where Q is aggregate output produced by the N firms and Q_{-i} is the aggregate output of all firms except firm i; that is,

$$Q_{-i} = Q - q_i$$

The profit function for firm i can then be written as

$$\pi_i(q_i, Q_{-i}) = q_i[A - B(q_i + Q_{-i}) - c]. \tag{15.3}$$

In a Cournot game, firms choose their output levels simultaneously to maximize profit. The resulting profit to each firm in a Cournot equilibrium is

$$\pi_i^C = \frac{(A - c)^2}{B(N + 1)^2} \tag{15.4}$$

Suppose now that $M \geq 2$ of these firms decide to merge. To exclude the case of merger to monopoly, we assume that $M < N$. Such a merger leads to an industry that contains $N - M + 1$ firms. Because all firms are the same, we can think of the merged firm as comprised of firms 1 through M.

The new merged firm picks its output q_m to maximize profit, which is given by

$$\pi_m(q_m, Q_{-m}) = q_m[A - B(q_m + Q_{-m}) - c] \tag{15.5}$$

where $Q_{-m} = q_{m+1} + q_{m+2} + \ldots + q_N$ denotes the aggregate output of the $N - M$ firms that have not merged. Each of the nonmerged firms chooses its output to maximize profit given, as before, by

$$\pi_i(q_i, Q_{-i}) = q_i[A - B(q_i + Q_{-i}) - c]. \tag{15.6}$$

In this case, the term Q_{-i} now denotes the sum of the outputs q_j of each of the $N - M$ nonmerging firms excluding firm i, plus the output of the merged firm q_m.

The only difference between equations (15.5) and (15.6) is that in the former we have a subscript m while in the latter we have a subscript i. In other words, a crucial implication of equations (15.5) and (15.6) is that, after the merger, *the merged firm acts just like any one of the other firms in the industry*. This means that all of these $N - M + 1$ firms, each having identical costs and producing the same product, must in equilibrium produce the same amount of output and therefore earn the same profit. In other words, in the post-merger Cournot equilibrium, it must be the case that the output and profit of the merged firm, q_m^C and π_m^C, are the same as the output and profit of each nonmerged firm. Using the Cournot output and profit equations for a market with $N - M + 1$ firms, these are, respectively:

$$q_m^C = q_{nm}^C = \frac{A - c}{B(N - M + 2)} \text{ and } \pi_m^C = \pi_{nm}^C = \frac{(A - c)^2}{B(N - M + 2)^2} \tag{15.7}$$

where the subscript m denotes the merged firm and nm a nonmerged firm.

Equations (15.4) and (15.7) allow us to compare the profit of the nonmerging firms before and after the merger. The first point to note is the free-riding opportunity afforded to the nonmerging firms when other firms merge. We know that in the Cournot model as the number of firms decreases industry output falls and price rises. Of course, a merger does just that. It reduces the number of firms. So the price rises for all firms, including those that did not merge. Moreover the merger allows those firms to gain market share while also benefiting from the increase in the market price.

What about the merging firms? There are M of these and, prior to the merger, each one earned the profit shown in equation (15.4). Hence, the aggregate profit of these firms taken together is M times that amount. After the merger, the profit of the merged firm is the profit shown in equation (15.7). Is the profit of the merged firm greater than the aggregate profit earned by the M firms before the merger? For the answer to be yes, it must be the case that

$$\frac{(A-c)^2}{B(N-M+2)^2} \geq M\frac{(A-c)^2}{B(N+1)^2} \tag{15.8}$$

This requires

$$(N+1)^2 \geq M(N-M+2)^2 \tag{15.9}$$

Note that equation (15.9) does not include any of the demand parameters or the firms' marginal costs. In other words, equation (15.9) tells us about the profitability of *any* M firm merger. All that is required is that demand is linear and that the firms each have the same, constant marginal costs.

In our example in which the number of firms is $N = 3$ and the number of firms merging is $M = 2$, it's easy to see then that the inequality in (15.9) is not satisfied. In other words, in a three-firm market satisfying our demand and cost assumptions *no two-firm merger is profitable*.

Condition (15.9) is much more general than this and turns out to be very difficult to satisfy even when more than two firms merge, as long as the merger does not result in a monopoly. To see this, suppose that we substitute $M = aN$ in equation (15.9), with $0 < a < 1$. That is, a is the fraction of firms in the industry that merge. We can then work out how large a has to be for the merger to be profitable. A little manipulation of condition (15.9) shows that for a merger to be profitable, we must have $a > a(N)$ where:[5]

$$a(N) = \frac{3+2N-\sqrt{5+4N}}{2N} \tag{15.10}$$

Table 15.1 gives $a(N)$ and the associated minimum number of firms $\underline{M}$ that have to merge for the merger to be profitable for a range of values of N, the number of firms in the industry.

Equation (15.10) and Table 15.1 illustrate what has come to be termed the 80 percent rule. For a merger to be profitable in our simple Cournot world of linear demand and identical constant costs, it is necessary that at least 80 percent of the firms in the market merge. The problem is that a merger of this magnitude would almost never be allowed by the antitrust authorities.

Table 15.1 Necessary condition for profitable merger

N	5	10	15	20	25
$a(N)$	80%	81.5%	83.1%	84.5%	85.5%
$\underline{M}$	4	9	13	17	22

[5] You can check this equation by direct substitution of $a(N)$ in equation (15.9).

15.1

Practice Problem

Suppose that demand for carpet-cleaning services in Dirtville is described by $P = 130 - Q$. There are currently twenty identical firms that clean carpets in the area. The unit cost of cleaning a carpet is constant and equal to \$30. Firms in this industry compete in quantities.

a. Show that in a Cournot–Nash equilibrium the profit of each firm is $\pi = 22.67$.
b. Now suppose that six firms in the industry merge. Show that the profit of each firm in the post-merger Cournot game is $\pi = 39.06$. Show that the profit earned by the merged firm is insufficient to compensate all the shareholders/owners who owned the six original firms and earned profit from them in the pre-merger market game.
c. Show that if fewer than seventeen firms merge, the profit of the merged firm is not great enough to buy out the shareholders/owners of the firms who merge.

The merger paradox is that many, if not most, horizontal mergers are unprofitable when viewed through the lens of our standard Cournot model. Yet, as the events of the 1990s and even of more recent years tell us, horizontal mergers appear to happen all the time. What aspect of real-world mergers has the simple Cournot model failed to capture? Alternatively, what aspect of the Cournot model is responsible for this prediction that seems at odds with reality?

The critical aspect of the Cournot model that gives rise to the merger paradox is not difficult to find. When firms merge in the Cournot model, the new combined firm behaves after the merger just like any of the remaining firms that did not merge. Thus, if two firms in a three-firm industry merge, the new firm competes as a duopolist. The nonmerging firm in this case has, after the merger, equal status to the merged firm even though it now faces the combined strength of both of its previous rivals.

One cannot help but suspect that, for a merger of any substantial size, either the newly merged firm is different in some material sense from its unmerged rivals, or the overall market has changed in a way that alters rivals' behavior. In the next three sections, we explore such possible modifications while staying within the basic homogeneous good Cournot framework. In the subsequent section, we consider mergers in a market with differentiated products.

15.2 MERGERS AND COST SYNERGIES

In presenting the merger paradox, we assumed that all firms in the market have identical costs and that there are no fixed costs. What happens if we relax these assumptions? It seems reasonable to suppose that if a merger creates sufficiently large cost savings it should be profitable. In this section, we develop an example to show that this can indeed be the case.[6]

[6] This is a special case of a much more sophisticated analysis by Farrell and Shapiro (1990) who show in a general setting that for consumers to benefit from a profitable horizontal merger of Cournot firms the merger has to create substantial cost synergies.

Suppose that the market contains three Cournot firms. Consumer demand is given by

$$P = 150 - Q \tag{15.11}$$

where Q is aggregate output, which pre-merger is $q_1 + q_2 + q_3$. Two of these firms are low-cost firms with a marginal cost of 30, so that total costs at each are given by

$$C_1(q_1) = f + 30q_1; C_2(q_2) = f + 30q_2 \tag{15.12}$$

The third firm is potentially high-cost with total costs given by

$$C_3(q_3) = f + 30bq_3 \tag{15.13}$$

where $b \geq 1$ is a measure of the cost disadvantage from which firm 3 suffers. In these cost functions f represents fixed costs associated with overhead expenses such as those for marketing or for maintaining corporate headquarters. We now consider the effect of a merger of firms 2 and 3.

15.2.1 The Merger Reduces Fixed Costs

Consider first the case in which $b = 1$ so that all firms have the same marginal cost of 30. Suppose, however, that after the merger, the merged firm has fixed costs af with $1 \leq a \leq 2$. What this means is that the merger allows the merged firms to economize on overhead costs, for example by combining the headquarters of the two firms, eliminating unnecessary overlap, combining R&D functions, and economizing on duplicated marketing efforts. These are, in fact, typical cost savings that most firms expect, or at least state they expect, to result from a merger.

