

ECN 453: Mid-term Exam 2 (Practice)

1. Short answer questions (30 points)

- (a) Hotelling
- (b) True.
- (c) True.
- (d) 20.
- (e) 80.
- (f) False.
- (g) Monopoly output.
- (h) 0.
- (i) 0.
- (j) Marginal cost.

2. Cournot Competition With Asymmetric Marginal Costs (30 points)

- (a) Given:

$$Q = 100 - p \implies$$
$$p = 100 - (q_1 + q_2)$$

For firms $i \in 1, 2$, profit maximization problem is (denoting by $-i$ the firm that is not i):

$$\pi_i = (100 - q_i - q_{-i})q_i - c_i q_i$$

$$MR = MC \implies$$
$$q_i(q_{-i}) = \frac{100 - c_i - q_{-i}}{2}$$

Therefore, the respective best responses for both firms can be derived as follows:

$$q_1(q_2) = 30 - q_2/2$$

$$q_2(q_1) = 35 - q_1/2$$

Both graphs are shown in part c

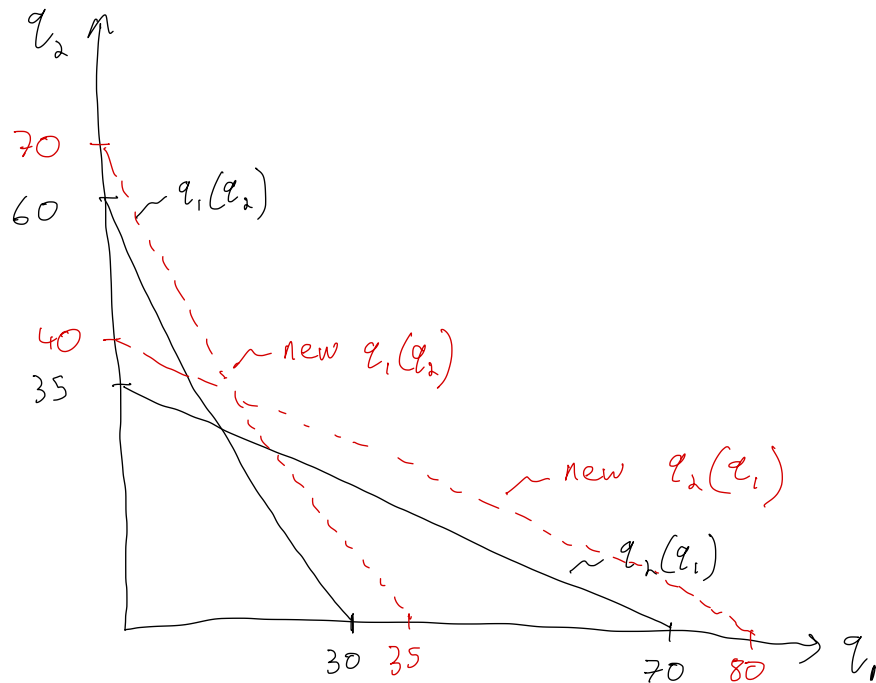
(b) Substitute one best response into another and solve:

$$q_1 = 16.66, q_2 = 26.67$$

(c) If marginal costs of both firms reduce by 10, then the new best responses can be written as:

$$q_1(q_2) = 35 - q_2/2$$

$$q_2(q_1) = 40 - q_1/2$$



3. Stackelberg (30 points)

(a) Solve by backward induction. Firm 2's profit maximization problem is:

$$\pi_2(q_1) = (100 - 2q_1 - 2q_2)q_2 - 20q_2$$

$$\begin{aligned} MR &= MC \implies \\ 100 - 2q_1 - 4q_2 - 20 &= 0 \implies \\ q_2(q_1) &= \frac{80 - 2q_1}{4} \end{aligned}$$

Plugging this into firm 1's problem:

$$\begin{aligned} \pi_1 &= (100 - 2q_1 - 2q_2(q_1))q_1 - 4q_1^2 \\ \pi_1 &= (60 - q_1)q_1 - 4q_1^2 \end{aligned}$$

Setting FOC = 0

$$\begin{aligned} 60 - 2q_1 - 8q_1 &= 0 \implies \\ q_1 &= 6 \implies q_2 = 17 \end{aligned}$$

3. Hotelling Model (30 points)

(a) Let's assume prices are set in such a way that the indifferent consumer will be in between the two firms, i.e. the indifferent consumer has address x' where $x' \in [0, 0.6]$. Therefore, all consumers to the right of firm 2 will choose firm 2 due to transport costs. The indifference consumer for consumer at x' is:

$$tx' + p_1 = t(0.6 - x') + p_2$$

So consumers to the left of x' buy from Firm 1, and consumers to the right of x' including those in between 0.6 and 1 buy from Firm 2. So Firm 1's demand is x' and Firm 2's demand is $(0.6 - x') + 0.4$

Solving the above equation for x' and substituting $t = 0.5$, we can get the demands for each firm:

$$\begin{aligned} q_1 &= 100 \times (0.3 + (p_2 - p_1)) \\ q_2 &= 100 \times (0.7 + (p_1 - p_2)) \end{aligned}$$

(b) Using demands, we can get the payoffs:

$$\pi_1 = q_1(p_1 - c_1) = 100(0.3 + (p_2 - p_1))p_1$$

Taking the derivative with respect to price and setting to zero:

$$\frac{0.3 + p_2}{2} = p_1(p_2)$$

At $p_2 = 0.5$, $p_1 = 0.4$.