changes in temperature.

- Transient Response of a Thermocouple - A thermocouple may be modeled as a first order system Rate at which sensor exchanges heat with the surroundings is equal to the rate of change of temperature of the sensor Assuming the that the sensor is a "lump" system, i.e, they are isothermal or their temperature do not vary with spacial coordinates.

$$hA(T - T_\infty) = mc\frac{\mathrm{d}T}{\mathrm{d}t}$$

  Solving this, we have

$$\frac{T(t) - T_\infty}{T_{initial} - T_\infty} = e^{-\frac{t}{\tau}}$$

  where

$$\tau = \frac{\rho V c_p}{hA}$$

7.1 Laws of Thermocouples

1. **Law of Homogeneous circuits**: It states that the thermal emf generated in the theromocouple is independent of the temperature distribution along the wire and only dependent on end temperatures.

2. **Law of Intermediate Temperatures**: The sum of emf generated by the junctions at T_1 and T_2 and emf generated by the junctions T_2 and T_3 is equal to emf generated by the junctions at T_1 and T_3.

$$e_{13} = e_{12} + e_{23}$$

3. **Law of Intermediate Metal**: A third metal wire inserted into the junction will have no effect on the emf produced by two metals iff the junctions with the third metal are kept at the same temperature.

8 RTD sensors

- In RTD sensors, we quantify the change in temperature using the change of resistance of the sensing element.

$$R = R_0(1 + \gamma_1 T + \gamma_2 T^2 + \ldots + \gamma_n T^n)$$

  where R_0 is the resistance of the sensising element at 0^oC

- Assuming linear variation of resistance wrt temperature, we can calculate temperature as

$$\Delta R = \gamma R_0 \Delta T$$

$$T = T_{ref} + \frac{\Delta R}{\gamma R_0}$$

- Usually, Pt, Ni, Cu are used as sensing elements. They representes as $X R_0$. For example, $Pt100$ is the RTD sensor which has 100Ω at 0^oC,

- They have a range of -250^oC to 1000^oC

- Sources of Errors -

1. **Self heating** (Joule's heating): Major source of error is , this is quantified using dissipation constant (P_d).
   Dissipation constant: It is the power to raise the temperature of RTD by 1K (or 1^oC).

   $$\text{Temperature change due to self heating } \Delta T = \frac{P}{P_d}$$

   Usually, excitation voltage should be limited to 0.25V and excitation power to 2mW. For higher sensitivity, voltage supplied should be higher. In this case, sensors with large surface area (more heat dissipation) should be employed.

2. **Lead wires resistance**: In the wheatstone bridge we actually measure the resistance of the RTD along with the resistance of lead wires. This causes a zero shift and reduction in sensitivity. Possible solution is to make the lead wires as short as possible.

3. **Stability**: Stability becomes an issue when the upper limit of sensor is exceeded.

- In the RTD, 3 lead wires (two are connected in parallel) are used instead of 2 lead wires because this will lead to reduction in internal resistance due to lead wires.

9 Thermistors

- Heat sensitive devices whose resistances changes rapidly with temperature.

- They *usually* have a negative temperature coefficient (NTC). (As temperature increases, resistance decreases)

- Resistance - temperature for thermistors can be expressed as

$$R = R_0 e^{\beta(\frac{1}{T} - \frac{1}{T_0})}$$

- Since the value of R_0 is high (around 3000Ω), lead wire resistance is negligible.

- Thermistors can measure upto 125^oC with an accuracy of 0.01^oC.

10 Basic Heat Transfer

- **Stefan - Boltzmann Law**: Rate of radiation emitted by a surface of area A at a temperature T is
$$\dot{Q} = \epsilon\sigma A T^4$$
where $\sigma = 5.67 \times 10^8 Wm^{-2}K^{-4}$ and $0 \le \epsilon \le 1$ is the emissivity of the surface.

- The sum of absortivity, reflectivity and transmissivity is always 1.

$$\alpha + \rho + \tau = 1$$

- **Blackbody Spectral Emissive Power**: It is the energy emitted by a blackbody at temperature T per unit time, per unit surface area and per unit wavelenght.

$$\boxed{E_{b\lambda}(T, \lambda) = \frac{c_1}{\lambda^5 (e^{\frac{c_2}{\lambda T}} - 1)}}$$

where $c_1 = 2\pi hc^2 = 3.742 \times 10^{-16} Wm^2$, $c_2 = \frac{hc}{k} = 1.439 \times 10^{-2} mK$

- **Wien's Displacement Law**: The blackbody radiation curve (spectral energy density vs wavelenght) for different temperatures has a peak at wavelengths inversely proportional to temperature.
$$\lambda_{max} = \frac{b}{T}$$

where $b = 2897.8 \mu mK$

11 Optical Pyrometer

- Optical pyrometer is non contact temperature measurement technique which can provide field measurement of the region of interest.

- In optical pyrometer, we compare the brightness of a filament whose brightness can be varied with the source brightness. The brightness of the filament is adjusted untill the brightness of the filament and the source is equal.

- The absorption screen is places before the filament to reduce the intensity of the light reaching the filament and appropriate calliberations are to made in the measured temperature.