Because the merger leaves marginal costs unaffected, this is similar to our first example, only now firms also have fixed costs. Accordingly, we know that in the pre-merger market each firm earns a profit of $\$900 - f$. In the post-merger market with just two firms, the nonmerged firm earns a profit of $\$1{,}600 - f$ while the merged firm earns $\$1{,}600 - af$. Hence, for this merger to be profitable, it must be the case that $\$1{,}600 - af > \$1{,}800 - 2f$ which requires that $a < 2 - 200/f$. What this says is that a merger is more likely to be profitable when fixed costs are relatively high and the merger gives the merged firm the ability to make "substantial" savings in these costs. Note, however, that even if the merger is profitable for the merging firms, consumers are actually worse off as a result of the higher equilibrium price. That same higher price also raises the profit of the nonmerged firm. Moreover, it is still the case that the merged firm loses market share after the merger.

15.2.2 The Merger Reduces Variable Costs

Now consider the case in which the source of the cost savings is not a reduction in fixed costs but instead a reduction in variable costs which we capture by assuming that $b > 1$. In other words, firm 3 is a high variable cost firm. It follows that after a merger of firms 2 and 3, production will be rationalized and the high-cost operations will be shut down (or redesigned to operate the low cost technology). To make matters as simple as possible, we assume that there are no fixed costs ($f = 0$).

Once again, we assume a Cournot framework. The outputs and profits of the three firms prior to the merger are:

$$q_1^C = q_2^C = \frac{90 + 30b}{4}; \quad q_3^C = \frac{210 - 90b}{4} \text{ and}$$
$$\pi_1^C = \pi_2^C = \frac{(90 + 30b)^2}{16}; \quad \pi_3^C = \frac{(210 - 90b)^2}{16} \tag{15.14}$$

The equilibrium pre-merger price is[7] $P^C = \frac{210 + 30b}{4}$. Total output is $Q = \frac{390 - 30b}{4}$ with each of the low cost firms, 1 and 2, producing a greater amount than their high cost rival, firm 3.

Now, as before, suppose that firms 2 and 3 merge. Because for any $b > 1$, it is always more expensive to produce a unit of output at firm 3 than it is at firm 2, all production will be transferred to firm 2's technology. The result is that the market now contains two identical firms, 1 and 2, each with marginal costs of \$30. Accordingly, in the post-merger industry, each firm produces 40 units, the product price is \$70 and each firm earns \$1,600.

Is this a profitable merger? For the merger to increase aggregate profit of the merged firms it must be the case that

$$1600 - \left(\frac{(90 + 30b)^2}{16} + \frac{(210 - 90b)^2}{16}\right) > 0 \tag{15.15}$$

You can check that this simplifies to

$$\frac{25}{2}(7 - 3b)(15b - 19) > 0 \tag{15.16}$$

The first bracketed term in equation (15.16) has to be positive for firm 3 to have been in the market in the first place. (See footnote 7.) So the merger is profitable provided that the second bracket is also positive, which requires that $b > 19/15$. In other words, *a merger between a high-cost and a low-cost firm is profitable provided that the cost disadvantage of the high-cost firm prior to the merger is "large enough."* In the case at hand, large enough means that firm 3's unit cost is at least 25 percent greater than firm 2's unit cost. However, as we have already demonstrated, whether the merger is profitable or not, price rises and consumers are made worse off.

Together, our analysis of a merger that generates fixed cost savings and one that generates variable cost savings makes clear that mergers can be profitable when the cost savings are great enough. However, there is no guarantee that consumers gain from such a merger. Admittedly, the merger removes a relatively inefficient technology but it also reduces competitive pressures between the remaining firms. Farrell and Shapiro (1990) demonstrate that in the Cournot setting used here, the cost savings necessary to generate a gain for consumers are much larger than those needed simply to make the merger profitable. In turn, this suggests that we should be skeptical of cost savings as a justification of the benefits to consumers of horizontal mergers.

[7] Note that this equilibrium exists only if there is a limit on the disadvantage b for firm 3. Specifically, firm 3's pre-merger output in equation (15.14) will be positive only if $b < 210/90 = 7/3$, otherwise it will not operate in this market in the first place.

Research by both Lichtenberg and Siegel (1992) and Maksimovic and Phillips (2001) finds that merger related productivity gains and therefore marginal cost savings, while real, are typically no more than 1 to 2 percent. Salinger (2005) expresses even more doubt that fixed cost savings are substantial. Beyond all this, it is also worth noting that even with cost savings, part of our initial paradox still remains because large profit gains continue to accrue to the firms that do not merge. Why should a firm incur the headaches of merging if it can enjoy many of the same benefits by free-riding on other mergers?[8]

15.2 **Practice Problem**

Return to the market for carpet-cleaning services in Dirtville, now described by the demand function $P = 180 - Q$. Suppose that there are currently three firms that clean carpets in the area. The unit cost of cleaning a carpet is constant and equal to \$30 for two firms and is \$30b for the third firm, where $b \geq 1$. In addition, all firms have fixed overhead costs of \$900. Firms in this industry compete in quantities.

a. What is the Cournot–Nash equilibrium price and what are the outputs and profits of each firm? What is the upper limit on b for the third firm to be able to survive?
b. Now suppose that a low-cost firm merges with the high-cost firm. In doing so, the fixed costs of the merged firm become \$900a with $1 \leq a \leq 2$. What is the post-merger equilibrium price? What are the outputs of the nonmerged and the merged firms?
c. Derive a relationship between a and b that is necessary to guarantee that the profit earned by the merged firm is sufficient to compensate all the shareholders/owners who owned the two original firms and earned profit from them in the pre-merger market game. Comment on this relationship.

15.3 THE MERGED FIRM AS A STACKELBERG LEADER

If cost efficiencies are not a promising way to resolve the merger paradox, then perhaps a resolution can be found in some other change that gives the merged firm an advantage. One possibility is that merged firms become Stackelberg leaders in the post-merger market.[9] Recall from our discussion in Section 11.1 that the source of a Stackelberg leader firm's advantage is its ability to commit to an output before output decisions are taken by the follower firms. This permits a leader to choose an output that takes into account the reactions of the followers.

Let us assume that a merged firm acquires a leadership role and see whether the commitment power associated with market leadership can resolve the merger paradox. Certainly, such a role seems plausible. After all, the new firm has a combined capacity twice that of any of its nonmerged rivals, and so might well be able to act as a Stackelberg leader. Will this be enough to make a merger profitable? If so, what will be the response of other firms? Will they also have an incentive to merge? If they do, will their merging undo the profitability of the first merger and thereby, if firms are foresighted, discourage them from merging in the first place?

[8] Perry and Porter (1985) assume that each firm's cost schedule declines with the total amount of capital it owns. Hence, by merging and gaining more capital, a firm lowers its costs. The scarcity of capital makes it difficult for other firms to do this and, because of rising costs, to free-ride as much on the merger of rivals.

[9] This analysis draws on the model of A.F. Daughety (1990), who suggested this role for the merged firms.

Suppose that demand is of the usual linear form: $P = A - BQ$. There are $N + 1$ firms in the industry and each of the $N + 1$ firms has a constant marginal cost of c. We know from the standard Cournot model that the equilibrium is described by the following equations:

$$q_i = \frac{A - c}{(N + 2)B} \Rightarrow Q = \frac{(N + 1)(A - c)}{(N + 2)B} \text{ and } P = \frac{A + (N + 1)c}{N + 2} \tag{15.17}$$

The profit of each firm, $(P - c)q_i$ is therefore:

$$\pi_i = \frac{(A - c)^2}{B(N + 2)^2} \tag{15.18}$$

Suppose now that two of these firms merge and, as a result, become a Stackelberg leader. The market now contains $F = N - 1$ follower firms and one leader firm so that we now have N firms in total. Of course, the Stackelberg leader is able to choose its output first in a two-stage game. In stage one, the leader chooses its output Q^L. In the second stage, the follower firms independently choose their outputs in response to that chosen by the leader.

