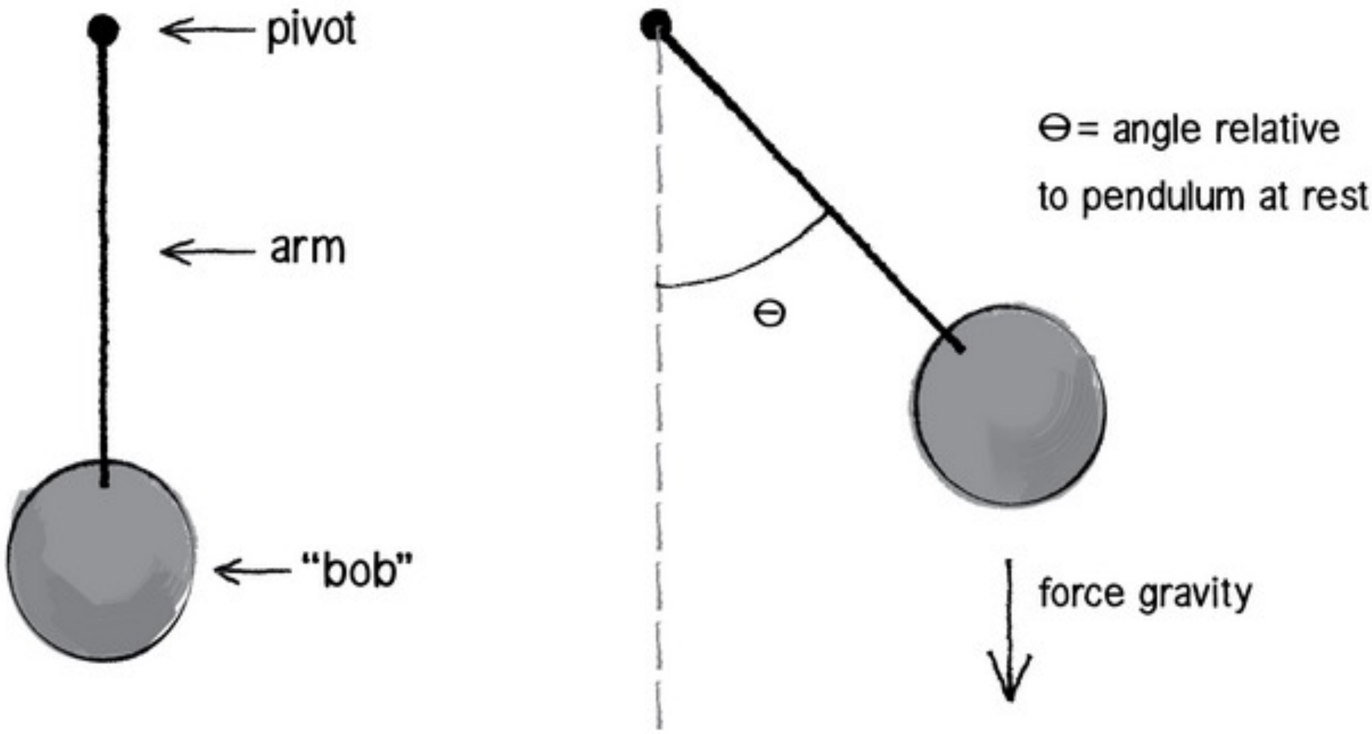
Pendulum State Estimation using Kalman Filtering & State Space Analysis

Prepared by:
Nishantkumar V Patel

Introduction:



The above picture is of simple pendulum with its different components which later on used in the simulink physical model from Mathworks.

The angle theta is the angle which is ultimately the state which we need to estimates and also the velocity during the pendulum oscillations.

The angle theta is our case considered at intial condition.

Parameter Values:

```
% gravity
g = 9.81; % (m/s^2)
% Pendulum mass
m = 2; % (kg)
% Pendulum length
l = 0.75; % (m)
% Process noise covariance
Q = 1e-3;
% Measurement noise covariance
R = 1e-4;
% Sampling time
Ts = 0.01; % (sec)




% State Space Matrices
% A
system_matrix= [0 1; -g/l 0];
% B
input_matrix = [0; 1/(m*l^2)];
% C
output_matrix = [1 0];
% D
direct_transmission_matrix = 0;
```

State Space Analysis for a Pendulum system behaviour

→ The simple pendulum system with no friction can be replaced by below equation of motion.

$$\boxed{J\frac{d^2\theta}{dt^2} + mgl\sin\theta = \tau} \text{ --- } (X)$$

where, $J = ml^2$ (moment of inertia)

→ To make the equation (X) into linear differential equation.

$$\frac{d^2\theta}{dt^2} + \frac{mgl\sin\theta}{ml^2} = \frac{\tau}{ml^2}$$

($\because$ divided by $J = ml^2$)

$$\boxed{\therefore \frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = \frac{\tau}{ml^2}} \text{ --- } (Y)$$

(Since the θ is very small $\therefore \sin\theta \simeq \theta$)

→ The state space representation,

Assume,

θ = output we want

τ = input we applied.

$\Rightarrow \theta = y = x_1$

$\Rightarrow x_2 = \dot{x}_1 = \dot{\theta} = (d\theta/dt)$

$\Rightarrow \dot{x}_2 = \ddot{x}_1 = \ddot{\theta} = (d^2\theta/dt^2)$

$\Rightarrow u = \tau$

$\Rightarrow \frac{\tau}{ml^2} - \frac{g}{l}\theta = \frac{d^2\theta}{dt^2}$

$\dot{x}_2 = \frac{1}{ml^2}(u) - \frac{g}{l}x_1$

→ State-Space Equation

$$\boxed{\begin{aligned} &\therefore \dot{X} = AX + Bu \\ &\therefore Y = CX + Du \end{aligned}}$$

($\because$ where $D=0$)

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g/l & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} \tau$$

