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A dynamic nonlinear model of online retail competition using Cusp Catastrophe Theory

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Abstract

While firms can take corrective actions to reverse gradually declining performance, sudden changes in performance are much more challenging to predict and handle. Specifically such patterns are difficult to model using traditional approaches. On the other hand, such incidents are likely to occur in complex economic systems such as the e-commerce domain. We propose a Catastrophe Theory model to capture the inherent nonlinearity and complexity that are associated with online retail competition. Theoretically and using numerical simulations, we identify conditions under which catastrophes can occur in the customer base of a less-established e-commerce firm (online retailer) competing with its more established competitor. We provide specific managerial guidelines about how the less-established e-commerce firm can strategically manage catastrophes in its customer base growth due to competition.
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Keywords: E-commerce; Catastrophe Model; Competition; Online retailing

1. Introduction

Electronic commerce has brought unique challenges to competitive processes and practices in inter-firm competitions (Heil and Montgomery, 2001). While competition in the online domain can exist in various subsectors such as between different search engines or online content providers (e.g., news sites), a common exemplification of electronic commerce occurs when products or services are sold to consumers by competing online retailers (e.g., amazon.com and bordersstores.com). In the present research on electronic commerce competition, our modeling focus is on online retail competition involving shopping and sales. The new cyberspace markets, while offering more choices to consumers, have created increased competition among firms participating in e-commerce (Brynjolfsson et al., 2003). Yet the competitive landscape of e-commerce and how firms are dealing with this unique

competitive market space are still not well understood. Many industry observers and pundits are still pondering about why many e-commerce firms suddenly failed and were driven out of the market while others succeeded (Green and Hof, 2002).

Above all, the issue of how relatively less-established e-commerce firms can successfully compete with well-established e-commerce firms is becoming increasingly relevant, especially in the global e-commerce setting. For example, a number of e-commerce giants such as eBay and Amazon are moving quickly into e-commerce markets in developing countries (Miller, 2004). It is reasonable to expect that the entry of these well-known, more resourceful e-commerce firms (e.g., amazon.com in online book sales) could conceivably exert significant influence on the customer growth of less-established, less resourceful e-commerce firms in the local market (e.g., jinqi.com—an online bookstore in China that caters primarily to the reading preferences of Chinese intellectuals).

While existing works on online competition (e.g., Goolsbee and Brown, 2002) typically employ the same techniques and frameworks developed in the traditional economy setting, many differences among the "old" and "new/digital" economy argue for the usage of new modeling tools that can accommodate the

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complexities inherent in the e-commerce system, such as network externality, interactivity, global access unconstrained by time and space, and the ability to conduct transactions in real time (Viswanathan, 2005). These unique features imply that e-commerce can be quite complex due to its inherent nonlinearity and dynamics in operating mechanisms, all of which may not be well captured by traditional economic models of competition (Heil and Montgomery, 2001; Whitby et al., 2001).

In particular, the rapid rise of e-commerce within a short period of time (less than 10 years) from a virgin space (Javalgi et al., 2004) makes it amenable to be studied under the organizational ecology rubric—an important research stream in management and marketing (e.g., Lambkin and Day, 1989; Boeker, 1991) that has a long history of utilizing nonlinear dynamic models to capture the dynamics of industry growth. Following this stream of research, we propose and develop a dynamic process model (Ferrier, 2001) of online competition using the Catastrophe Theory modeling approach, a special subset of nonlinear dynamics models (Hufford et al., 2003). The strengths of Catastrophe Models are that they are parsimonious and can capture complex behavior by using fewer nonlinear equations than the number of equations needed to describe the same phenomena (Oliva et al., 1992). Thus, Catastrophe Theory is uniquely positioned to capture the inherent nonlinearity and complexity in e-commerce system. Lange et al. (2004) argue that the Catastrophe Model can capture the evolution of network externalities as they develop in nascent or changing markets, such as the case of e-commerce (Javalgi et al., 2004).

Our proposed model focuses on the competitive influence that an established e-commerce firm can exert on its less-established rival competing in the same online industry (e.g., online book sales). This modeling perspective allows us to offer useful insights for the numerous less-established, typically small-to-medium size e-commerce organizations. The existence and success of those less-established e-commerce firms (in the face of competitive assaults from their more established rivals) is essential for maintaining entrepreneurial spirit, and keeping competition in cyberspace healthy and alive (Downie, 2003). While the media and academics tend to showcase e-commerce giants such as eBay or amazon.com, it is those less-established-yet-growing firms that pose one of the most interesting possibilities for online retailing's future (Grewal et al., 2004). Accordingly, a practical goal of this study is to derive useful insights of strategic importance for the numerous less-established e-commerce firms fighting to stay in this competitive game of e-commerce.

Our modeling analysis demonstrate that the competitive dynamics between a well-established e-commerce firm and its less-established rival can be modeled by a Cusp Catastrophe Model in which the web site visit rate and a unique competitive advantage factor (e.g., price or service quality) serve as components of two control variables. These control variables are conceptualized to affect the dynamic growth of customer base of the less-established e-commerce firm, and eventually its sales. We derive an equilibrium solution for the competition process, and delineate conditions under which catastrophes in

the growth of customer base (e.g., sudden drops) may occur for the less-established e-commerce firm. The study represents improvements in the field in two ways. First, it explicitly models the process aspect of competitive dynamics in the e-commerce system. Second, it adopts the Catastrophe Theory approach in handling in a parsimonious manner the inherent nonlinearity that is characteristic of e-commerce system behaviors.