To find the equilibrium, we work through the game backwards. Consider the second stage of the game in which the follower nonmerged firms make their output decisions in response to the output choice Q^L of the leader or merged firm. We use the notation Q_{F-f} to denote the aggregate output of the follower firms *other than* f, and denote the output of follower firm f by q_f. Then aggregate output of *all* firms is $Q = Q^L + Q_{F-f} + q_f$. Moreover, the residual demand for firm f, which is the demand left after taking into account the outputs of the leader and the followers other than firm f is:

$$P = [A - B(Q^L + Q_{F-f})] - Bq_f \tag{15.19}$$

Marginal revenue for firm f is, therefore,

$$MR_f = [A - B(Q^L + Q_{F-f})] - 2Bq_f. \tag{15.20}$$

Equating this with marginal cost gives the best response function for firm f:

$$A - 2Bq_f - BQ^L - BQ_{F-f} = c \Rightarrow q_f^* = \frac{A - c}{2B} - \frac{Q^L}{2} - \frac{Q_{F-f}}{2} \tag{15.21}$$

Equation (15.21) is the best response of a follower firm to both the output of the leader and the output of all the other follower firms. Because all follower firms are identical, symmetry demands that in equilibrium the output of each of the follower firms must be identical. The group of followers excluding firm f has $F - 1 = N - 2$ firms. Therefore, $Q^*_{F-f} = (N - 2)q^*_f$. Substituting this into equation (15.21) and simplifying gives the optimal output for each nonmerged follower firm as a function of the aggregate output of the leader pair of merged firms:

$$q_f^* = \frac{A - c}{BN} - \frac{Q_L}{N} \tag{15.22}$$

The aggregate output of all followers as a function of the output of the leader is then

$$Q^F = (N-1)q_f^* = \frac{(N-1)(A-c)}{BN} - \frac{(N-1)Q^L}{N} \tag{15.23}$$

We can use the same basic technique to determine the output for the leader firm in stage one of the game. The residual inverse demand function for the leader firm is dependent upon the output of all the other firms, which is given by equation (15.23). So, the demand function facing leader firm l is:

$$P = A - B(Q^F + Q^L) = A - B\left[\frac{(N-1)\,(A-c)}{BN} - \frac{(N-1)Q^L}{N}\right] - BQ^L$$
$$P = A - \frac{(N-1)(A-c)}{N} - \frac{B}{N}Q^L \tag{15.24}$$

Its associated marginal revenue function is:

$$MR_l = A - \frac{(N-1)(A-c)}{N} - \frac{2B}{N}Q^L \tag{15.25}$$

Equating this marginal revenue with marginal cost allows us to solve for the leader firm's optimal output:

$$MR_l = c \Rightarrow Q^L = \frac{A-c}{2B} \tag{15.26}$$

You should by now recognize that the output level in equation (15.26) is just the output level chosen by a uniform-pricing monopolist. This is, of course, a standard result for a single leader model with linear demand and constant costs. In turn, this implies the following industry equilibrium values:

$$q_f^* = \frac{A-c}{2BN};\ Q^F = \frac{(N-1)(A-c)}{2BN};\ Q = Q^L + Q^F = \frac{(2N-1)(A-c)}{2BN};$$
$$P = \frac{A + (2N-1)c}{2N} \tag{15.27}$$

Profits for the leader and for each follower firm are then:

$$\pi^L = \frac{(A-c)^2}{4BN} \text{ and } \pi^F = \frac{(A-c)^2}{4BN^2} \tag{15.28}$$

Comparison of equation (15.28) with (15.18) reveals that for any industry initially comprising three or more firms and characterized by symmetric Cournot competition, a two-firm merger that creates a Stackelberg leader is profitable for the merged firms. This seems to resolve the merger paradox. However, equations (15.28) and (15.18) also show that the unmerged firms who have become followers are definitely worse off as a result of the merger. We may therefore expect some response from these firms.

Furthermore, if we compare the market price and output in (15.17) with that in (15.27), we find that while the merger has raised the profit of the merging parties, it also has lowered price. Hence, the merger is good for consumers. We seem to have replaced one paradox with another. We now have a model in which a merger is profitable, but that model also removes a principal reason why the antitrust authorities should object to such a merger.

However, we need to consider the response of other firms to the merger. Because leadership confers additional profit, they, too, have an incentive to merge and become leaders. This raises the question as to what happens if there is more than one two-firm merger. Daughety's (1990) model answers this question by assuming that there can be more than one leader firm and merging is the ticket to entry into the club of such leaders. That is, imagine a market that may be divided into two groups of firms: followers and leaders. The first of these groups acts just as the followers did in the preceding analysis. They compete as Cournot rivals over the demand remaining after the leaders make their output decisions. The group of leaders understands this reaction. They compete as Cournot rivals *against each other* in the knowledge that they act first and the followers take their production decisions as given.

To analyze this two-stage competition, we can use the model derived above. In particular, instead of assuming N firms with one leader and $N - 1$ followers, we can assume that there are N firms with L leaders and $N - L = F$ followers. The detailed calculations for this version of the model are presented in the Appendix. Here we concentrate on the resulting price and profit equations. These imply that in an industry comprised of N firms in total, L of which are leaders, the price-cost margin $(P - c)$, the profits for the typical leader firm $(P - c)\, q_l^*$, and typical follower firm $(P - c)q_f^*$ are:

$$P(N, L) - c = \frac{A - c}{(L + 1)(N - L + 1)} \tag{15.29}$$

$$\pi^L(N, L) = \frac{(A - c)^2}{B(L + 1)^2(N - L + 1)} \tag{15.30}$$

$$\pi^F(N, L) = \frac{(A - c)^2}{B(L + 1)^2(N - L + 1)^2} \tag{15.31}$$

You can readily confirm that the profit values shown in equations (15.30) and (15.31) for the general case of N total firms with L leaders yields the same profits as those given in equation (15.28) for the special case of N total firms and $L = 1$ leader.

It is clear from these profit equations that the leader firms are individually more profitable than the nonmerged followers. However, that is not the real issue facing two firms that are contemplating merger. The question is whether *one more merger* is profitable, given that there will then be one more leader, two fewer followers, and one less firm in total. This is why we have written the profit expressions as functions of N and L. The point is that an additional merger creates two countervailing forces. On the one hand, there are fewer firms in total, which ought to increase profits, but there are also more leaders, which ought to decrease the profits of the leaders. Which force is greater?

Suppose there is an additional merger of two followers, so that the newly merged firm and all other leaders earn profit given by equation (15.30) with N replaced by $N - 1$ and L replaced by $L + 1$ to give $\pi^L(N - 1, L + 1)$. For there to be an incentive to merge, this profit must exceed the combined profit earned by the two follower firms prior to the merger. This latter profit is $2\pi_j^F(N, L)\ 2\pi_f^F(N, L)$. So, the merger will be profitable if the following condition is satisfied:

$$\pi^L(N-1, L+1) = \frac{(A-c)^2}{B(L+2)^2(N-L-1)} > 2\pi^F(N, L) = \frac{2(A-c)^2}{B(L+1)^2(N-L+1)^2} \tag{15.32}$$

This simplifies to the condition

$$(L+1)^2(N-L+1)^2 - 2(L+2)^2(N-L-1) > 0 \tag{15.33}$$

Note that this condition does not include the demand parameters A and B or the marginal cost c. In other words, the profitability or otherwise of this type of merger depends only on the number of leaders and followers, not on the precise demand and cost conditions.

We show in the Appendix that the condition in (15.33) is always met.[10] In other words, starting from any configuration of leaders and followers, *an additional two follower firms always wish to merge*.

This result is encouraging. It says that the Daughety model offers one way to resolve the merger paradox. A merger raises the profit of the two merging firms by allowing them to take a position as one of, perhaps several, industry leaders. Moreover, the fact that such a merger is always profitable also helps us to understand better the domino effect so often observed within an industry. Once one firm merges and becomes a leader, the remaining firms will wish to do the same rather than watch their output and their profits be squeezed.

15.3 **Practice Problem**

Return again to the town of Dirtville where the inverse demand for carpet-cleaning services is described by $P = 130 - Q$. Once again assume that there are twenty identical firms that clean carpets in the area, and the unit cost of cleaning a carpet is constant and equal to \$30. Firms in this industry compete in quantities.

a. Show that in a Cournot equilibrium the aggregate number of carpets cleaned is $Q = 95.24$. What is the equilibrium price?
b. Suppose that five two-firm mergers occur, that these five merged firms become leader firms, and the remaining ten nonmerged firms are followers. Now there are fifteen firms in the industry. Work through the model just described and show that in the two-stage game a leader firm cleans 16.67 carpets and each follower firm cleans 1.51 carpets. Leadership certainly has its benefits! Show that the total industry output in this case will be $Q = 98.45$. What is the equilibrium price now?
c. If after the five two-firm mergers took place there were no leadership advantages conferred to the merged firms, then we would have fifteen firms competing like Cournot firms in the market. Show that in this case aggregate output is $Q = 93.75$.

[10] We are grateful to our colleague Professor Boris Hasselblatt of the Tufts University Mathematics Department for this proof.

While the Daughety model can resolve the merger paradox, it does leave unanswered the question as to whether such mergers are in the public interest. Is there some point at which further mergers are harmful to consumers? The answer to this question can be most easily derived from the price-cost margin $P(N, L) - c$ shown in equation (15.29). Because marginal cost c is constant, any rise or fall in $P(N, L)$ will be reflected in a rise or fall of $P(N, L) - c$.