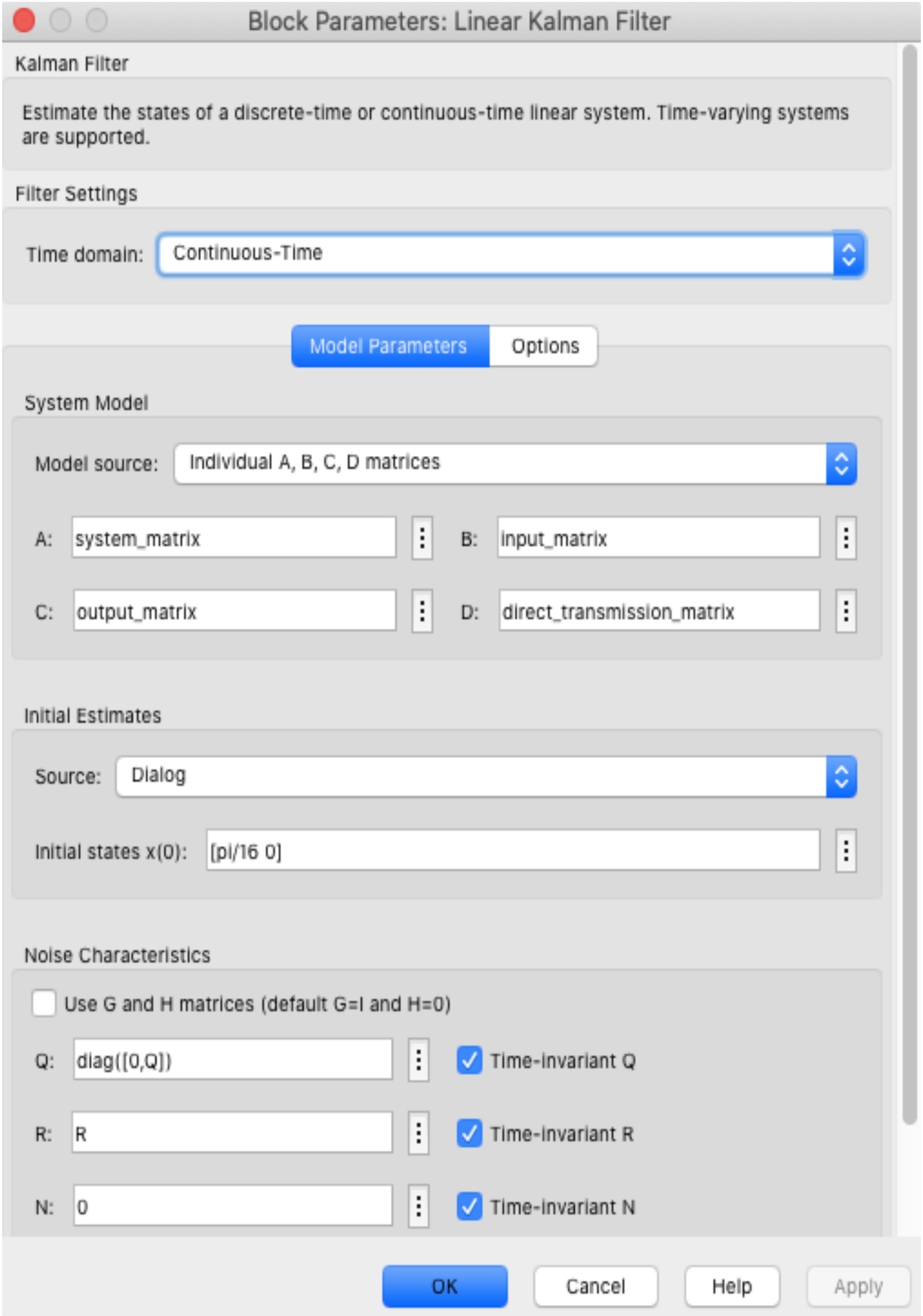
$$[y] = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

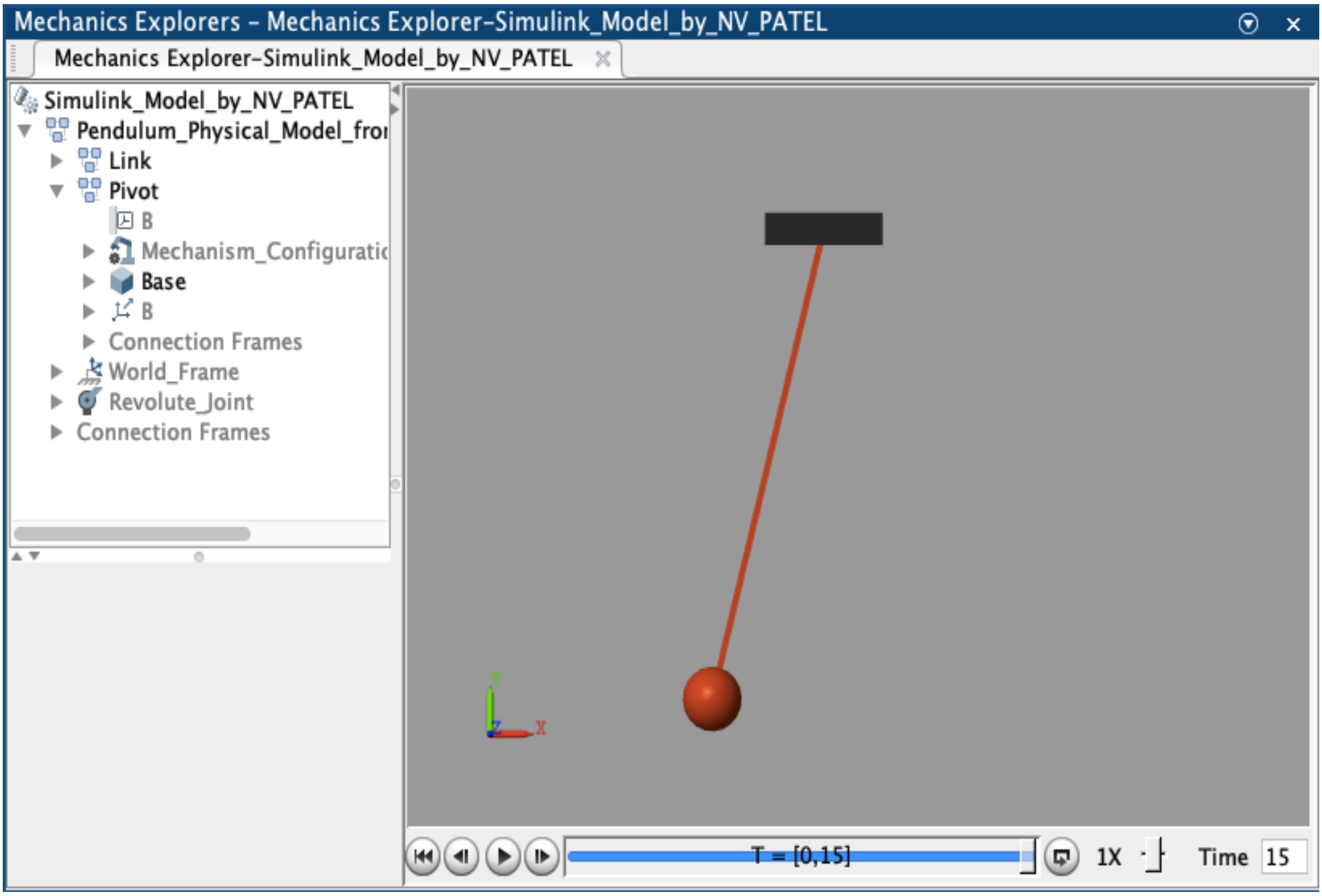
→ 9. So, we get the A, B, C and D matrices.