In the next section, we review the fundamentals of Catastrophe Theory and its applications in the business literature. Then in Section 3 we derive a mathematical model which shows that customer base growth of the less-established e-commerce firm can be described by a Cusp Catastrophe Model. In Section 4, we show the results of numerical simulations for our derived Catastrophe Model that explores the patterns and scenarios associated with this model. In particular, we investigate how the equilibrium values of customer base size change with regard to the changes in the independent factors of the Catastrophe Model. We also identify ranges of model parameters that are associated with the occurrences of catastrophes. Lastly, in Section 5, we explore the implications of the model results for e-commerce organizations and for research on e-commerce competition.

2. The Catastrophe Theory approach

Research in Information Systems and E-commerce (e.g., Hoffman and Novak, 1996; Bakos, 1997) has highlighted the fundamental differences between the Internet Economy and the Traditional Economy including such features as network externality, interactivity, global access unconstrained by time and space, and the ability to conduct transactions in real time (Viswanathan, 2005). According to Farrell and Saloner (1986), when network externalities exist, structural anomaly can be exhibited in firm behaviors (e.g., in discontinuous jump of adoptions of new products). In addition, Vengerov (2003) identified specific sources of complexity in the e-commerce system, namely, value structure and dynamics, self-adaption capability, and sensitive systems. These unique features imply that e-commerce can be more complex in its inherent nonlinearity and dynamics in operating mechanisms, which may not be well captured by traditional economic models.

Both marketing (e.g., Oliva et al., 1992) and organizational researchers (e.g., Garud and Van de Ven, 1992) have suggested that traditional modeling approaches may not work well particularly during highly ambiguous and uncertain periods of the organizational development process, such as in the case of the rapid development of e-commerce from nonexistent to substantial (Javalgi et al., 2004). Indeed, the rapid rise of e-commerce within a short period of time (less than 10 years) from a virgin space (Stone, 2005) makes it amenable to be studied under the organizational ecology rubric that has a long history of application in management (e.g., Boeker, 1991) and marketing (e.g., Lambkin and Day, 1989) research. Such research utilizes nonlinear dynamic models to capture the dynamics of industry growth.

Therefore, this study applies the Catastrophe Theory, a special subset of the nonlinear dynamics models, to capture the inherent nonlinearity and complexity in the e-commerce system. In particular, there are two advantages in adopting the Catastrophe Modeling approach. First, Catastrophe Models are parsimonious (Gresov et al., 1993) so they can capture complex system behaviors by using fewer nonlinear equations than the number of equations needed to describe the same phenomena (Oliva et al., 1992). Further, Lange et al. (2004) argue that the Cusp Catastrophe Model can capture the evolution of network externalities as they develop in nascent or changing markets, such as the case of e-commerce (Javalgi et al., 2004).

3. Introduction to Catastrophe Theory

Catastrophe Theory is a mathematical tool developed by French mathematician Rene Thom in the early 1970s (Thom, 1975). It can deal with complex systems (e.g., economies, societies) with properties of discontinuities directly without reference to any specific underlying mechanism (Saunders, 1980). This attractive property makes it especially appropriate for the modeling of systems whose inner workings may not be known, as is usually the case in the study of complex economical systems (Kauffman and Oliva, 1994); an example of such a system would be the e-commerce domain.

3.1. Cusp Catastrophe Model

For a basic tutorial on Cusp Catastrophe Model, the reader is referred to look at Woodcock and Davis (1978). A classical graphical illustration of the Cusp Catastrophe Model is provided in Fig. 1, which shows how discontinuous changes in the dependent behavioral responses can occur with smooth changes in the independent control and splitting factors. Of particular interest to our study model are the areas in the catastrophe system where sudden changes in the responses (e.g., sudden rise or drop) can occur in regions comprised of particular values of control and splitting factors.

The Cusp model describes a three-dimensional response surface consisting of one dependent and two independent variables. The dynamics of the system is recorded in the vertical movements of the dependent variable as a result of the changes in its two control dimensions. In marketing, researchers have used the Cusp Catastrophe Model to study problems such as service satisfaction (Oliva et al., 1992) and innovation adoption (Herbig, 1991; Lange et al., 2004).

In mathematical terminology, the three-dimensional phase space of the Cusp model or system can be described by the following potential equation (Saunders, 1980):

$$V(x) = x^4 + ux^2 + vx$$

where x and u are the two control dimensions and V represents the behavioral surface. The equilibrium surface M is determined by

$$12x^2 + 2u = 0$$

To obtain the bifurcation (or splitting) set, project the equilibrium surface M into the control space (i.e., $x=0$) by eliminating x from the above two equations
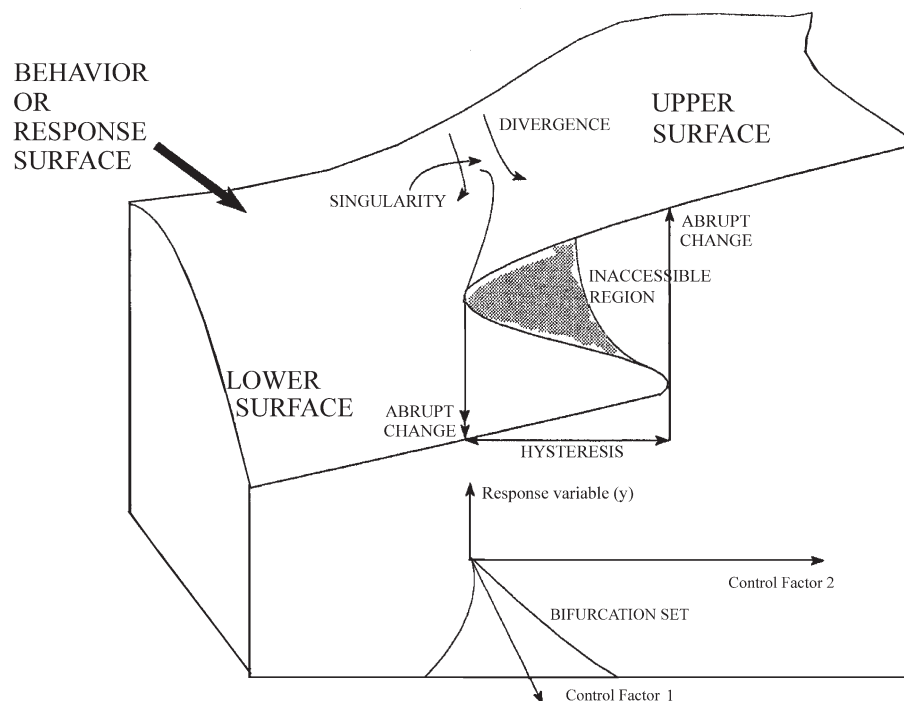
$$8u^3 + 27v^2 = 0.$$