With L leader merged firms and $N - L$ follower nonmerged firms, the price-cost margin is $\dfrac{A-c}{(L+1)(N-L+1)}$. An additional two-firm merger increases L to $L + 1$ and decreases N to $N - 1$, so that the price-cost margin is now $\dfrac{A-c}{(L+2)(N-L-1)}$. Thus for this additional merger to benefit consumers it must be the case that:

$$\frac{A-c}{(L+2)(N-L-1)} < \frac{A-c}{(L+1)(N-L+1)}$$
$$\Rightarrow (L+1)(N-L+1) < (L+2)(N-L-1) \Rightarrow N - 3(L+1) > 0 \tag{15.34}$$

What this tells us is that an additional two-firm merger benefits consumers only if $N > 3(L + 1)$ or, equivalently, $L < N/3 - 1$. In other words, *a two-firm merger that increases the number of leaders benefits consumers only if the current group of leaders contains fewer than a third of the total number of firms in the industry*. We know from equation (15.33) that a two-firm merger that creates a leader will always be profitable. Yet, as we have also just shown, such a merger will be harmful to consumers once the leader group includes one-third or more of the industry's firms. In other words, some mergers are bad—at least for consumers. Accordingly, we now have a model that both resolves the merger paradox and explains why the antitrust authorities are correct to worry about anticompetitive mergers.

For example, return to Practice Problem 15.3 in which we had five leader firms and ten follower firms cleaning carpets in Dirtville. In that scenario, we know that the equilibrium price for cleaning a carpet is \$31.55. Now suppose that two additional firms merge to join the leadership group. We then have a market structure of six leaders and eight followers. In this case, the equilibrium price for cleaning a carpet is \$31.60. This merger harms the consumers in Dirtville.

Daughety's model solves the merger paradox and gives rise to a merger wave by assuming an asymmetry between newly merged firms and their remaining unmerged rivals. The former gain membership in the club of industry leaders. However, this is a rather strong assumption. While some mergers may create corporate giants with an ability to commit to large production levels, it is far from obvious that every two-firm merger should have this leadership role regardless of which two firms are joined and irrespective of the number of leaders already present. In principle, Daughety's model implies that in an industry of ten firms there could be, say, eight leaders. It seems odd to imagine a configuration with so many leaders and so few followers. Moreover, it leaves unanswered the question as to what happens if two leaders merge. Does this merger create a super-leader?

It is also worth noting that while output decisions are sequential in Daughety's model, merging is not. While leader firms choose production first, it is not accurate to describe the decision to merge in a sequential way. The model simply says that for any market configuration, if a two-firm merger creates an industry leader, all follower firm pairs will

Reality Checkpoint

At First Gush: Merger Mania and Spinoffs in the Oil Industry

In August of 1998, British Petroleum or BP announced plans to merge with Amoco, another large oil firm although not quite as large as BP. The price tag was $48.2 billion making it, at the time, the biggest industrial merger ever. The new BP-Amoco would control more oil and gas production within North America than any other firm. It would also be the third-largest publicly traded oil firm in the world. (The largest firm of all, Saudi Aramco, is not publicly traded.)

Reaction from the rest of the oil industry came swiftly. Within a year, Exxon and Mobil merged in a deal worth $73.7 billion to become the largest publicly traded firm on earth. That was quickly followed by the merger of Phillips Petroleum and Conoco. Almost simultaneously, Paris-based Total acquired both PetroFina and Elf to create TotalFinaElf. Soon after, Chevron acquired Texaco for $36 billion. BP then went a step further and acquired Arco for $27 billion. The oil merger wave subsided with the economic decline of 2000–2001 but, even then, did not die altogether. Chevron acquired Unocal in 2005.

This wave of merger activity concentrated oil and gas refining and marketing into the hands of a noticeably smaller number of firms relative to the situation prior to BP's purchase of Amoco. The BP-Amoco merger was then the catalyst for a major wave of mergers and consolidations. In turn, this suggests that a common motive must be behind all these mergers. Yet whether this common factor was the naked pursuit of market power or simply the profit-maximizing response of firms to similar problems is difficult to say. The mergers were taken at a time when energy prices were quite low. Oil, for example, was selling at less than $12 per barrel in 1998. Oil prices and profits have risen dramatically since that time and, correspondingly, energy sector merger activity has stalled and, in come cases, been reversed. Indeed, in 2011, ConocoPhillips de-merged itself by spinning off its oil exploration and production activities from its oil refining and distribution activities—selling off the latter as an independent firm. In so doing, ConocoPhillips was following the decision of Marathon Oil to do the same thing earlier that year. This pattern of oil mergers and de-mergers across the energy sector suggests that the major motivation is a common technical factor—not market power.

Sources: Jim Wells, "Energy Markets: Factors Contributing to Higher Gasoline Prices," Statement of Director of Natural Resources and Environment, General Accounting Office, to US Senate Judiciary Committee, 1 February 2006; and B. Bahree, C. Cooper, and S. Liesman, "BP to Buy Amoco in Biggest Industrial Merger Ever," *Wall Street Journal*, August 12, 1998, p. A1; R. Beales and J. Bush, "The Advantages of Breaking Up," *New York Times*, August 31, 2011, p. B2.

wish to merge as well. One pair does not merge only after it sees another pair merge. Instead, at any single point in time, merging is a dominant strategy and, absent any antitrust intervention, all follower firms will pursue it. Again, this is not because of any new cost savings or product development. It is simply because merging confers leadership status. Daughety's model does not give rise to the sporadic merger waves that we often see as much as it suggests an ever-present tendency for the industry to become more concentrated.

15.4 SEQUENTIAL MERGERS

To capture the idea that merger decisions may be explicitly sequential, i.e., that the decision of one firm pair to merge is a catalyst for another pair to do the same, a number of papers including Nilssen and Sørgaard (1998), Fauli-Oller (2000) and Salvo (2010) have presented models in which a sudden change in cost or product qualities gives rise to merger opportunities that are only profitable if other mergers also occur. It is difficult for this to happen in a simultaneous game because each potential merger pair cannot be sure if others will also merge. However, in a sequential game, some firms get to make their merger decision knowing for certain that others have already merged. This greatly enhances the likelihood of a successful merger.

We illustrate the sequential merger model with a simplification of the Fauli-Oller (2000) model. Consider a four-firm industry characterized by Cournot competition. Initially, all of these firms are high-cost firms with constant unit cost $c^h = c$. Suppose that two firms have had a technical breakthrough that allows them to become low-cost firms with low constant unit cost $c^l = 0$. Industry demand is described by: $P = A - Q$.

Now consider the following sequential scenario. In period 1, low-cost firm 1 decides whether to merge with one of the high-cost firms 3 and 4. Without loss of generality, suppose that firm 1 chooses whether or not to merge with firm 3. In period 2 low-cost firm 2, having observed firm 1's decision, decides whether or not to merge with the remaining high-cost firm 4. The game ends after period 2. To determine the subgame perfect equilibrium we need the pay-offs detailed in Table 15.2. Clearly, for these to make sense we must have $A > 3c$ otherwise a high-cost firm cannot be active in the no-merger case.

Rather than work with this general case, we consider a specific example in which $A = 100$ and $c = 10$. This gives the profits reported in Table 15.3.

First note that a merger in period 1 by firms 1 and 3 *not* followed by a merger of firms 2 and 4 is unprofitable for the merged firms. Merger gives them a combined profit of \$756.25 whereas not merging gives aggregate profit to these two firms of $\$(576 + 196) = \$772$.

Table 15.2 Pay-offs for the sequential merger game

Number of Mergers	*Profit of Low-Cost Firm*	*Profit of High-Cost Firm*
No Mergers	$\frac{(A+2c)^2}{25}$	$\frac{(A-3c)^2}{25}$
One Merger	$\frac{(A+c)^2}{16}$	$\frac{(A-3c)^2}{16}$
Two Mergers	$\frac{A^2}{9}$	NA

Table 15.3 Pay-offs for the sequential merger game

Number of Mergers	*Profit of Low-Cost Firm*	*Profit of High-Cost Firm*
No Mergers	\$576	\$196
One Merger	\$756.25	\$306.25
Two Mergers	\$1,111.11	NA

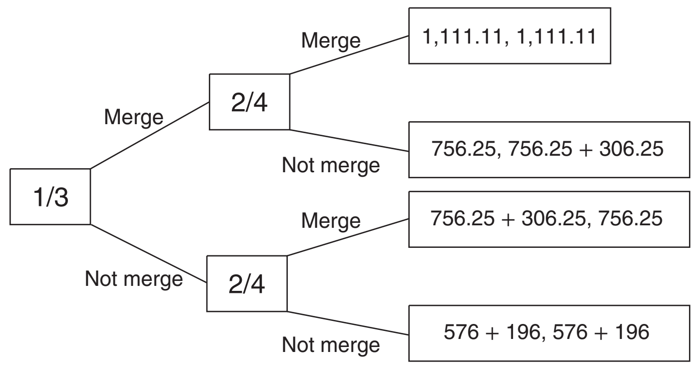

Figure 15.1 Sequential mergers

What we now show is that assuming mergers to be sequential actually leads to merger being profitable in our example.

