ECN 594: Consumer Surplus, IIA, and Price Discrimination

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Plan for today

1. **Consumer surplus: the log-sum formula**
2. The IIA problem: Red Bus / Blue Bus
3. From demand to supply

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4. Types of price discrimination
5. Selection by indicators
6. Worked example: optimal pricing across markets

Why do we care about consumer surplus?

- Policy analysis requires measuring welfare
- Questions we want to answer:
  - How much do consumers gain from a new product?
  - How much do consumers lose from a merger?
  - What is the welfare cost of a price increase?
- Need a way to compute consumer surplus from our demand model

Consumer surplus in logit: the log-sum formula

- For consumer i, expected utility from choosing among J products:

$$E[\max_j u_{ij}] = \ln\left[\sum_{j=0}^{J} \exp(\delta_j + \mu_{ij})\right] + \text{constant}$$

- This is the "log-sum" or "inclusive value"

- Consumer surplus (in dollars):

$$CS_i = \frac{1}{\alpha} \ln\left[\sum_{j=0}^{J} \exp(\delta_j + \mu_{ij})\right]$$

- Divide by α (price coefficient) to convert to dollars

Intuition for the log-sum

- Think of choosing the best option as a lottery
- Each product gives you a random utility draw
- **Expected value of the BEST draw** is the log-sum
- Key insights:
  - More options $\rightarrow$ higher CS (more lottery tickets)
  - Better options $\rightarrow$ higher CS (higher δ_j)
  - Higher price sensitivity $\rightarrow$ divide by larger α

Worked example: CS change from price increase

- **Question:**
- Two products with $\delta_1 = 2$ and $\delta_2 = 1$. Outside option $\delta_0 = 0$.
- Price coefficient $\alpha = 0.8$.
- If product 1's price increases by \$2 (so δ_1 falls to 0.4), what is the CS loss?

Take 3 minutes to solve this.

Worked example: CS change from price increase (solution)

Solution

- Note: $\Delta\delta_1 = -\alpha \times \Delta p = -0.8 \times 2 = -1.6$, so new $\delta_1 = 2 - 1.6 = 0.4$
- **Before:**

$$CS^{\text{before}} = \frac{1}{0.8}\ln(e^0 + e^2 + e^1) = 1.25\ln(1 + 7.39 + 2.72) = 1.25 \times 2.41 = 3.01$$

- **After:**

$$CS^{\text{after}} = \frac{1}{0.8}\ln(e^0 + e^{0.4} + e^1) = 1.25\ln(1 + 1.49 + 2.72) = 1.25 \times 1.65 = 2.06$$

- **Loss:** $3.01 - 2.06 = \$0.95$ per consumer

Worked example: CS change from removing a product

- **Question:** Market has 2 products with $\delta_1 = 1$, $\delta_2 = 0.5$. Outside option has $\delta_0 = 0$. Suppose $\alpha = 0.5$.

- What is the consumer surplus loss if product 1 is removed?

Take 3 minutes to solve this.

Worked example: CS change (solution)

Solution

- **Before removal:**

$$CS^{\text{before}} = \frac{1}{0.5} \ln(e^0 + e^1 + e^{0.5}) = 2 \ln(1 + 2.72 + 1.65) = 2 \ln(5.37) = 3.36$$

- **After removal:**

$$CS^{\text{after}} = \frac{1}{0.5} \ln(e^0 + e^{0.5}) = 2 \ln(1 + 1.65) = 2 \ln(2.65) = 1.95$$

- **Loss:** $3.36 - 1.95 = 1.41$ dollars per consumer
- This is on HW1!

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The IIA problem: Red Bus / Blue Bus

- **Setup:** Consumers choose how to commute
- Choices: Car, Red Bus
- Suppose: half choose Car, half choose Red Bus
- So: $\delta_{\text{car}} = \delta_{\text{red bus}} = 0$

Red Bus / Blue Bus: introducing a new option

- Now introduce a **Blue Bus**
- But consumers are color-blind!
- Blue Bus is identical to Red Bus in every way
- **Reality:** Welfare should NOT change
  - It's the same bus, just different color
  - No real new option

What does logit predict?

- **Before Blue Bus:**

$$\text{Inclusive value} = \ln(e^0 + e^0) = \ln(2)$$

- **After Blue Bus:** $\delta_{\text{blue bus}} = \delta_{\text{red bus}} = 0$

$$\text{Inclusive value} = \ln(e^0 + e^0 + e^0) = \ln(3)$$

- Logit says welfare **increased**!
- But nothing real changed...