$$\therefore A = \begin{bmatrix} 0 & 1 \\ -g/l & 0 \end{bmatrix}, \quad \therefore B = \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix}$$

$$\therefore C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \therefore D = 0.$$

The state space matrices A, B, C and D are later on used for the block parameters of Kalman filter by selecting the State Space analysis method.


Block Parameters: Linear Kalman Filter
Kalman Filter
Estimate the states of a discrete-time or continuous-time linear system. Time-varying systems are supported.
Filter Settings
Time domain:
Continuous-Time
Model Parameters
Options
System Model
Model source:
Individual A, B, C, D matrices
A:
system_matrix
B:
input_matrix
C:
output_matrix
D:
direct_transmission_matrix
Initial Estimates
Source:
Dialog
Initial states x(0):
[pi/16 0]
Noise Characteristics
Use G and H matrices (default G=I and H=0)
Q:
diag([0,Q])
Time-invariant Q
R:
R
Time-invariant R
N:
0
Time-invariant N
OK
Cancel
Help
Apply


Mechanics Explorers - Mechanics Explorer-Simulink_Model_by_NV_PATEL
Mechanics Explorer-Simulink_Model_by_NV_PATEL
Simulink_Model_by_NV_PATEL
Pendulum_Physical_Model_fro
Link
Pivot
B
Mechanism_Configurati
Base
B
Connection Frames
World_Frame
Revolute_Joint
Connection Frames
T = [0,15]
1X
Time
15

1. With Linear Kalman Filter:

Usig linear kalman filter we can only estimate the position of pendulum with small value of angle at initial condition.

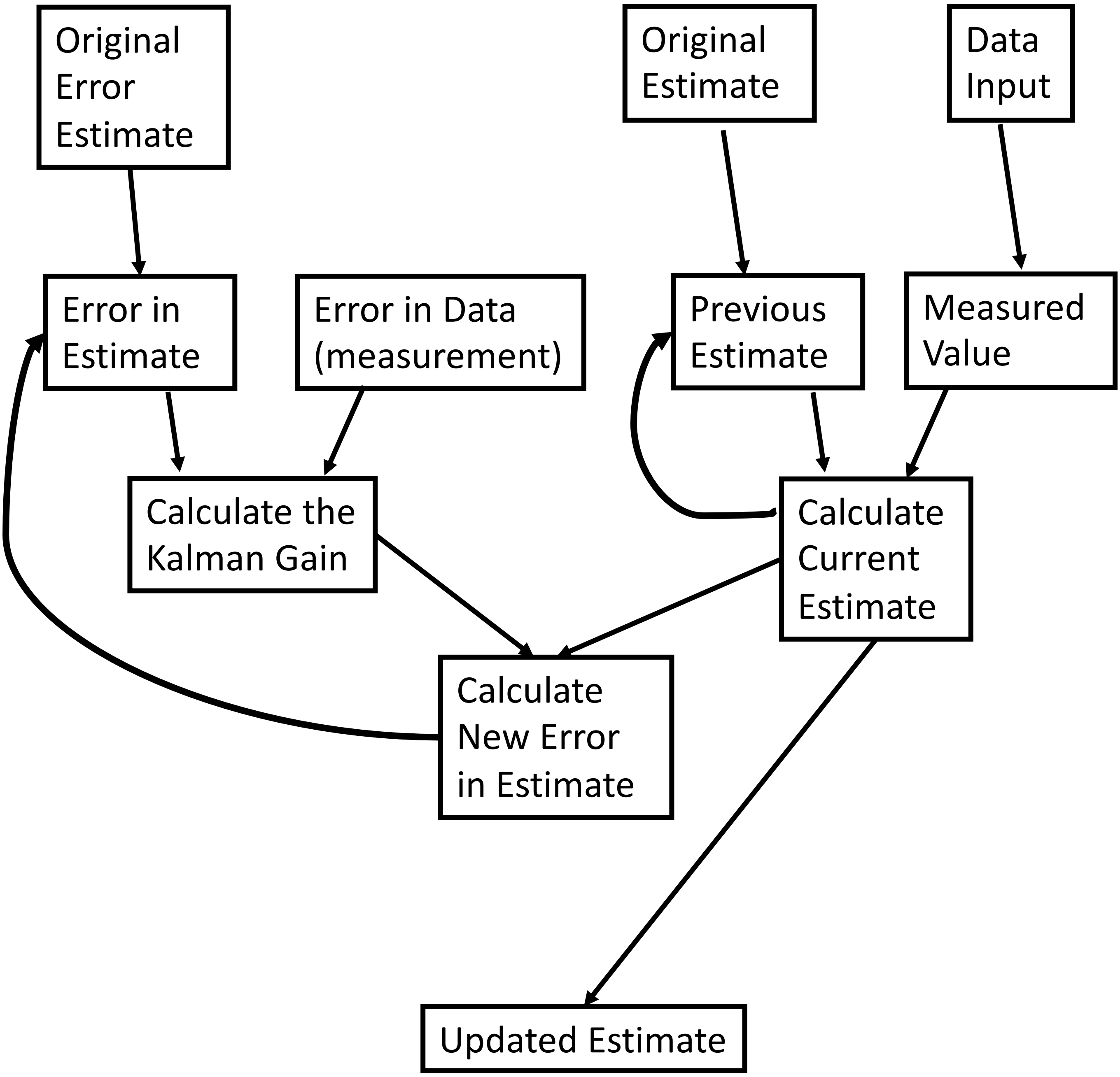
As in this case I used the initial condition of angle theta equal to 100.