Fig. 1. A graphical illustration of the classical Cusp Catastrophe Model (adapted from Saunders, 1980).

4. A Catastrophe Model of competition among online retailing firms

The rising trend of e-commerce sales, fueled by improving e-commerce infrastructure and more "Online Shopping Ready" Internet users (Greenspan, 2003), ensures that e-commerce firms (henceforth in the paper for ease of exposition, the usage of the term e-commerce is used to refer more specifically to online retailing) are benefiting from an increasingly larger "pie" of business opportunities. Consequently, it is reasonable to assume that such favorable external factors will provide a positive source of customer growth for companies that are involved in e-commerce activities (i.e., online sales).

On the other hand, the positive gains of e-commerce sales growth are likely to be distributed unevenly among participating e-commerce firms. In particular, less-established e-commerce firms may not be able to get a "fair" slice of the growing "pie" as more powerful competitors can lure potential customers easily. Indeed, the competitive landscape in some of the relatively consolidated "online industries" (e.g., book selling) is quickly taking shape. Some players have already emerged as market share leaders—such as amazon.com (Elstein, 2003). An e-commerce behemoth, armed with its superior marketing and financial resources, poses substantial threats to the less-established e-commerce firm, whose potential customers may be enticed away through targeted competitive promotions. The range of competitive actions that can be utilized by the well-established e-commerce firm may vary from tactical moves, such as price cuts and service improvements, to strategic maneuvers such as industry alliances, new product or service introductions, etc. (Ferrier, 2001).

To capture the opposing forces that may affect the growth of customers for less-established e-commerce firms, we develop a formal mathematical model based on the predator–prey model commonly used in mathematical biology (Beltrami, 1998). Our model will show that in a typical two-player e-commerce system consisting of a well-established and a less-established e-commerce firm, sudden or unpredictable changes in the customer base growth of the less-established firm can occur simply as a result of sharing the same system with the well-established firm. By explicitly modeling the net growth of the less-established firm in a two-firm online industry, our model depicts the outcome of the competition between the two e-commerce firms.

4.1. Model setting

Assume without loss of generality that the online industry under investigation consists of a well-established e-commerce firm and its less-established rival. An example of the former would be amazon.com and that of the latter would be books-for-cooks.com. The basic underlying interpretation is that the less-established firm is in a weaker position in the market in terms of many factors such as site visit rate, market power, name recognition, synergy with other business units, promotion efficiency, and so on.

We define the customer base (S) of the less-established e-commerce firm as consisting of those customers who are aware of the firm's existence, and who consider buying products from it. This broad definition would most likely include regular buyers, one-time buyers (who may come back), registered users, etc. In addition, we propose that the growth in the customer base for the less-established e-commerce firm, which would eventually translate into sales gains, results from the difference between two opposite forces: one is the positive source factor (P) maintained through such factors as higher adoption rate of Internet shopping; the other is the competitive dissipation effect (Q) that measures the loss of customers to its more resourceful competitor due to competitive attacks. Thus, net growth rate over time in customer base $\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)$ for the less-established e-commerce firm is

$$\frac{\mathrm{d}S}{\mathrm{d}t} = P - Q$$

4.1.1. Source of natural growth in customer base (P)

Customer base is a dynamic concept in that the size of the base may be changing over time. New customers may get to know the e-commerce firm and consider buying products there. On the other hand, existing potential customers may no longer be interested in its offerings so that they decide to "drop out" from the firm's customer base. The difference between the adding of "new customers" and the dropping out of "existing customers" thus comprises the net growth of customer base.

Let r (in proportion) denote the average difference between the "adding" and "drop-out" rate per unit time. In this model, we assume that the growth of potential customer base for the less-established e-commerce firm follows an S-shaped growth curve. Initially, the customer base is small as the site may not be widely known and only a handful of customers can manage to find it. As the firm grows in popularity and size, it attracts more customers either through increased advertising or word-of-mouth effect. Thus, this stage will oversee a rapid growth in the customer base. Finally, growth in the customer base will not be as frenzied as before once the firm has tapped deep into its potential users. For instance, for books-for-cooks.com, its customers are likely to be niche buyers (i.e., cookbook enthusiasts who patronize specialty cookbook stores). As a result, attracting more of these niche users will be tougher once the majority of the users have received its marketing messages.

We use the following functional form for the growth component

$$P = rS\left(1 - \frac{S}{B}\right)$$

where S is the total number of customers of the less-established e-commerce firm; defined as those who know its existence and also consider buying products from its online store. B is the number of its potential customers if the less-established firm's marketing expenditure goes to infinity (Kotler, 1996), which may be a varying parameter if the total industry market size expands (Kalish and Lilien, 1986).

