ECN 453: Mid-term Exam 2 (Practice)

1. Short answer questions (30 points)

(a) Hotelling

(b) True.

(c) True.

(d) 20.

(e) 80.

(f) False.

(g) Monopoly output.

(h) 0.

(i) 0.

(j) Marginal cost.

2. Cournot Competition With Asymmetric Marginal Costs (30 points)

(a) Given:

$$Q = 100 - p \implies$$
$$p = 100 - (q_1 + q_2)$$

For firms $i \in 1, 2$, profit maximization problem is (denoting by $-i$ the firm that is not i):

$$\pi_i = (100 - q_i - q_{-i})q_i - c_i q_i$$

$$MR = MC \implies$$
$$q_i(q_{-i}) = \frac{100 - c_i - q_{-i}}{2}$$

Therefore, the respective best responses for both firms can be derived as follows:

$$q_1(q_2) = 30 - q_2/2$$
$$q_2(q_1) = 35 - q_1/2$$

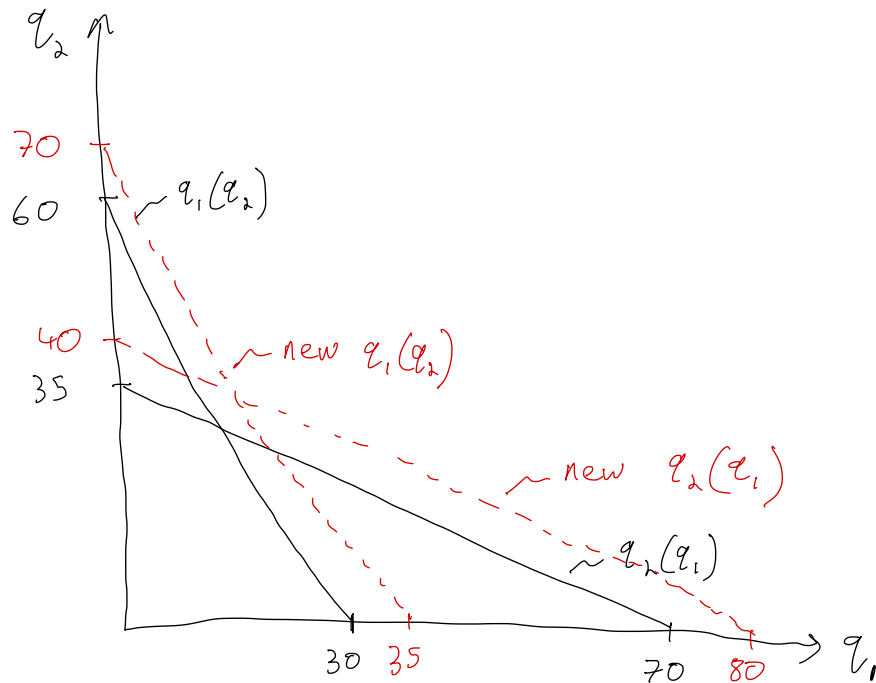Both graphs are shown in part c

(b) Substitute one best response into another and solve:

$$q_1 = 16.66, q_2 = 26.67$$

(c) If marginal costs of both firms reduce by 10, then the new best responses can be written as:

$$q_1(q_2) = 35 - q_2/2$$
$$q_2(q_1) = 40 - q_1/2$$

3. Stackelberg (30 points)

(a) Solve by backward induction. Firm 2's profit maximization problem is:

$$\pi_2(q_1) = (100 - 2q_1 - 2q_2)q_2 - 20q_2$$

$$MR = MC \implies$$
$$100 - 2q_1 - 4q_2 - 20 = 0 \implies$$
$$q_2(q_1) = \frac{80 - 2q_1}{4}$$

Plugging this into firm 1's problem:

$$\pi_1 = (100 - 2q_1 - 2q_2(q_1))q_2 - 4q_1^2$$
$$\pi_1 = (60 - q_1)q_1 - 4q_1^2$$

Setting FOC $= 0$

$$60 - 2q_1 - 8q_1 = 0 \implies$$
$$q_1 = 6 \implies q_2 = 17$$

3. Hotelling Model (30 points)

(a) Let's assume prices are set in such a way that the indifferent consumer will be in between the two firms, i.e. a the indifferent consumer has address x' where $x' \in [0, 0.6]$. Therefore, all consumers to the right of firm 2 will choose firm 2 due to transport costs. The indifference consumer for consumer at x' is:

$$tx' + p_1 = t(0.6 - x') + p_2$$

So consumers to the left of x' buy from Firm 1, and consumers to the right of x' including those in between 0.6 and 1 buy from Firm 2. So Firm 1's demand is x' and Firm 2's demand is $(0.6 - x) + 0.4$

Solving the above equation for x' and substituting $t = 0.5$, we can get the demands for each firm:

$$q_1 = 100 \times (0.3 + (p_2 - p_1))$$
$$q_2 = 100 \times (0.7 + (p_1 - p_2))$$

(b) Using demands, we can get the payoffs:

$$\pi_1 = q_1(p_1 - c_1) = 100(0.3 + (p_2 - p_1))p_1$$

Taking the derivative with respect to price and setting to zero:

$$\frac{0.3 + p_2}{2} = p_1(p_2)$$

At $p_2 = 0.5$, $p_1 = 0.4$.