For larger amount of angle theta linear kalman filter is not able to track the precise position of pendulum over the period of time.

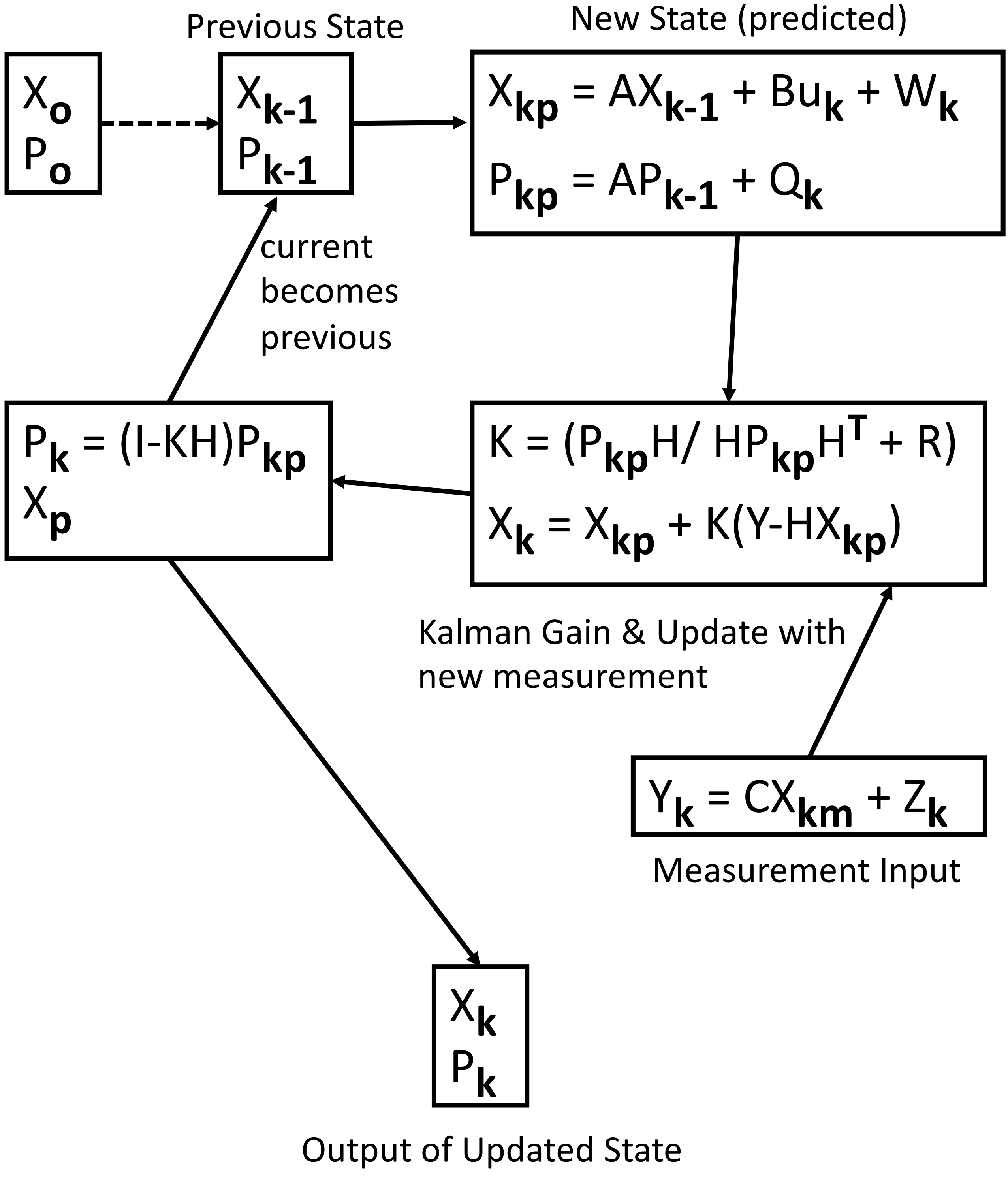
Suppose, I would take the larger angle for example theta of value 300.

I will compare the both results with small and large theta using linear kalman filter.