4.1.2. Competitive dissipation in customer base (Q)

While the well-established e-commerce firm commands a bigger customer base than its less-established rival, the overlapping of potential target customers is often possible as the well-established firm's online store typically covers customers from broader segments. For instance, some customers of books-for-cooks.com could also be customers of amazon.com when buying cookbooks online. The overlapping of customer bases often provides an incentive for the dominant, more-established firm to exert its competitive influence on the less-established firm's more specific customer base.

To model the effect of the well-established firm's predation on the less-established firm's customer base, we use the following "predation" term to represent the "competitive dissipation" effect, which impacts the less-established firm's natural customer growth rate negatively

$$\frac{bS^2}{a^2 + S^2}$$

where a and b are model parameters that relate to certain characteristics of the less-established firm, which will be discussed in further details below.

The S-shaped response function for customer base is commonly used in the marketing literature to model sales responses (e.g., Lilien et al., 1992). Our formulation of the "predation" term here ensures a negligible predation for small S and an increasingly effective one as S increases. The latter effect occurs as the well-established firm sees more incentives to attack a less-established firm that it considers as a more lucrative target with a relatively bigger following (or customer base S).

Parameter a in the above expression establishes that the predation term is larger for smaller a. In this model, we propose a to be proportional to v (or $a = kv$ where k is a constant parameter)—the visit rate to the online store of the less-established e-commerce firm (i.e., how many visitors come to visit its online store during a given time period). Given other things constant, a lower visit rate v (which is proportional to the parameter a) at the less-established firm would likely make it easier for the more-established firm to leverage its dominant market position and attract customers away from the relatively weaker rival. Parameter b in this model formulation stipulates that the magnitude of predation is smaller for smaller b. Hence, the higher the b value for the less-established firm, the more potential customers it will lose to the big firm. Parameter b can be considered as a measure of relative disadvantage of the less-established firm as compared to the well-established firm (e.g., limited services, or higher prices). Without loss of generality, we choose to use "price" to represent b in this model to denote that the higher the price at the less-established firm, the more customers it will lose to the dominant firm given other things constant.

4.2. Model formulation

Based on the above discussion of two sources of factors (natural growth and competitive dissipation) affecting growth in the customer base of the less-established e-commerce firm, we deduce that the resultant growth rate of the potential customer base for the less-established firm can be captured by the following dynamic equation

$$\frac{dS}{dt} = r \cdot S \cdot \left(1 - \frac{S}{B}\right) - \frac{c \cdot p \cdot S^2}{k^2 \cdot v^2 + S^2} \tag{1}$$

where c is a unit conversion constant that converts the unit for the predation term to that of the left-hand side of the equation; the other parameters have been defined previously.

4.3. Model equilibrium solution

The above model constitutes a mathematical representation of a dynamic system (i.e., the less-established e-commerce firm, its online customers, and the well-established e-commerce firm). In this model, we focus on the non-trivial equilibrium solution ($S \neq 0$) to this dynamic system equation. The following two theorems were derived to characterize the equilibrium solution properties.

Theorem 1. *The solution for the dynamic e-commerce system conforms to a standard Cusp model (Proof: see Appendix A).*

This theorem suggests that the (converted) customer base size of the less-established firm s (deviated from the true S by a constant) at the equilibrium, depends on the value of parameters a and b—which in turn depend on price p, site visit rate v, market potential B, natural growth rate r, and two constants k and c. In other words, the normal and splitting factors for the Catastrophe Model are functions of price and visit rates of the less-established firm.

Theorem 2. *Catastrophe solutions to the above Cusp model of e-commerce competition dynamics will occur when the following conditions are met.*

$$8 \cdot \left(B \cdot k^2 \cdot v^2\right)^2 + 4 \cdot \left(B^2 \cdot k \cdot v\right)^2 + 4 \cdot \left(k \cdot v\right)^6 + \frac{12 \cdot c \cdot p \cdot B \cdot (k \cdot v)^4}{r}$$
$$+ \frac{12 \cdot (k \cdot v \cdot c \cdot p \cdot B)^2}{r^2} + \frac{4 \cdot (c \cdot p \cdot B)^3}{r^3} - \frac{(c \cdot p \cdot B)^2}{r^2}$$
$$- \frac{20 \cdot c \cdot p \cdot B^3 \cdot (k \cdot v)^2}{r} < 0$$

(Proof: see Appendix A).

Theorem 2 prescribes the necessary conditions for catastrophe solutions to occur based on model parameters. It suggests that sudden changes in the customer base of a less-established e-commerce firm may occur only under specific conditions when certain combinations of environmental (e.g., B, r) and organization factors (e.g., v, p) generate a negative condition set. Thus, when these conditions are not met, the less-established firm will not witness sudden changes in its customer base growth.

5. Application and validation of the Cusp Catastrophe Model

The analytic solutions for the Catastrophe Model are highly complex (e.g., higher order equations of v and p) to render clear-

cut interpretations. To illustrate how the customer base growth of the less-established firm may exhibit sudden fluctuations in the two-firm e-commerce system, this section uses numerical simulation (see, e.g., McKelvey, 1999) to investigate (1) under what conditions catastrophes will occur in customer base growth for the less-established e-commerce firm with regard to its site visit rate v and price p; and (2) how this pattern of catastrophe solutions varies depending on two key model parameters: market potential B and natural growth rate of its customer base r.

We start our simulation with a baseline scenario in which we examine when catastrophes will occur for certain ranges of site visit rate v and price p. We then explore how such patterns may change with regard to B and r through a sensitivity analysis for four modified scenarios.

