ECN 453: Game Theory 1

Nicholas Vreugdenhil

Strategic decision making

- So far, we have seen models of *independent* decision making.
- In other words, when firms made their optimal choices (for example, choosing optimal prices) they did not need to think about what other firms were doing.
  - For example, in monopoly, there were no other firms in the market!
- Today, we will study **game theory**.
- These are models where optimal choices are *strategic*: a firm's optimal choice depends on what other firms are doing.
- Much of what we will see in this lecture is review from previous courses. We will build on this review in future lectures.

Example of strategic decision making

- Two hollywood studios in 2010: Warner Bros and Fox, deciding when to release their blockbuster movies: Harry Potter, Chronicles of Narnia.

Example of strategic decision making (continued)

- Which month should the studios release their movie: December or November?
- Consider the decision of Warner Bros who are thinking about when to release the Harry Potter movie.

Example of strategic decision making (continued)

- Which month should the studios release their movie: December or November?
- Consider the decision of Warner Bros who are thinking about when to release the Harry Potter movie.
  - December is in the holidays so it's a better month to release Harry Potter if it is the only movie because it will get a bigger audience.
  - But: if Fox decides to release Narnia in December, the audience for the Harry Potter movie will be split.
  - So, if Fox decides to release Narnia in December, it will be better to release the Harry Potter movie in November.
- So: the decision of one movie studio about when to release the movie depends on the choices of the *other* movie studio. It is *strategic*.
  - Game theory can be used to model and understand strategic decisions like these.

Plan

1. **Simultaneous games: setup and Nash equilibrium**
2. Simultaneous games: dominant and dominated strategies

Simultaneous games: setup

- We will start by studying **simultaneous games**.
- These are games where choices are made at the same time.
  - By contrast, we will next study *sequential games* where firms makes choices one after the other.
- I will begin by defining some important language that economists use when talking about game theory.

Simultaneous games: setup

It's very important that you can identify the following components from the figure on the right.

Player 2

Player 1		L	R
	T	5 5	6 3
	B	3 6	4 4

- **Players**:

Simultaneous games: setup

It's very important that you can identify the following components from the figure on the right.

Player 2

Player 1		L	R
	T	5, 5	3, 6
	B	6, 3	4, 4

- **Players**: Player 1 and Player 2
- **Strategies**

Simultaneous games: setup

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- **Players**: Player 1 and Player 2
- **Strategies**
  - Player 1 has strategies 'T' and 'B'
  - Player 2 has strategies 'L' and 'R'
- **Payoffs**:

Simultaneous games: setup

It's very important that you can identify the following components from the figure on the right.

		Player 2	
		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

- **Players**: Player 1 and Player 2
- **Strategies**
  - Player 1 has strategies 'T' and 'B'
  - Player 2 has strategies 'L' and 'R'
- **Payoffs**: the numbers in the matrix
  - Higher payoffs are better
  - Payoffs depend on the choices of *both players*
  - E.g. if Player 1 chooses 'T', Player 2 chooses 'R' then:
    - Player 1 receives payoff $= 3$
    - Player 2 receives payoff $= 6$.

Simultaneous games: setup

Player 2

Player 1		L	R
	T	5 5	6 3
	B	3 6	4 4

- Representing the elements (players, strategies, payoffs) in this form is called the ‘normal form’ of a game.

Simultaneous games: best responses

Player 2

		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

- **Best response**:

Simultaneous games: best responses

Player 2

Player 1		L	R
	T	5, 5	3, 6
	B	6, 3	4, 4

- **Best response**: the optimal strategy for a player *given* the choice of the other player.
- Notation: "best response of player 1 given that player 2 chooses R" written as:

$$BR_1(R)$$

- We can find the best response by finding the strategy with the highest payoff given a choice by the other player.
- **Example:** $BR_1(R) = B$.
- Why? Given player 2 plays R, player 1 gets:
  - Payoff=3 if play T $<$ payoff=4 if play B.

Simultaneous games: best responses

Player 2

Player 1

	L	R
T	5 5	6 3
B	3 6	4 4

- Let's find some more best responses.
- $BR_1(L) = B$
  - Payoff=5 if play T $<$ payoff=6 if play B.
- $BR_2(T) = R$
  - Payoff=5 if play L $<$ payoff=6 if play R.
- $BR_2(B) = R$
  - Payoff=3 if play L $<$ payoff=4 if play R.

