ECN 453: Bertrand Competition

Nicholas Vreugdenhil

Static Models of Oligopoly

- We will start the second part of the course today.
- In this part of the course, we will study different **static models of oligopoly**

Static Models of Oligopoly

- What are **static models of oligopoly**?
  - 'Static': this means that the game is only played once
    - Note that there might be a sequential element to it e.g. players take turns to choose like in the entry deterrence example we saw in previous lectures, but ultimately the game is played only once
    - (In the third part of the course, we will contrast 'static' games with 'dynamic' or 'repeated' games which are played again and again)
  - 'Oligopoly': this means when we have (potentially) more than one firm competing in the market
- These models (arguably) the central building blocks of industrial organization, and the models will correspond to different types of competition we observe in real-world markets.

Plan

1. Bertrand competition
2. Bertrand competition with capacity constraints

Plan

1. **Bertrand competition**
2. Bertrand competition with capacity constraints

Bertrand Competition

- Named after Joseph Bertrand
- 11 March 1822 – 5 April 1900
- He was a mathematician

Bertrand Competition

- **Players**: two firms (firm 1 and firm 2)
- **Strategies**
  - Firms choose prices
  - Prices are set simultaneously
- **Payoffs**
  - Each firm has a constant marginal cost (for now, assume they have the *same* marginal cost)
  - Consumers buy from the firm which sets the lower price
  - If the prices are the same, consumers split their demand equally between the firms
  - So: total demand is $Q = D(p)$ where $p = min\{p_1, p_2\}$

Bertrand Competition

- Before getting into the implications of Bertrand competition, first consider: what kind of real-world competitive situations does Bertrand competition correspond to?

- It is a model of **price competition** (the decision is 'what price do I choose'?)

- Best suited to markets where firms offer a **homogeneous** (identical) product
  - In other words, the products are **perfect substitutes**
  - E.g. gas stations next to each other offer the same gas
  - If the price is just a few cents lower, all demand will go to the station with the lowest price
  - If the products were *not* homogeneous, then the strong assumption that 'consumers buy from the firm which sets the lower price' would probably be violated and we would need to use a different model.

Bertrand Competition: Continuous Strategies

- Unlike in the previous lectures, where strategies were limited to just a few discrete alternatives (e.g. choose a high price/low price), here firms can choose *any* price ('continuous strategies').

- All of our definitions about best responses, Nash equilibrium, etc still work!

- I will first show you the mathematical definitions (which are pretty abstract), but they will hopefully make more sense when applied to the Bertrand competition example.

Bertrand Competition: Continuous Strategies - Best Responses

- Here, the **best responses** are prices (where $p_1^*(p_2)$ is just another way of writing $BR_1(p_2)$ and $p_2^*(p_1)$ is just another way of writing $BR_2(p_1)$):

$$p_1^*(p_2) = BR_1(p_2) \in argmax_{p_1} \Pi_1(p_1, p_2)$$
$$p_2^*(p_1) = BR_2(p_1) \in argmax_{p_2} \Pi_2(p_1, p_2)$$

- Notation:
    - $BR_1(p_2)$: the best response of firm 1 given that firm 2 chooses the price p_2
    - $BR_2(p_1)$: the best response of firm 2 given that firm 1 chooses the price p_1
    - *argmax*: the argument(s) (i.e. prices) that maximize profit
    - $\in$: 'in' the set of best responses
    - p_1, p_2: prices that firm 1 and firm 2 set, respectively
    - $\Pi_1(p_1, p_2)$: profit of firm 1 given that firm 1 and firm 2 set the prices p_1, p_2.

Bertrand Competition: Continuous Strategies - Nash Equilibrium

- The idea behind the Nash equilibrium is the same as before: ‘a Nash equilibrium is two prices (p_1, p_2) where each firm has no incentive to unilaterally deviate by choosing a different price’

- Equivalently: a Nash equilibrium is a price for each firm (p_1, p_2) that is the best response to the price of the other firm.

$$p_1 \in p_1^*(p_2)$$
$$p_2 \in p_2^*(p_1)$$

Bertrand Competition: Best Responses - Firm 1

- Let's apply the definitions on the previous two slides to Bertrand competition, starting with firm 1.

- We want to find the best response $p_1^*(p_2)$
  - i.e. the optimal price for firm 1 given firm 2 plays p_2.