- The bandpass filter allows only certain wavelength of light to pass through, so that we can compare the intensity at only that wavelength.

- When the brightness of the actual surface and the brightness of the filament are equal then,

$$(E_\lambda(\lambda, T))_{source} = (E_\lambda(\lambda, T))_{filament}$$

$$\frac{\epsilon_s c_1}{\lambda^5 (e^{\frac{c_2}{\lambda T_s}} - 1)} = \frac{c_1}{\lambda^5 (e^{\frac{c_2}{\lambda T_f}} - 1)}$$

Assuming the filament to be a black body and ignoring 1 in the denominator, since in the temperature range we are working, it is negligible

$$\frac{1}{T_s} - \frac{1}{T_f} = \frac{\lambda}{c_2} \log(\epsilon_s)$$

12 Infrared Camera Based Temperature Measurement

- A photon detector produces an electrical signal after being irradiated with photon flux.

- Based on the type of material of the sensor, their sensivity to the spectral range differs. This is due to difference in the band gap between conduction band and valence band between the materials.

- Based on their sensitivity to range they can be classified as

    - Near infrared (0.75-3 μm)
    - Mid infrared (3-6 μm)
    - Long infrared (7.5 - 15 μm)

- Energy striking the camera (per unit area and per unit time) from a surface is given by

$$E_c = E_{emitted} + E_{reflected}$$
$$= \epsilon_s E^b_{\lambda_1 - \lambda_2}(T_s) + \rho \epsilon_\infty E^b_{\lambda_1 - \lambda_2}(T_\infty)$$
$$= \epsilon_s F_{\lambda_1 - \lambda_2} \sigma T_s^4 + \rho_s \epsilon_\infty F_{\lambda_1 - \lambda_2} \sigma T_\infty^4$$

13 Strain Gauge

- Resistance $R = \frac{\rho L}{A}$, as L increases R increases. This principal is used to measure the strain.

- **Gauge Factor**: It is the ratio of relative change in resistance of the strain gauge to its strain.

$$S_G = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = \frac{\frac{\Delta R}{R}}{\epsilon}$$

$$S_G = \frac{\frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}}{\frac{dL}{L}}$$
$$= \frac{\frac{d\rho}{\rho} + \frac{dL}{L} + 2\nu \frac{dL}{L}}{\frac{dL}{L}}$$

$$\boxed{S_G = 1 + 2\nu + \frac{\frac{\Delta \rho}{\rho}}{\epsilon}}$$

where $\frac{\frac{\Delta \rho}{\rho}}{\epsilon}$ is called the piezo resitive sensitivity

- In metal piezo resitive sensitivity, piezo resitive sensitivity is very small. Hence, for metals, $S_G = 1 + 2\nu$

- For a wheat stone bridge consisting of resistances R_1, R_2 and R_3, R_4,

$$E_o = \frac{R_1 R_4 - R_3 R_2}{(R_1 + R_2)(R_3 + R_4)} E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2} \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} + \frac{\Delta R_4}{R_4} \right) (1 - \eta) E_i$$

where η denotes the non linearity and $r = \frac{R_2}{R_1}$

- **Bridge sensitivity**: (Assuming that the change in resistance is only in R_1)

$$S_B = \frac{\Delta E_o}{\epsilon}$$
$$= \frac{r}{(1+r)^2} E_i$$

$$\boxed{S_B = \frac{r}{1+r} \sqrt{P_1 R_1}}$$

where $r = \frac{R_2}{R_1}$

- The ratio $\frac{r}{1+r}$ is called circuit efficiency.

- **Sensitivity**:

$$S_{sg} = \frac{\Delta E_o}{\frac{\Delta R}{R}} = S_G S_B$$

14 Interference

- For constructive interference, path difference should be intergal of wavelength, λ

$$\delta = m\lambda, \qquad m = 0, \pm 1, \pm 2, \ldots$$

- For destructive interference, path difference should be intergal of wavelength plus half the wavelength (for destruction)

$$\delta = (m + \frac{1}{2})\lambda, \qquad m = 0, \pm 1, \pm 2, \ldots$$

- In a double slit experiment, the path difference between the two waves is

$$\delta = d \sin \theta$$

- An electromagnetic wave traveling from a medium of refractive index n_1 toward a medium of refractive index n_2 undergoesa 180° phase change on reflection when $n_2 > n_1$ and does not undergoe any phase change if $n_2 < n_1$
  i.e, incident refactive index < refractive index of surface on which it is reflected $\rightarrow$ 180° phase change
  i.e, incident refactive index > refractive index of surface on which it is reflected $\rightarrow$ no phase change

- Optical path length of a light wave travelling a distance of t in a medium of refractive index n is
$$\text{Optical path length} = nt$$

15 Data Sampling

- Fourier transform : This changes any function from time domain to frequency domain. This means that in the fourier transformed graph, we get the peaks at frequencies of which the original wave is composed of.
$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt$$

- In discrete fourier transform

$$f_j = \sum_{k=0}^{N-1} x_k e^{-i(2\pi \frac{j}{N})k}$$

- **Nyquist theorem**: To capture all information in the analog signal into a digital signal, the sampling frequency must be greater 2 times maximum frequency occuring in the original signal.

$$f_s > 2f_{max}$$