Simultaneous games: Nash equilibrium

- Where do we expect the game to end up? A **Nash equilibrium**:

Simultaneous games: Nash equilibrium

- Where do we expect the game to end up? A **Nash equilibrium**:

  A pair of strategies constitutes a Nash equilibrium if no player can unilaterally change its strategy in a way that improves its payoff.

- So, a Nash equilibrium occurs when each player is choosing a best response to the other player's choice.

Simultaneous games: Nash equilibrium

- In practice we can find a Nash equilibrium using the following steps:

1. Find the best responses for player 1 and circle them in the payoff matrix.
2. Find the best responses for player 2 and circle them in the payoff matrix.
3. If a box in the payoff matrix has two circles, it is a Nash equilibrium.

- We will now go through these steps for our example game in the previous slide.

Simultaneous games: Nash equilibrium

- Step 1: find the best responses for player 1 and circle them in the payoff matrix.

Player 2

Player 1		L	R
	T	5 5	6 3
	B	3 6	4 4

Simultaneous games: Nash equilibrium

- Step 1: find the best responses for player 1 and circle them in the payoff matrix.

- Step 2: find the best responses for player 2 and circle them in the payoff matrix.

		Player 2	
		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

Simultaneous games: Nash equilibrium

- Step 1: find the best responses for player 1 and circle them in the payoff matrix.
- Step 2: find the best responses for player 2 and circle them in the payoff matrix.
- Step 3: if a box in the payoff matrix has two circles, it is a Nash equilibrium.
- So, (B, R) is a Nash equilibrium.
  - The notation (B, R) means player 1 chooses B, and player 2 chooses R.

		Player 2	
		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

Simultaneous games: Nash equilibrium

- It's possible to have *multiple* Nash equilbria in a game.
- When this happens, our simple model makes no predictions about which equilibrium will be observed - all we can say is that the game will end up in *one* of the equilbria.
- An example of a game with multiple equilibria is on the right.
  - I have circled the best responses and the two Nash equilibria are at (T, L) and (B, R)

Player 2

Player 1		L	R
	T	2 1	0 0
	B	0 0	1 2

Simultaneous games: prisoner's dilemma

- Let's look at the first game that we looked at today (rewritten on the right). This game is an example of the *prisoner's dilemma*.

- Let's look at it again with the players and strategies relabelled (but with the same payoffs).
  - The strategies are now to choose prices: a high price 'H' or to choose a low price 'L'.

  - The interpretation of the payoffs is now profits.

  - (Note: we could come up with demand, costs, specific prices, that generate these profits, but I will ignore this for now.)

		Firm 2	
		H	L
Firm 1	H	5 5	6 3
	L	3 6	4 4

Simultaneous games: prisoner's dilemma

- What can we learn from the prisoner's dilemma?

- From the *joint* perspective of both firms, the optimal strategies are to choose high prices (H,H). Joint profits will be $= 10$.

- But, from the *individual* perspective of each firm, there is an incentive to *deviate* from (H, H).
  - A firm gets payoff $6 > 5$ from choosing L when the other firm chooses H.

- Therefore, the game results in a nash equilibrium of (L,L): here, joint profits are $= 8$. This is lower than what is *jointly* best for the firms!

		Firm 2	
		H	L
Firm 1	H	5 5	6 3
	L	3 6	4 4

Simultaneous games: prisoner's dilemma

- The prisoners dilemma shows the *'conflict between individual incentives and joint incentives'*.

- It is an example of why competition might be good for consumers, but bad for firms.
    - If both firm 1 and firm 2 were the same firm (e.g. a monopoly) they would choose high prices - this would maximize profits but consumers would have to pay high prices.

    - When the firms are choosing individually ('competing') they undercut each other and choose lower prices - this lowers joint profits for firms but consumers do better because they pay lower prices.

    - We will study competition a lot more in the second part of the course.

Simultaneous games: finding Nash Equilibria in complicated games, p164-p165

		Player 2		
		L	M	R
Player 1	T	2, 1	0, 2	0, 3
	M	1, 1	1, 1	1, 0
	B	0, 1	2, 0	2, 2

Simultaneous games: finding Nash Equilibria in complicated games, p164-p165

- Nash equilibrium at (B,R).