The IIA problem: what went wrong?

- Logit gives an extra “lottery ticket” for each product
- It doesn’t know that buses are close substitutes
- **IIA:** The ratio s_j / s_k doesn’t depend on other options

$$\frac{s_{\text{car}}}{s_{\text{red bus}}} = \frac{e^0}{e^0} = 1 \quad \text{(before and after!)}$$

- Adding Blue Bus steals equally from Car and Red Bus
- But Car and Red Bus are NOT equally similar to Blue Bus

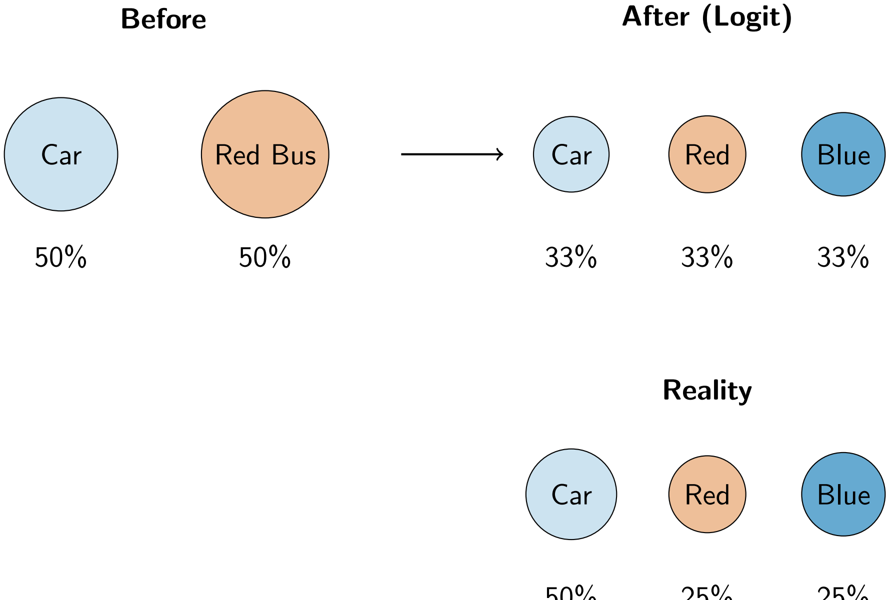
Red Bus / Blue Bus: The math

- **Before** (Car, Red Bus): $s_{\text{car}} = s_{\text{red}} = 0.5$
- **After** (Car, Red Bus, Blue Bus), logit predicts:

$$s_{\text{car}} = \frac{e^0}{e^0 + e^0 + e^0} = \frac{1}{3}$$

- Car's share dropped from 0.5 to 0.33!
- **Reality:** Car share should stay at 0.5
  - Car commuters don't care about bus color
  - Blue Bus should only steal from Red Bus

Red Bus / Blue Bus: Visual



Why IIA matters

- IIA affects:
  1. **Valuing new products:** May overstate welfare gains
  2. **Merger analysis:** May mispredict substitution patterns
  3. **Cross-elasticities:** All products same cross-elasticity with any given product
- **When is logit "good enough"?**
  - Products are genuinely similar (e.g., brands of cereal)
  - You're not analyzing entry/exit of close substitutes

Worked example: Cross-elasticity and IIA

- **Question:**
- Market has 3 products: Luxury Car ($s = 0.1$), Economy Car ($s = 0.2$), Bus ($s = 0.3$).
- Outside option $s_0 = 0.4$. Price coefficient $\alpha = 0.5$.
- Luxury Car price = \$50K. Calculate cross-elasticity of Luxury Car with respect to Economy Car price.
- What's wrong with this prediction?

Take 2 minutes to solve this.

Worked example: Cross-elasticity and IIA (solution)

Solution

- Cross-price elasticity formula: $\eta_{jk} = \alpha p_k s_k$
- Cross-elasticity of Luxury Car w.r.t. Economy Car:

$$\eta_{\text{Lux, Econ}} = \alpha \times p_{\text{Econ}} \times s_{\text{Econ}}$$

- But notice: this is the SAME as cross-elasticity w.r.t. Bus!
- **IIA problem:** Logit says Luxury Car responds equally to price changes by Economy Car and by Bus
- **Reality:** Luxury Car buyers probably substitute more with Economy Car than with Bus

How demographics help (partial solution)

- With demographics: different consumer types have different substitution patterns
- Bus riders vs car commuters substitute differently
- Aggregate substitution is richer
- But IIA still holds *within* each consumer type
- **Mixed logit** (random coefficients) fully relaxes IIA
  - Beyond our scope, but important to know

When is IIA "good enough"?

