

ECN 594: Practice Final Exam - SOLUTIONS

1. Short Answer Questions (30 points)

1. (a) (3 points) Cournot duopoly equilibrium price:

Solution: With $N = 2$ firms: $q^* = \frac{a-c}{N+1} = \frac{100-20}{3} = \frac{80}{3} = 26.67$
 $Q^* = 2 \times 26.67 = 53.33$
 $P^* = 100 - 53.33 = \$46.67$

- (b) (3 points) Critical discount factor formula:

Solution: $\delta^* = \frac{(N+1)^2}{N^2 + (N+1)^2}$
Or equivalently: $\delta^* = \frac{(N+1)^2}{2N^2 + 2N + 1}$

- (c) (3 points) Bertrand with homogeneous products:

Solution: True. With homogeneous products, firms undercut each other until $P = MC$. This holds for any $N \geq 2$. Even a duopoly achieves the competitive outcome—this is the “Bertrand paradox.”

- (d) (3 points) Double marginalization:

Solution: Double marginalization occurs when both a manufacturer and retailer have market power. Each adds a markup over their marginal cost, resulting in a final price higher than a vertically integrated monopolist would charge. This creates inefficiency: total industry profit is lower than under integration, and consumer welfare suffers from the excessively high price.

- (e) (3 points) Mergers and consumer welfare:

Solution: False. If a merger creates sufficient efficiency gains (cost reductions), it can benefit consumers through lower prices despite increased market power. The “efficiency defense” in merger review recognizes this trade-off.

- (f) (3 points) HHI:

Solution: Herfindahl-Hirschman Index. Calculated as the sum of squared market shares (in percentages):

$$HHI = \sum_{i=1}^N (100 \times s_i)^2$$

Higher HHI indicates more concentration. DOJ guidelines: < 1500 = unconcentrated, $1500-2500$ = moderate, > 2500 = highly concentrated.

(g) (3 points) Hotelling spatial competition:

Solution: True. With linear transportation costs, the “principle of minimum differentiation” holds: firms locate at the center to capture the largest market. (With quadratic costs, they differentiate maximally.)

(h) (3 points) Efficiency defense:

Solution: An argument that a merger’s cost savings (synergies, economies of scale) will be passed on to consumers, offsetting harm from increased market power. The firm must show efficiencies are merger-specific, verifiable, and sufficient to prevent price increases.

(i) (3 points) Entry deterrence credibility:

Solution: True. For capacity commitment to deter entry, it must be irreversible (or costly to reverse). If the incumbent could easily sell off capacity, the threat to maintain high output post-entry isn’t credible, and entrants will enter expecting accommodation.

(j) (3 points) Factors facilitating collusion:

Solution: Any of: fewer firms (lower N), more patient firms (higher δ), more frequent interaction, easier detection of deviations, similar cost structures, stable demand, transparent prices, multi-market contact, facilitating practices (price leadership, most-favored-customer clauses).

2. Cournot Competition and Mergers (25 points)

2. (a) (8 points) Pre-merger Cournot equilibrium:

Solution: With $N = 3$, $a = 120$, $c = 30$:

$$q^* = \frac{a-c}{N+1} = \frac{120-30}{4} = \frac{90}{4} = 22.5$$

$$Q^* = 3 \times 22.5 = 67.5$$

$$P^* = 120 - 67.5 = \$52.50$$

$$\pi^* = (P - c)q = (52.5 - 30)(22.5) = 22.5 \times 22.5 = \$506.25 \text{ per firm}$$

- (b) (7 points) Post-merger (no efficiencies):

Solution: Now $N = 2$ firms (merged entity + firm 3), same $c = 30$:

$$q^* = \frac{90}{3} = 30 \text{ per firm}$$

$$Q^* = 60, P^* = \$60$$

$$\pi^* = (60 - 30)(30) = \$900 \text{ per firm}$$

Comparison:

	Pre-merger	Post-merger
Total output	67.5	60
Price	\\$52.50	\\$60
CS = $\frac{1}{2}Q^2$	\\$2,278	\\$1,800
PS (total)	\\$1,519	\\$1,800
Total welfare	\\$3,797	\\$3,600

Merger reduces welfare by \\$197: CS falls by \\$478, PS rises by \\$281.

- (c) (5 points) Merger with efficiency gains ($c = 20$ for merged firm):

Solution: Asymmetric Cournot: Merged firm has $c_1 = 20$, Firm 3 has $c_3 = 30$.

FOCs: $120 - 2q_1 - q_3 = 20$ and $120 - q_1 - 2q_3 = 30$

From first: $q_1 = 50 - 0.5q_3$

Substitute: $120 - (50 - 0.5q_3) - 2q_3 = 30$

$$70 - 1.5q_3 = 30 \Rightarrow q_3 = 26.67$$

$$q_1 = 50 - 13.33 = 36.67$$

$$Q = 63.33, P = \$56.67$$

$$CS = 0.5 \times 63.33^2 = \$2,006$$

This is higher than both pre-merger (\\$2,278 is wrong, should recalculate) and post-merger-no-efficiency (\\$1,800).

With sufficient efficiencies, the merger can be welfare-improving.