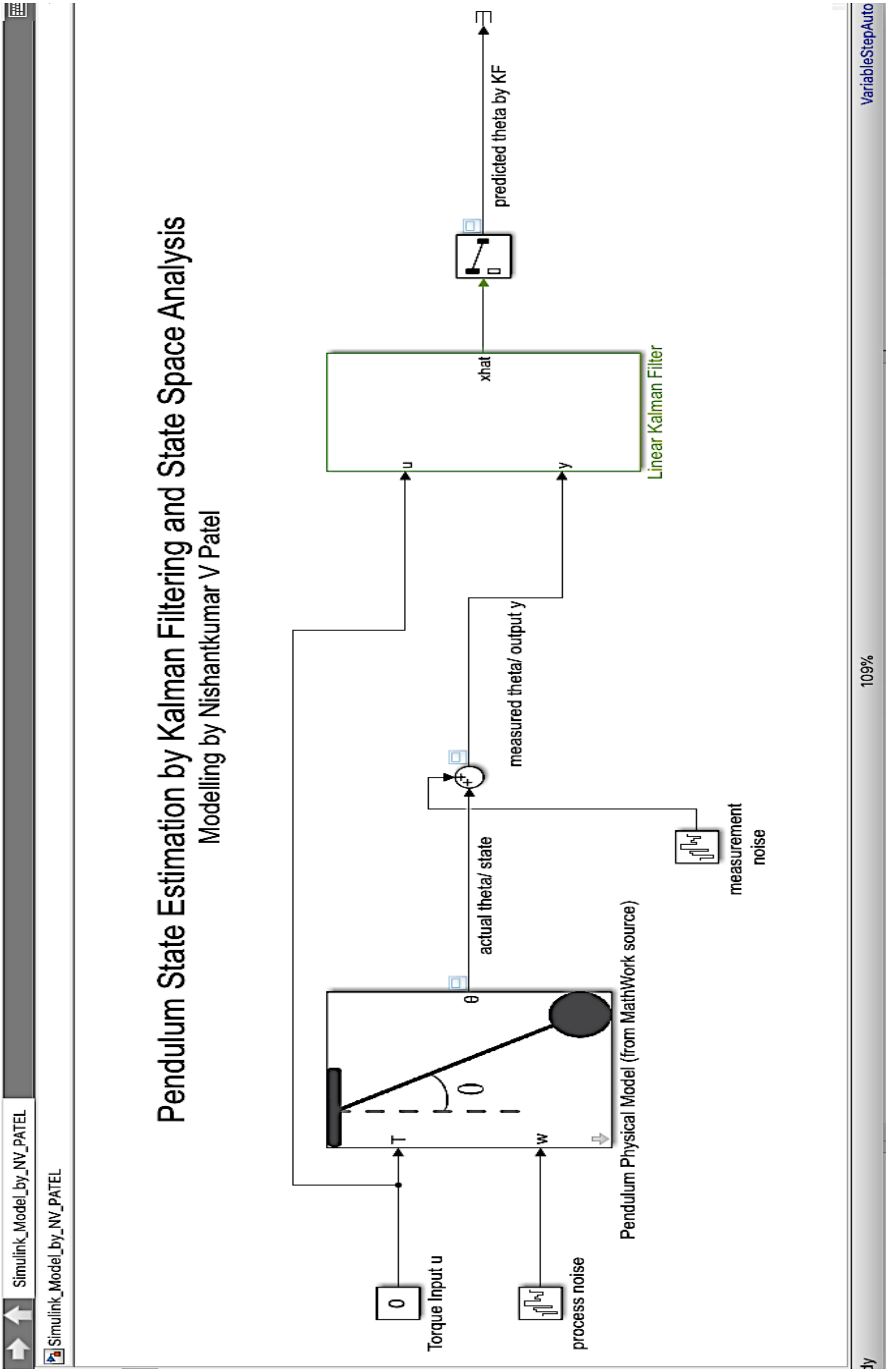
Basically how the kalman filter actually works are described as below using process flow chart.



Kalman Filtering Equations

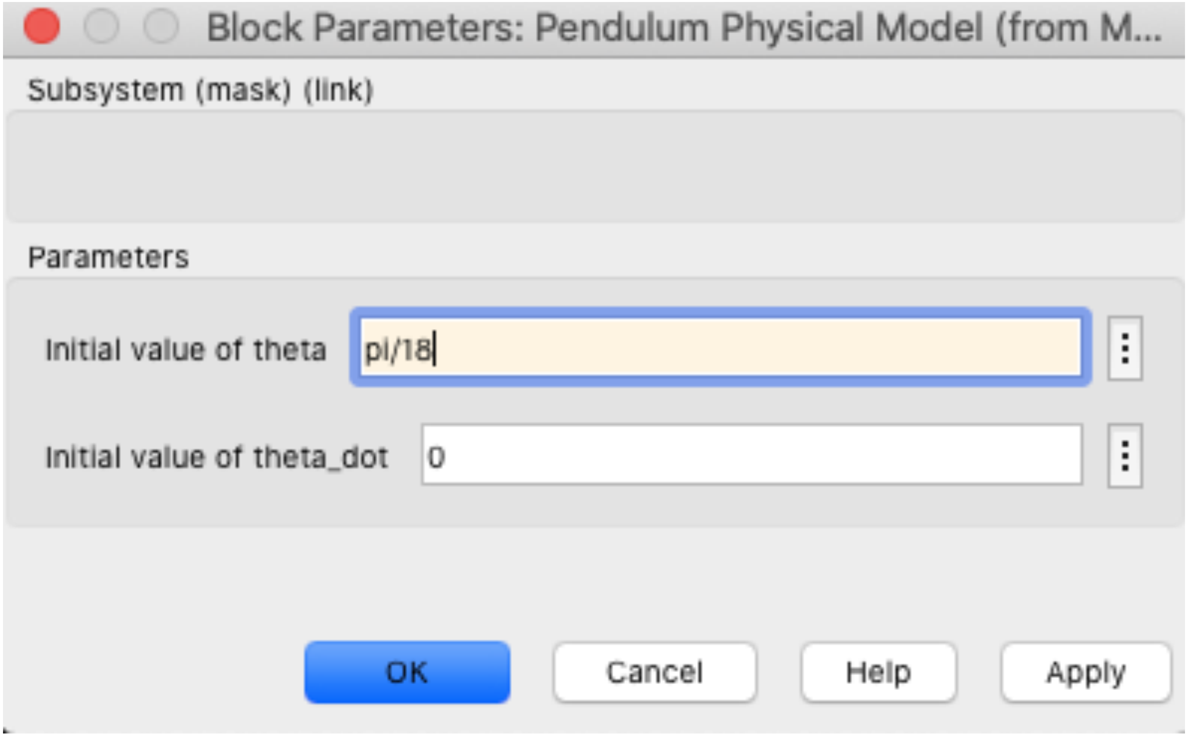


Simulink Model (linear KF):