Player 2

Player 1		L	M	R
	T	2, 1	0, 2	0, 3
	M	1, 1	1, 1	1, 0
	B	0, 1	2, 0	2, 2

Plan

1. Simultaneous games: setup and Nash equilibrium

2. **Simultaneous games: dominant and dominated strategies**

Dominant and dominated strategies

- We give names to certain types of strategies.
- One is a **dominant strategy**:
  - *A dominant strategy yields a player the highest payoff regardless of the other players choices.*
- One is a **dominated strategy**:
  - *A dominanted strategy yields a player a payoff which is lower than that of a different strategy, regardless of what the other players do.*
- Note: for many games, there are no dominant or dominated strategies.

Dominant and dominated strategies: prisoner's dilemma

		Firm 2	
		H	L
Firm 1	H	5 5	6 3
	L	3 6	4 4

- As an example of dominant and dominated strategies, let's revisit the prisoner's dilemma.
- Are there dominant and dominated strategies in this game?

Dominant and dominated strategies: prisoner's dilemma

		Firm 2	
		H	L
Firm 1	H	5 5	6 3
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- As an example of dominant and dominated strategies, let's revisit the prisoner's dilemma.
- Are there dominant and dominated strategies in this game?
- Yes!
- For Firm 1: L is the dominant strategy, H is the dominated strategy
- For Firm 2: L is the dominant strategy, H is the dominated strategy

Dominated strategies: example

		Player 2		
		L	C	R
Player 1	T	1, 2	1, 0	1, 1
	M	0, 0	0, 3	0, 0
	B	2, 0	−2, 1	2, 2

- Let's solve the game from before using *iterated deletion of dominated strategies*.
  - Essentially, keep deleting dominated strategies until we reach a Nash equilibrium.

Dominated strategies: example

		Player 2		
		L	C	R
Player 1	T	2 1	0 1	1 1
	M	0 0	3 0	0 0
	B	0 2	1 -2	2 2

- Let's solve the game from before using *iterated deletion of dominated strategies*.
  - Essentially, keep deleting dominated strategies until we reach a Nash equilibrium.
  - For player 1, M is a dominated action. Eliminate it (cross it out) of the game.
  - Given the above, C is dominated for player 2. Eliminate it from the game.
  - Given the above, T is dominated for player 1. Eliminate it.
  - Finally, given the above, L is a dominated strategy for player 2. Eliminate it.
  - What is left: Nash equilbrium at (B,R).

Dominant and dominated strategies

- Why is identifying dominant strategies useful?

- Computing Nash equilibria requires a player to make predictions about what the *other* player will do.

- A key assumption behind these predictions is that the other player is *rational*.
  - Specifically, we assume that the other players are sophisticated, can accurately weight up their payoffs, and choose the optimal action etc

  - But what if we are wrong and the other player is not choosing optimally? Then, our best responses might be wrong, and our predictions about the equilibrium of the game will be wrong.

- But, playing a dominant strategy is the best thing to do even if we get the assumption about rationality wrong.
  - Another way of saying this is that our predictions about the game are 'robust' to the rationality assumption.

Dubious application of dominated strategies (p 163)

Player 2

Player 1		L	R
	T	0 1	1 1
	B	0 -100	1 2

- Deleting *dominated* strategies, however, does rely on strong assumptions about rationality.
- Consider the game on the left.
  - Player 2 has a dominated strategy: L.
  - If player 1 believes player 2 is rational, then player 1 will choose their optimal action assuming that player 2 does not play L.
  - So, player 1 chooses B, gets payoff=2.

Dubious application of dominated strategies (p 163)

Player 2

Player 1		L	R
	T	0 1	1 1
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- Deleting *dominated* strategies, however, does rely on strong assumptions about rationality.
- Consider the game on the left.
  - Player 2 has a dominated strategy: L.
  - If player 1 believes player 2 is rational, then player 1 will choose their optimal action assuming that player 2 does not play L.
  - So, player 1 chooses B, gets payoff=2.
- But what if player 2 is not rational?
  - Then, perhaps player 1 should play T to ensure they do not get the payoff -100.

Summary of key points*

- Know how to identify the components of a simultaneous game: players, strategies, payoffs
- Know how to compute best responses
- Compute Nash equilibrium from best responses, and understand the the predictions of a Nash equilibrium rely on assumptions about whether the players are choosing *rationally*
- Understand the prisoner's dilemma illustrates the 'conflict between individual incentives and joint incentives'.
- Know what a 'dominant' and 'dominated' strategy are.
- Solve games using iterated deletion of dominated strategies.

*To clarify, all the material in the slides, problem sets, etc is assessable unless stated otherwise, but I hope this summary might be a useful place to start when studying the material.