5.1. Determination of catastrophe sets in Scenario 1

We consider Scenario 1 to be like a baseline scenario so that we can compare other scenarios with it. Catastrophe solutions for our proposed model will occur, mathematically, when $\Delta < 0$ in Eq. (10) (see Appendix A). The simulations are presented for different levels of p. Thus, for each price level, we identify the v values associated with the catastrophe solutions (i.e., where Δ's are negative).

Without loss of generality, the following values are assigned to the baseline scenario parameters: $r = 0.3$, $c = 2$, $k = 1$, $B = 800$. The values for p are set from 1 to 15 and the site visit rates are set from 1 to 50. The results of the simulated catastrophe solutions, which correspond to sudden changes in customer base of the less-established firm, are shown in Fig. 2 (catastrophe solutions are indicated by the *shaded* area).

Fig. 2 shows that as price p increases, the range of v values that are associated with catastrophes also widens. Thus, it can be inferred that for the less-established firm, catastrophe solutions may occur over a broader range of web site visit rates when its price is higher. Alternatively, if price is lower, the possible range of v values associated with the catastrophe solutions is narrower.

That is, the less-established firm is generally well-guarded against the onslaught of its well-established rival if its price is low. On the other hand, Fig. 2 shows that if the less-established e-commerce firm only attracts a very small group of visitors, then it may be quite vulnerable even with competitive pricing; this happens presumably because its small visit base cannot withstand much competitive influence from the more established firm.

5.2. Scenario 2: smaller market potential

This scenario (see Fig. 3) differs from the baseline scenario (Scenario 1) in that market potential B is half as large ($B = 400$). While we still observe a similar pattern as seen in Fig. 2 in terms of how catastrophes may be associated with v and p values, we now observe that full range of catastrophes may result from relatively lower p values compared to those seen in Scenario 1. For instance, in Scenario 1, catastrophes may occur for all ranges of visit rates when p is greater than 14.5. On the other hand, catastrophe solutions may occur for all ranges of visit rates when p is greater than 12.5. Therefore, in the smaller market potential scenario, the less-established firm is much more vulnerable to competitive attacks from the more-established firm (i.e., growth in the customer base of the less-established e-commerce firm may be disrupted even when it tries to stay competitive with the more established firm by featuring lower prices). Given that market potential B symbolizes the ultimate achievable customer base for the less-established firm, smaller potential can be related to a distinct niche market (e.g., online sales of biography books on medieval figures). The less-established firm has to deal with this niche segment without much diversification possibilities (e.g., venturing into books on renaissance figures). Consequently, it is likely to have less maneuvering room in reacting to the more-established firm's assault so higher prices will cause more damages in this situation. In other words, a less-established firm operating in a niche market is more vulnerable to competitive attacks that cause abrupt fluctuations in its customer base growth.
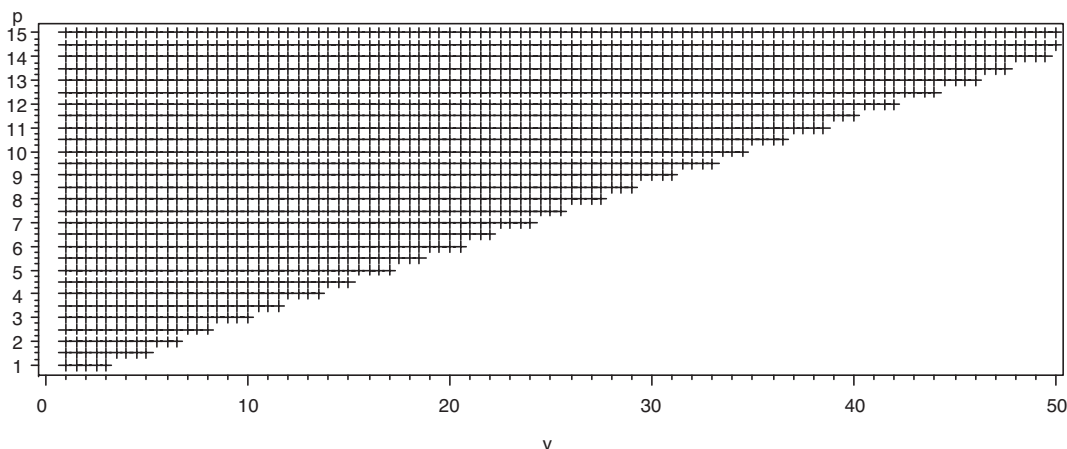

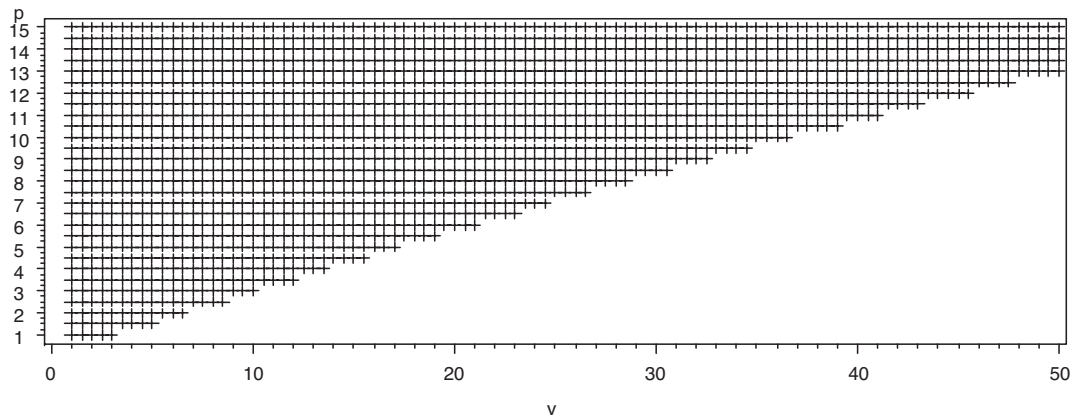Fig. 2. Catastrophe set for Scenario 1 (baseline).