The extensive form of the game is illustrated in Figure 15.1. In formulating the game, we assume that a merger offer will not be made unless the post-merger profit of the merged entity is greater than the aggregate profit of the two firms pre-merger. If this were not the case, no merger offer could be made that is satisfactory to both firms. Thus the pay-offs in Figure 15.1 for a pair of unmerged firms are aggregate profits As usual we solve this game backwards.

Suppose first that firms 1 and 3 choose to merge. If firms 2 and 4 choose not to merge they earn aggregate profit $\$(756.25 + 306.25) = \$1062.50$ whereas if they merge they earn profit \$1,111.11. They will choose to merge. Suppose instead that firms 1 and 3 choose not to merge. If firms 2 and 4 also choose not to merge they earn aggregate profit $\$(576 + 196) = \$772$ whereas if they merge they earn profit \$756.25. They will choose not to merge.

Firms 1 and 3 can now see that if they merge this will be followed by a merger of firms 2 and 4, with the result that firms 1 and 3 as a merged entity earns profit \$1,111.11. By contrast, if firms 1 and 3 do not merge, firms 2 and 4 will also choose not to merge, with the result that firms 1 and 3 earn aggregate profit of \$772. Firms 1 and 3 will therefore choose to merge, in the knowledge that their merger will be followed in the next period by a merger of firms 2 and 4. As a result, we can expect a merger wave in which first one pair merges and then the second pair merges. In other words, conditional on the first merger taking place, the second merger is profitable. In effect, the sequential nature of the game allows the first pair to commit credibly to merging. In turn, this means that the second merger pair does not have to worry that in merging they may be acting alone.

Such a merger wave is, however, not good for consumers. Prior to either merger talking place price is $(A + 2c)/5 = \$24$ whereas after the merger wave it is $A/3 = \$33.33$.

The foregoing story is not limited to just two mergers or to models of Cournot competition. Once cost asymmetries or product quality differences are introduced, we can construct sequential merger models that lead to merger waves for a large number of firms in a variety of settings, e.g., Nilssen and Sørgaard (1998) and Salvo (2006), and these mergers are also anticompetitive. This approach offers another resolution to the merger paradox not simply because it demonstrates why mergers may happen but, in addition, why they often happen in sequential waves. As with Daughety's (1990) model, these models also justify concern over the impact that mergers may have on consumer prices.

15.5 HORIZONTAL MERGERS AND PRODUCT DIFFERENTIATION

Our analysis of mergers has so far been set in the Cournot framework of identical products and quantity competition. However, many firms expend considerable effort differentiating their products and this differentiation gives them some latitude in setting their price. Accordingly, we also need to consider the incentives for and the impact of mergers in industries in which firms produce and market differentiated products.

It is particularly important to explore the merger phenomenon in differentiated product markets for at least two reasons. First, firms are often price setters in such markets and the nature of competition is different with price competition than with quantity competition. In quantity competition, firms' best response functions are downward sloping, i.e., quantities are strategic substitutes. Thus, when merging occurs, the nonmerged firms want to *increase* their outputs in response to the lower output produced by the merger. This response undermines the effectiveness of the merger. By contrast, with price competition, best response functions are upward sloping: prices are strategic complements. A merger leading to an increase in the merged firms' price(s) will encourage the nonmerged firms also to increase their prices, potentially strengthening the effectiveness of the merger.

Second, we saw that one reason for the merger paradox under Cournot competition is that the merged firm looks no different from a nonmerged firm. If m firms merge, effectively $m - 1$ of them disappear. This is not the case with differentiated products. Merger allows coordination of the prices of the products offered by the merged firm *and also* allows the merged firm to keep all m products on the market.

We develop this intuition more explicitly using two different approaches to product differentiation. The first approach is to extend our standard linear demand representation of consumer preferences to incorporate product differentiation. The second is to adopt the spatial model of horizontal differentiation, which we first introduced in Chapter 4 and then revisited in Chapter 10.[11]

15.5.1 Bertrand Competition and Merger with Linear Demand Systems

Suppose that there are three firms in the market, each producing a single differentiated product.[12] Inverse demand for each of the three products is assumed to be given by:

$$\begin{aligned} p_1 &= A - Bq_1 - s(q_2 + q_3) \\ p_2 &= A - Bq_2 - s(q_1 + q_3) \qquad (0 \leq s < B) \\ p_3 &= A - Bq_3 - s(q_1 + q_2) \end{aligned} \tag{15.35}$$

In these inverse demand functions, the parameter s measures how similar the three products are to each other. If $s = 0$ the products are totally differentiated. In this case, each firm

[11] The spatial model was first formulated in Hotelling (1929), and subsequently extended in Schmalensee (1978) and Salop (1979). We saw in Chapters 4, 7, and 10 that this sort of spatial model has proven insightful in analyzing a variety of topics in industrial organization, including brand proliferation in the ready-to-eat breakfast cereal industry, Schmalensee (1978), and the effects of deregulation of transport services such as airlines or passenger buses, Greenhut, Norman, and Greenhut (1991). It is not surprising that the spatial model is also useful in analyzing mergers of firms selling differentiated products.

[12] An excellent development of the full analysis can be found in Deneckere and Davidson (1985).

is effectively a monopolist. By contrast, as s approaches B the three products become increasingly identical, moving us closer to the homogeneous product case. We also assume that the three firms have identical marginal costs of c per unit. Finally, assume that the three firms are Bertrand competitors, i.e., they compete in prices and set their prices simultaneously.

We show in the Appendix to this chapter that when these firms compete they each set a price of $p^*_{nm} = \dfrac{A(B-s)+c(B+s)}{2B}$ and each sell quantity $q^*_{nm} = \dfrac{(A-c)(B+s)}{2B(B+2s))}$. Profit of each firm is

$$\pi^*_{nm} = \frac{(A-c)^2(B-s)(B+s)}{4B^2(B+2s)} \tag{15.36}$$

Now suppose that firms 1 and 2 merge but that the merged and nonmerged firms continue to set their prices simultaneously. The two previously independent, single-product firms are now product divisions of a two-product merged firm, coordinating their prices to maximize the joint profit of the two divisions. The result is that the merged firm sets its product prices to $p^m_1 = p^m_2 = \dfrac{A(2B+3s)(B-s)+c(2B+s)(B+s)}{2(2B^2+2Bs-s^2)}$ while the remaining nonmerged firm 3 sets its product price as $p^{nm}_3 = \dfrac{A(B+s)(B-s)+cB(B+2s)}{(2B^2+2Bs-s^2)}$.

It is straightforward to confirm that the merger increased the prices of all three products, as we might have expected because the merger reduces competitive pressures in the market. However, there remains the question of the merger's profitability. The profits of each product division (1 and 2) of the merged firm, and the profit of the independent nonmerged firm 3 are, respectively:

$$\pi^m_1 = \pi^m_2 = \frac{(A-c)^2B(B-s)(2B+3s)^2}{4(B+2s)(2B^2+2Bs-s^2)^2};\ \pi^m_3 = \frac{(A-c)^2(B-s)(B+s)^3}{(B+2s)(2B^2+2Bs-s^2)^2} \tag{15.37}$$

In comparing equations (15.37) and (15.36), we can simplify matters by normalizing $A-c=1$ and $B=1$, so that profits are functions solely of the degree of product differentiation s. It is then easy to confirm that this two-firm merger is profitable for the merged firm *and* for the nonmerged firm. More generally, Deneckere and Davidson (1985) show that in a market containing N firms, any merger of $M \geq 2$ firms is profitable for the merged firms and for the nonmerged firms. This simple framework of price setting in a product differentiated market avoids the merger paradox, suggesting that mergers are both profitable and of potential concern to antitrust authorities unless accompanied by cost efficiencies.

15.5.2 Mergers in a Spatial Market

In the spatial model, a merger between two firms may well bring increased profit for reasons similar to those in the previous section. Although merging means that the firms lose their separate identity, they do not lose the ownership or control of the product varieties they offer. For example, the merger of two major banks, Bank of America and Fleet Bank, results in a single new corporate entity. Yet it does not require that the new firm give up any of the locations at which either Bank of America or Fleet currently operate—or that it lose control over the choice of moving some of those locations. Similarly, the acquisition many years ago of American Motors by Chrysler did not mean that the Jeep product line disappeared.