- There are three main cases to consider, corresponding to different potential p_2
  - **Case 1**: Firm 2 plays $p_2 > p^M$ (where p^M is the monopoly price)
  - **Case 2**: Firm 2 plays $MC < p_2 \leq p^M$
  - **Case 3**: Firm 2 plays $p_2 \leq MC$

Bertrand Competition: Best Responses - Firm 1

- **Case 1**: Firm 2 plays $p_2 > p^M$ (where p^M is the monopoly price)
- The best response: $p_1^*(p_2) = p^M$.
- Why?
  - Since $p_2 > p^M$, for any $p_1 \leq p^M$ firm 1 gets all of the total demand.
  - So, firm 1 can just set prices like it is a monopoly for the entire market and can set the monopoly price.

Bertrand Competition: Best Responses - Firm 1

- **Case 2**: Firm 2 plays $MC < p_2 \leq p^M$
- The best response: $p_1^*(p_2) = p_2 - \epsilon$
  - Define ϵ: a tiny change in the price
  - Idea: undercut firm 2 by a tiny amount.
- Why? (Intuition)
  - Firm 1 should undercut firm 2 by a tiny amount, and receive the entirety of total demand, rather than setting a price equal to firm 2 (and splitting the market), or setting a price higher than firm 2 (and receiving 0 demand).

Bertrand Competition: Best Responses - Firm 1

- **Case 2**: Firm 2 plays $MC < p_2 \leq p^M$

- The best response: $p_1^*(p_2) = p_2 - \epsilon$
  - Define ϵ: a tiny change in the price

  - Idea: undercut firm 2 by a tiny amount.

- Why? (Math)
  - Note that the profit at $p_1 = p_2 - \epsilon$ is:

$$\Pi_1(p_1 = p_2 - \epsilon, p_2) = D(p_2 - \epsilon)(p_2 - \epsilon - MC)$$

  - Since firm 1 already gets all of total demand at $p_1 = p_2 - \epsilon$, any price lower than this will just reduce profit.

  - If firm 1 were to set a price that exactly matched p_2, the profit would be $0.5D(p_2)(p_2 - MC) < D(p_2 - \epsilon)(p_2 - \epsilon - MC)$ if ϵ is sufficiently small

  - If firm 1 were to set a price $p_1 > p_2$ then firm 1's profits would be $= 0$ since noone would buy from them, but firm 1's profits would be positive if ϵ is small enough because $p_2 > MC$.

Bertrand Competition: Best Responses - Firm 1

- **Case 3**: Firm 2 plays $MC \geq p_2$
- The best response: $p_1^*(p_2) = MC$
- Why?
  - Any price $p_1 < p_2$ would give firm 1 negative profit, since $p_2 < MC$.
  - So, set $p_1 = MC$: here you get 0 profit.
  - (Note: technically any $p_1 > p_2$ is a best response if $MC > p_2$ and any $p_1 \geq MC$ is a best response if $p_2 = MC$, since for all these p_1 prices profits are zero for firm 1. But, dealing with these situations just complicates the proof and does not change the equilibrium; the textbook just ignores them and so will I).

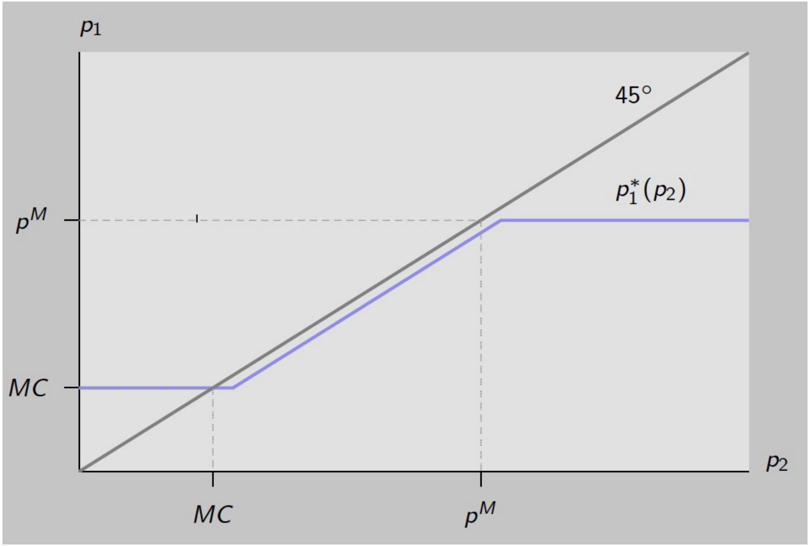
Bertrand Competition: Best Responses - Firm 1, summary