- **IIA is usually fine when:**
  - Products are genuinely similar (cereal brands, gas stations)
  - You're not analyzing entry/exit of close substitutes
  - You have rich demographics capturing key preference heterogeneity
- **IIA is problematic when:**
  - Products form clear "nests" (cars vs buses, luxury vs economy)
  - Analyzing new product entry (especially into a crowded segment)
  - Computing welfare from removing specific products
- **Solution:** Nested logit or mixed logit (beyond this course)

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5. Selection by indicators
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From demand to supply

- We've focused on demand estimation
- **Key insight:** Demand gives us the hard part
  - Elasticities
  - Substitution patterns
  - Consumer welfare
- Costs can often be *backed out* from pricing behavior
- Using the Lerner index: $mc = p - p/|\varepsilon|$

Worked example: Backing out marginal cost

- **Question:**
- You estimate demand and find own-price elasticity $\varepsilon = -4$.
- Observed price is $p = 100$.
- Assuming Nash-Bertrand pricing, what is the implied marginal cost?

Take 2 minutes to solve this.

Worked example: Backing out marginal cost (solution)

Solution

- Lerner index: $\frac{p - mc}{p} = \frac{1}{|\varepsilon|}$
- Rearranging: $mc = p - \frac{p}{|\varepsilon|} = p\left(1 - \frac{1}{|\varepsilon|}\right)$
- Plug in:

$$mc = 100 \times \left(1 - \frac{1}{4}\right) = 100 \times 0.75 = 75$$

- Implied marginal cost is \$75
- **Markup:** $(100 - 75)/100 = 25\%$
- This technique is used extensively in merger simulation

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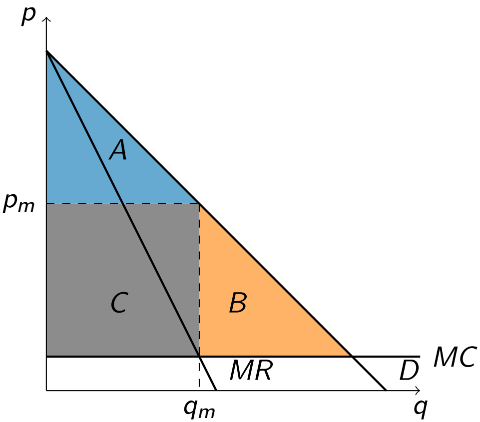
Price Discrimination

- Price discrimination: **setting different prices for the same good**
- Examples: airline tickets, software, pharmaceuticals, student discounts
- We will look at different ways firms price discriminate

Why price discriminate?



- **Area A:** Consumers WTP $> p_m$
  - Could charge them more!
- **Area B:** Consumers WTP between MC and p_m
  - Could sell to them at lower price
- **Area C:** Current profit

Types of price discrimination (Cabral terminology)

1. **Perfect price discrimination**
   - Charge each consumer their exact WTP
   - Extracts all surplus; unrealistic benchmark
2. **Selection by indicators**
   - Divide buyers into groups by observable characteristics
   - Set different price for each group
3. **Self-selection**
   - Cannot observe type directly
   - Design menu to induce consumers to reveal type
   - (Covered next lecture)

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Selection by indicators

- Divide buyers into groups based on **observable characteristics**
- Set different price for each group
- Examples:
  - Student discounts (show student ID)
  - Senior discounts
  - Geographic pricing (different prices in different countries)
  - Time-based pricing (matinees vs evening shows)

Real-world examples of selection by indicators

1. **Geographic:**
   - New car prices differ by region (Arizona vs California)
   - Software priced differently in US vs India
2. **Age-based:**
   - Senior discounts (more elastic, retired, fixed income)
   - Student discounts (more elastic, lower income)
3. **Time-based:**
   - Happy hour (flexible drinkers are more elastic)
   - Black Friday (patient shoppers are more elastic)

Selection by indicators: setup

- Two markets: market 1 and market 2
- Demand: $q_1 = D_1(p_1)$ and $q_2 = D_2(p_2)$
- Cost: $C(q_1 + q_2)$, with constant MC
- **Goal:** Find optimal price in each market

Selection by indicators: solution

- Apply optimal pricing rule in each market:

$$MR_1 = MC \quad \text{and} \quad MR_2 = MC$$

- Equivalently, using elasticity rule:

$$\frac{p_1 - MC}{p_1} = \frac{1}{|\varepsilon_1|} \quad \text{and} \quad \frac{p_2 - MC}{p_2} = \frac{1}{|\varepsilon_2|}$$

- **Key implication:** Charge higher price in market with more inelastic demand

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Worked example: Optimal pricing across markets

- **Question:**
- Two markets with $\varepsilon_1 = -2$ and $\varepsilon_2 = -4$
- $MC = 6$
- Find optimal prices in each market.