When we consider a firm's product lines, there is a second source of potential profit increase. The merged firms can now coordinate not just the prices but also the design of their product line, or in the context of the spatial model, their location choices. Chrysler can redesign the Jeep line to fit better in its overall range of models. Similarly, Bank of America and Fleet can change the locations of their branches in those areas where each formerly operated an outlet quite close to the other.

To investigate the impact of a merger in the spatial model, we begin by recalling the basic setup of the model.[13] There is a group of consumers who are uniformly distributed over a linear market of length L. Again, we can think of this as Main Street in Littlesville. However, one small problem with the Main Street analogy is that outlets at either end of the market can only reach consumers on one side. This restriction introduces an asymmetry in the model, which we would like to avoid. To make the product differentiated market symmetric, we can bend the ends of the line around until they touch each other, and replace our straight line of length L with a circle of circumference L. For example, if we use the spatial model to represent departure times in the differentiated passenger airline market, the circle represents the twenty-four hours of the day over which consumers differ in terms of their most preferred time of departure. In all other respects, the spatial model remains as before.

Each consumer has an "address" indicating the consumer's location on the circle and, hence, the consumer's most preferred product type. Each consumer is also willing to buy at most one unit of a particular good. The consumer's reservation price for the most preferred good is denoted by V. Different varieties of the good are offered by the firms that are also located on Main Street—or, more appropriately, Main Circle.[14] A consumer buys from the firm that offers the product at the lowest price, taking into account the costs of transporting the good from the firm's address to the consumer's. We assume that these transport costs are linear in distance. If the distance between a firm and a consumer is d, the transport costs from the firm to the consumer are td, i.e., t is the transport cost per unit distance. Recall that in the nongeographic interpretation of the model, transport costs become the consumer's valuation of the loss of utility incurred by consuming a product with characteristics that are not the consumer's most preferred characteristics.

Suppose that there are five firms selling to a group of N consumers who are distributed evenly around the circle of circumference L. A firm is differentiated only by its location on the circle, and we assume that the distance between any two neighboring firms is the same and equal to $L/5$. Each firm has identical costs given by $C(q) = F + cq$, where F is fixed cost and c is (constant) marginal cost. In contrast to our earlier merger analysis, we do not set F, the fixed cost, equal to zero, but instead set unit cost $c = 0$. This simplifies the analysis without losing any generality. What it does do is make it easy to talk about the price-cost margin, which is now just price, denoted by m for mill price.[15]

No Price Discrimination

We start by considering the case in which firms do not engage in price discrimination. This means that each firm sets a single mill price m that consumers pay at the firm's store

[13] A more general, but much more complicated version of this analysis can be found in Brito (2003).

[14] It bears repeating that the spatial or geographic interpretation of this model is only the most obvious one.

[15] If the reader is interested in working out the outcome for the case of $c \neq 0$, then we note here that in each case that we examine, the equilibrium price m^* that we derive should be replaced by $c + m^*$.

or mill location. The consumer then pays the fee for transporting the product back to the consumer's location. The full price paid by a consumer who buys from firm i is $m_i + td_i$, where m_i is firm i's mill price and td_i is the consumer's transport cost (or the utility lost by this consumer in buying a product that is not "ideal"). Because marginal cost is zero, the net revenue or profit margin earned by firm i on every such sale is m_i. Consumers buy from the firm offering the product at the lowest full price. As a result, for any set of mill prices across our five hypothetical firms $(m_1, m_2, m_3, m_4, m_5)$ the market is divided between the firms, as illustrated in Figure 15.2. The dotted lines indicate the market division between the firms. Firm 1, for example, supplies all consumers in the region (r_{15}, r_{12}).

When the firms set their prices noncooperatively and the maximum willingness to pay V is relatively large, the market is completely covered. That is, every consumer buys from some firm. Hence, the marginal consumer for any firm is the one who is just indifferent between buying from that firm and buying from one of the firm's neighbors.[16] We show in the Appendix to this chapter that in equilibrium the mill price set by each firm is $m_i^* = tL/5$. At this price, the profit earned by each firm is

$$\pi_i^* = \frac{NtL^2}{25} - F \tag{15.38}$$

The market outcome is illustrated in Figure 15.3, in which we have "flattened out" the circular market to simplify the geometry; that is, firm 1 is to the left of firm 5 and firm 5 is to the right of firm 1. In Figure 15.3, the vertical distance is the effective price—mill price plus transport cost—that each buyer pays. The sloped lines show that this price rises for consumers who live farther from a firm.

Now consider a merger between some subset of these firms. The first point to note is that, taking store locations or product choice as given, *such a merger will have no effect*

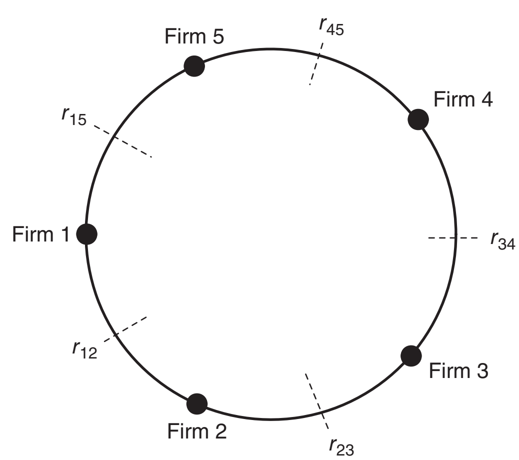

Figure 15.2 Product differentiation—no price discrimination

[16] We assume no firm prices so low as to lure buyers from beyond its two immediate neighbors. See the Appendix to this chapter.

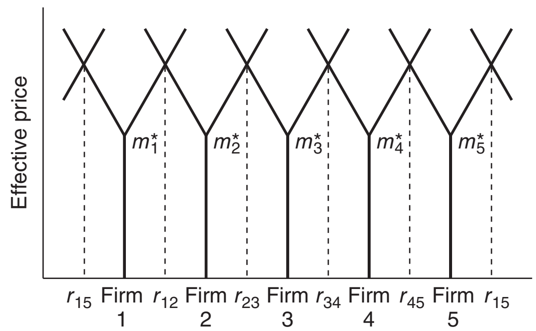

Figure 15.3 Price equilibrium without a merger

unless it is made between neighboring firms. A merger, for example, between firms 2 and 4 leaves prices and market shares unaffected. More generally, this suggests that a merger has no effect on the market outcome unless the market areas of the merging firms have a common boundary. The reason is straightforward. The merging firms hope to gain by softening price competition between them. This will happen only if, prior to the merger, they actually compete directly for some of the same consumers. For example, the merger of the two investment firms Wells Fargo and Morgan Stanley was not for the most part regarded as anticompetitive because the two firms market their services to different, or non-neighboring customers of households and businesses.

Mergers between neighboring firms, however, *do* alter the market outcome. Consider a merger between firms 2 and 3. Suppose that after the merger, the firms do not change either the locations of their existing products or the number of products they offer. Acting now as a single corporate firm with stores in two locations, the merged firm has an incentive to set prices to maximize the joint profits of both products 2 and 3, while the remaining firms continue to price noncooperatively. Of course, firms 1, 4, and 5 also take account of the fact that the merger has taken place. Because firms 2 and 3 are now cooperating divisions of the merged firm they no longer compete for the consumers located between them and so have an incentive to raise the prices of products 2 and 3.[17] This will likely lead to the loss of some consumers, namely, those just on the boundaries identified by the points r_{12} and r_{34}. But provided that the merged firm does not raise prices too much, the loss of market share will be more than offset by the increased profit margins on their "captive" consumers—the consumers between the two merging firms. Moreover, the increased prices set by the merged firm will induce a similar increase in prices set by firms 1, 4, and 5. Such a response reduces the loss of market share that the merged firm actually suffers making the price increase all the more profitable.

Again we show in the Appendix that the merger leads to a new equilibrium with the following prices:

$$m_2^* = m_3^* = \frac{19tL}{60}; m_1^* = m_4^* = \frac{14tL}{60}; m_5^* = \frac{13tL}{60} \tag{15.39}$$

[17] If the merger leaves products 2 and 3 under the control of separate, competing product divisions, prices will not change. The merged firms need to exploit the opportunity they now have to coordinate their prices.

Profits to each product are

$$\pi_2^* = \pi_3^* = \frac{361NtL^2}{7{,}200} - F;\, \pi_1^* = \pi_4^* = \frac{49NtL^2}{900} - F;\, \pi_5^* = \frac{169NtL^2}{3{,}600} - F \qquad (15.40)$$

This equilibrium is illustrated in Figure 15.4. Comparison with equation (15.38) confirms that this merger is profitable for the merging firms.