- $p_1^*(p_2) = MC$ if $p_2 \leq MC$
- $p_1^*(p_2) = p_2 - \epsilon$ if $MC < p_2 \leq p^M$
- $p_1^*(p_2) = p^M$ if $p_2 > p^M$

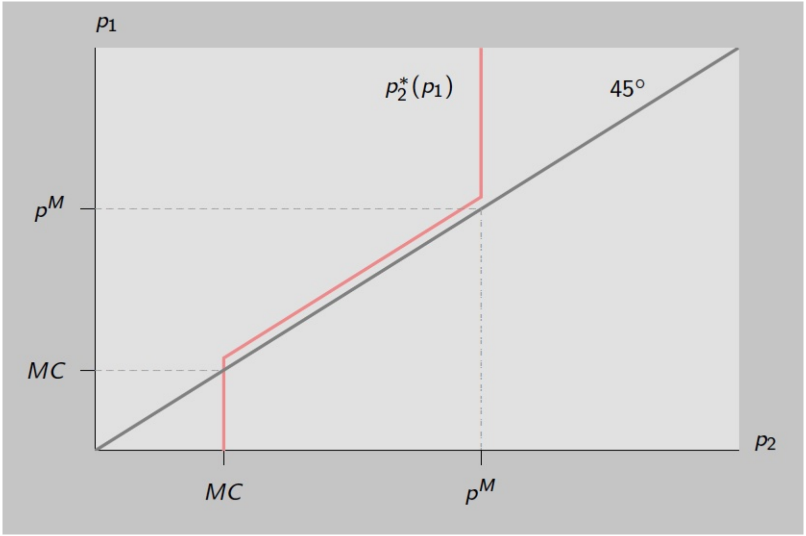
Bertrand Competition: Best Responses - Firm 2, summary

- By exactly the same arguments, we can find the best responses for firm 2:
- $p_2^*(p_1) = MC$ if $p_1 \leq MC$
- $p_2^*(p_1) = p_1 - \epsilon$ if $MC < p_1 \leq p^M$
- $p_2^*(p_1) = p^M$ if $p_1 > p^M$

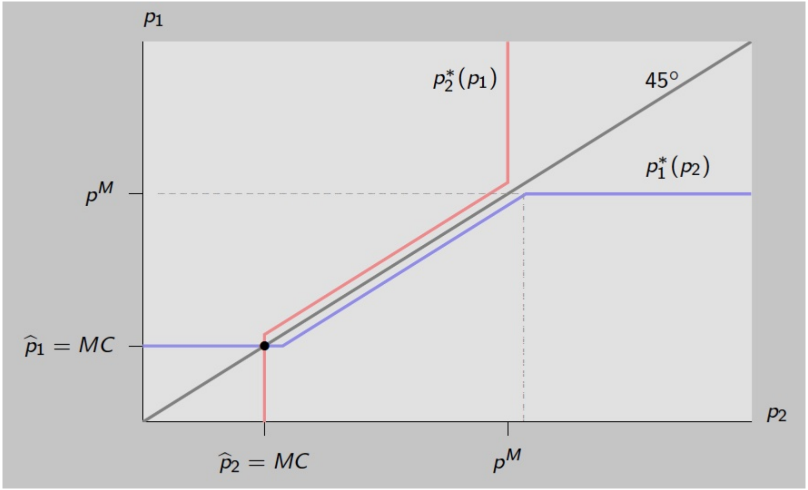
Bertrand Competition: Best Responses - Firm 1, graph



Bertrand Competition: Best Responses - Firm 2, graph



Bertrand Competition: Nash Equilibrium, graph



- Nash equilibrium is where the two best response curves cross.
- This is at $p_1 = p_2 = MC$.

Bertrand Competition: recipe for how to solve it

1. Find the best responses for firm 1 (i.e. find the optimal prices p_1 for all prices p_2 that firm 2 could set)

2. Find the best responses for firm 2 (i.e. find the optimal prices p_2 for all prices p_1 that firm 1 could set)

3. Find where the two best responses cross: this is a Nash equilibrium!

- Note: This recipe is exactly the same as in the simultaneous games we saw before, the 'trick' is splitting the best responses into different cases.