Fig. 3. Smaller market potential (Scenario 2).

5.3. Scenario 3: bigger market potential

This scenario (see Fig. 4) differs from the baseline scenario (Scenario 1) only in that market potential B is twice as large ($B=1600$). Again a similar pattern as seen in Fig. 2 emerges but we now spot that full range of catastrophes develop for relatively higher p values (i.e., when p is greater than 15 instead of 14.5). Therefore, in this bigger market potential scenario, the less-established firm seems to be less vulnerable to competitive attacks from the well-established firm (i.e., customer base growth may be disrupted relatively less even when it charges higher prices). It is conceivable that, in this scenario, the less-established firm endowed with a bigger market potential probably enjoys more maneuvering room in defending attacks from the more-established firm. Thus, it can afford to charge higher prices without incurring disruptions in growth of customers at certain levels of site visit rates.

5.4. Scenario 4: lower natural growth rate

This scenario is different from Scenario 1 only in that the rate of natural growth is lower ($r=0.1$). Differences in the simulation results (as shown in Fig. 5) indicate that catastrophe sets that occur for all levels of visit rates start from $p=5$ instead of 14.5 in the baseline scenario (Scenario 1). Since the natural growth rate

reflects the business prospects with respect to the less-established firm's target market, a lower growth rate implies that the less-established firm does not have much maneuvering room in fighting off the well-established firm's assault. This observation is consistent with the conclusion reached in Scenario 2 on the impact of lower market potential on catastrophe solutions. As a result, the less-established firm is more vulnerable to catastrophe scenarios unless its prices are considerably lower.

5.5. Scenario 5: higher natural growth rate

This scenario is different from Scenario 1 only in that the rate of natural growth is higher ($r=0.9$). Fig. 6 indicates that catastrophe sets are still more likely to occur for higher p values. However, the visit rate range that may induce catastrophes is narrower. For instance, at $p=15$, catastrophes occur for visit rates ranging from 1 to 17; yet there are no catastrophes for visit rates higher than 17. This observation is in contrast to what happens in Scenario 4 yet in agreement with the conclusion reached in Scenario 3 regarding the effect of bigger market potential on catastrophe solutions. High growth rate implies that the less-established firm enjoys more flexibility in fending away the more-established firm's offensive moves. Thus the less-established firm, with high visit rates (e.g., $v>17$) could manage
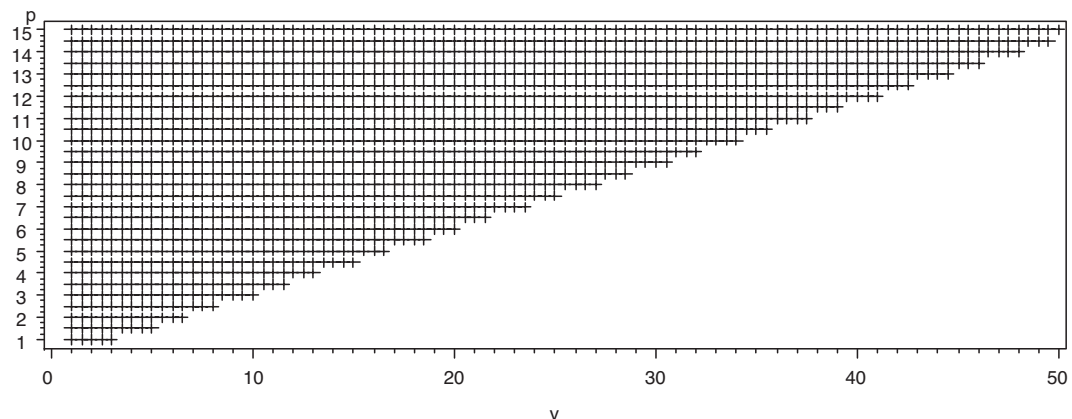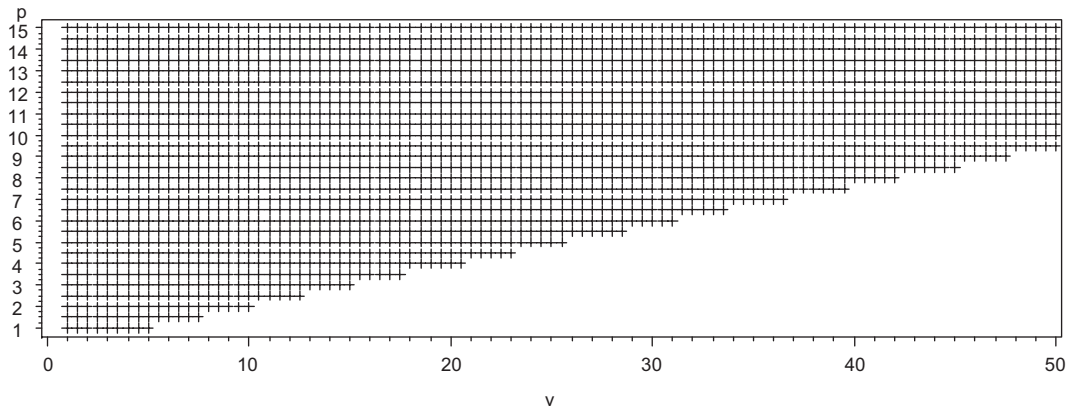

Fig. 4. Bigger market potential (Scenario 3).

