ECN 594: Practice Midterm Exam - SOLUTIONS

1. Short Answer Questions (30 points)

1. (a) (3 points) Optimal monopoly price with constant elasticity:

   Solution: Using the Lerner Index: $\frac{P-MC}{P} = \frac{1}{|\varepsilon|}$
   $\frac{P-20}{P} = \frac{1}{3}$
   $3(P-20) = P$
   $3P - 60 = P$
   $2P = 60$
   $\boxed{P = \$30}$

   (b) (3 points) Own-price elasticity formula:

   Solution: $\boxed{\eta_{jj} = \alpha p_j(1 - s_j)}$

   (c) (3 points) True/False: Higher prices = more elastic demand in logit.

   Solution: True. From the formula $\eta_{jj} = \alpha p_j(1 - s_j)$, since $\alpha < 0$ and $(1 - s_j) \approx 1$ for small shares, elasticity is roughly proportional to price. This is a mechanical relationship imposed by the functional form.

   (d) (3 points) IIA definition:

   Solution: Independence of Irrelevant Alternatives. The ratio of choice probabilities between any two products is independent of other alternatives. Limitation: Substitution patterns don't depend on how similar products are—e.g., removing a red bus affects blue buses and trains equally, regardless of product characteristics.

   (e) (3 points) OLS bias direction:

   Solution: True. OLS underestimates the magnitude of α (makes it less negative) because unobserved quality ξ_j is positively correlated with price. High-quality products charge higher prices, and OLS attributes this to lower price sensitivity rather than quality.

   (f) (3 points) Perfect price discrimination and DWL:

   Solution: True. Under perfect price discrimination, the monopolist captures all consumer surplus but produces the efficient quantity (where $P = MC$), so there is no deadweight loss.

   (g) (3 points) Instruments for price endogeneity:

Solution: Any of: (1) Cost shifters (input prices, exchange rates), (2) BLP instruments (characteristics of other products), (3) Hausman instruments (prices in other markets), (4) Supply-side exclusion restrictions.

(h) (3 points) Two-part tariff efficiency:

Solution: True. With $p = MC$, consumers face efficient marginal prices and consume the efficient quantity. The fee F extracts surplus but doesn't distort consumption. Compared to monopoly pricing ($p > MC$), total surplus increases because DWL is eliminated.

(i) (3 points) Selection by indicators:

Solution: Selection by indicators is price discrimination based on observable customer characteristics. Examples: student discounts (student ID), senior discounts (age), geographic pricing (location), business vs. personal rates (company affiliation).

(j) (3 points) IC constraint in self-selection:

Solution: True. The incentive compatibility (IC) constraint ensures each type prefers their designated option. For the high type: $v_H(q_H) - p_H \geq v_H(q_L) - p_L$. If violated, high-type consumers would buy the low-type product, undermining the pricing strategy.

2. Demand Estimation (30 points)

2. (a) (5 points) Verify market share for product 1:

Solution: $v_j = \delta_j + \alpha p_j$
$v_1 = 2.0 + (-0.5)(10) = 2.0 - 5.0 = -3.0$
$v_2 = 1.5 + (-0.5)(8) = 1.5 - 4.0 = -2.5$
$v_3 = 2.5 + (-0.5)(12) = 2.5 - 6.0 = -3.5$
$\exp(v_1) = \exp(-3.0) = 0.0498$
$\exp(v_2) = \exp(-2.5) = 0.0821$
$\exp(v_3) = \exp(-3.5) = 0.0302$
Denominator: $1 + 0.0498 + 0.0821 + 0.0302 = 1.1621$
$s_1 = \frac{0.0498}{1.1621} = 0.0429 \approx 0.04$ (not 0.25)
Note: The shares given in the problem are illustrative. In a real exam, the numbers would be consistent.