The equilibrium we have identified is based upon the assumption that the merged firms leave their product lines unchanged after the merger. What do we expect to happen if we relax this assumption? It turns out that the answer to this question depends upon the precise nature of transport costs. Consider the product location choice facing the newly merged firm 2 and firm 3. The firm faces a trade-off. On the one hand, relocating products 2 and 3 nearer to products 1 and 4, respectively, gives the merged firm two advantages. First, it softens the competition between the merged firm's own two product lines, so that when the firm tries to reach out to customers near the boundary with a lower price, there is less of a fear of simply "robbing Peter to pay Paul." Second, the move also makes it easier for the firm to steal some customers away from its true rivals, firms 1 and 4.[18] On the other hand, relocating products 2 and 3 further from products 1 and 4 offers potential advantages. Admittedly this gives up market share to the rivals but such a move also softens price competition, leading to increased prices by all firms. In our example with linear transport

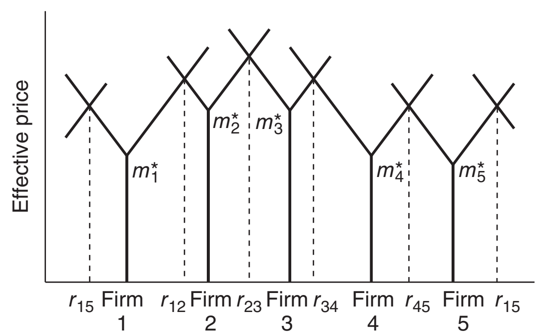

Figure 15.4 Price equilibrium—after merger of firms 2 and 3

18 There is one complication that we have ignored in this discussion. Judd (1985) argues that a merger that creates a multiproduct firm, as for example a merger of firms 2 and 3, may not be sustainable. The intuition is as follows: Assume that an entrant comes in exactly at firm 3's location after the merger of firms 2 and 3. Price competition will drive the price for this product down to marginal cost, in which case the entrant and the incumbent earn zero profits at this location (ignoring fixed costs). But the merged firm also loses money at the neighboring location 2 because the price war with the new entrant forces it to reduce the price there as well. If the merged firm were to close down its location 3 product, the entrant will raise price above marginal cost, and so the merged firm can raise the price at location 2. There is, in other words, a stronger incentive for the merged firm to exit location 3 than for the entrant to do so. Hence, this kind of multiproduct merger may not be sustainable because it is not credible. This argument turns, however, on two important assumptions: that entry costs are not recovered on exit and that the merged firm has no incentive to try to develop a reputation for toughness. Note that in a vertically differentiated context, it may turn out that the merging firms cease producing some versions of the good. See Norman, Pepall, and Richards (2002).

costs, it turns out that the merged firm will wish to relocate its products closer to the rivals 2 and 3. On the other hand, if transport costs were quadratic, of the form td_i^2 the merged firms would actually want to relocate *further* from their rivals.[19]

A merger between two firms in our spatial market is clearly advantageous to the merging firms but is disadvantageous to consumers because a merger tends to raise prices throughout the industry. Both merged and unmerged firms enjoy greater profit and consumers obtain less surplus. There is a possible gain that could benefit consumers: when the merger leads to cost savings that permit lower prices. Remember that the two products that are merged, while not identical, are close substitutes. They could be, for example, low-sugar and high-sugar versions of a soft drink. We might expect there to be some cost complementarities in the production of these products. If so, then production of both goods by one firm will be cheaper than production of both by two separate firms. In short, we should not be surprised if in a product-differentiated market, production of many closely related product lines exhibits economies of scope.[20]

Scope economies provide a strong incentive to merge. The merger allows the new firm to operate as a multiproduct company and thereby exploit the cost-savings opportunities this generates. These savings may be reflected in a reduction in fixed costs. For example, the firms can combine their headquarters, research and development, marketing, accounting, and distribution operations. If, in addition, the merger leads to a reduction in variable costs of production, then this change will be reflected in lower prices. Moreover, even if scope economies are not present, it is still possible that one of the merging firms has a more effective purchasing division or a superior production technology that, following the merger, will be extended to its new partners. The greater such cost synergies are, the more likely it is that consumers will benefit from the merger.

Price Discrimination

Firms that operate in a spatial or product-differentiated setting clearly have some monopoly power. Yet if firms have monopoly power, we might expect them to use discriminatory pricing strategies to exploit this power. In particular, we might expect these firms to adopt some of the price discrimination strategies that we developed in earlier chapters. We now turn to the analysis of how price discrimination affects the incentives for and the impact of mergers in a product differentiated market.

Suppose that firms adopt first-degree or personalized discriminatory pricing policies but maintain at the same time the remaining assumptions of our spatial model in the no–price-discrimination case. The noncooperative price equilibrium is then easy to identify. Remember that firms compete in prices for customers. Accordingly, they set the price as low as need be—at the margin—to attract customers, so long as that price covers their marginal cost. As a result, the equilibrium must be characterized by the following condition. Suppose that firm i is the firm that can supply consumer location s at the lowest unit cost, say $c + ts$ (the marginal production cost plus transport fee), and that firm j is the firm that can supply this location at the next cheapest unit cost, $c + ts + e$, where e is a measure of how much closer the consumer is to firm i than it is to firm j. The Bertrand–Nash equilibrium price to consumer s will be for firm i to charge one cent or epsilon less than the cost of firm j to supply consumer s; that is, to charge just less than $c + ts + e$.

19 A formal proof for the case of three firms is provided by Posada and Straume (2004).
20 Refer to Section 4.3 of Chapter 4 for a definition and explanation of economies of scope.

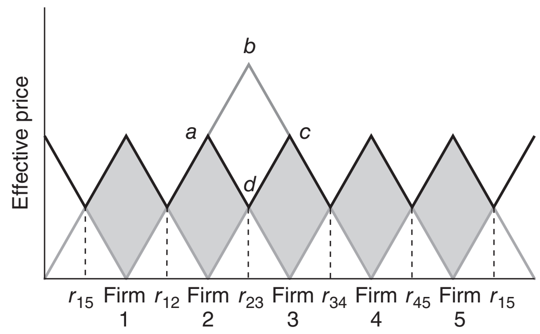

Figure 15.5 Price equilibrium with price discrimination

The heavy shaded line in Figure 15.5 illustrates this price equilibrium. Firm 2 is the lowest-cost supplier (including transport cost) for all consumers in the region (r_{12}, r_{23}). Therefore, firm 2 supplies all consumers in this market region, charging its consumers on the left one cent less than firm 1's costs of supplying them, and its consumers to the right one cent less than firm 3's costs of supplying these consumers. By adopting this pricing strategy, each firm earns a gross profit (profit before deducting fixed cost) given by the shaded areas for their market regions in Figure 15.5.

An interesting feature of this set of discriminatory prices is that the highest price now paid by any consumer is $c + tL/5$. This was the *lowest* price paid by any consumer when firms did not practice price discrimination! Price discrimination in this oligopolistic market unambiguously benefits consumers. Why is this?[21] With nondiscriminatory pricing, when a firm reduces the price to one consumer, it has to reduce the price to every consumer—an expensive prospect. With discriminatory pricing, by contrast, a firm can lower price in one location without having to lower its prices elsewhere. But this means that price discrimination weakens each firm's ability to commit to a set of higher prices, making price competition between the firms much fiercer and so leading to the lower prices that we have just identified.

Now consider the effect on this equilibrium of a merger between two of these firms, say firms 2 and 3, as before. Two points should be clear. First, as in the no–price discrimination case, a merger of non-neighboring firms has no effect. Second, the merged firm's ability to coordinate the formerly separate pricing strategies is particularly valuable in this discriminatory setting. This is because prior to the merger these firms were engaged in what is nearly cutthroat price competition. By merging, the two firms can avoid this expensive conflict, at least with respect to each other.[22] From the perspective of the merged

[21] This is discussed in Norman and Thisse (1996). They show that with a given number of firms, discriminatory pricing always benefits consumers. They also show, however, that the much more competitive environment of discriminatory pricing may cause enough firms to want to leave the market that prices actually increase for some consumers. See also Reitzes and Levy (1995).

[22] There is a further important feature of this type of merger. The potential problems that we discussed previously in footnote 18 cannot arise when firms charge discriminatory prices. Consider, for example, an entrant coming in at product 3's location. This would lead to the pre-merger price equilibrium in Figure 15.5 whether or not the merged firm removes its product at this location. There is, in other words, no benefit to the merged firm in exiting from this market. Because a potential entrant can correctly anticipate this, no such entry will take place.

firm, the nearest competitor for consumers in the region between firm 2's location and r_{23} is now firm 1. Similarly, for consumers in the region between firm 3's location and r_{23}, the nearest competitor is now firm 4. As a result, the merged firm can raise prices to all consumers located between firms 2 and 3, as indicated by the line *abc* in Figure 15.5. A merger of firm 2 and firm 3 increases the profits of the merging firms by an amount given by the area *abcd*. One further effect of this type of merger, which is not quite so intuitive, is that when firms practice price discrimination the merger only benefits the merging firms. Prices and profit increase only for those consumers who were served by the merged firm prior to the merger. All other prices are unaffected, and so the profits of the nonmerging firms are unaffected by the merger.