Take 3 minutes to solve this.

Worked example: Optimal pricing (solution)

Solution

- Using Lerner index: $\frac{p-MC}{p} = \frac{1}{|\varepsilon|}$
- **Market 1** ($\varepsilon_1 = -2$):

$$\frac{p_1 - 6}{p_1} = \frac{1}{2} \quad \Rightarrow \quad p_1 - 6 = 0.5p_1 \quad \Rightarrow \quad p_1 = 12$$

- **Market 2** ($\varepsilon_2 = -4$):

$$\frac{p_2 - 6}{p_2} = \frac{1}{4} \quad \Rightarrow \quad p_2 - 6 = 0.25p_2 \quad \Rightarrow \quad p_2 = 8$$

- Price is higher in more inelastic market (market 1)

Worked example: Student discount pricing

- **Question:**
- A software company can distinguish students from professionals.
- Students: $\varepsilon_s = -3$; Professionals: $\varepsilon_p = -1.5$
- $MC = \$20$
- Calculate optimal prices for each group.

Take 3 minutes to solve this.

Worked example: Student discount pricing (solution)

Solution

- Using Lerner index: $p = \frac{MC}{1+1/\varepsilon}$
- **Students** ($\varepsilon_s = -3$):

$$p_s = \frac{20}{1 + 1/(-3)} = \frac{20}{1 - 0.33} = \frac{20}{0.67} = \$30$$

- **Professionals** ($\varepsilon_p = -1.5$):

$$p_p = \frac{20}{1 + 1/(-1.5)} = \frac{20}{1 - 0.67} = \frac{20}{0.33} = \$60$$

- Students pay \$30 (50% discount); professionals pay \$60
- Students more elastic $\rightarrow$ lower price

Welfare effects of selection by indicators

- **Producer surplus:** Increases (that's why firms do it)
- **Consumer surplus:** Ambiguous
  - Some consumers pay more (inelastic market)
  - Some consumers pay less (elastic market)
  - Some consumers now served who weren't before
- **Total welfare:** Depends on whether new markets are served
  - If discrimination opens new markets $\rightarrow$ welfare may increase
  - If just redistributes $\rightarrow$ welfare may decrease

When is price discrimination welfare-improving?

- **Welfare improves when:**
  - Discrimination opens up new markets (serves consumers who otherwise wouldn't be served)
  - Example: Student discounts let students afford textbooks
- **Welfare may decrease when:**
  - Just redistributes from consumers to firm
  - No expansion of output
- **Key insight:** Output matters!
  - If total quantity sold goes up, welfare likely increases
  - If total quantity stays same, welfare likely decreases

Connection: Demand estimation and price discrimination

- **Demand estimation gives us:**
  - Elasticities by consumer group (if demographics used)
  - Which groups are more/less price-sensitive
- **This informs pricing strategy:**
  - High elasticity groups $\rightarrow$ lower price
  - Low elasticity groups $\rightarrow$ higher price
- **Example:** Nevo (2001) found that families with children are less price-sensitive for kid cereals
- Cereal companies can use this for targeted promotions

Key Points

1. **Log-sum formula**: $CS_i = \frac{1}{\alpha} \ln \left[\sum_j \exp(\delta_j)\right]$
2. **Red Bus / Blue Bus**: Logit overcounts value of similar products
3. **IIA**: Substitution proportional to share, not similarity
4. Demographics partially help; mixed logit fully relaxes IIA
5. **Price discrimination**: Different prices for same good
6. **Perfect PD**: Charge each consumer their WTP (benchmark)
7. **Selection by indicators**: Group pricing based on observables
8. Charge higher price in **more inelastic** market: $p = MC/(1 + 1/\varepsilon)$

Next time

- **Lecture 5:** Two-Part Tariffs and Self-Selection
  - Two-part tariffs: $F + p \times q$
  - Self-selection: menu design, versioning
  - Incentive compatibility constraints