(b) (5 points) Own-price elasticities:

Solution: $\eta_{jj} = \alpha p_j(1 - s_j)$
$\eta_{11} = (-0.5)(10)(1 - 0.25) = (-0.5)(10)(0.75) = -3.75$
$\eta_{22} = (-0.5)(8)(1 - 0.20) = (-0.5)(8)(0.80) = -3.20$
$\eta_{33} = (-0.5)(12)(1 - 0.15) = (-0.5)(12)(0.85) = -5.10$
Product 3 has the most elastic demand ($|\eta_{33}| = 5.10$), driven by its higher price.

(c) (5 points) Cross-price elasticity and IIA:

Solution: Cross-price elasticity: $\eta_{jk} = -\alpha p_k s_k = -(-0.5)(8)(0.20) = 0.80$
$\eta_{12} = 0.80$ (how much demand for 1 changes when price of 2 increases)
$\eta_{13} = -(-0.5)(12)(0.15) = 0.90$
IIA implication: The cross-price elasticity depends only on the other product's price and share, NOT on how similar the products are. If products 1 and 2 were close substitutes (e.g., Coke and Pepsi), we'd expect $\eta_{12} > \eta_{13}$, but logit doesn't capture this.

(d) (5 points) Berry inversion:

Solution: From logit: $\frac{s_j}{s_0} = \exp(\delta_j + \alpha p_j)$
Taking logs: $\ln(s_j) - \ln(s_0) = \delta_j + \alpha p_j$
Berry inversion: $\boxed{\delta_j + \alpha p_j = \ln(s_j) - \ln(s_0)}$
This allows us to compute mean utilities directly from observed shares.

(e) (10 points) Consumer surplus calculation:

Solution: Using $v_j = \delta_j + \alpha p_j$:
$v_1 = 2.0 - 5.0 = -3.0$, $v_2 = 1.5 - 4.0 = -2.5$, $v_3 = 2.5 - 6.0 = -3.5$
$IV = \ln(1 + e^{-3.0} + e^{-2.5} + e^{-3.5}) = \ln(1.1621) = 0.150$
$CS = \frac{1}{|\alpha|} \cdot IV = \frac{1}{0.5} \times 0.150 = \$0.30$ per consumer
Without product 3:
$IV' = \ln(1 + e^{-3.0} + e^{-2.5}) = \ln(1.1319) = 0.124$

$CS' = \frac{1}{0.5} \times 0.124 = \$0.248$

$\Delta CS = 0.248 - 0.30 = \boxed{-\$0.052}$ per consumer

Consumers lose about 5 cents per person from removing product 3.

3. Price Discrimination by Indicators (20 points)

3. (a) (5 points) Optimal prices under price discrimination:

Solution: For each segment, maximize profit separately.
Business: $Q_B = 100 - P_B$, so $P_B = 100 - Q_B$
$\pi_B = (P_B - 10)Q_B = (100 - Q_B - 10)Q_B = (90 - Q_B)Q_B$
FOC: $90 - 2Q_B = 0 \Rightarrow Q_B = 45$, $P_B = 100 - 45 = \$55$
Students: $Q_S = 50 - 2P_S$, so $P_S = 25 - 0.5Q_S$
$\pi_S = (P_S - 10)Q_S = (25 - 0.5Q_S - 10)Q_S = (15 - 0.5Q_S)Q_S$
FOC: $15 - Q_S = 0 \Rightarrow Q_S = 15$, $P_S = 25 - 7.5 = \$17.50$
$\boxed{P_B = \$55, \quad P_S = \$17.50}$

(b) (5 points) Total profit under price discrimination:

Solution: $\pi_B = (55 - 10) \times 45 = 45 \times 45 = \$2,025$
$\pi_S = (17.50 - 10) \times 15 = 7.50 \times 15 = \$112.50$
$\boxed{\pi_{total} = 2,025 + 112.50 = \$2,137.50}$

(c) (5 points) Uniform pricing:

Solution: Total demand: $Q = Q_B + Q_S = (100 - P) + (50 - 2P) = 150 - 3P$
Inverse demand: $P = 50 - \frac{Q}{3}$
$\pi = (P - 10)Q = (50 - \frac{Q}{3} - 10)Q = (40 - \frac{Q}{3})Q$
FOC: $40 - \frac{2Q}{3} = 0 \Rightarrow Q = 60$
$P = 50 - 20 = \$30$
Check: At $P = 30$: $Q_B = 100 - 30 = 70$, $Q_S = 50 - 60 = -10 < 0$
Students are priced out! Only businesses buy.
With only businesses: $\pi = (30 - 10) \times 70 = \$1,400$
Better to set $P = \$25$ (students just indifferent): $Q_B = 75$, $Q_S = 0$
$\pi = (25 - 10) \times 75 = \$1,125$
Optimal: $P = \$30$, $Q = 70$ (businesses only), or compare to $P = \$55$ for businesses only: $\pi = \$2,025$ which is better.
$\boxed{P^* = \$55}$ (serve only businesses under uniform pricing)

(d) (5 points) Consumer surplus comparison:

Solution: Under discrimination:
$CS_B = \frac{1}{2}(100 - 55)(45) = \frac{1}{2}(45)(45) = \$1,012.50$
$CS_S = \frac{1}{2}(25 - 17.50)(15) = \frac{1}{2}(7.50)(15) = \$56.25$
Total CS = \$1,068.75
Under uniform ($P = \$55$, businesses only):
$CS_B = \frac{1}{2}(100 - 55)(45) = \$1,012.50$
$CS_S = \$0$ (priced out)
Total CS = \$1,012.50
Students benefit from price discrimination (get positive surplus vs. zero). Business CS is the same either way.

4. Two-Part Tariff (20 points)

4. (a) (5 points) Uniform pricing:

Solution: Demand: $q = 20 - p$, so $P = 20 - q$

Per-consumer profit: $\pi = (p-2)q = (20-q-2)q = (18-q)q$

FOC: $18 - 2q = 0 \Rightarrow q = 9$, $p = 20 - 9 = \$11$

Per-consumer profit: $(11-2)(9) = \$81$

Total profit: $100 \times 81 = \boxed{\$8,100}$

(b) (5 points) Optimal two-part tariff:

Solution: Set $p = MC = \$2$ to maximize quantity consumed.

At $p = 2$: $q = 20 - 2 = 18$ visits per consumer.

Consumer surplus at $p = 2$: $CS = \frac{1}{2}(20-2)(18) = \frac{1}{2}(18)(18) = \$162$

Set $F = CS = \$162$ to extract all surplus.

Optimal two-part tariff: $\boxed{F = \$162, \quad p = \$2}$

Total profit: $100 \times 162 = \$16,200$

(c) (5 points) Total surplus comparison:

Solution: Uniform pricing:

$CS = \frac{1}{2}(20-11)(9) \times 100 = \frac{1}{2}(9)(9)(100) = \$4,050$

$PS = \$8,100$

$TS = 4,050 + 8,100 = \$12,150$

Two-part tariff:

$CS = 0$ (all extracted by fee)

$PS = \$16,200$

$TS = 0 + 16,200 = \$16,200$

Difference: Two-part tariff increases total surplus by $4,050.

Explanation: Under uniform pricing, $p > MC$ creates deadweight loss. Under the two-part tariff, $p = MC$ means efficient quantity is consumed. The fee transfers surplus but doesn't distort consumption.

(d) (5 points) Heterogeneous consumers constraint:

Solution: With two types, the gym faces a participation constraint. If F is too high, light users won't join.

Light users at p: $q_L = 10 - p$, $CS_L = \frac{1}{2}(10-p)(10-p) = \frac{(10-p)^2}{2}$

For light users to participate: $F \leq CS_L = \frac{(10-p)^2}{2}$

The binding constraint is the light users' participation constraint.

The fee must be low enough that light users are willing to pay it given their smaller consumer surplus. This limits how much surplus can be extracted.

If $p = MC = 2$: $CS_L = \frac{(8)^2}{2} = \$32$, so $F \leq \$32$

(Much less than $162 from heavy users, leaving money on the table)