We could also consider issues regarding the merged firm's product location strategies. However, the basic point has been made. Our conclusions for the no–price discrimination case hold all the more strongly when firms engage in discriminatory pricing practices. Prices to consumers rise and the merging firms are more profitable. There is absolutely no paradox about merging in this price discrimination case. From the viewpoint of the merging firms a merger can be a highly profitable venture.

There is one final point to emphasize. Why is it that mergers with price competition in a product-differentiated market do not run into the merger paradox that so bedeviled our earlier analysis with homogenous products and quantity-setting firms? The first part of the answer has already been suggested. Prices are strategic complements whereas quantities are strategic substitutes. With price competition, therefore, the strategic responses of nonmerged firms are potentially beneficial to the merged firms whereas with quantity competition they are potentially harmful.

The second part of the answer is equally important and is related to the notion of credible commitment. The reason why mergers are profitable in the spatial or differentiated products context is that the merged firms can credibly commit to produce some particular *range* of products—that is, the commitment required in the spatial context is a commitment to particular locations or to continue marketing the products of the previously independent firms. By contrast, the commitment necessary with homogenous products and quantity competition must be in terms of production *levels*. The merging firms must be able to commit to a high volume of output following the merger. Generally, this is not credible because such a high volume of production is not the merged firm's best response to a Cournot output decision by the other firms. If, however, the merged firm becomes a Stackelberg leader then the commitment to a high level of post-merger output is credible.

15.6 PUBLIC POLICY TOWARD HORIZONTAL MERGERS

US public policy with respect to horizontal mergers has changed dramatically over the last forty years. To a large extent, this change is reflected in the differences between the first Merger Guidelines issued by the Justice Department in 1968 and the Merger Guidelines currently in force. While it is tempting to summarize these differences as a move from a very strict regime to a more permissive one, it is more accurate to describe the evolution of merger policy as one that has increasingly become more sophisticated and that gives greater recognition to the complexity of corporate organizations in the real world.

The 1968 Merger Guidelines relied heavily on market structure—particularly the four-firm concentration ratio—to determine the legality of a proposed merger. Mergers would be challenged in any industry in which the four-firm concentration ratio exceeded 75 percent and the merging firms each had a market share of as little as 4 percent. In markets with a four-firm concentration ratio below 75 percent, mergers would be challenged if the two firms each had market shares of 5 percent or more. Thus, under the 1968 Guidelines, a combined share of as little as 10 percent would be sufficient in many cases for the government to challenge a merger.

While the approach taken in 1968 reflected many years of empirical work within the Structure-Conduct-Performance framework, its rigidity led to increasingly questionable decisions culminating in perhaps one of the most controversial merger cases ever, *U.S. v. Von's Grocery* (1966) in which the Supreme Court upheld the government's prohibition of a merger between two grocery store chains in Los Angeles that, in combination, had less than 10 percent of the market.

Ironically, courts began to deviate from the rigid, structure-based guidelines of 1968 almost as soon as they were adopted. One early such case was the acquisition by General Dynamics of another coal producer, which was ultimately allowed by the Supreme Court in 1974 despite the fact that the combined market shares of the two firms clearly exceeded the permissible levels set forth by the then 1968 Merger Guidelines. As the courts permitted a number of similar mergers, it soon became clear that the 1968 Guidelines were no longer compelling. This eventually led to the Justice Department issuing of a new set of Merger Guidelines in 1982.

Under the new rules, reliance on the four-firm concentration ratio was abandoned in favor of the Herfindahl-Hirschman Index (HI). The threshold for intervention now became an HI of 1,800 (a little more concentrated than an industry comprised of six, equally large firms). Mergers in less concentrated industries would only be challenged if they raised the HI by more than one hundred points and even then, only if the industry HI already exceeded 1,000. Subsequent amendments to the Guidelines in 1984, 1992, and 1997 relaxed even more the constraints on mergers by specifying and enlarging the ability of merger-generated cost efficiencies as a merger justification.[23]

Underlying these developments was an increasing awareness of modern industrial organization theory as well as a growing body of empirical data that suggested many mergers did not threaten competition as much as the Structure-Conduct-Performance paradigm implied. Moreover the evidence on profitability of mergers was mixed as well. Quite a long list of studies including Mueller (1982); Ravenscraft and Scherer (1989); Lichtenberg and Siegel (1992); Loughran and A. Vijh (1997); Andrade, Mitchell, and Stafford (2001), and Maskimovic and Phillips (2001) have found that mergers are not terribly profitable—especially for the acquiring firm. Indeed, many acquisitions are later reversed by "spin-offs."

The change in attitude reflected by the 1982 guidelines has led to many more mergers being permitted. These have included such major consolidations as Union Pacific and Southern Pacific (railroads), AOL and Time Warner (telecommunications), Chase Manhattan and J.P. Morgan (finance), Exxon and Mobil and also British Petroleum and Amoco (both petroleum mergers), Westinghouse and Infinity Broadcasting (radio), Aetna and US Healthcare (health services), MCI and WorldCom (telecommunications), Maytag and Whirlpool (laundry machines), and XM and Sirius (satellite radio) among others. Many of these mergers were controversial and virtually all raised some competitive concerns, nevertheless these were approved.

[23] For work on a theoretical approach to formulating generalized merger policy rules, see Nocke and Whinston (2013) and works cited therein.

Public policy on mergers has increasingly made use of sophisticated empirical techniques to estimate key market parameters and then to use these parameters to model the most likely post-merger scenario. We briefly describe this process of merger simulation in the next section. As developed by Werden and Froeb (1994, 2002) and extended by Epstein and Rubinfeld (2002) merger simulation has become an important, albeit somewhat controversial tool in merger policy. [See Slade (2007).]

In addition to a greater reliance on econometric evidence and economic modeling, policy makers have taken two additional steps that permit horizontal mergers to be approved despite some clear antitrust concerns. First, the antitrust authorities have increasingly used a "fix-it-first" approach regarding proposed mergers. This procedure usually centers on divestiture of some of the assets of the merging parties to another, third firm so as to ensure that competitive pressures are maintained. If, for example, the two firms operate in several towns across the country, but in one town they are the only two such suppliers, then the government may permit the merger so long as one of the firms sells off its operations in the town in question to a new, rival entrant firm. This principle was applied to approve the 2013 beer industry merger of Anheuser Bush Inbev and Grupo Modelo. It is often used in the case of media mergers where newspaper and broadcasting firms have ben required to sell their operations in certain locations before being permitted to conclude a merger.

Divestiture does have some problems. Cabral (2003) notes that divestiture allows the merging firms to dictate the entry position for new rivals. If we think of the circle spatial model described above, if two firms merge but sell the location of some of their stores to a formerly excluded entrant, it means that entrant enters at the same location at which the initial stores existed rather than at other locations on the circle that would be better for consumers. Further, firms can act strategically to reduce the competitive threat presented by divested stores. This occurred in 1995 when Schnucks Markets, a supermarket chain, acquired National Food Markets, which was the major competitor of Schnucks in the St. Louis area. The merger was approved when Schnucks agreed to divest twenty-four supermarkets in the St. Louis area over the next year. However, no immediate buyer was named. Schnucks then took the stores to be divested and proceeded to run them into the ground. It closed departments. It kept the stores understaffed, and referred customers to the other Schnucks stores that were not being divested. Soon, sales at the divesting stores had declined by about one-third and, as a result, they posed less of a competitive threat to the stores that the new Schnucks/National firm continued to operate. It was partly a response to this case that led the FTC to require that the buyer of the divested plants be named in advance and that the firm be one that has the industry knowledge to be an effective competitor. This remedy does not, however, correct for the problems identified by Cabral (2003).

A second, alternative procedure has been to approve mergers subject to behavioral constraints on the merging firms, and then to follow this agreement with active monitoring by government agents. Typically, these consent agreements require the firms to take specific actions and to avoid engaging in certain practices. In monitoring these agreements, the regulatory agencies can always count on a reliable source of outside help, namely, the customers of the merged firms and all parties who opposed the merger. They are always quick to report violations of the consent agreement. Since 1992, the number of consent decrees issued by the FTC and the Justice Department has dramatically increased.

In addition to these procedural changes, the FTC and the Justice Department have also continued to adjust the merger guidelines themselves. In this connection, an important recent modification is the 1997 expansion of Section 4 of the guidelines to permit greater