ECN 453: Homework 3 - solutions

1. New technology and market structure (30 points)

Consider an industry with market demand $Q = a - p$ and an infinite number of potential entrants with access to the same technology. Initially, the technology is given by $C = F + cq$. A new technology allows for a lower marginal cost $c' < c$ at the expense of a higher fixed cost $F' > F$.

Given $a = 10, F = 2, F' = 3, c = 2, c' = 1$.

a. Use the following formula (from the lecture slides) for the number of firms n:

$$n = \left[(a - c)\sqrt{\frac{S}{F}} - 1 \right]$$

Then, use the following formula (from the lecture slides) for the price, given the above number of firms:

$$p = \frac{a + nc}{n + 1}$$

<u>Under the old technology</u>:

Here, $a = 10, c = 2, S = 1, F = 2$. So: $n = \left[(10 - 2)\sqrt{1/2} - 1 \right] = 4$.

Then, equilibrium price under old technology:

$$p = \frac{10 + 4 \times 2}{4 + 1} = 3.6$$

<u>Under the new technology</u>:

Here, $a = 10, c = 1, S = 1, F = 3$. So: $n = \left[(10 - 1)\sqrt{1/3} - 1 \right] = 4$.

Then, equilibrium price under new technology:

$$p = \frac{10 + 4 \times 1}{4 + 1} = 2.8$$

2. Repeated games (50 points)

Consider the following game and suppose that it is repeated an infinite number of times. Players have a discount value of δ.

Player 2

Player 1	L	R
T	10, 10	0, 12
B	12, 0	1, 1

a. Equilibrium payoff:

$$\Pi = 10 + \delta 10 + \delta^2 10 + \delta^3 10 + ... = \frac{10}{1-\delta}$$

Deviation payoff:

$$\Pi' = 12 + \delta + \delta^2 + \delta^3 + ... = 12 + \frac{\delta}{1-\delta}$$

Therefore, for collusion to sustain, we need:

$$\begin{aligned} \Pi &\geq \Pi' \\ \frac{10}{1-\delta} &\geq 12 + \frac{\delta}{1-\delta} \\ 10 - \delta &\geq 12(1-\delta) \\ \delta &\geq \frac{2}{11} \end{aligned}$$

b. Equilibrium payoff is the same.

Deviation payoff:

$$\Pi' = 12 + 0\delta + 0\delta^2 + 0\delta^3 + ... = 12$$

Therefore, for collusion to sustain, we need:

$$\begin{aligned} \Pi &\geq \Pi' \\ \frac{10}{1-\delta} &\geq 12 \\ 10 &\geq 12(1-\delta) \\ \delta &\geq \frac{2}{12} \end{aligned}$$

c. The players sustain collusion on (T,L) using the grim trigger punishment of playing (B,R) in all future periods, and the discount factor δ indexes how much agents care about this future punishment. Since in Part b, the punishment is harsher than in Part a, (since the players get (0,0) for all future periods rather than (1,1)), collusion can be sustained for lower discount factors.