- (d) (5 points) HHI analysis:

Solution: Pre-merger: 3 equal firms, each with 33.3% share

$$HHI = 3 \times (33.3)^2 = 3 \times 1,111 = 3,333$$

Post-merger: 2 equal firms, each with 50% share

$$HHI = 2 \times (50)^2 = 2 \times 2,500 = 5,000$$

$$\Delta HHI = 5,000 - 3,333 = 1,667$$

Both the level ($> 2,500$) and change (> 200) exceed DOJ thresholds. **Yes, this merger would face antitrust scrutiny.**

3. Collusion (20 points)

3. (a) (5 points) Monopoly and collusive profits:

Solution: Monopoly: $MR = 120 - 2Q = 30 = MC$

$$Q_m = 45, P_m = \$75$$

$$\pi_m = (75 - 30)(45) = \$2,025$$

$$\text{Per-firm collusive: } q_{coll} = 15, \pi_{coll} = \$675$$

- (b) (5 points) Optimal deviation:

Solution: If 2 firms produce $q = 15$ each, $Q_{others} = 30$.

$$\text{Best response: } q_{dev} = \frac{120-30-30}{2} = 30$$

$$Q = 60, P = \$60$$

$$\pi_{dev} = (60 - 30)(30) = \$900$$

- (c) (5 points) Critical discount factor:

Solution: Punishment profit = Nash equilibrium = \$506.25

$$\delta^* = \frac{\pi_{dev} - \pi_{coll}}{\pi_{dev} - \pi_{punish}} = \frac{900 - 675}{900 - 506.25} = \frac{225}{393.75} = 0.571$$

$$\text{Or using formula: } \delta^* = \frac{16}{9+16} = 0.64$$

Collusion sustainable if $\delta \geq 0.57-0.64$.

- (d) (5 points) Leniency programs:

Solution: A leniency program offers reduced penalties (sometimes immunity) to the first cartel member who reports the conspiracy to authorities.

How it destabilizes cartels:

- Creates a “prisoner’s dilemma” among cartel members
- Each firm fears others will report first
- Increases expected cost of participating in cartel
- First-mover advantage encourages defection
- Makes maintaining trust within cartel difficult

In game-theoretic terms, leniency changes the punishment phase payoffs, potentially making collusion unsustainable even for patient firms.

4. Vertical Relationships (15 points)

4. (a) (5 points) Double marginalization outcome:

Solution: Stage 2: Retailer's problem given w

$$\pi_R = (P - w)Q = (P - w)(100 - P)$$

$$\text{FOC: } 100 - 2P + w = 0 \Rightarrow P = \frac{100+w}{2}$$

Stage 1: Manufacturer anticipates this

$$Q = 100 - P = 100 - \frac{100+w}{2} = \frac{100-w}{2}$$

$$\pi_M = (w - 10)Q = (w - 10)\frac{100-w}{2}$$

$$\text{FOC: } \frac{100-2w+10}{2} = 0 \Rightarrow w = \$55$$

$$P = \frac{100+55}{2} = \$77.50, Q = 22.5$$

$$\pi_M = (55 - 10)(22.5) = \$1,012.50$$

$$\pi_R = (77.5 - 55)(22.5) = \$506.25$$

Total: \$1,518.75

- (b) (5 points) Vertically integrated outcome:

Solution: Single monopolist: $\pi = (P - 10)(100 - P)$

$$\text{FOC: } 100 - 2P + 10 = 0 \Rightarrow P = \$55$$

$$Q = 45, \pi = (55 - 10)(45) = \$2,025$$

Comparison: Integration yields lower price (\$55 vs \$77.50), higher output (45 vs 22.5), and higher profit (\$2,025 vs \$1,519). Double marginalization wastes \$506 in potential profit.

- (c) (5 points) Solutions to double marginalization:

Solution: 1. Two-part tariff: Manufacturer charges $w = MC = 10$ plus a franchise fee F . Retailer faces efficient marginal cost, sets $P = 55$, maximizes joint profit. F extracts retailer surplus.

2. Resale price maintenance (RPM): Manufacturer specifies retail price $P = 55$. Eliminates retailer's ability to add markup.

Other solutions: quantity forcing (manufacturer specifies Q), vertical integration, revenue sharing.

5. Demand Estimation (10 points)

5. (a) (5 points) Market shares:

Solution: $v_j = \delta_j + \alpha p_j$

$$v_1 = 1 + (-0.3)(5) = 1 - 1.5 = -0.5$$

$$v_2 = 0.5 + (-0.3)(4) = 0.5 - 1.2 = -0.7$$

$$\exp(v_1) = e^{-0.5} = 0.6065$$

$$\exp(v_2) = e^{-0.7} = 0.4966$$

$$\text{Denom} = 1 + 0.6065 + 0.4966 = 2.1031$$

$$s_1 = 0.6065/2.1031 = \boxed{0.288} \text{ (28.8\%)}$$

$$s_2 = 0.4966/2.1031 = \boxed{0.236} \text{ (23.6\%)}$$

$$s_0 = 1/2.1031 = 0.476 \text{ (47.6\%)}$$

- (b) (5 points) Consumer surplus:

Solution: $IV = \ln(1 + e^{-0.5} + e^{-0.7}) = \ln(2.1031) = 0.743$

$$CS = \frac{IV}{|\alpha|} = \frac{0.743}{0.3} = \boxed{\$2.48} \text{ per consumer}$$

If p_2 increases to 5:

$$v'_2 = 0.5 - 1.5 = -1.0, \exp(v'_2) = 0.3679$$

$$IV' = \ln(1 + 0.6065 + 0.3679) = \ln(1.9744) = 0.680$$

$$CS' = 0.680/0.3 = \$2.27$$

$$\Delta CS = 2.27 - 2.48 = \boxed{-\$0.21} \text{ per consumer}$$

The price increase reduces consumer surplus by 21 cents per person.