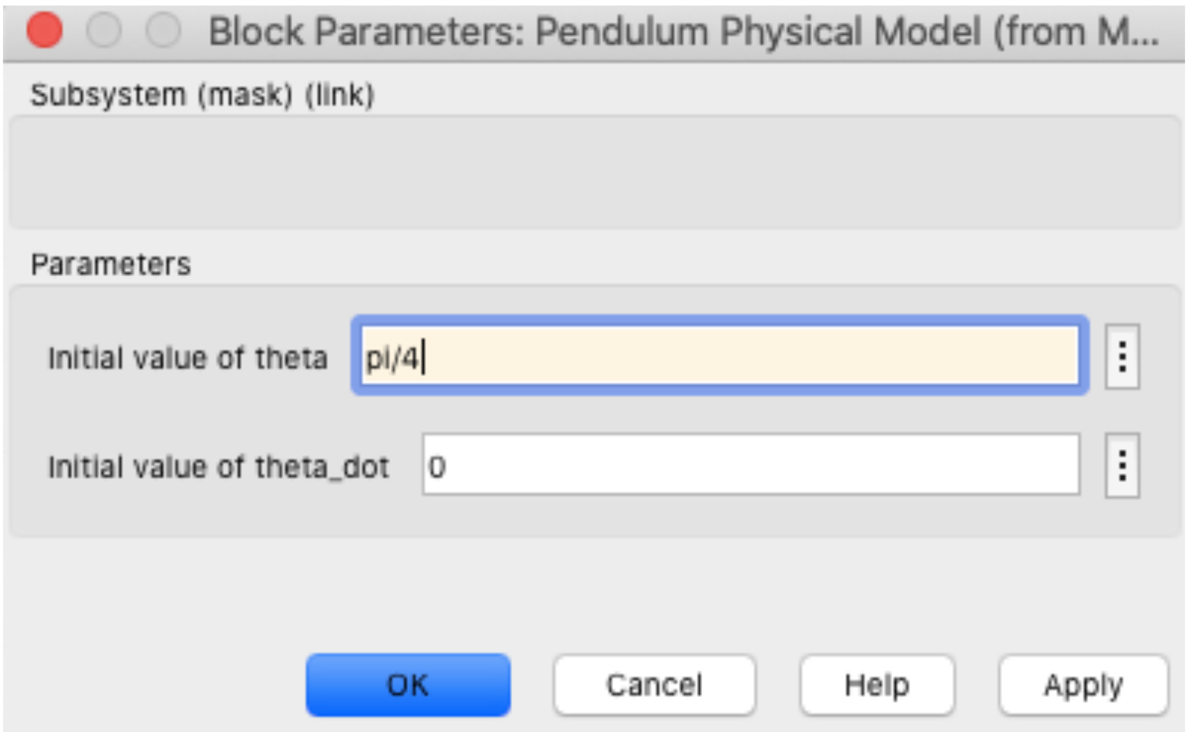


Pendulum Block Parameters with Small & Large Angle at Initial Condition:

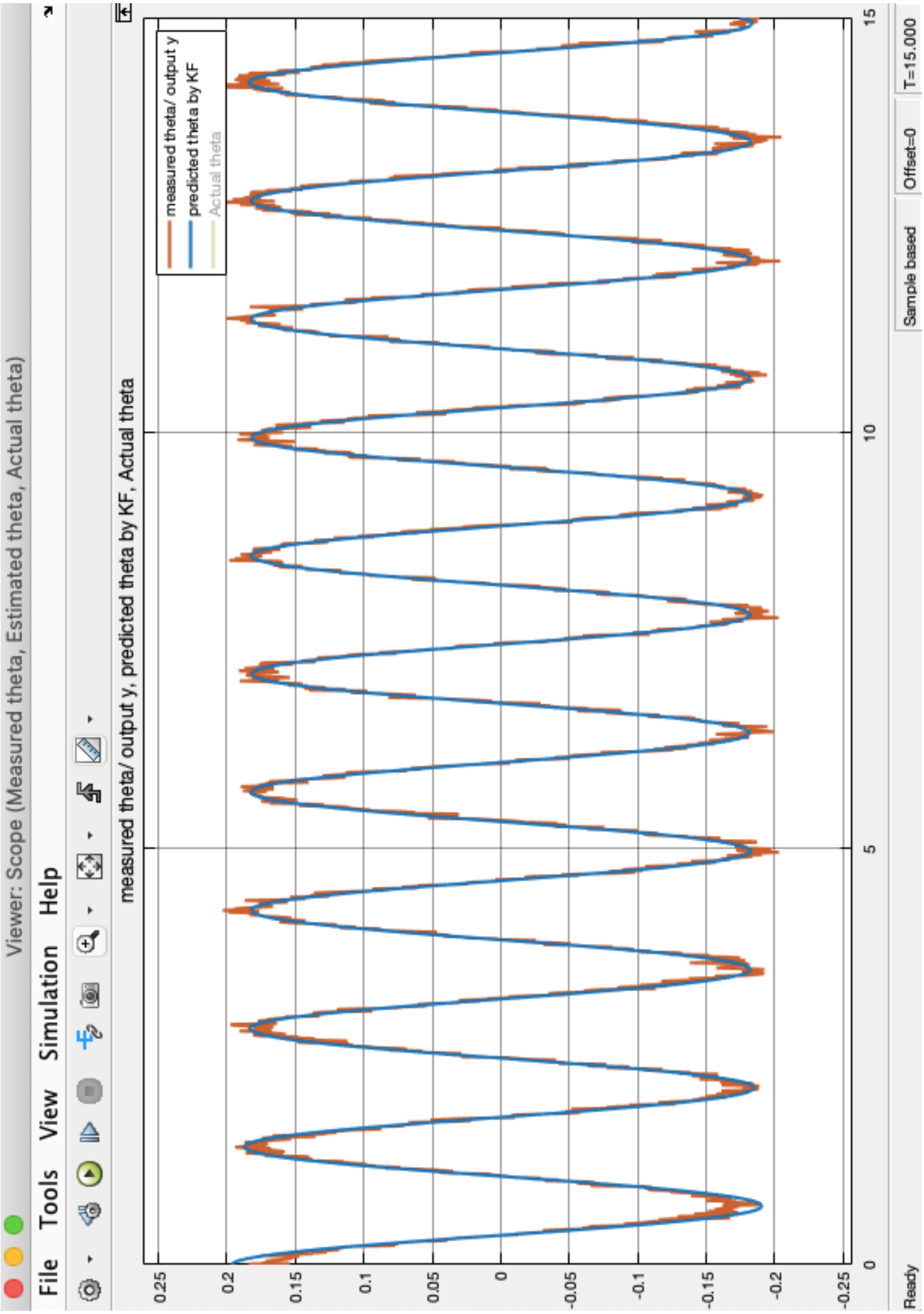
With small angle 10^0:



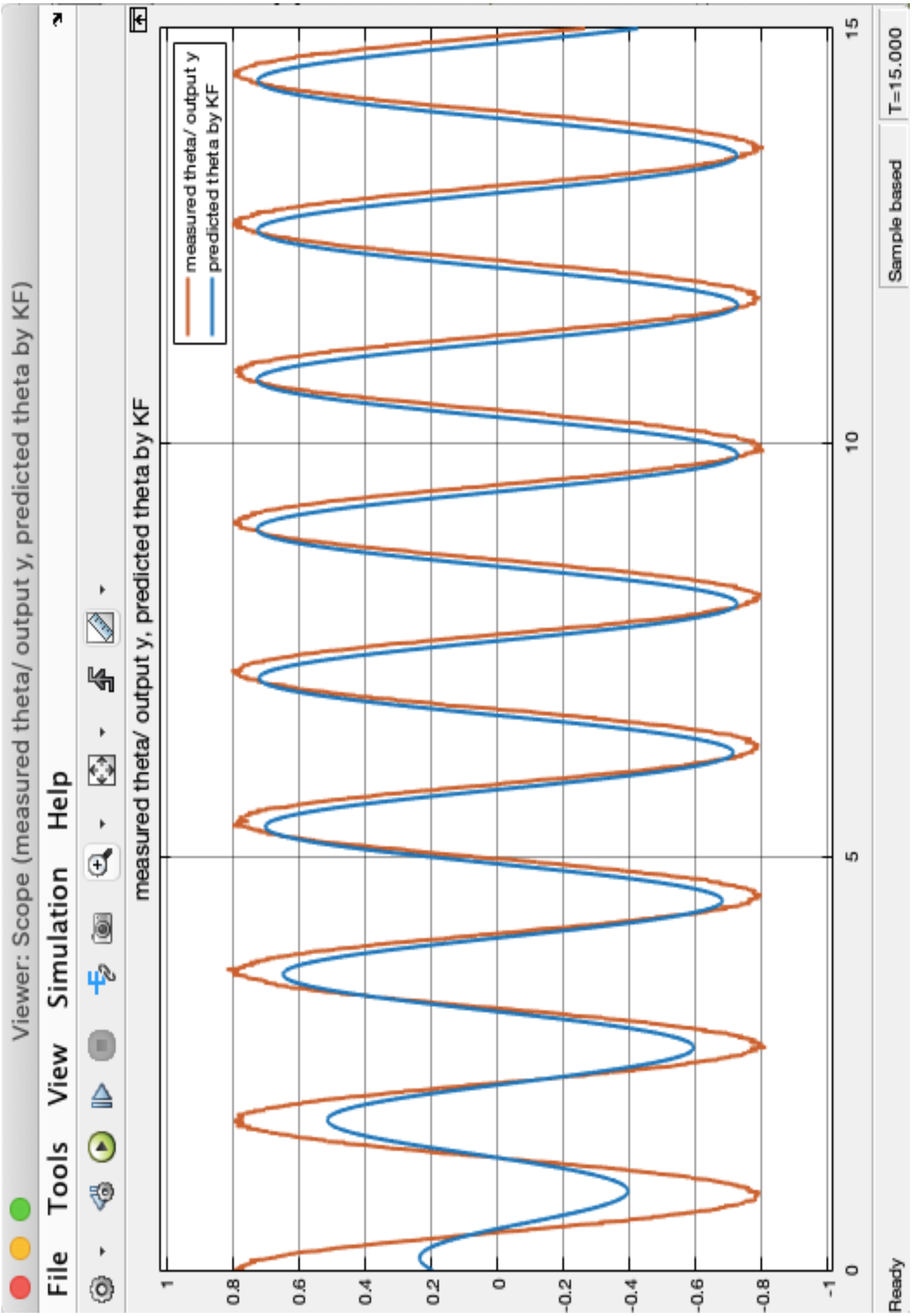
With large angle 45^0:



Results Comparison (with small angle):



Results Comparison (with large angle):



2. With Extended Kalman Filter:

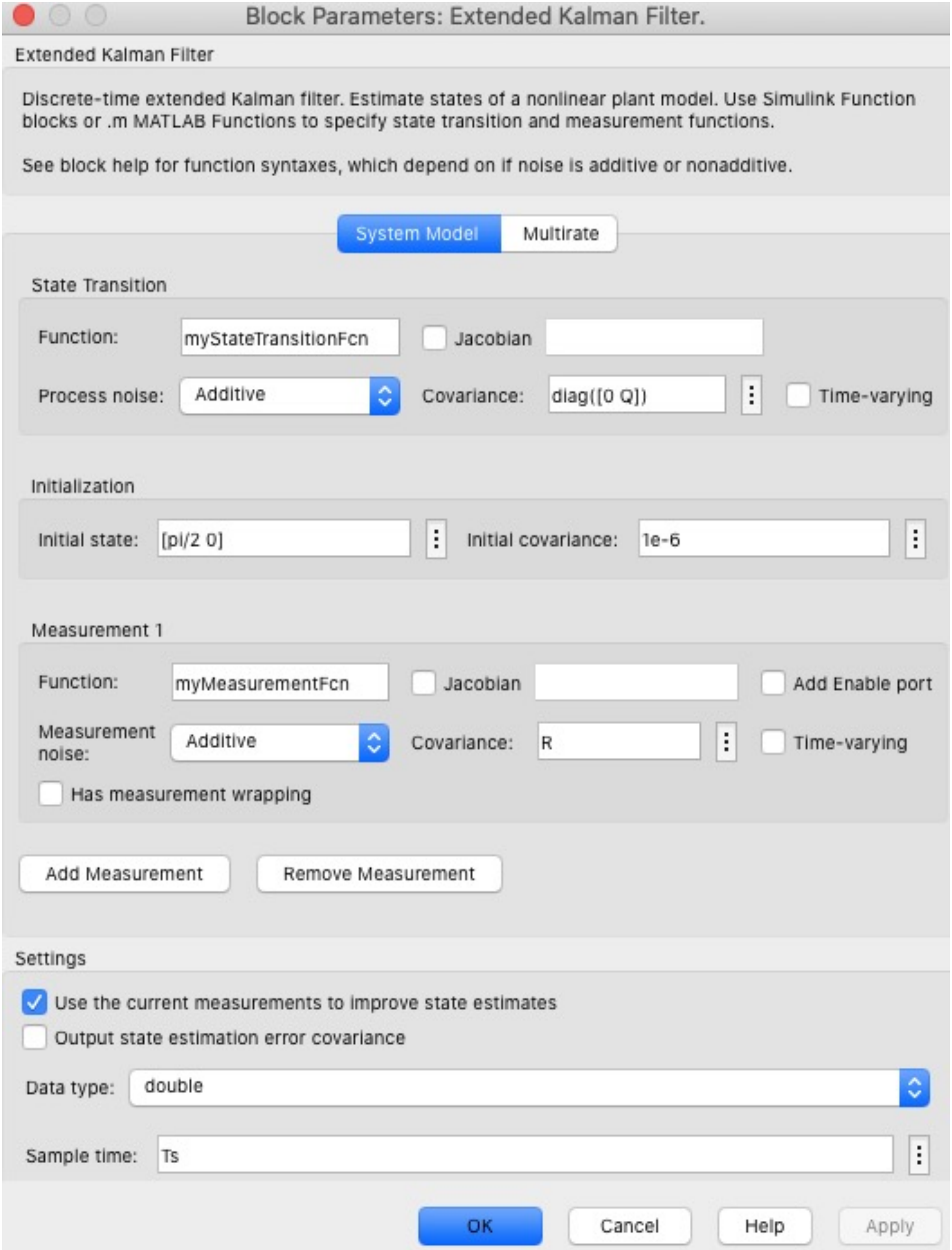
Why use?

