Predicting Federal Funds Rate: Taylor's Rule Regression Model Validation and the Role of Target and Unemployment

by Nicholas Pham

Introduction

In this project we will construct models for the Federal Funds Effective Rate. This is a rate determined by the market, similar to stock prices in the stock market. The Federal Funds Rate is a key tool used in monetary policy that influences economic activity. The Federal Reserve sets a Target for the Federal Funds Rate, then performs operations such as trading bonds to adjust the Federal Funds Rate, bringing it closer to the Target. Predicting the Federal Funds Rate benefits educators, economists, investors, financial institutions, and policy planners.

This project aims to first reproduce regression models predicting the Federal Funds Rate using Taylor's Rule, a policy guideline by John Taylor from Stanford in 1993 and a modification of this used by researcher Alper D. Karakas, the equations of which are derived in Karkas' (2023) paper, "Reevaluating the Taylor Rule with Machine Learning."

We will then attempt to construct other models by adding the Target and Unemployment Rate to the Taylor Model to see if the addition of new features can improve the performance of regression models in predicting the Federal Funds Effective Rate. We choose to build off the Taylor Model as this is the foundational model and Karakas (2023) found little difference in performance between this model and their model.

We will also check the assumptions of regression for each model. We will also construct simple Neural Network Models for the regression equations.

Possible stakeholders for this project include policy makers, economists, investors, and lending institutions.

Setup

Import Libraries, Functions, and Classes

In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import random
import math
```

```python
import seaborn as sns
import statsmodels.api as sm
import tensorflow as tf
from sklearn.metrics import mean_squared_error, mean_absolute_error, r2_score
from sklearn.preprocessing import MinMaxScaler
from statsmodels.stats.outliers_influence import variance_inflation_factor
from statsmodels.stats.diagnostic import het_breuschpagan, linear_rainbow
from statsmodels.stats.stattools import durbin_watson, jarque_bera
from scipy.stats.mstats import winsorize
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Input, Dropout
from tensorflow.keras.regularizers import l2
from tensorflow.keras.callbacks import EarlyStopping
```

Define Functions

In [2]:

```python
def fit_ols_model(X, y, model_name):
    """Fits an OLS Regression model for the given variables and returns predictions."""

    # Fit the model
    model = sm.OLS(y, X).fit()

    # Get predictions
    y_pred = model.predict(X)
    return model, y_pred


def calculate_vif(X, model_name):
    """Calculates Variance Inflation Factors (VIFs)."""

    vif_data = pd.DataFrame()
    vif_data["Feature"] = X.columns
    vif_data["VIF"] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
    return vif_data


def error_metrics(y, y_pred):
    """Computes error metrics."""

    mse = round(mean_squared_error(y, y_pred), 3)
    rmse = round(np.sqrt(mse), 3)
    mae = round(mean_absolute_error(y, y_pred), 3)
    mpe = round(np.mean((y - y_pred) / y) * 100, 3)
    mape = round(np.mean(np.abs((y - y_pred) / y)) * 100, 3)
    r2 = r2_score(y, y_pred)
    return {
```

```python
        "Mean Squared Error": mse,
        "Root Mean Squared Error": rmse,
        "Mean Absolute Error": mae,
        "Mean Percentage Error": mpe,
        "Mean Absolute Percentage Error": mape,
        "R-Squared": r2
    }


def find_resid_sum(y, y_pred):
    """Function to compute the Sum of Residuals."""

    rs = np.sum(y - y_pred)
    return rs


def find_sae(y, y_pred):
    """Function to compute the Sum of Absolute Errors."""

    sae = np.sum(np.abs(y - y_pred))
    return sae


def fit_nn_model(X, y, model_name):
    """Fits a Neural Network model for the given variables and returns predictions and model."""

    # Set the input dimension
    input_dim = X.shape[1]

    # Define the neural network with dropout and l2 regularization
    model = Sequential([
        Input(shape=(input_dim,)), # Input layer
        Dense(32, activation='relu', kernel_regularizer=l2(0.01)), # Hidden Layer
        Dropout(0.2), # Dropout to prevent overfitting
        Dense(1, activation='linear', kernel_regularizer=l2(0.01)) # Output Layer
    ])

    # Compile the model and fit with early stopping
    model.compile(optimizer='adam', loss='mse', metrics=['mae'])
    early_stopping = EarlyStopping(monitor='val_loss', patience=5, restore_best_weights=True)
    history = model.fit(
        X, y,
        epochs=100,
        batch_size=32,
        validation_split=0.2,
        callbacks=[early_stopping],
        verbose=0
    )
```

```
    # Get predictions
    y_pred = model.predict(X).flatten()
    return model, y_pred, history
```

Random Seeds

In [3]:
```
# Set random seeds for reproducibility
random.seed(42)
np.random.seed(42)
tf.random.set_seed(42)
```

Data Wrangling

In [4]:
```
# Define a dictionary with datasets and their URLs
datasets = {
    "ffer": "https://raw.githubusercontent.com/nvpham12/Capstone-Project/refs/heads/main/FFER.csv",
    "pgdp": "https://raw.githubusercontent.com/nvpham12/Capstone-Project/refs/heads/main/PGDP.csv",
    "rgdp": "https://raw.githubusercontent.com/nvpham12/Capstone-Project/refs/heads/main/RGDP.csv",
    "cpi": "https://raw.githubusercontent.com/nvpham12/Capstone-Project/refs/heads/main/CPI.csv",
    "fftr_lower": "https://raw.githubusercontent.com/nvpham12/Capstone-Project/refs/heads/main/FFTR_lower.csv",
    "fftr_upper": "https://raw.githubusercontent.com/nvpham12/Capstone-Project/refs/heads/main/FFTR_upper.csv",
    "fftr_old": "https://raw.githubusercontent.com/nvpham12/Capstone-Project/refs/heads/main/FFTR_old.csv",
    "unrate": "https://raw.githubusercontent.com/nvpham12/Capstone-Project/refs/heads/main/UNRATE.csv"
}

# Iterate and Load datasets
for name, url in datasets.items():
    globals()[name] = pd.read_csv(url, parse_dates=["observation_date"])
    print(f"{name} dataset loaded successfully.")
```

```
ffer dataset loaded successfully.
pgdp dataset loaded successfully.
rgdp dataset loaded successfully.
cpi dataset loaded successfully.
fftr_lower dataset loaded successfully.
fftr_upper dataset loaded successfully.
fftr_old dataset loaded successfully.
unrate dataset loaded successfully.
```

In [5]:
```
# Iterate through the dataset names in the dictionary
for name in datasets.keys():
    # Access the DataFrame using globals()
    dataframe = globals()[name]
```

```python
    # Find missing values
    missing_val = dataframe.isna().sum()
    total_missing = missing_val.sum()

    # Find duplicates
    duplicates = dataframe[dataframe.duplicated(keep=False)]

    # Print a message indicating if there are any missing values or not
    if total_missing > 0:
        print(f"'{name}' has missing values:")
        print(missing_val[missing_val > 0])
    else:
        print(f"'{name}' has no missing values.")

    # Print a message indicating if there are any duplicate values or not
    if not duplicates.empty:
        print(f"'{name}' has {len(duplicates)} duplicate rows:")
        print(duplicates)
    else:
        print(f"'{name}' has no duplicate rows.\n")
```

```
'ffer' has no missing values.
'ffer' has no duplicate rows.

'pgdp' has no missing values.
'pgdp' has no duplicate rows.

'rgdp' has no missing values.
'rgdp' has no duplicate rows.

'cpi' has no missing values.
'cpi' has no duplicate rows.

'fftr_lower' has no missing values.
'fftr_lower' has no duplicate rows.

'fftr_upper' has no missing values.
'fftr_upper' has no duplicate rows.

'fftr_old' has no missing values.
'fftr_old' has no duplicate rows.

'unrate' has no missing values.
'unrate' has no duplicate rows.
```

Feature Extraction

In [6]:
```
# The data for CPI can be used to find inflation rates.
# This is done to obtain a seasonally adjusted inflation dataset that isn't available on FRED.
inflation = pd.DataFrame()
inflation["observation_date"] = cpi["observation_date"]
inflation["Inflation"] = cpi["CPIAUCSL"].pct_change(periods=12) * 100
inflation.dropna(inplace=True)

# The Federal Funds Target Rate (FFTR) is set by the Federal Reserve.
# The Fed used to set a single value as the target, but they shifted to setting a range.
# Find the midpoint of the range.

fftr_midpoint = pd.DataFrame()
fftr_midpoint["observation_date"] = fftr_upper["observation_date"]
fftr_midpoint["Target"] = fftr_upper["DFEDTARU"] - fftr_lower["DFEDTARL"]

# Combine the midpoint with the old FFTR to get a complete FFTR dataset.
fftr_old = fftr_old.rename(columns = {"observation_date": "observation_date", "DFEDTAR": "Target"})
fftr = pd.concat([fftr_old, fftr_midpoint])
```

Resample and Merge Data

In [7]:
```
# Resample datasets to daily frequency.
inflation = inflation.set_index("observation_date").resample("D").ffill().reset_index()
pgdp = pgdp.set_index("observation_date").resample("D").ffill().reset_index()
rgdp = rgdp.set_index("observation_date").resample("D").ffill().reset_index()
unrate = unrate.set_index("observation_date").resample("D").ffill().reset_index()

# Merge the dataframes
df = ffer.merge(inflation, on="observation_date", how="outer") \
        .merge(pgdp, on="observation_date", how="outer") \
        .merge(rgdp, on= "observation_date", how="outer") \
        .merge(unrate, on="observation_date", how="outer") \
        .merge(fftr, on="observation_date", how="outer")

# Set date as an index and rename the columns
df = df.set_index("observation_date")
df.columns = ["Federal Funds Rate", "Inflation (%)", "Potential GDP", "GDP", "Unemployment", "Target"]

# Print the dataframe
df.head()
```

Out[7]:

observation_date	Federal Funds Rate	Inflation (%)	Potential GDP	GDP	Unemployment	Target
1947-01-01	NaN	NaN	NaN	2182.681	NaN	NaN
1947-01-02	NaN	NaN	NaN	2182.681	NaN	NaN
1947-01-03	NaN	NaN	NaN	2182.681	NaN	NaN
1947-01-04	NaN	NaN	NaN	2182.681	NaN	NaN
1947-01-05	NaN	NaN	NaN	2182.681	NaN	NaN

All datasets were obtained from the Federal Reserve Economic Database (FRED). Links to each dataset are in the reference list at the bottom of the Notebook.

Further Feature Extraction and Range Selection

In [8]:
```
# Find the inflation gap and output gap and add them to the dataframe
df["Inflation Gap"] = df["Inflation (%)"] - 2
df["Output Gap"] = df["GDP"] - df["Potential GDP"]

# Find Inflation Lag and Output Gap Lag for Karakas Model
df["Inflation Lag"] = df["Inflation (%)"].shift(1)
output_gap_lag = df["Output Gap"].shift(1)

# Create a percentage versions of Output Gap Lag and Inflation Lag for Karakas Model
df["Output Gap Lag %"] = (output_gap_lag / df["Potential GDP"]) * 100
```

In [9]:
```
# Drop rows with missing values from table
df.dropna(inplace=True)

# Drop Potential GDP and GDP columns as they are not needed
df = df.drop(["Potential GDP", "GDP"], axis=1)
```

Check for Duplicates and Missing Values

In [10]:
```
num_duplicates = df.reset_index().duplicated().sum()
print(num_duplicates)
```

```
0
```

We checked for duplicate rows earlier, but there were none. We check again for dupliucates after merging the data sets to ensure that they were merged correctly into a single dataframe. With 0 duplicates, the merge went through without any complications.

In [11]:
```
print(df.isna().sum())
```

```
Federal Funds Rate    0
Inflation (%)         0
Unemployment          0
Target                0
Inflation Gap         0
Output Gap            0
Inflation Lag         0
Output Gap Lag %      0
dtype: int64
```

These missing values were not present when we checked each individual data set. They have appeared now when they weren't present previously because each dataset has different ranges of available data. We dealt with missing values from the dataset by removing them completely.

Outlier Handling

In [12]:
```
# Define the percentage of extreme values to cap
winsor_limits = (0.05, 0.05)

# Apply Winsorization to all numeric columns except the dependent variable
for col in df.columns:
    if col != "Federal Funds Rate":
        lower = np.percentile(df[col], winsor_limits[0] * 100)
        upper = np.percentile(df[col], 100 - winsor_limits[1] * 100)
        df[col] = np.clip(df[col], lower, upper)
```

Here, we capped extreme values using Winsorization. We set the percentage of extreme values to cap at 5%, which should remove most outliers with massive gaps from the rest of the data.

Check Data Types

In [13]:
```
# Check the types of each variable
print(df.dtypes)
```

```
Federal Funds Rate    float64
Inflation (%)         float64
Unemployment          float64
Target                float64
Inflation Gap         float64
Output Gap            float64
Inflation Lag         float64
Output Gap Lag %      float64
dtype: object
```

The data is entirely in float64, a numerical data type, with data types consistent across feature. This type is perfect for our models and does not require any further action.

Exploratory Data Analysis (EDA)

Data

In [14]:
```
# Print the dataframe
df
```

Out[14]:

observation_date	Federal Funds Rate	Inflation (%)	Unemployment	Target	Inflation Gap	Output Gap	Inflation Lag	Output Gap Lag %
1982-09-27	10.18	4.940924	9.5	9.0625	2.940924	-538.588377	4.940924	-4.532054
1982-09-28	9.70	4.940924	9.5	9.0625	2.940924	-538.588377	4.940924	-4.532054
1982-09-29	9.88	4.940924	9.5	9.0625	2.940924	-538.588377	4.940924	-4.532054
1982-09-30	12.17	4.940924	9.5	9.0625	2.940924	-538.588377	4.940924	-4.532054
1982-10-01	10.87	5.032120	9.5	9.0625	3.032120	-596.113113	4.940924	-4.532054
...	...	...	...	...	...	...	...	...
2024-09-27	4.83	2.432541	4.1	0.2500	0.432541	311.848000	2.432541	1.867123
2024-09-28	4.83	2.432541	4.1	0.2500	0.432541	311.848000	2.432541	1.867123
2024-09-29	4.83	2.432541	4.1	0.2500	0.432541	311.848000	2.432541	1.867123
2024-09-30	4.83	2.432541	4.1	0.2500	0.432541	311.848000	2.432541	1.867123
2024-10-01	4.83	2.571403	4.1	0.2500	0.571403	311.848000	2.432541	1.867123

15346 rows × 8 columns

Federal Funds Rate, Inflation, Unemployment, Target, Inflation Gap, Inflation Lag, and Output Gap Lag % are percentage rates.

Output Gap is a numerical value in billions of dollars.

Feature Information:

- **Federal Funds Rate**: The interest rate that banks use when lending to each other overnight. This is determined by the market.
- **Inflation**: The percentage rate at which prices increase.
- **Unemployment**: The ratio of people without jobs to the total labor force.
- **Target**: The Federal Funds Rate that the Federal Reserve wants to set as part of economic and monetary policy.
- **Inflation Gap**: The difference between Inflation and the Inflation Target (the Inflation Target is treated as a constant 2% in this project).
- **Output Gap**: The difference between Real and Potential Gross Domestic Product (GDP). Measures the difference between how many goods and services a country produces each year vs how much it can produce in theory when using all available resources.

- **Inflation Lag**: The 1st lagged values of Inflation.

- **Output Gap Lag %**: The 1st lagged percentage of Output Gap.

In [15]:
```
# Print the shape of the dataframe, showing the number of rows (observations) and columns
print(df.shape)
```

```
(15346, 8)
```

Our dataset has 15,346 observations and 8 columns/features.

Summary Statistics

In [16]:
```
# Compute Summary Statistics
df.describe()
```

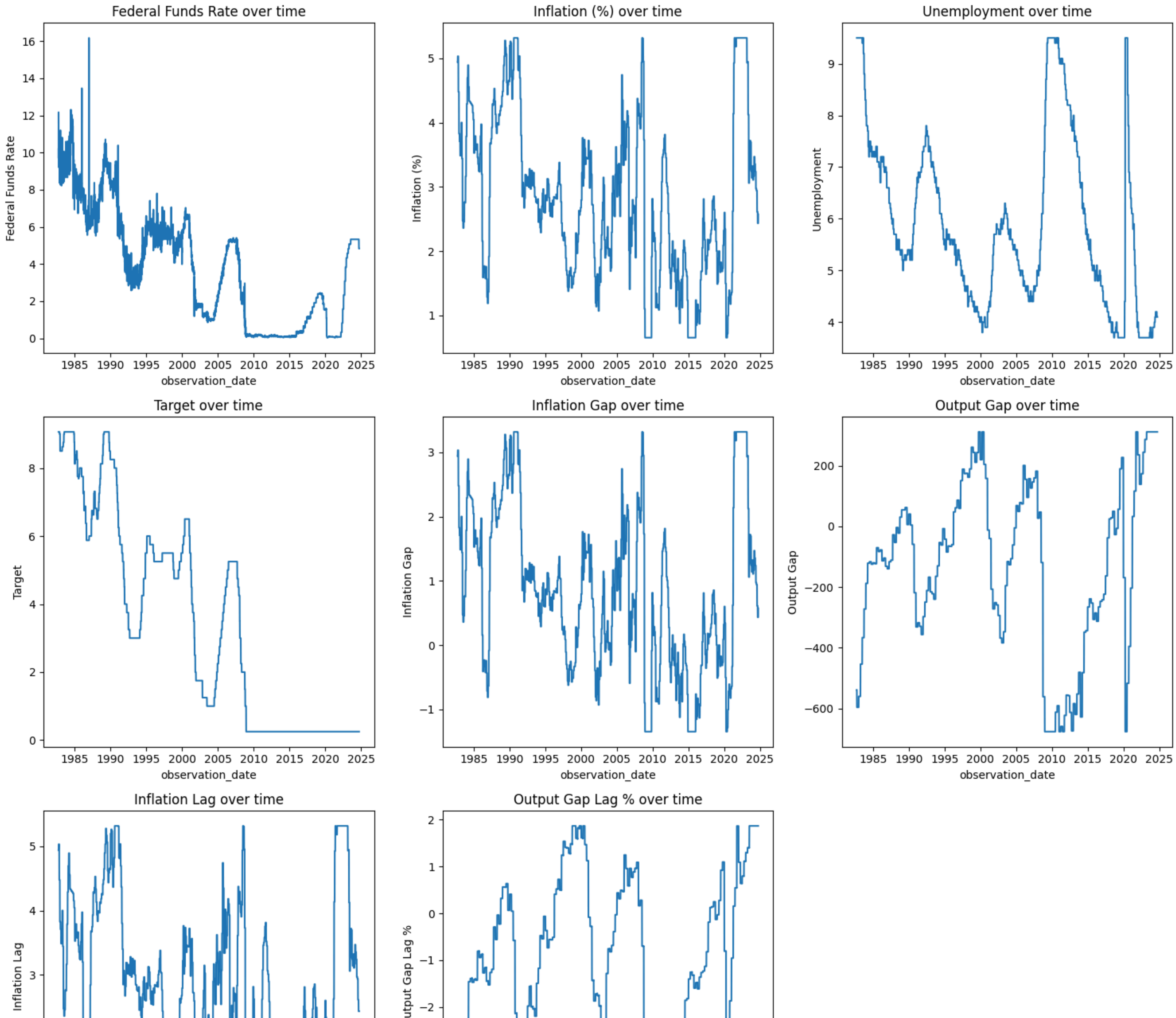
Out[16]:

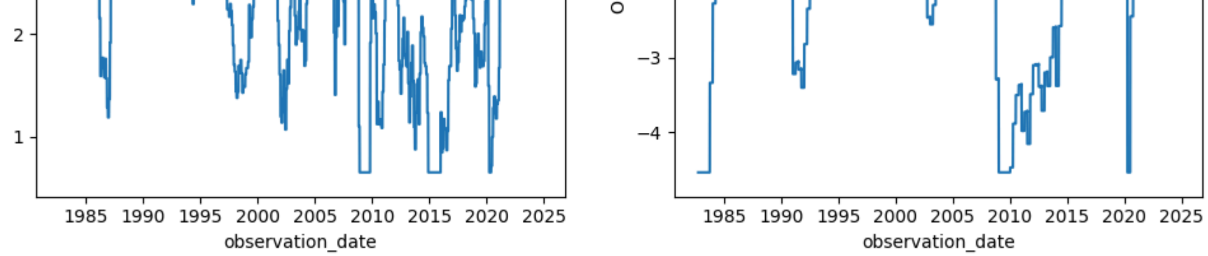
	Federal Funds Rate	Inflation (%)	Unemployment	Target	Inflation Gap	Output Gap	Inflation Lag	Output Gap Lag %
count	15346.000000	15346.000000	15346.000000	15346.000000	15346.000000	15346.000000	15346.000000	15346.000000
mean	3.770183	2.829089	5.912003	3.393910	0.829089	-128.803404	2.829244	-0.919336
std	3.041177	1.265226	1.648995	3.069481	1.265226	279.093430	1.265339	1.809887
min	0.040000	0.653121	3.700000	0.250000	-1.346879	-676.427716	0.653121	-4.532054
25%	0.660000	1.795911	4.600000	0.250000	-0.204089	-297.319722	1.795911	-2.138757
50%	3.770000	2.773597	5.600000	3.000000	0.773597	-105.140969	2.781922	-0.808115
75%	5.740000	3.669725	7.000000	5.750000	1.669725	68.930863	3.669725	0.543627
max	16.170000	5.317419	9.500000	9.062500	3.317419	311.848000	5.317419	1.867123

Plots of Data Over Time

In [17]:
```
# Set subplots
fig, axes = plt.subplots(3, 3, figsize=(15, 15))
axes = axes.flatten()

# Make the line plots
for i, column in enumerate(df.columns):
    sns.lineplot(data=df, x=df.index, y=df[column], ax=axes[i])
    axes[i].set_title(f"{column} over time")
    axes[i].set_ylabel(column)
```

```python
# Hide any unused subplots
for j in range(i + 1, len(axes)):
    axes[j].set_visible(False)

plt.tight_layout()
plt.show()
```


Federal Funds Rate over time
Federal Funds Rate
observation_date
Inflation (%) over time
Inflation (%)
observation_date
Unemployment over time
Unemployment
observation_date
Target over time
Target
observation_date
Inflation Gap over time
Inflation Gap
observation_date
Output Gap over time
Output Gap
observation_date
Inflation Lag over time
Inflation Lag
Output Gap Lag % over time



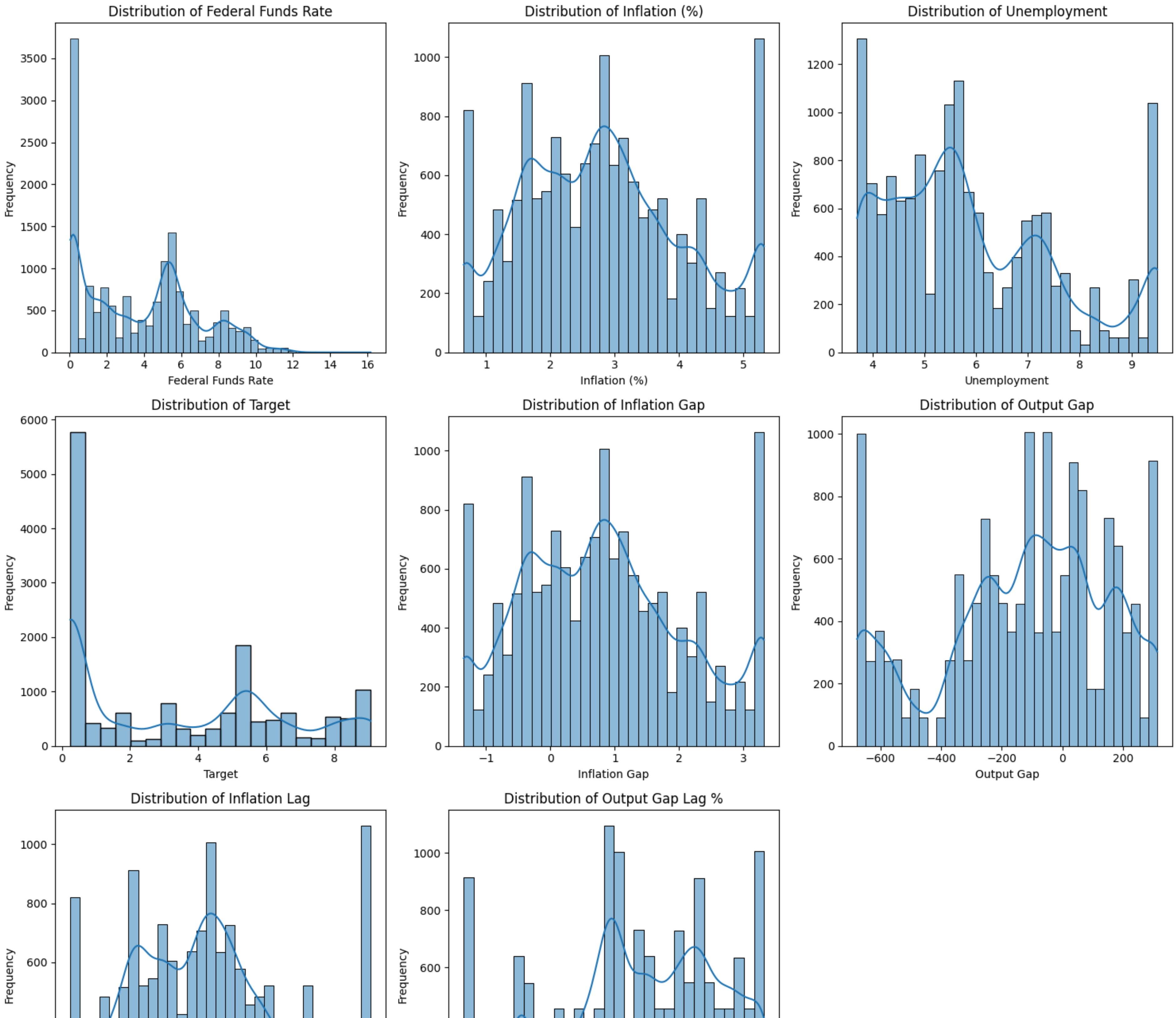
The Federal Funds Rate and Target have been steadily decreasing since the 1980s.

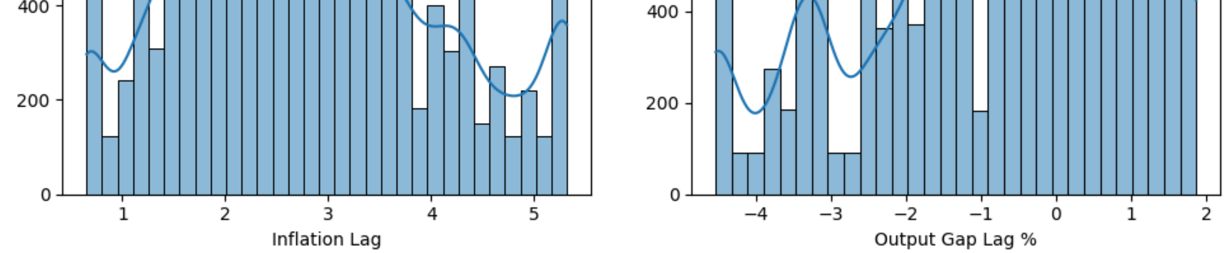
Inflation, Output Gap, and Unemployment have roughly remained around a constant level over time, despite having some sharp rises or drops.

Distribution Plots

In [18]:

```
# Determine the number of columns
num_cols = len(df.columns)
rows = math.ceil(num_cols / 3)  # Adjust to set 3 plots per row (change as needed)

# Create subplots with independent scales
fig, axes = plt.subplots(3, 3, figsize=(15, 15))

# Flatten axes array for iteration
axes = axes.flatten()

for i, column in enumerate(df.columns):
    sns.histplot(df[column], kde=True, ax=axes[i])
    axes[i].set_title(f"Distribution of {column}")
    axes[i].set_xlabel(column)
    axes[i].set_ylabel("Frequency")

# Hide any unused subplots (if num_cols isn't a perfect multiple of 3)
for j in range(i + 1, len(axes)):
    axes[j].set_visible(False)

plt.tight_layout()  # Adjust spacing
plt.show()
```


Distribution of Federal Funds Rate
Frequency
Federal Funds Rate
Distribution of Inflation (%)
Frequency
Inflation (%)
Distribution of Unemployment
Frequency
Unemployment
Distribution of Target
Frequency
Target
Distribution of Inflation Gap
Frequency
Inflation Gap
Distribution of Output Gap
Frequency
Output Gap
Distribution of Inflation Lag
Frequency
Distribution of Output Gap Lag %
Frequency

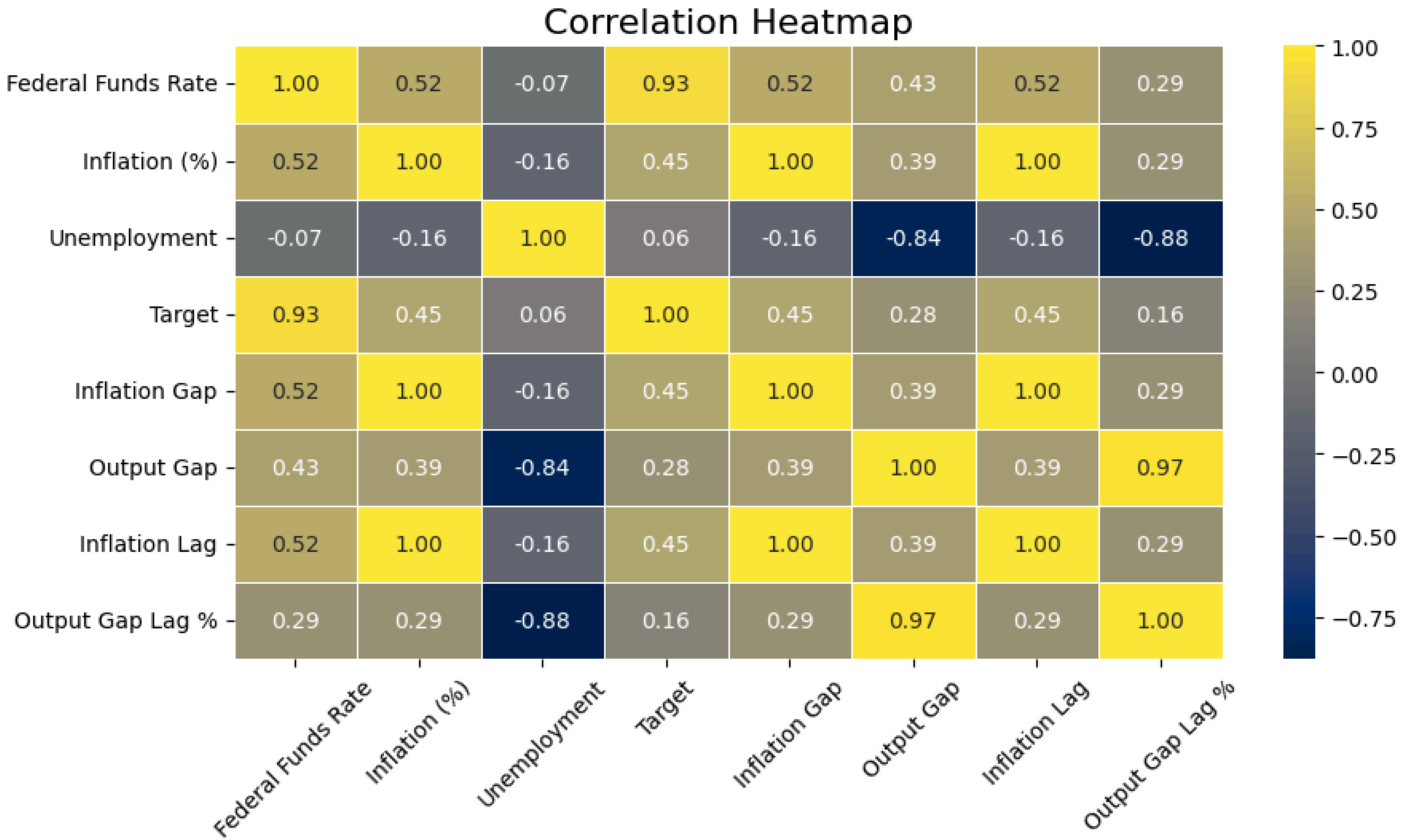


None of the distributions are normal. Some are skewed and others are multimodal.

Correlation Heatmap

In [19]:

```
# Construct a Correlation Heatmap
plt.figure(figsize=(10, 5))
sns.heatmap(
    df.corr(),
    annot=True,
    cmap="cividis",
    fmt=".2f",
    linewidths=0.5,
)
plt.title("Correlation Heatmap", fontsize=16)
plt.xticks(rotation=45)
plt.show()
```



Inflation, Inflation Gap, and Inflation Lag are perfectly correlated to each other, while Output Gap and Output Gap Lag (%) are very strongly correlated, which is expected. We will not be trying any models that use more than 1 in each set as predictors at a time.

Unemployment and Output Gap are strongly correlated, so we will need to watch out for these when checking the Variance Inflation Factors (VIFs) for multicollinearity issues.

Target is highly correlated with our dependent variable, Federal Funds Rate, and we expect it to have the biggest impact on predictive performance of the model. Other features generally have between 40% and 50% correlation with the dependent variable, which is moderate. Unemployment has almost 0 correlation to the dependent variable which could mean low impact on predictive performance or a non-linear relationship.

OLS Modeling

Model Features

The variables used for each regression Model are listed as follows:

Taylor's Rule Model:

```
Dependent variable: Federal Funds Rate.
Independent variables: Output Gap, Inflation Gap
```

Karakas Model:

```
Dependent variable: Federal Funds Rate.
Independent variables: Output Gap Lag %, Inflation Lag (%)
```

Target Model:

```
Dependent variable: Federal Funds Rate.
Independent variables: Output Gap, Inflation Gap, Target
```

Unemployment Model:

```
Dependent variable: Federal Funds Rate.
Independent variables: Output Gap, Inflation Gap, Unemployment
```

Both Model (uses both Unemployment and Target):

```
Dependent variable: Federal Funds Rate.
Independent variables: Output Gap, Inflation Gap, Target, Unemployment
```

Data Scaling

In [20]:
```
# Apply MinMaxScaler to independent variables
scaler = MinMaxScaler()
df_vars = ["Unemployment", "Target", "Inflation Gap", "Output Gap", "Output Gap Lag %", "Inflation Lag"]
df[df_vars] = scaler.fit_transform(df[df_vars])
```

The data should be scaled since we have numerical values of varying magnitudes. MinMaxScaler is chosen since it doesn't make the assumption that the distribution is normal, which StandardScaler does. RobustScaler is not used because we handled outliers already.

Fitting the Models

We will be using Ordinary Least Squares (OLS) to fit the models.

In [21]:

```
# Set dependent variable
y = df["Federal Funds Rate"]

# Define independent variables for each model
model_features = {
    "Taylor": df[["Output Gap", "Inflation Gap"]],
    "Karakas": df[["Output Gap Lag %", "Inflation Lag"]],
    "Target": df[["Output Gap", "Inflation Gap", "Target"]],
    "Unemployment": df[["Output Gap", "Inflation Gap", "Unemployment"]],
    "Both": df[["Output Gap", "Inflation Gap", "Target", "Unemployment"]],
}

# Initialize dictionaries to store results
ols_fitted_models = {}
ols_predictions = {}
ols_vif_results = {}
ols_error_metrics = {}

# Loop to fit, calculate VIFs, Calculate error metrics, and extract model stats for all models
for model_name, X in model_features.items():
    X = sm.add_constant(X)

    # Fit the model
    model, y_pred = fit_ols_model(X, y, model_name)
    ols_fitted_models[model_name] = model
    ols_predictions[model_name] = y_pred

    # Calculate VIF
    vif_data = calculate_vif(X, model_name)
    ols_vif_results[model_name] = vif_data

    # Calculate error metrics
    metrics = error_metrics(y, y_pred)
    ols_error_metrics[model_name] = metrics
```

OLS Regression Assumptions

In [22]:

```
# Dictionary to store assumption test results
ols_assumption_tests = {}
ols_residuals = {}

# Iterate through models
for model_name, model in ols_fitted_models.items():
    residuals = model.resid
    X = model.model.exog

    # Normality Tests (Jarque Bera)
    jb_test = jarque_bera(residuals)

    # Homoscedasticity Test (Breusch-Pagan)
    bp_test = het_breuschpagan(residuals, X)

    # Independence Test (Durbin-Watson)
    dw_stat = sm.stats.durbin_watson(residuals)

    # Linearity Tests (Rainbow)
    rainbow = linear_rainbow(model)

    # Store results
    ols_assumption_tests[model_name] = {
        "Durbin-Watson Test Statistic": f"{dw_stat:.4f}",
        "Jarque-Bera p-value": f"{jb_test[1]:.4f}",
        "Breusch-Pagan p-value": f"{bp_test[1]:.4f}",
        "Rainbow Test p-value": f"{rainbow[1]:.4f}"
    }

# Print assumption test statistics for all models
ols_assumption_tests_df = pd.DataFrame(ols_assumption_tests).T.round(4)
ols_assumption_tests_df
```

Out[22]:

	Durbin-Watson Test Statistic	Jarque-Bera p-value	Breusch-Pagan p-value	Rainbow Test p-value
Taylor	0.0071	0.0000	0.0000	0.0000
Karakas	0.0069	0.0000	0.0000	0.0000
Target	0.0445	0.0000	0.0000	0.0000
Unemployment	0.0109	0.0000	0.0000	0.0000
Both	0.0459	0.0000	0.0000	0.0000

The assumptions of regression include:

1. Normality of residuals
2. Homoscedasticity (constant variance of residuals)
3. Independence (autocorrelation)
4. Linearity
5. No Multicollinearity

These assumptions can be tested using the Jarque-Bera test (Normality), Breusch-Pagan test(Homoscedasticity), Durbin-Watson test (Independence), and Rainbow test (Linearity). We perform a simple hypothesis test for these where the hypotheses are as follows:

Null Hypothesis, H0: The model does not violate the regression assumption

Alternative Hypothesis, H1: The model does violate the regression assumption

Using the 95% confidence level (significance level 0.05), the Jarque-Bera, Breusch-Pagan, and Rainbow test statistics all have p-values of around 0, which is less than the significance level. Therefore, we would reject the null hypothesis, H0, in favor of the alternative and conclude that each model violates the assumptions for these tests.

For the Durbin-Watson test, any test statistics less than 1 or greater than 3 indicate strong autocorrelation and would violate the regression assumption of independence. Since the Durbin Watson test statistics for every model lie between 0 and 0.5, we would reject the null hypothesis and conclude the assumption of independence has been violated for each model.

All models violate the first 4 regression Assumptions. When regression assumptions are violated any metrics become biased and unreliable.

In [23]:

```
# Combine VIF results into a single DataFrame
all_vifs = []
```

```python
for model_name, vif_df in ols_vif_results.items():
    vif_df = vif_df.copy()
    vif_df["Model"] = model_name
    all_vifs.append(vif_df)

# Concatenate all into one DataFrame and round
ols_vif_results_df = pd.concat(all_vifs, ignore_index=True)
ols_vif_results_df = ols_vif_results_df.round(4)

# Pivot table
ols_vif_results_df = ols_vif_results_df.pivot(index="Feature", columns="Model", values="VIF").round(4)

# Rearrange columns
ols_vif_results_df = ols_vif_results_df[["Taylor", "Karakas", "Target", "Unemployment", "Both"]]

# Print table of VIFs for all models
ols_vif_results_df
```

Out[23]:

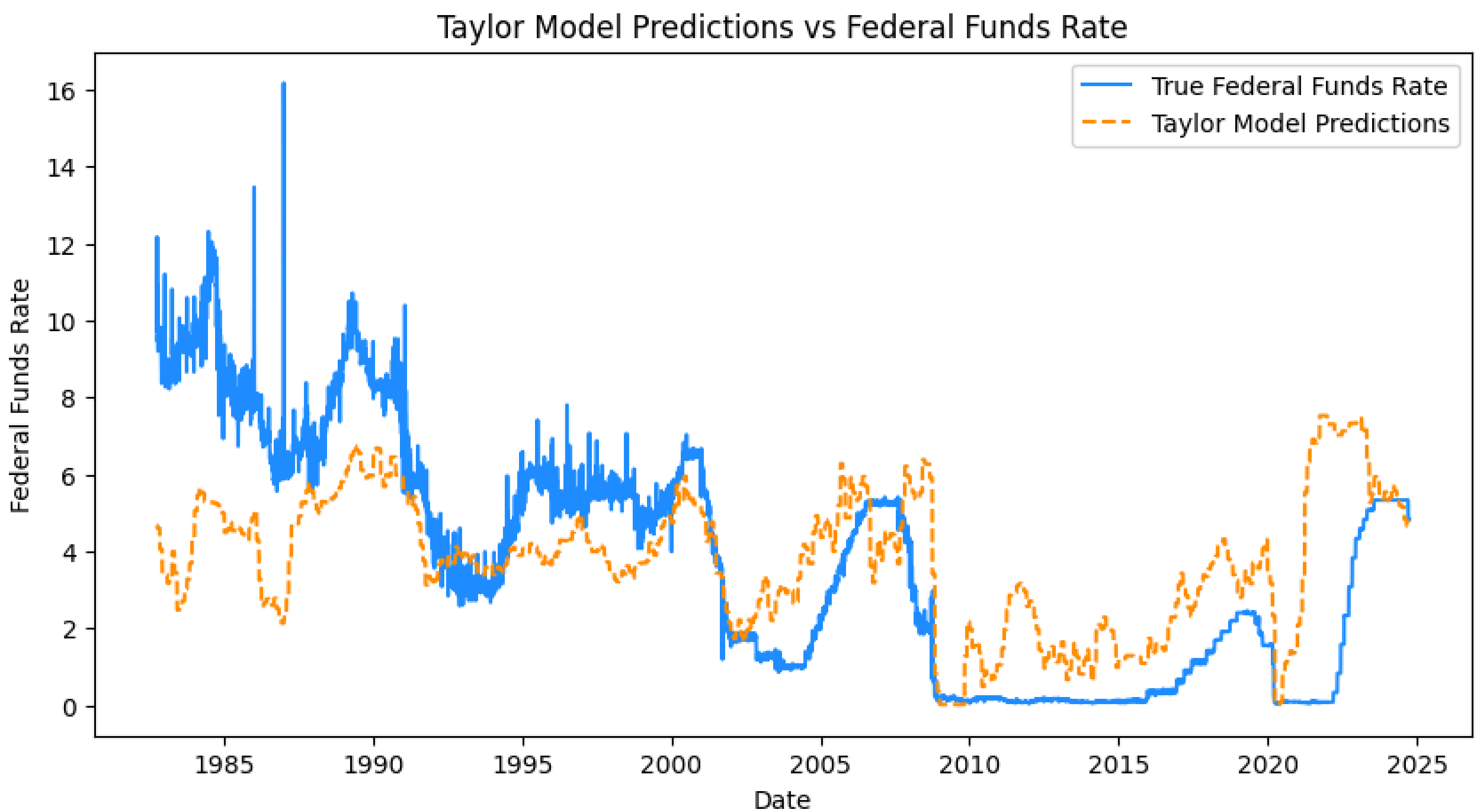
Model	Taylor	Karakas	Target	Unemployment	Both
Feature					
Inflation Gap	1.1778	NaN	1.3848	1.3082	1.4085
Inflation Lag	NaN	1.0887	NaN	NaN	NaN
Output Gap	1.1778	NaN	1.1964	4.2165	5.5812
Output Gap Lag %	NaN	1.0887	NaN	NaN	NaN
Target	NaN	NaN	1.2739	NaN	1.7020
Unemployment	NaN	NaN	NaN	3.6800	4.9166
const	5.9301	6.4245	5.9413	34.7962	43.7499

Variance Inflation Factors (VIFs) are used to test for multicollinearity. Any null values here are due to the feature not being included in the model. The threshold is that VIFs below 5 have no issues. From the VIFs, we do not have serious problems with multicollinearity except for the Output Gap in the Both Model. The best practice would be to remove that from the model entirely, but since all other regression assumptions have been violated we will leave it and run the regression anyway. Unemployment, the other variable of interest from our correlation matrix analysis, has acceptable multicollinearity levels.

Model Comparison of Taylor and Karakas Models to Karakas' Work

While all the regression assumptions except for Multicollinearity were violated and the results are unreliable, we can still compare our visualizations and metrics to Karakas's results and check if we reproduced their regression models, which is one objective of this project.

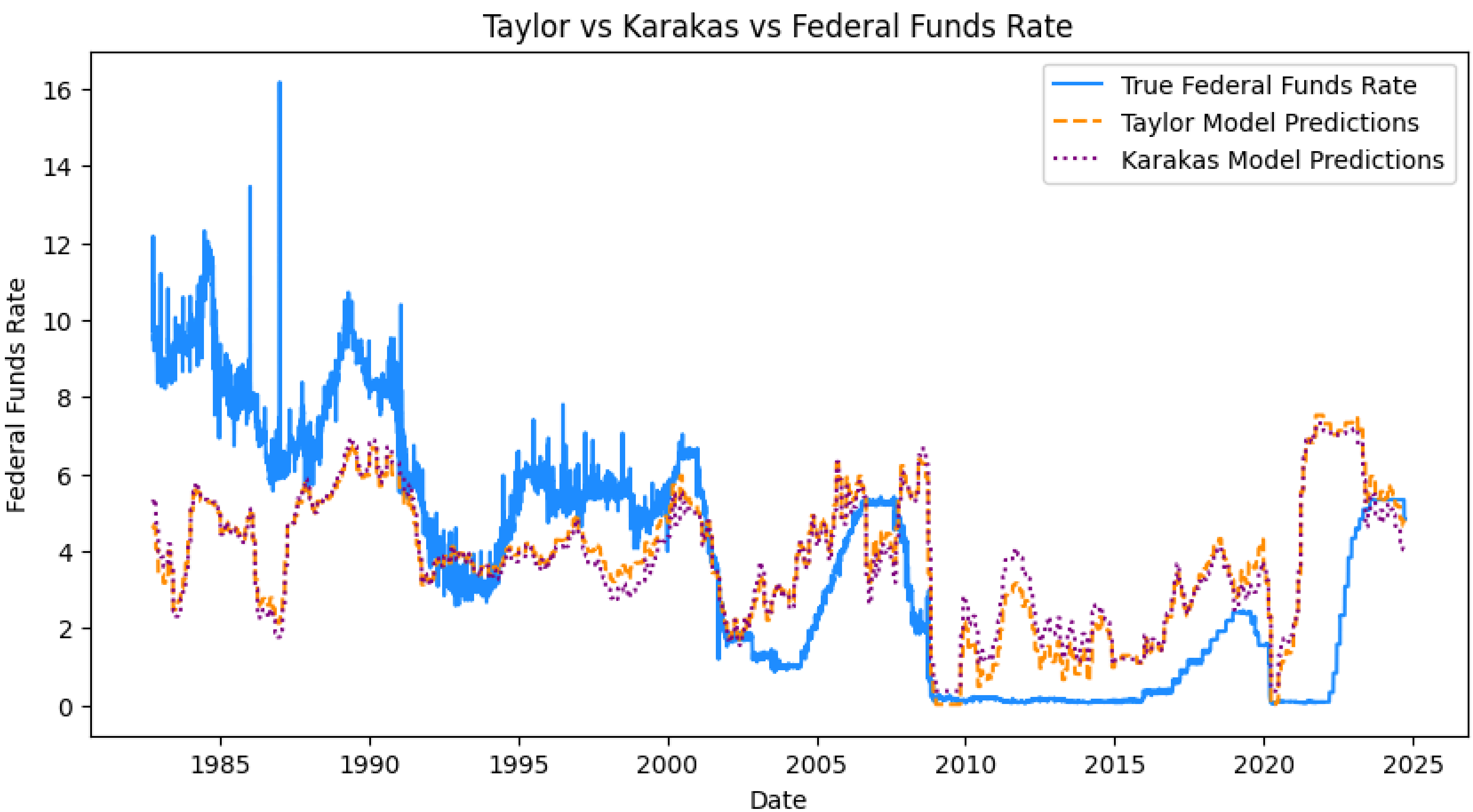
In [24]:
```
# Taylor Predictions vs Actual Plot
plt.figure(figsize=(10, 5))
plt.plot(df.index, y, label="True Federal Funds Rate", color="dodgerblue")
plt.plot(df.index, ols_predictions["Taylor"], label="Taylor Model Predictions", color="darkorange", linestyle="--")
plt.title("Taylor Model Predictions vs Federal Funds Rate")
plt.xlabel("Date")
plt.ylabel("Federal Funds Rate")
plt.legend()
plt.show()
```



Consistent with Karakas' plots, we see our Taylor Model underpredicting in year 1990, briefly overpredicting between 1990 and 1995, before underpredicting until a bit past 2000. Around the year 2002, the model begins overpredicting for the rest of the years, except between 2005 and 2010 where it underpredicts where the actual values form a peak and around 2008 or 2009. The plots may not be an exact match, but they look similar enough to the ones shared in Karakas' paper.

In [25]:

```
# Taylor Predictions vs Karakas Predictions vs Actual Plot
plt.figure(figsize=(10, 5))
plt.plot(df.index, y, label="True Federal Funds Rate", color="dodgerblue")
plt.plot(df.index, ols_predictions["Taylor"], label="Taylor Model Predictions", color="darkorange", linestyle="--")
plt.plot(df.index, ols_predictions["Karakas"], label="Karakas Model Predictions", color="purple", linestyle=":")
plt.title("Taylor vs Karakas vs Federal Funds Rate")
plt.xlabel("Date")
plt.ylabel("Federal Funds Rate")
plt.legend()
plt.show()
```



We capture similar patterns such as the Karakas Model underpredicting more than Taylor Model between 1997 to 2001 and mostly between 2010 and 2015, where there are 2 cross overs as they switch between overpredicting or underpredicting each other.

In [26]:

```
taylor_and_karakas = ["Taylor", "Karakas"]
for model in taylor_and_karakas:
    rs = find_resid_sum(y, ols_predictions[model])
```

```
    sae = find_sae(y, ols_predictions[model])
    print(f"{model}: ")
    print(f"\tSum of Residuals: {rs}")
    print(f"\tSum of Absolute Errors: {sae:.4f}")
```

```
Taylor:
        Sum of Residuals: -1.8189894035458565e-11
        Sum of Absolute Errors: 29358.7040
Karakas:
        Sum of Residuals: 1.4551915228366852e-11
        Sum of Absolute Errors: 31029.3788
```

We got much smaller Sum of Residuals than Karakas did and obtained larger Sum of Absolute Errors for both models. Additionally, we received lower values for each in the Taylor Model than the Karakas Model.

Both metrics for each model are very close to each other, which implies that there isn't much of a difference in performance between the two models and is consistent with Karakas' findings.

OLS Model Metrics

In [27]:
```
ols_error_metrics_df = pd.DataFrame(ols_error_metrics).T.round(4)
ols_error_metrics_df
```

Out[27]:

	Mean Squared Error	Root Mean Squared Error	Mean Absolute Error	Mean Percentage Error	Mean Absolute Percentage Error	R-Squared
Taylor	6.217	2.493	1.913	-473.933	504.281	0.3277
Karakas	6.574	2.564	2.022	-550.486	579.147	0.2892
Target	0.884	0.940	0.605	-78.489	117.299	0.9044
Unemployment	4.573	2.138	1.669	-416.232	502.970	0.5056
Both	0.864	0.930	0.600	-85.274	119.014	0.9066

The error metrics show the Taylor Model having better performance over the Karakas Model. Karakas (2023) claimed that their model had more accurate predictions, but not by much.

We, on the other hand, found that the Taylor model has better predictions. Note that we have a different date range than Karakas, and this could mean that Karakas' model better predicts older data but performs poorly on more recent data. Overall, their model does not perform as well as the Taylor Model, having larger average errors and percentage errors. The difference, however, is not very big, which is consistent with Karakas' findings.

All models have negative Mean Percentage Errors, and thus they all underpredict the Federal Funds Rate. The percentage error metrics seem to be very high. This is likely because due to our data having small values. Average error metrics would be better for analysis.

In terms of performance, the models from worst to best are:

Karakas < Taylor < Unemployment < Target < Both

Models with lower error metrics and higher R-squared are considered better performing than others.

Adding Target or Unemployment alone to the Taylor Model increases performance, but there are marginal differences in performance when adding Unemployment alongside the Target. This suggests that Unemployment contributes little to the model, consistent with the findings from the correlation heatmap. Note that metrics are likely biased and unreliable since regression assumptions have been violated.

In [28]:

```python
# Initialize a list to store model statistics
model_statistics = []

# Loop through the fitted models to extract key statistics
for model_name, model in ols_fitted_models.items():
    # Extract key values
    results = {
        "model": model_name,
        "adj_r_squared": round(model.rsquared_adj, 3),
        "aic": round(model.aic, 3),
        "bic": round(model.bic, 3),
        "f_stat": round(model.fvalue, 3),
        "f_p_value": round(model.f_pvalue, 3),
        "t_p_values": model.pvalues.round(3).tolist(),
    }
    # Append the results for each model
    model_statistics.append(results)

# Convert results to a DataFrame
statistics = pd.DataFrame(model_statistics)
statistics
```

Out[28]:

	model	adj_r_squared	aic	bic	f_stat	f_p_value	t_p_values
0	Taylor	0.328	71598.369	71621.285	3739.751	0.0	[0.728, 0.0, 0.0]
1	Karakas	0.289	72453.922	72476.837	3120.973	0.0	[0.0, 0.0, 0.0]
2	Target	0.904	41668.118	41698.673	48379.575	0.0	[0.0, 0.0, 0.0, 0.0]
3	Unemployment	0.505	66885.511	66916.065	5228.964	0.0	[0.0, 0.0, 0.0, 0.0]
4	Both	0.907	41308.066	41346.259	37240.048	0.0	[0.0, 0.0, 0.0, 0.0, 0.0]

A model with lower values for AIC and BIC is better than a model with higher values for them. The AIC and BIC show a similar pattern to our findings from the error metrics and R-squared, with the models listed from worst performing to best performing being:

Karakas < Taylor < Unemployment < Target < Both.

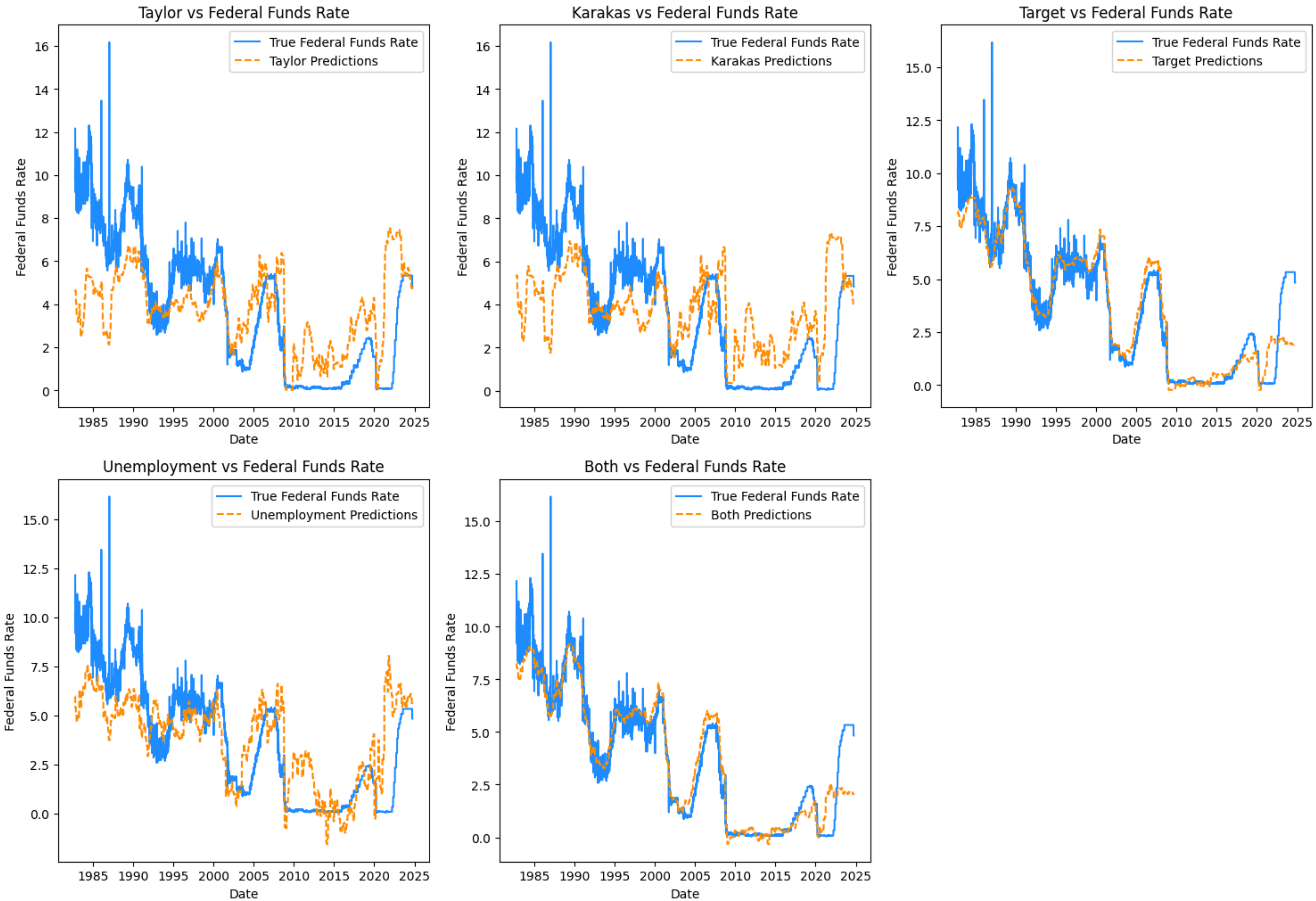
The p-values for the f-statistic and t-statistic suggests the models and coefficients are almost all statistically significant from a simple hypothesis test where the null hypothesis, H0, is that the models/coefficients are not significant. The exception is the constant from the Taylor Model which would not be statistically significant.

Note that metrics are likely biased and unreliable since regression assumptions have been violated.

Plots of All OLS Models

In [29]:

```
# Set subplots and flatten
fig, axes = plt.subplots(rows, 3, figsize=(15, 15))
axes = axes.flatten()

# Plot the neural network model predictions
for i, (model_name, y_pred) in enumerate(ols_predictions.items()):
    axes[i].plot(df.index, y, label="True Federal Funds Rate", color="dodgerblue")
    axes[i].plot(df.index, y_pred, label=f"{model_name} Predictions", color="darkorange", linestyle="--")
    axes[i].set_title(f"{model_name} vs Federal Funds Rate")
    axes[i].set_xlabel("Date")
    axes[i].set_ylabel("Federal Funds Rate")
    axes[i].legend()

# Hide any unused subplots
for j in range(i + 1, len(axes)):
    axes[j].set_visible(False)

plt.tight_layout()  # Avoid overlapping
plt.show()
```



The plot of predictions remains consistent with our findings from the error metrics. Adding Target and Unemployment improve model predictions, but they tend to more closely follow earlier years of the data.

Neural Network Modeling

Fitting the Models

In [30]:
```
# Initialize dictionaries to store results
nn_fitted_models = {}
nn_predictions = {}
nn_error_metrics = {}
nn_history = {}
# Loop to fit, calculate VIFs, Calculate error metrics, and extract model stats for all models
for model_name, X in model_features.items():
    # Fit the model
    model, y_pred, history = fit_nn_model(X, y, model_name)

    nn_fitted_models[model_name] = model

    # Obtain predictions
    nn_predictions[model_name] = y_pred

    # Find model metrics
    metrics = error_metrics(y, y_pred)
    nn_error_metrics[model_name] = metrics

    # Save model training history
    nn_history = history
```

```
480/480 ━━━━━━━━━━━━━━━━━━━━ 0s 499us/step
480/480 ━━━━━━━━━━━━━━━━━━━━ 0s 533us/step
480/480 ━━━━━━━━━━━━━━━━━━━━ 0s 518us/step
480/480 ━━━━━━━━━━━━━━━━━━━━ 0s 489us/step
480/480 ━━━━━━━━━━━━━━━━━━━━ 0s 501us/step
```

Neural Network Model Metrics

In [31]:
```
nn_error_metrics_df = pd.DataFrame(nn_error_metrics).T.round(4)
nn_error_metrics_df
```

Out[31]:

	Mean Squared Error	Root Mean Squared Error	Mean Absolute Error	Mean Percentage Error	Mean Absolute Percentage Error	R-Squared
Taylor	6.978	2.642	2.191	-635.237	661.376	0.2455
Karakas	7.104	2.665	2.168	-693.999	715.020	0.2319
Target	1.011	1.005	0.613	-72.375	129.459	0.8907
Unemployment	6.426	2.535	2.166	-696.731	720.984	0.3051
Both	1.063	1.031	0.657	-118.800	156.094	0.8851

Our models have returned worse results for each metric compared to the OLS models. The regression assumptions being violated may have inflated the OLS model metrics. Our model performance ranking differs slightly here.

We now have from worst performing to best performing:

Karakas < Taylor < Unemployment < Both < Target.

Like with the OLS models, we find that there is little difference in performance between the Karakas and Taylor Models.

All models have negative Mean Percentage Errors, meaning that all of the models underpredict the Federal Funds Rate.

The inclusion of both the Target and Unemployment into the Taylor Model raised the R-squared from around 0.24 to around 0.89, which is very strong performance for a model using real economic data. However, there is not much of a difference between the performance of the Target Model and the Both Model in average errors or R-squared. On the other hand, the MPE and MAPE have differences of around 27% to 40%. These values are likely high due to our data having small values. Average error metrics would be better for analysis.

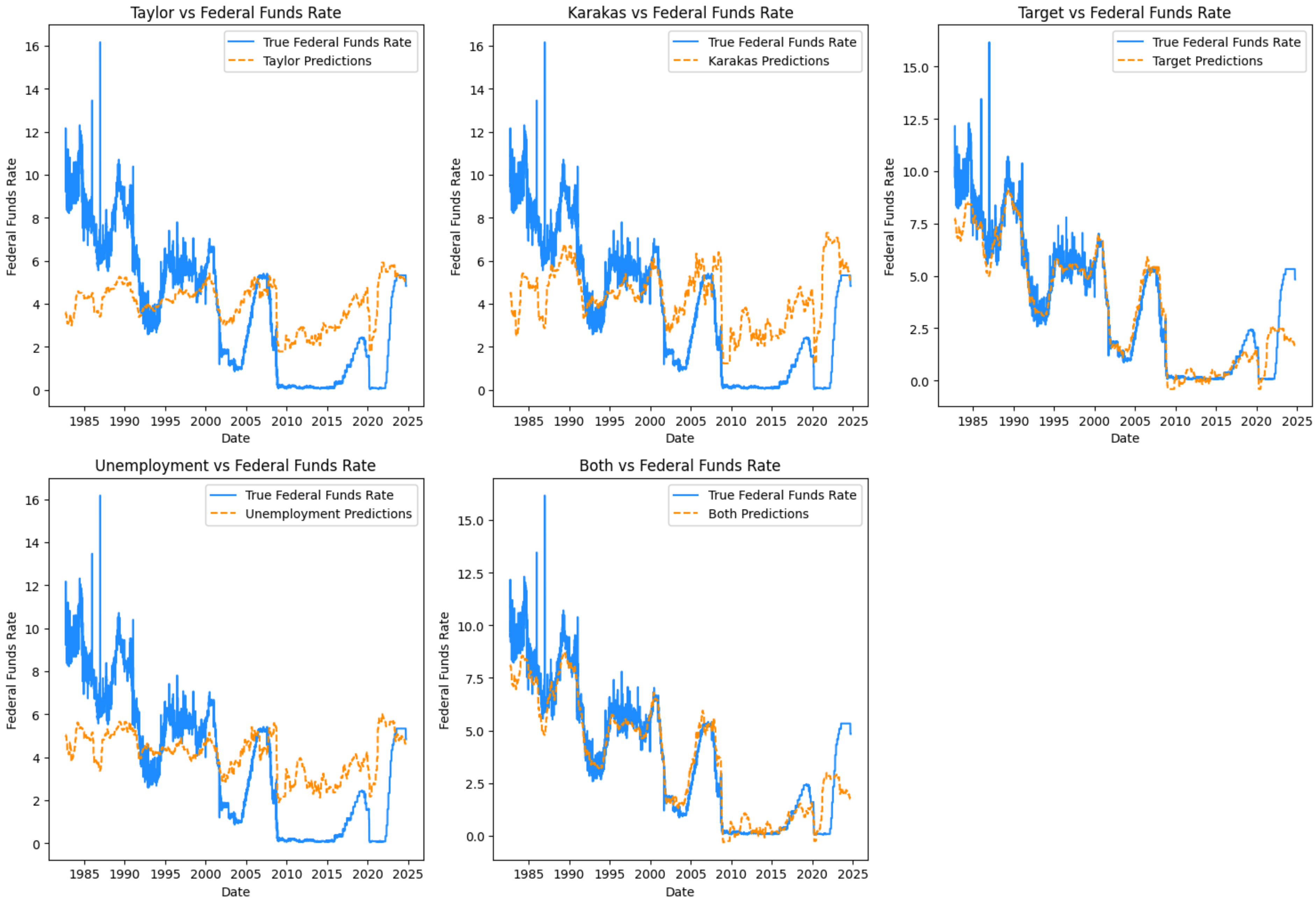
Adding Unemployment to the Taylor Model alone reduces errors and explains more variance but increases percentage errors. Unemployment contributes little when added with the Target.

Neural Network Prediction Plots

In [32]:

```
# Set subplots and flatten
fig, axes = plt.subplots(rows, 3, figsize=(15, 15))
axes = axes.flatten()

# Plot the neural network model predictions
for i, (model_name, y_pred) in enumerate(nn_predictions.items()):
    axes[i].plot(df.index, y, label="True Federal Funds Rate", color="dodgerblue")
    axes[i].plot(df.index, y_pred, label=f"{model_name} Predictions", color="darkorange", linestyle="--")
    axes[i].set_title(f"{model_name} vs Federal Funds Rate")
```

```
    axes[i].set_xlabel("Date")
    axes[i].set_ylabel("Federal Funds Rate")
    axes[i].legend()

# Hide any unused subplots
for j in range(i + 1, len(axes)):
    axes[j].set_visible(False)

plt.tight_layout()  # Avoid overlapping
plt.show()
```



Each model tends to underpredict the Federal Funds Rate in the first half of the date range (around 1980 to 2000). Afterwards, the models tend to overpredict.

The Taylor, Karakas, and Unemployment Models tend to predict Federal Funds Rates of around 4. The Target and Both Models follow the true Federal Funds Rates more closely, having smaller gaps between them and their predictions.

Conclusion

Although the results may differ due to different date ranges in our data, our models captured similar predictive patterns to those presented in Karakas' paper from our examination of the plots as well as the Sum of Residuals and Sum of Absolute Errors.

Karakas (2023) noted that the Taylor Model (and their transformation of it) did not predict the Federal Funds Rate well and mentioned their model having predictions closer to the actual Federal Funds Rate than the Taylor Model, but not by much. However, our models' metrics show that it is the Taylor Model, rather than Karakas' Model that has smaller average errors and percentage errors. We suspect this may be due to how the data we use contains more recent data. Also, our plot of the Taylor Model predictions does not have any predictions below 0 unlike Karakas' plot. This may be because Karakas did not handle outliers well or at all.

We noticed that Karakas did not mention regression assumptions in their paper when discussing their OLS models, so we decided to check them as part of the validation process. We found that almost all classical regression assumptions were violated, with the exception being multicollinearity.

Karakas later created neural network models for using Taylor's Rule, getting better predictions than their OLS models. From our findings, the OLS models had inflated metrics due to regression assumption violations. Our neural network models, on the other hand, had lower performance metrics than our OLS models. Since Karakas got better results from their neural network model than their OLS model which should also have inflated metrics, their neural network is likely to have issues with overfitting given that they did not mention usage of techniques such as regularization, dropout, or early stopping like we had for our models.

This casts serious doubts about the credibility, rigor, and professionalism of their work.

We believe that non-linear models are better suited for predicting the Federal Funds Rate given that no feature we used had a linear relationship with the Federal Funds Rate.

It is worth noting that the inclusion of Target and Unemployment to the Taylor Model, individually, did improve performance metrics in both the OLS and Neural Network Models. However, the Target has a much larger effect on model performance than Unemployment and Unemployment has little effect on performance when it is alongside the Target.

While performance metrics look strong for the Target Model, there is room for improvement. Some next steps would be to explore interaction terms with Unemployment or taking lags of Unemployment to avoid multicollinearity issues with Output Gap. The addition of other features such as Global Economic Growth, Prices, Interest rates, Foreign Currency Exchange Rates, Consumer Confidence, and Business Confidence are also worth exploring.

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Data obtained from:

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Karakas' Paper:

Karakas, A. D. (2023). Reevaluating the Taylor Rule with Machine Learning. ArXiv.org. https://arxiv.org/abs/2302.08323