Bài tập Các pp phân tích A=LU

1–5 DOOLITTLE'S METHOD

Show the factorization and solve by Doolittle's method.

1.
$$4x_1 + 5x_2 = 14$$
$$12x_1 + 14x_2 = 36$$

2.
$$2x_1 + 9x_2 = 82$$
$$3x_1 - 5x_2 = -62$$

3.
$$5x_1 + 4x_2 + x_3 = 6.8$$
$$10x_1 + 9x_2 + 4x_3 = 17.6$$
$$10x_1 + 13x_2 + 15x_3 = 38.4$$

4.
$$\begin{aligned} 2x_1 + x_2 + 2x_3 &= 0 \\ -2x_1 + 2x_2 + x_3 &= 0 \\ x_1 + 2x_2 - 2x_3 &= 18 \end{aligned}$$

5.
$$\begin{aligned} 3x_1 + 9x_2 + 6x_3 &= 4.6 \\ 18x_1 + 48x_2 + 39x_3 &= 27.2 \\ 9x_1 - 27x_2 + 42x_3 &= 9.0 \end{aligned}$$

6. TEAM PROJECT. Crout's method factorizes $\mathbf{A} = \mathbf{LU}$, where $\mathbf{L}$ is lower triangular and $\mathbf{U}$ is upper triangular with diagonal entries $u_{jj} = 1, j = 1, \cdots, n$.

(a) Formulas. Obtain formulas for Crout's method similar to (4).

(b) Examples. Solve Prob. 5 by Crout's method.

(c) Factor the following matrix by the Doolittle, Crout, and Cholesky methods.

$$\begin{bmatrix} 1 & -4 & 2 \\ -4 & 25 & 4 \\ 2 & 4 & 24 \end{bmatrix}$$

(d) Give the formulas for factoring a tridiagonal matrix by Crout's method.

(e) When can you obtain Crout's factorization from Doolittle's by transposition?

7–12 CHOLESKY'S METHOD

Show the factorization and solve.

7.
$$\begin{aligned} 9x_1 + 6x_2 + 12x_3 &= 17.4 \\ 6x_1 + 13x_2 + 11x_3 &= 23.6 \\ 12x_1 + 11x_2 + 26x_3 &= 30.8 \end{aligned}$$

8.
$$\begin{aligned} 4x_1 + 6x_2 + 8x_3 &= 0 \\ 6x_1 + 34x_2 + 52x_3 &= -160 \\ 8x_1 + 52x_2 + 129x_3 &= -452 \end{aligned}$$

9.
$$\begin{aligned} 0.01x_1 + 0.03x_3 &= 0.14 \\ 0.16x_2 + 0.08x_3 &= 0.16 \\ 0.03x_1 + 0.08x_2 + 0.14x_3 &= 0.54 \end{aligned}$$

10.
$$\begin{aligned} 4x_1 \qquad\quad + 2x_3 &= 1.5 \\ 4x_2 + x_3 &= 4.0 \\ 2x_1 + x_2 + 2x_3 &= 2.5 \end{aligned}$$

11.
$$\begin{aligned} x_1 - x_2 + 3x_3 + 2x_4 &= 15 \\ -x_1 + 5x_2 - 5x_3 - 2x_4 &= -35 \\ 3x_1 - 5x_2 + 19x_3 + 3x_4 &= 94 \\ 2x_1 - 2x_2 + 3x_3 + 21x_4 &= 1 \end{aligned}$$

12.
$$\begin{aligned} 4x_1 + 2x_2 + 4x_3 \qquad &= 20 \\ 2x_1 + 2x_2 + 3x_3 + 2x_4 &= 36 \\ 4x_1 + 3x_2 + 6x_3 + 3x_4 &= 60 \\ 2x_2 + 3x_3 + 9x_4 &= 122 \end{aligned}$$

13. Definiteness. Let $\mathbf{A}, \mathbf{B}$ be $n \times n$ and positive definite. Are $-\mathbf{A}, \mathbf{A}^{\mathsf{T}}, \mathbf{A} + \mathbf{B}, \mathbf{A} - \mathbf{B}$ positive definite?

14. CAS PROJECT. Cholesky's Method. **(a)** Write a program for solving linear systems by Cholesky's method and apply it to Example 2 in the text, to Probs. 7–9, and to systems of your choice.

15–19 INVERSE

Find the inverse by the Gauss–Jordan method, showing the details.

15. In Prob. 1

16. In Prob. 4

17. In Team Project 6(c)

18. In Prob. 9

19. In Prob. 12

20. Rounding. For the following matrix $\mathbf{A}$ find $\det \mathbf{A}$. What happens if you roundoff the given entries to **(a)** 5S, **(b)** 4S, **(c)** 3S, **(d)** 2S, **(e)** 1S? What is the practical implication of your work?

$$\mathbf{A} = \begin{bmatrix} \frac{1}{3} & \frac{1}{4} & 2 \\ -\frac{1}{9} & 1 & \frac{1}{7} \\ \frac{4}{63} & -\frac{3}{28} & \frac{13}{49} \end{bmatrix}$$