This notebook corresponds to part (d) of the REU plan. The focus will now be on more complicated dynamics involving 4 wave processes as opposed to 3, and will borrow from the Photonics West paper provided[2], and at times from *Boyd*[1].

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy as sp
from scipy.integrate import RK45, odeint, solve_ivp
from scipy.special import expit
import math
```

Moses, et al.[2] presents a four-wave mixing process through optical parametric amplification (OPA) with simultaneous second-harmonic generation (SHG). This procedure is dominated by dynamics surrounding respective wave amplitudes: (A_{signal}, A_{pump}, A_{idler}, $A_{2ndharmonic}$), with initial conditions (B, C, 0, 0) - B and C both being complex numbers - and for interesting results, $|C| >> |B|$ (increasing the gain).

Shown below:

$$\frac{dA_s}{dz} = \frac{i\omega_s d_{eff}}{n_s c} A_p A_i^* e^{i\Delta k_{OPA} z}$$

$$\frac{dA_p}{dz} = \frac{i\omega_p d_{eff}}{n_p c} A_s A_i e^{-i\Delta k_{OPA} z}$$

$$\frac{dA_i}{dz} = \frac{i\omega_i d_{eff}}{n_i c} [A_p A_s^* e^{i\Delta k_{OPA} z} + A_{2i} A_i^* e^{i\Delta k_{SHG} z}]$$

$$\frac{dA_{2i}}{dz} = \frac{i\omega_{2i} d_{eff}}{2n_{2i} c} A_i^2 e^{-i\Delta k_{SHG} z}$$

With Δk_{OPA} = $k(\omega_p) - k(\omega_s) - k(\omega_i)$, and Δk_{SHG} = $k(\omega_{2i}) - 2k(\omega_i)$. We take both of these quantities to be 0. This system is solved below, and the importance of "arbitrary" variables will be described.

In [2]:
```
ws = 3.0*10**7; wp = 5.5*10**7; wi = 2/5*ws; w2i = 2*wi #Frequencies
ns = 5; n_p = 3; ni = 2; n2i = 2 #Indexes of refraction.
#I believe these should be ~2 (thank you noah!).

d = 1; c = 3.0*10**8

Ks = 1j*ws*d/(ns*c)
Kp = 1j*wp*d/(n_p*c)
Ki = 1j*wi*d/(ni*c)
K2i = 1j*w2i*d/(2*n2i*c)
```

Initial conditions are of importance for OPA processes. The local intensity of the pump must necessarily be larger than that of the signal to observe damping (graph below), and to converge to a solution for all intensities. Besides the above, initial conditions seemingly have few salient effects on the behavior of relevant intensity plots.

Variables of interest, however, seem to be the indexes of refraction and frequencies associated with each amplitude.

Consider the intensity for each wave, given by the magnitude of the time-averaged Poynting vector [1]: $I_j = 2n_j\epsilon_0 c|A_j|^2$, with index $j = s, p, i, 2i$. It is clearly seen that $I_j \propto n_j|A_j|^2$.

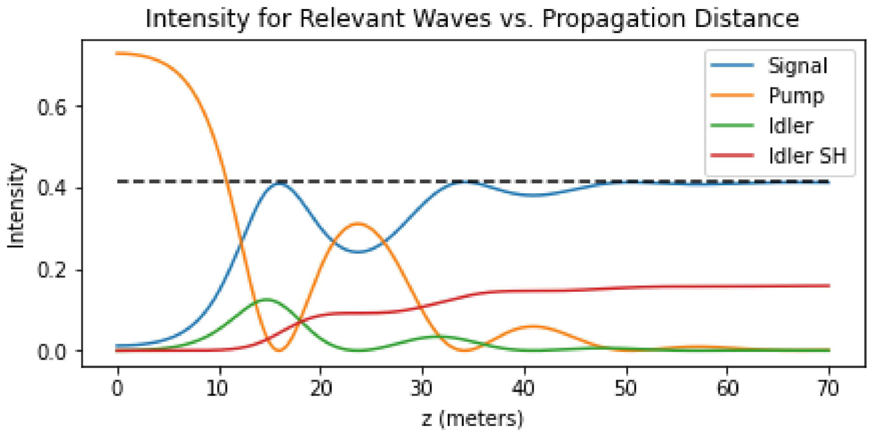
While the intensity/amplitude will not be solved for analytically, a clear behavior is observed corresponding to that of a damped oscillator. This will be further discussed with the plot below.

In [3]:

```
def dAdz(z,S):
    As, Ap, Ai, A2i = S
    return (Ks*Ap*np.conj(Ai),
            Kp*As*Ai,
            Ki*(Ap*np.conj(As)+A2i*np.conj(Ai)),
            K2i*Ai**2)

z_max = 70
As = 1+0.0j; Ap = 10+0.0j; Ai = 0.0+0.0j; A2i = 0.0+0.0j #Gain is pretty much the minim
y0 = [As, Ap, Ai, A2i]

sol = solve_ivp(dAdz,np.array([0,z_max]),y0, t_eval = np.linspace(0,z_max,1000))

#Coeff of I (discounting amplitude squared).
I_s = 2*ns*c*8.9**-12
I_p = 2*n_p*c*8.9**-12
I_i = 2*ni*c*8.9**-12
I_2i = 2*n2i*c*8.9**-12

z = sol.t
As_list = I_s*(np.abs(sol.y[0])**2)
Ap_list = I_p*(np.abs(sol.y[1])**2)
Ai_list = I_i*(np.abs(sol.y[2])**2)
A2i_list = I_2i*(np.abs(sol.y[3])**2)

zero = [np.amax(As_list)]*len(z)
```

In [4]:

```
plt.figure(figsize=(7,3))
plt.plot(z,As_list)
plt.plot(z,Ap_list)
plt.plot(z,Ai_list)
plt.plot(z,A2i_list)
plt.plot(z,zero,'k--')
plt.xlabel("z (meters)")
plt.ylabel("Intensity")
plt.legend(["Signal", "Pump", "Idler", "Idler SH"])
plt.title("Intensity for Relevant Waves vs. Propagation Distance")
plt.show()
```



The plot above mimicks the behavior found in *Moses, et al.* (figure 4c.), with perhaps a notable difference that the idler SH graph is shifted downwards. I believe this is because of the default frequencies suggested ($\omega_s = \frac{5}{2}\omega_i$), since the relative intensities should primarily be proportional to the frequencies of each wave.

If the "arbitrary" variables above are modified (n, ω), one can make a reasonable estimate that a relation for the damping ratio may follow such that $\zeta \propto \frac{n_j}{\omega_j}$.

This can be observed if one increases the index of refraction of either the pump, or signal; where something closer to an over-damped oscillator behavior will be seen. The **opposite is true**, however, if one increases the index of either the idler, or idler SH amplitude. This points to a different relation for both the idler, and idler SH waves:

$$\zeta \propto \frac{1}{\omega_j n_j},$$

The choice of index of refraction, and frequency is therefore of importance to the aforementioned dynamics.

References:

[1]: Boyd, Robert W. Nonlinear Optics. San Diego, CA: Academic Press, 2003. Print.

[2]: Jeffrey Moses, Noah Flemens, and Xiaoyue Ding "Back-conversion suppressed parametric frequency conversion for ultrawide bandwidth and ultrahigh efficiency devices", Proc. SPIE 11264, Nonlinear Frequency Generation and Conversion: Materials and Devices XIX, 112640B (2 March 2020); https://doi.org/10.1117/12.2548361