The extended kalman filter eliminates the inefficiency of linear kalman filter and it can be used for the large angle values.

Extended kalman filter is able to track the non liear behaviour of the system quite well.

I will show the same value of angle 300 using extended kalman filter and we will compare the scope graph that how fairly it can predict the states of pendulum.

Block & Pendulum Parameters



Editor - /Users/jamesbond/Desktop/Pendulum State Estimation by Kalman Filt

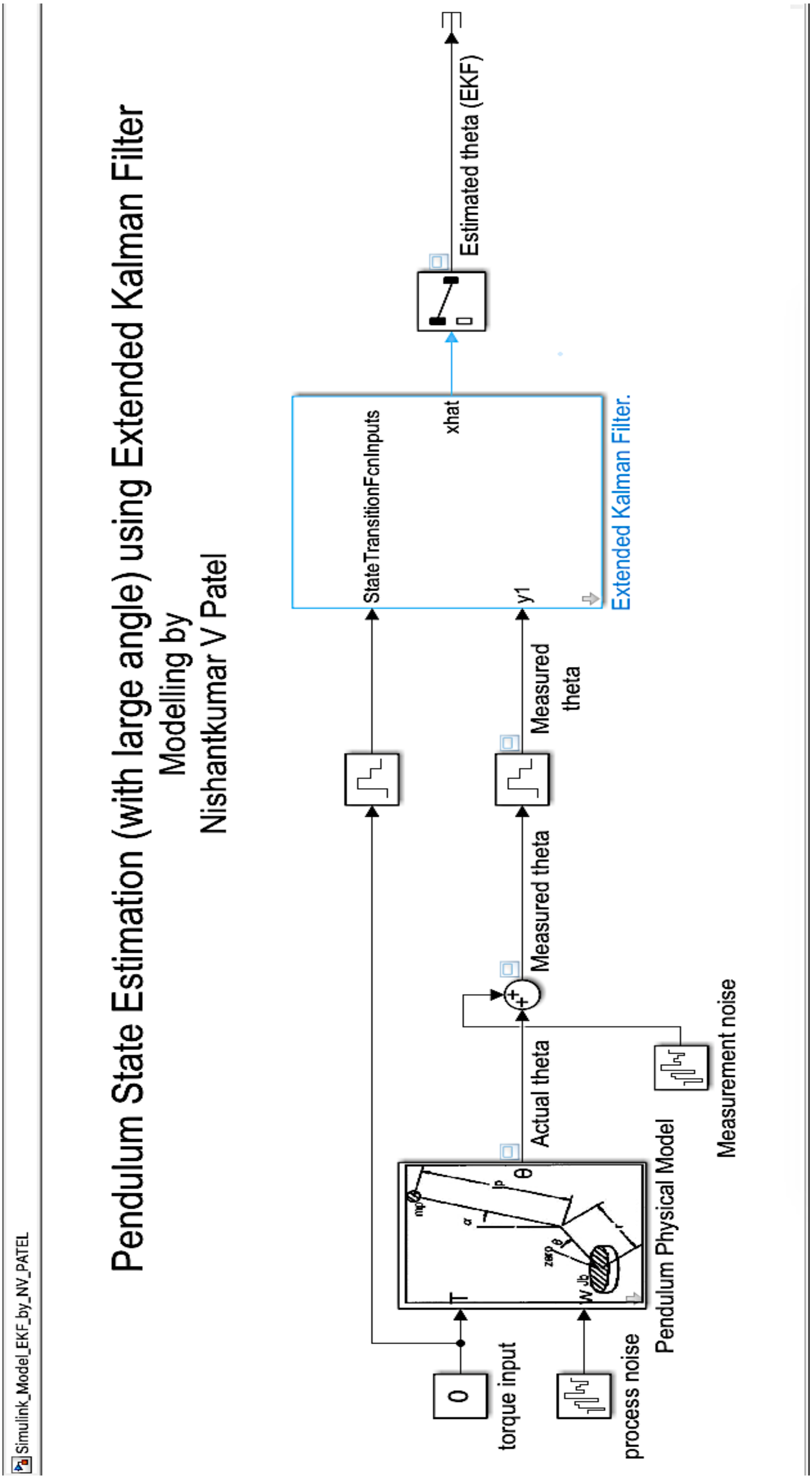
myStateTransitionFcn.m * | myMeasurementFcn.m | +

```

function x = myStateTransitionFcn(x,u)
% Sample time [s]
dt = 0.01;

x = x + [x(2); -9.81/0.5*sin(x(1)) + u]*dt;
end
```

Simulink Model (extended KF):



Result Comparison: