Optimizing Organ matching markets

Venkata Praveen Naradasu,Sneha Dasgupta,Minsung Kang,Prathyush Reddy Raghava Reddy
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Abstract

We studied the possibility of barter systems in organ transplantation by centralizing the exchange mechanism using a mixed integer programme approach.Kidney exchange is special instance that is studied here.We used synthetic data to simulate and showcase the complexity of mapping the right matches (Maximizing the number of matches) and also maximize the total compatibility weightage of the matched pairs to improve life expectancy after matching.

Introduction

One of the most important and lingering problems in healthcare has been the shortage of organs for transplants. For instance about 121,678 people are awaiting lifesaving organ transplants in the US alone, of which around 100,791 comprises just for kidney transplants as per the national kidney foundation. Monetary markets for organs are illegal across the world in spite of a significant shortage of organs. Iran is a rare exception where it is legalized and the government is also involved in this by setting up a registry for buyers and sellers, setting up the prices and facilitating the matchmaking. This does bring into question a lot of ethical issues and thus primarily most of the world sticks to the universal donor list where recipients are ranked and waitlisted according to their health conditions and clinical needs.In countries like the US where kidney transplants go as per a list which is dependent on factors such as health of the patient, compatibility with the donor, and availability of the organs, one can estimate an average wait time of about 3.6 years for a patient's first kidney transplant. To make the process more cumbersome there is an exponential rise in the waiting list every month and every year with around 3000 patients being added each month to the waiting list for kidney organ recipients. A large number of patients die waiting for a kidney transplant or their clinical condition worsens to a point where they are no longer eligible to be an organ recipient for kidney transplant.

In this project we have primarily focused on kidney transplants since out of all life saving organ transplants kidney and liver transplants comprise the highest number of domino transplants taking place. Domino transplants refers to the process where the patient has a willing yet incompatible donor and a compatible altruist donor helps with the match, further the incompatible donor for that particular patient is paired up with the next appropriate patient match from the UNOS (United Network of Organ Sharing) list which sets a chain of patient-donor organ transplant pairs. Altruistic donors are crucial to the success of Kidney paired Donation transplants as they make the donor pool bigger by adding their name just to the recipient list.

The altruistic donor acts as the first step towards these domino transplants followed by the bridge donor which is usually the incompatible family or friend match from the recipient of the first round of transplant. Thus an altruist not only helps start the first round of transplants but also facilitates the next round and rounds after that, in a way helping multiple patients at once by this selfless act of organ donation. Thus, this is an important issue to address since the waiting times keep getting longer and longer with the demand of kidneys being much higher than the amount of donors. Though, only about one third of the organs received are from living donors, a well processed matchmaking process to facilitate domino transplant operations will be a huge benefit to the transplant community.

The modeling approach used in this project was Mixed integer programming. In this type of approach we consider certain variables as integers, these are known as the different constraints, and on the other hand the remaining variables are set as non-integers. We can find various examples where mixed integer programming models have been used to address issues in the healthcare industry. For instance, to ensure efficient use of all resources (especially in case of emergencies) the floor plan for the Intensive Care units were redesigned in a hospital in Spain using this kind of model and technique. We have used this approach to address the problem of domino organ kidney transplants since previous methods such as the Hungarian Algorithm focus on patient success match as an individual while in the case of this problem we would like optimal matching between donors and recipients overall as a group which is easier approached using the Mixed integer technique.

Methodology

A simple example of kidney paired donation is shown below in which there are two pairs of donors,patients which is the smallest number of pairs required to effectively carry out the exchange based on compatibility as a simple ‘Bipartite Graph’.(‘A bipartite graph with a set of vertices decomposed into two disjoint sets and the matches are represented by edges). On the left side, Volunteer donor 1 wishes to donate to Recipient A also Donor 2 to Recipient B but because of the difference various factors (Maybe as simple as blood type as mentioned in figure) they won't be able to make the final match.Here the dotted line represents the intent to donate.Yet if we observe carefully, we can see that the Donor 1 is true match for the Recipient 2 and vice versa hence final matches are explained in right side of figure 2. Thick lines indicate the final matches.

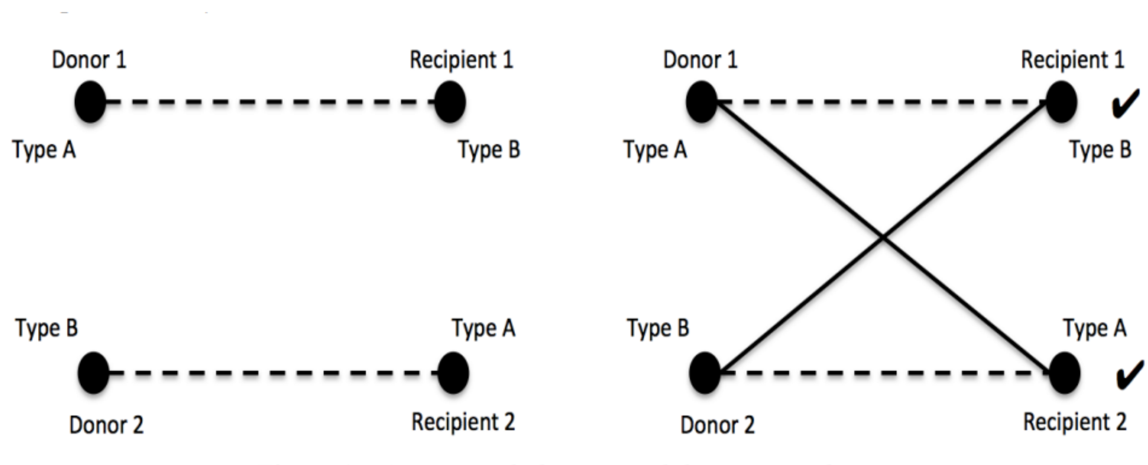

Figure 1: A two-way kidney paired donation exchange

When we increase the thickness of the market by considering 4 pairs with random match case scenarios (i.e edges represent potential matches) as showcased in figure 2.

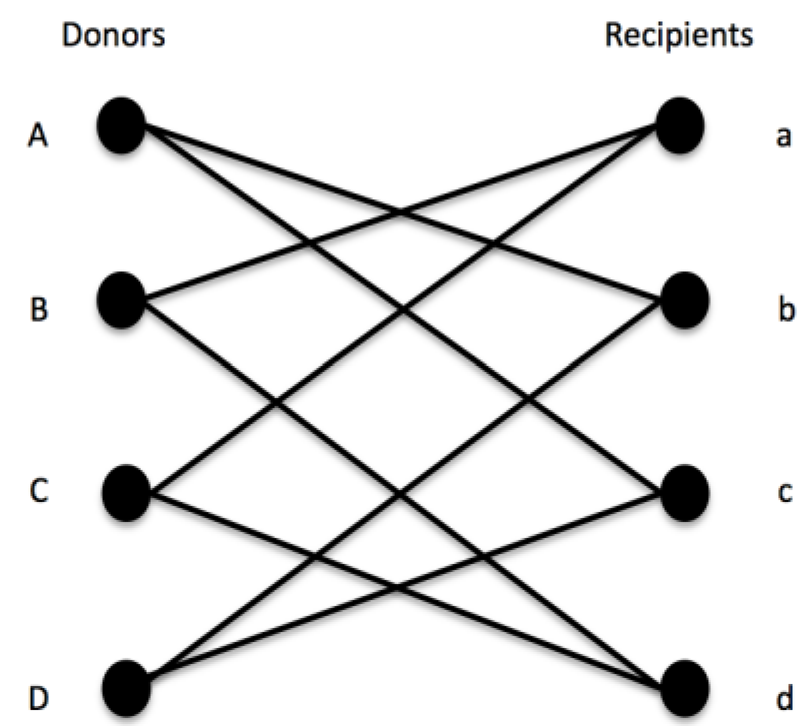

Figure 2. Potential matches in a four-way kidney paired donation exchange

From the above graph, we want to find a subgraph with every vertex in the donor list to be paired with only one vertex in the group of recipients (Constraint used later in modeling). We come to a final matching which is called ‘Perfect matching’. In this scenario we end up with 2 such solutions as explained in figure 3 below.

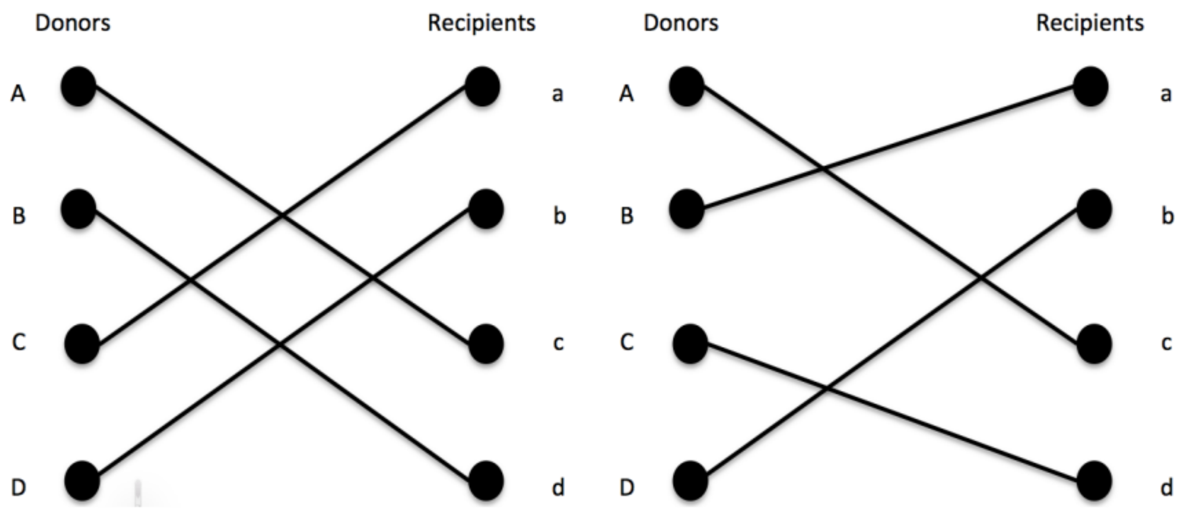

Figure 3. Two possible matchings from the graph in Figure 2

As number of solutions for a bipartite increases, it is important to choose the best fit out of these by looking at maximizing the ‘Good of fit’ to increase the life expectancy of as many recipients as possible and Doctors usually use number of factors (HLA type,age,antigens,Blood types,previous sensitization) to come to a conclusion on the ‘Good of fit’ of all possible Donors and Recipients, a higher weightage of this indicates higher probability for a patient body to accept the organ from that particular donor.

When we look at the above scenario from Figure 2, by adding in the weights of compatibility,we can come to the conclusion of choosing one of the two scenarios.

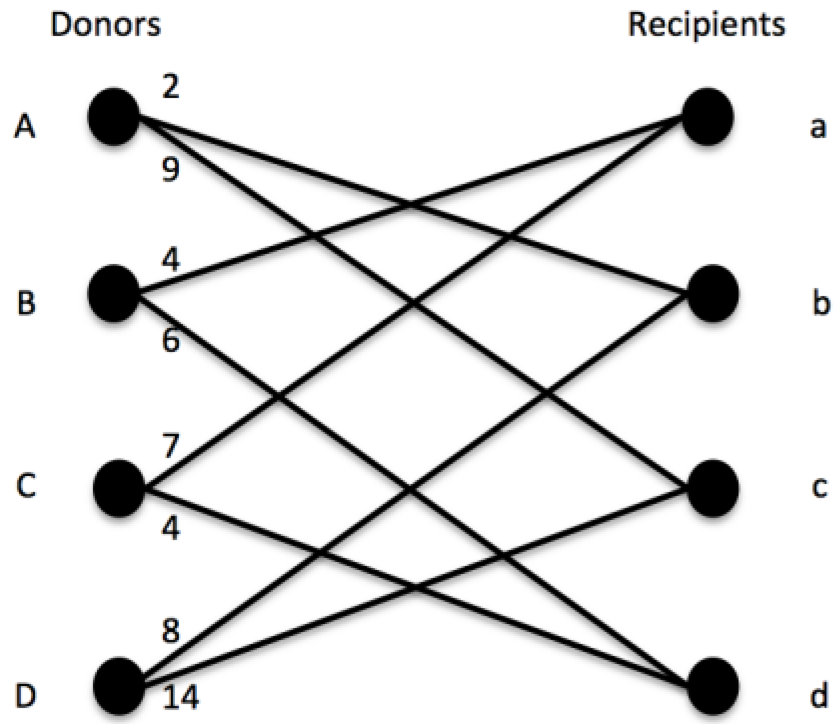

Figure 4. Graph from Figure 2 with weights added

By solving the above Bipartite as above, we have subgraphs with perfect solutions represented below. But when we observe the total weightage in each case, we see that the left side matches set has a total weightage of 30 and the right side scenario has a total match of 25. Hence we prefer going ahead with scenario 1 with an increased life expectancy over all.

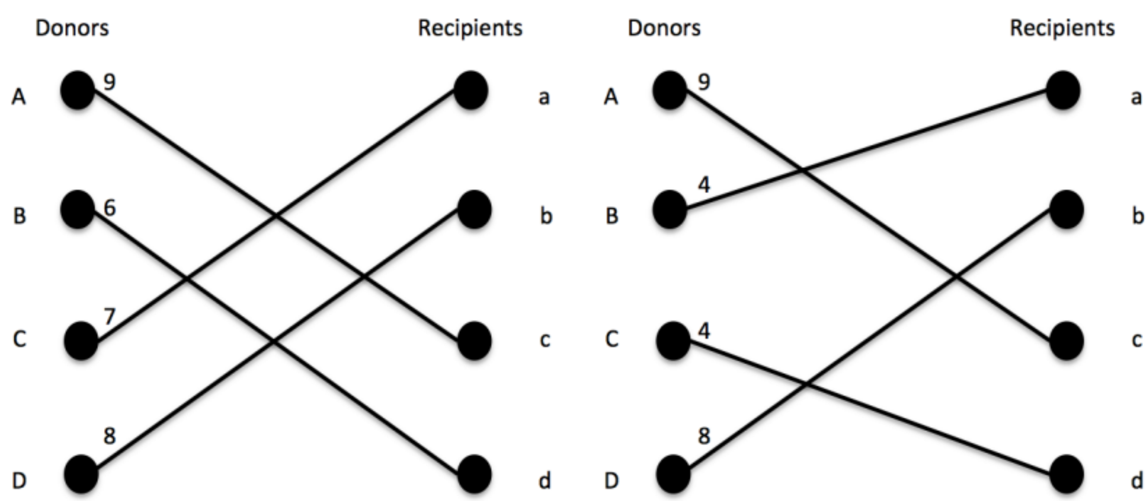

Figure 5. Two potential matchings with weights from the graph in Figure 4

In these scenarios, we also have another agent called ‘Altruist’, who enters the market with no other intention and hence doesn't expect to limit to any constraints (3rd constraint in below model). These altruists make the model a lot easier to solve and increase the possible edges in a bipartite graph. Altruistic donors are crucial to the success of Kidney paired Donation transplants as they make the donor pool bigger by adding their name just to the recipient list.

The altruistic donor acts as the first step towards these domino transplants followed by the bridge donor which is usually the incompatible family or friend match from the recipient of the first round of transplant. Thus an altruist not only helps start the first round of transplants but also facilitates the next round and rounds after that, in a way helping multiple patients at once by this selfless act of organ donation.

In practice, in regions of low thickness in exchange markets, traditional ‘Dynamic exchange’ model is used in which an agent arrives into the exchange pool with an ‘indivisible’ object to trade through barter exchange. In this type of exchanges there is no centralized approach to see if there is a better alternative of exchanges and oftentimes limited to certain groups of people,hospital networks. Though the increase in thickness of markets help, complexity in solving these bipartites increases.Hence we modeled this problem as a Mixed integer problem and with the help of solver ‘Gurobipy’ in python solved the equations.

Mathematical Model:

By mathematical programming approach we turn this matching problem into a math model and then find the best match case with the objective of maximizing match count and possible weightage of final matches.

Data:

Set of patients P $= \{P_1, P_2, ..., P_n\}$ i $\in N$

Set of Altruists A $= \{A_1, A_2, ..., A_n\}$ i $\in M(M << N)$

$Set of donors D = \{\mathrm{D}_1, D_2, ..., D_n\}$

Compatibility (weightage) $= W_{d,p}, \forall p \in P, d \in D$

Variable:

Decision variable : Match or not

$X_{d,p} \forall d \in D, p \in P$

Constraints:

→ A donor can only give to one patient alone. $\sum_{d \in D} X_{d,p} \leq 1, \forall p \in P$

→ A patient can only receive from one donor. $\sum_{p \in P} X_{d,p} \leq 1, \forall d \in D$

→ Donor 'i' will give his organ only when patient 'i' receive a organ [$X_{i,n} = 1 only when X_{m,i} = 1$]

Objective: Max($\sum X_{d,p}.W_{d,p}$), $\forall d \in D, p \in P$

Data:

For the purpose of solving the above explained mathematical model we generated a set of synthetic data.

- Number of Donor,Patient sets =200 (Owing to the limitation of solver license)
- Number of Altruists=10
- Assumed a possible match from each donor to be a maximum of 3. (Owing to the complexity of finding the right compatibility in a pool). This number indicates that every donor will have a compatible weightage of more than 0 only to 3 patients in the whole set of 200. Oftentimes in reality the number is far less.
- Compatibility weightage between all donors to the possible patient pool is a random integer between 1-100.

Tools:

Gurobipy solver,Python

Due to the easy usage of gurobi in Python and its powerful mathematical optimization, solving makes it a right choice in solving our mathematical model.

Results

For a total 210 donors and 200 recipients we attained a total of 148 matches in a matter of seconds which is usually complex to solve in a generalized dynamic approach.
A result of the final subgraph of Bipartite is showcased in figure 6.Here each edge indicates the successful final match between the donor and altruists (Left side disjoint set) and patients (Right side set).