Fig. 5. Lower natural growth rate (Scenario 4).

to protect itself from sudden swings in its customer growth even when it charges high prices (e.g., when $p=15$). This happens presumably because a strong and fast growing customer base makes it hard for its more resourceful rival to launch disruptive and catastrophic attacks.

6. Discussions and conclusions

Major results from our simulations as shown in Figs. 2–6, are summarized and discussed below:

6.1. The range of visit rate v that is associated with catastrophe solutions is narrower, when price p is lower than when p is higher

This means that the less-established e-commerce firm is at a greater risk of witnessing disturbances in growth of its customer base if it is at a competitive disadvantage (higher price) with regard to its well-established rival.

This pattern, as exhibited across Figs. 2–6, can be explained as follows: if the less-established e-commerce firm has established a truly competitive advantage (i.e., low price), then its customers are likely to be loyal, and its customer base is likely to fluctuate less often (i.e., exhibit

catastrophe properties). This is true because the more-established firm will then have to face an uphill and costly battle in grabbing those "loyal" customers through vehicles such as costly promotion incentives. However, if price is high, the less-established firm, now without its protective armor of competitive price, will open itself to the possibility of more intensive attacks by the well-established firm. Therefore, for a wide variety of visit rate scenarios, discontinuities in customer base may occur as natural growth of consumers is disrupted. In contrast, when price is low, discontinuities in customer base growth (i.e., catastrophes) only occur only for a limited range of visit rate scenarios, such as when the less-established firm's visit rate is quite low.

6.2. If market potential for the small firm is low, then its customer base growth is likely to be more disruptive

This finding was illustrated in Figs. 3 and 4. The explanation is that the less-established firm, having less maneuvering ability due to its limited market base (i.e., less potential for promising market niches to grow), will have a more difficult time fending off the well-established firm's attack. As a result, disruptions in customer base growth are more likely to occur.
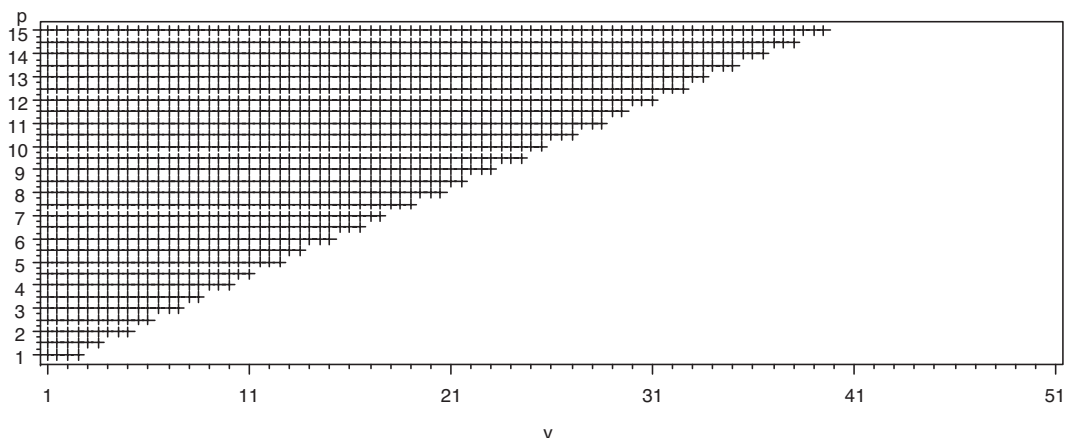

Fig. 6. Higher natural growth (Scenario 5).

6.3. If natural growth rate r is smaller, then catastrophe solutions are more likely to occur

This conclusion was demonstrated in Figs. 5 and 6. The less-established firm with a lower natural growth rate of potential customers sells to a less promising market and thus lacks the flexibility to plot against its more resourceful competitor. This situation will get even worse if the less-established firm does not enjoy a strong competitive advantage (e.g., low price).

6.4. Managerial implications of the Cusp Catastrophe Model

The derived Cusp Catastrophe Model of online competition provides three important managerial implications to the managers of less-established e-commerce firms in understanding the dynamics of online competition. We discuss each in details below with our modeling scope in mind; as our model, just like other Catastrophe Models (e.g., Oliva et al., 1992; Lange et al., 2004) provides qualitative insights about the outcomes of competition.

First, it demonstrates the complexity and inherent unpredictability of customer growth for the less-established e-commerce firm competing with a more-established rival in a two-firm online industry. In fact, the model shows that even for well-known market conditions (v) and operational characteristics (p), disturbances such as sudden drops in customer base (i.e., catastrophes) can still occur due to complexities and nonlinearity inherent in the e-commerce system. While managers cannot control the outcomes of catastrophes, they should at least be informed of such possibilities and be prepared for potential consequences. For instance, if customer base growth is expected to be turbulent and wide swinging for certain ranges of p and v, a flexible inventory plan might be put in place so as to meet unexpected orders or lack of orders.

Second, it is discovered that at any given price level only certain ranges of visit rates may induce catastrophes in customer base. Thus, to avoid wide-swings in customer base growth, managers should be informed of this principle so that they can make their choices accordingly. For instance, since only low visit rates are associated with catastrophes at the less-established e-commerce firm with competitive pricing, managers can worry less about catastrophes once its online store has built up a decent amount of visits. Also, management of the less-established firm needs to carefully select the target market it intends to compete head-on with its more resourceful rival and avoid unpromising markets such as those with small market potential or lower natural growth rate. Even though this "niching" strategy may sound like an interesting idea at first, our analysis indicates that this orientation may actually result in disruptions in customer growth for the less-established firm. On the other hand, if the less-established firm picks a target market that either has a big potential or a higher natural growth rate, it will face fewer disruptions in its customer growth as long as it maintains a competitive advantage (e.g., low price or superior services). Thus, according to this line of reasoning, an online book store focusing on Braille books (most likely with a smaller market potential) may have a tougher time competing with e-commerce behemoths (e.g., amazon.com) than one focusing on diet books (presumably has a larger market growth potential). Lastly, since the catastrophe range is wider at higher price, a less-established e-commerce firm should strive to keep its price low so as to avoid sudden changes in its customer base. This conclusion can be extended to other competitive weapons that may be possessed by the less-established e-commerce firm. For example, if books-for-cooks.com can establish a competitive advantage in offering personalized book recommendations to its buyers, then it will be more difficult for more established rivals such as amazon.com to launch disruptive competitive attacks.

6.5. Future studies

The current research, we believe, represents an important first attempt in modeling the complexities of dynamic competition in the e-commerce setting using Catastrophe Theory. We acknowledge that the proposed Cusp model is based on a number of assumptions (e.g., a pricing factor and a visit rate factor; the industry consisting of a dominant player and a less-established player) which future model extensions can expand. Further, our present work is just a first step towards understanding the complexity of e-commerce competition and not all competitive behaviors are applicable to the modeling approach of Catastrophe Theory. Indeed, since the Catastrophe Model is only a special type of model in the nonlinear dynamic system models, it cannot capture other types of complexities such as self-organizations (Dooley and Van de Ven, 1999).

We propose that further extensions of the current work be carried along the following two directions. First, while the validity of our usage of the S-shaped growth functional form is also corroborated in a recent forecasting study (Modis, 2005) that investigates the growth curve for the number of Internet users worldwide, we recognize that in each different industry, customer growth curve may take shapes other than the S-shaped curve. Future studies may use different functional forms in the growth rate or even use a general functional form and investigate how the catastrophe situation may be different. Second, researchers should empirically examine the validity of this Cusp model across a variety of different types of online industries (e.g., travel, apparel). These empirical applications will allow possible modifications of the current model and establish its practical significance for e-commerce firms.

With the rapid growth and consolidation among e-commerce firms in key online industries (e.g., books, music), the competition among well-established, more resourceful firms and the thousands of less-established, mom-and-pop type online operations is likely to become more intense. Our model offers a new perspective in understanding the complex competition process, and we believe that our model offers valuable insights to the numerous less-established e-commerce firms fighting for their share in the growing e-commerce market.

Appendix A

In Eq. (1), let $\dfrac{\mathrm{d}S}{\mathrm{d}t}=0$ then we obtain the following

$$r\cdot\left(1-\frac{S}{B}\right)=\frac{c\cdot p\cdot S}{k^2\cdot v^2+S^2} \qquad (2)$$

By re-arranging terms in (2), we obtain

$$-\frac{r}{B}\cdot S^3+r\cdot S^2-\left(\frac{r\cdot k^2\cdot v^2}{B}+c\cdot p\right)\cdot S+r\cdot k^2\cdot v^2=0 \qquad (3)$$

To transform the above expression, we introduce variable s, which is defined as

$$S=s+\frac{B}{3} \qquad (4)$$

Then, Eq. (3) can be simplified (using Eq. (4)) to the following

$$s^3+\left(k^2\cdot v^2+\frac{c\cdot p\cdot B}{r}-\frac{B^2}{3}\right)\cdot s+\left(\frac{c\cdot p\cdot B^2}{3\cdot r}-\frac{2\cdot B\cdot k^2\cdot v^2}{3}-\frac{2\cdot B^3}{27}\right)$$
$$=0 \qquad (5)$$

Let

$$a=k^2\cdot v^2+\frac{c\cdot p\cdot B}{r}-\frac{B^2}{3} \qquad (6)$$

and

$$b=\frac{c\cdot p\cdot B^2}{3\cdot r}-\frac{2\cdot B\cdot k^2\cdot v^2}{3}-\frac{2\cdot B^3}{27} \qquad (7)$$

then we obtain the following cubic equation that conforms to the functional form of the standard Cusp model

$$s^3+a\cdot s+b=0 \qquad (8)$$

For the catastrophe solution in the general form of a cubic equation such as

$$x^3+a\cdot x+b=0 \qquad (9)$$

the catastrophe solutions occur when multiple solutions to the above equation are found under the following condition:

$$\Delta=27\cdot b^2+4\cdot a^3 \qquad (10)$$

If $\Delta=0$, there are three real roots. Further, if $27b^2=-(4a^3)\neq 0$, two of the three real roots are equal.
If $\Delta<0$, there are three distinct real roots to (8). In this case, a must be negative.

Thus, we can claim that inequality (10) defines the whole *catastrophe set* where the multiple states exist. Further, by letting (3.19) equal to zero, we obtain the so-called *bifurcation set* that forms the boundary of the catastrophe:

$$27\cdot b^2+4\cdot a^3=0 \qquad (11)$$

If we plug (6) and (7) into (11), we obtain the following catastrophe set solution

$$8\cdot(B\cdot k^2\cdot v^2)^2+4\cdot(B^2\cdot k\cdot v)^2+4\cdot(k\cdot v)^6+\frac{12\cdot c\cdot p\cdot B\cdot(k\cdot v)^4}{r}$$
$$+\frac{12\cdot(k\cdot v\cdot c\cdot p\cdot B)^2}{r^2}+\frac{4\cdot(c\cdot p\cdot B)^3}{r^3}-\frac{(c\cdot p\cdot B)^2}{r^2}$$
$$-\frac{20\cdot c\cdot p\cdot B^3\cdot(k\cdot v)^2}{r}<0 \qquad (12)$$

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