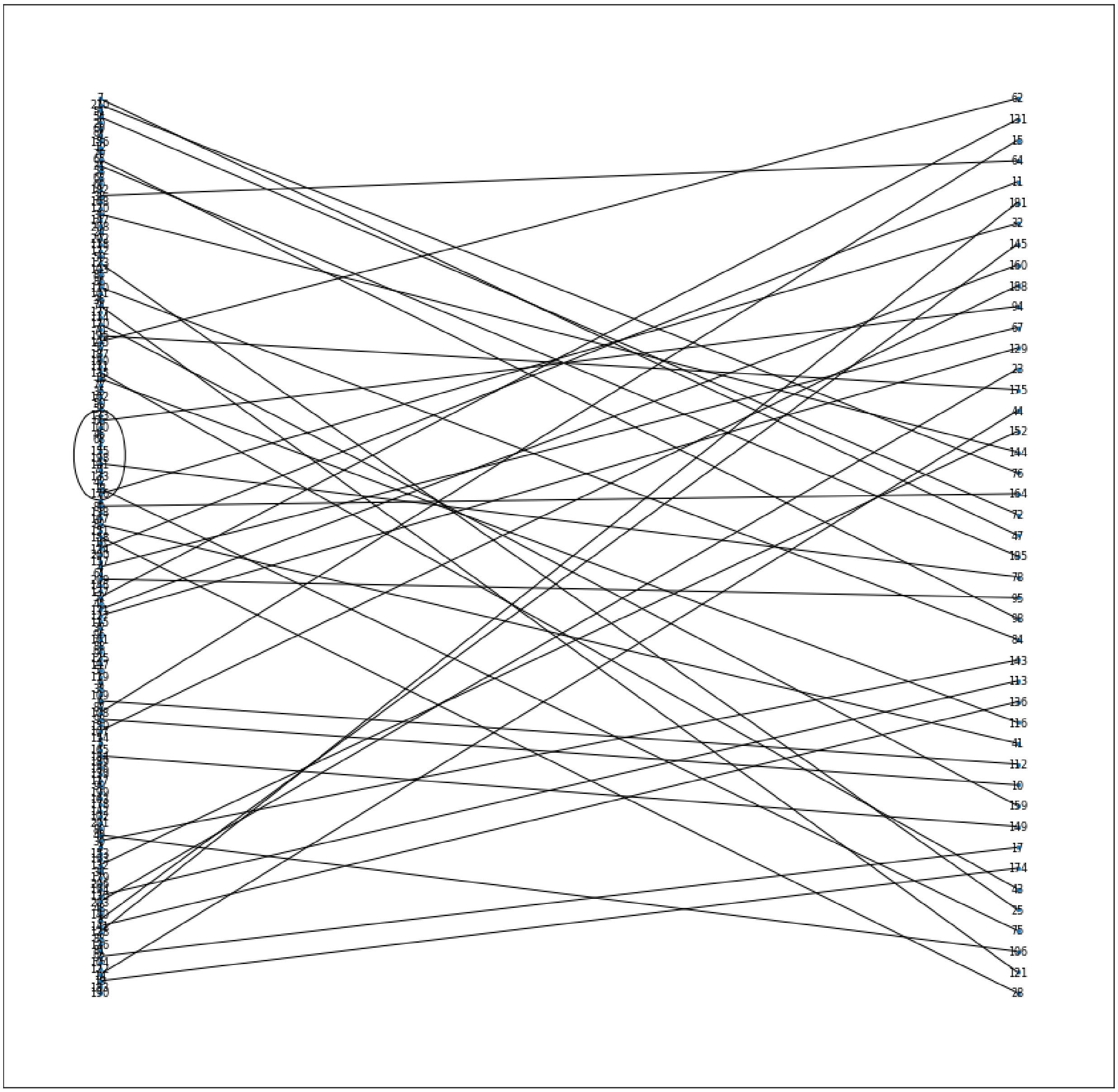

Figure 6. Bipartite Graph showing the final result

Discussion

Mathematical modeling approach solved using the solver provides the optimal result by which the solution is a perfect match because none of the agents in the market are willing to find a match outside the pool and also they only lose by trying to game the system.

The above mathematical model approach makes it easier for Governments/Organizations to form and run a centralized exchange mechanism which has potential to save many lives and also prevent illegal markets to overcome the scenarios where these exchanges are done without any regularization. Further study on ‘Weightage matrix’ calculations helps in making better decisions of final matches. As the thickness of markets increases we might even look at adding extra constraints of ‘breaking point’ where compatibility weightage less than particular breakpoint can be avoided to check the possibility of match to reduce the complexity and still assure the patients to get a right match.

Currently the model is studied without any ‘monetary’ aspect. Further scope can include incentivizing certain kinds of transplants when the right matches are rare to find in these exchange markets to encourage and increase certain types of donors.

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