ECN 594: Logit Demand and Identification

Nicholas Vreugdenhil

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Announcements

- **Homework 1 released today**
- Due: Feb 4 (before Lecture 6)
- Demand estimation using Python and `pyblp`
- Start early!

Plan

1. **The Logit Demand Model**
2. Identification and Instrumental Variables

Recap: Random utility

- Consumer i chooses among J products
- Utility:

$$u_{ij} = x_j\beta - \alpha p_j + \xi_j + \varepsilon_{ij}$$

- Consumer chooses the product with highest utility
- Last time: we left ε_{ij} unspecified
- Today: we assume ε_{ij} has a specific distribution

The logit assumption

- Assume ε_{ij} is i.i.d. **Type I Extreme Value**
- Also called Gumbel distribution
- CDF: $F(\varepsilon) = \exp(-\exp(-\varepsilon))$
- Why this assumption?
  - Gives us **closed-form** choice probabilities!
  - Computationally tractable

Logit choice probabilities

- Define **mean utility**:

$$\delta_j = x_j\beta - \alpha p_j + \xi_j$$

- So utility is: $u_{ij} = \delta_j + \varepsilon_{ij}$
- With Type I Extreme Value errors, the probability that consumer chooses j:

$$P(\text{choose } j) = \frac{\exp(\delta_j)}{\sum_{k=1}^{J} \exp(\delta_k)}$$

- This is the **logit** formula

The outside option

- Problem: Our formula doesn't allow consumers to "not buy"
- We need an **outside option** (product $j = 0$)
- Utility of outside option:

$$u_{i0} = \varepsilon_{i0}$$

- We normalize: $\delta_0 = 0$
- All other utilities are *relative* to this outside option

Logit with outside option

- With the outside option, the share of product j is:

$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{J} \exp(\delta_k)}$$

- And the share of the outside option is:

$$s_0 = \frac{1}{1 + \sum_{k=1}^{J} \exp(\delta_k)}$$

- Note: $s_0 + \sum_{j=1}^{J} s_j = 1$ (shares sum to 1)

Berry (1994) inversion: the key insight

- We observe: market shares s_j
- We want: mean utilities δ_j (to estimate β, α)
- **Problem:** How do we get δ_j from s_j?
- **Berry's insight:** Take the log of shares!

Berry (1994) inversion

- Start with:

$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{J} \exp(\delta_k)}, \quad s_0 = \frac{1}{1 + \sum_{k=1}^{J} \exp(\delta_k)}$$

- Take logs:

$$\ln(s_j) = \delta_j - \ln\left(1 + \sum_{k=1}^{J} \exp(\delta_k)\right)$$
$$\ln(s_0) = -\ln\left(1 + \sum_{k=1}^{J} \exp(\delta_k)\right)$$

- Subtract:

$$\ln(s_j) - \ln(s_0) = \delta_j$$

Berry (1994) inversion: the estimating equation

- We have: $\ln(s_j) - \ln(s_0) = \delta_j$
- Substitute $\delta_j = x_j\beta - \alpha p_j + \xi_j$:

$$\ln(s_j) - \ln(s_0) = x_j\beta - \alpha p_j + \xi_j$$

- This is a **linear regression**!
- LHS: can compute from observed shares
- RHS: product characteristics, price, and an error term

Logit elasticities

- Given shares $s_j = \frac{\exp(\delta_j)}{1+\sum_k \exp(\delta_k)}$
- We can derive price elasticities:

$$\eta_{jj} = \frac{\partial s_j}{\partial p_j}\frac{p_j}{s_j} = -\alpha p_j(1 - s_j) \quad \text{(own-price)}$$

$$\eta_{jk} = \frac{\partial s_j}{\partial p_k}\frac{p_k}{s_j} = \alpha p_k s_k \quad \text{(cross-price)}$$

- Note: $\alpha > 0$, so own-price elasticity is **negative** (as expected)

Worked example: Logit elasticities

- **Question:**
- Suppose $\alpha = 0.5$, product j has price $p_j = 20$ and market share $s_j = 0.1$.
- Compute the own-price elasticity for product j.

Take 2 minutes to solve this.

Worked example: Logit elasticities (solution)

Solution

- Own-price elasticity formula:

$$\eta_{jj} = -\alpha p_j (1 - s_j)$$

- Plug in: $\alpha = 0.5$, $p_j = 20$, $s_j = 0.1$

$$\begin{aligned} \eta_{jj} &= -0.5 \times 20 \times (1 - 0.1) \\ &= -0.5 \times 20 \times 0.9 \\ &= -9 \end{aligned}$$

- Interpretation: A 1% price increase $\Rightarrow$ 9% decrease in quantity

Worked example: Cross-price elasticity

- Now compute the cross-price elasticity with product k

- Given: $\alpha = 0.5$, $p_k = 25$, $s_k = 0.05$

- Cross-price elasticity formula:

$$\eta_{jk} = \alpha p_k s_k$$

- Plug in:

$$\begin{aligned}\eta_{jk} &= 0.5 \times 25 \times 0.05 \\ &= 0.625\end{aligned}$$

- Interpretation: A 1% increase in $p_k \Rightarrow 0.625\%$ increase in s_j

Worked example: Berry inversion

- **Question:**
- Market has 2 products plus outside option. Observed shares:

$$s_0 = 0.5, \quad s_1 = 0.3, \quad s_2 = 0.2$$

- Compute the mean utilities δ_1 and δ_2.

Take 2 minutes to solve this.

Worked example: Berry inversion (solution)

Solution

- Berry inversion: $\ln(s_j) - \ln(s_0) = \delta_j$
- For product 1:

$$\delta_1 = \ln(0.3) - \ln(0.5) = \ln(0.3/0.5) = \ln(0.6) \approx -0.51$$

- For product 2:

$$\delta_2 = \ln(0.2) - \ln(0.5) = \ln(0.2/0.5) = \ln(0.4) \approx -0.92$$

- Interpretation: Both products have negative mean utility relative to outside option
- Product 1 is "better" than product 2 (higher δ)

What drives the IIA property?

- IIA comes from the **logit error structure**
- Key: ε_{ij} are independent across products
- This means: your taste shock for BMW is unrelated to your taste shock for Mercedes
- **Implication:** Model doesn't "know" that BMW and Mercedes are similar
- When BMW's price rises:
    - Logit: BMW consumers spread evenly across ALL other products
    - Reality: BMW consumers mostly switch to Mercedes, Audi
- We'll see how to fix this with demographics (Lecture 3) and nested logit

The IIA problem (preview)

- Look at the cross-price elasticity again:

$$\eta_{jk} = \alpha p_k s_k$$

- This doesn't depend on product j at all!

- Implication: All products have the **same** cross-elasticity with product k

- Is this realistic?

- Suppose BMW raises its price. Logit says: same fraction go to Mercedes as to Honda Civic!

- This is the **IIA** (Independence of Irrelevant Alternatives) property

- We'll discuss this in detail in Lecture 4

Plan

1. The Logit Demand Model
2. **Identification and Instrumental Variables**

The estimating equation (reminder)

- From Berry inversion:

$$\ln(s_j) - \ln(s_0) = x_j\beta - \alpha p_j + \xi_j$$

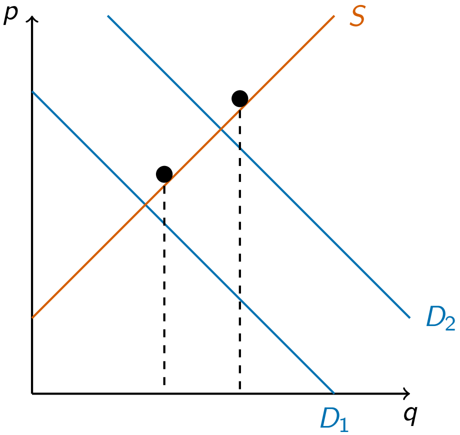
- This looks like a regression we can run with OLS

- **But there's a problem...**

The classic identification problem

- We observe equilibrium prices and quantities

- Problem: can't tell if demand shifted or supply shifted



The classic identification problem

- If we just regress q on p, what do we get?
- Neither demand nor supply!
- **Key insight:** We need variation that shifts ONE curve but not the other
  - To identify demand: need **supply shifters**
  - To identify supply: need **demand shifters**

Price endogeneity in demand estimation

- Our estimating equation:

$$\ln(s_j) - \ln(s_0) = x_j\beta - \alpha p_j + \xi_j$$

- ξ_j = unobserved product quality
- **Problem:** Firms observe ξ_j when setting prices!
- High quality products (ξ_j high) tend to have high prices
- $\Rightarrow \text{Cov}(p_j, \xi_j) > 0$
- OLS gives biased estimates

Direction of bias

- If high-quality products have high prices...
- OLS sees: high price, but demand still high (because of ξ)
- OLS concludes: price doesn't hurt demand much
- **Result:** $\hat{\alpha}$ biased toward zero (less negative than truth)

Worked example: Bias direction

- **Question:** You estimate a logit demand model using OLS and get $\hat{\alpha} = -0.3$. A colleague says the true α is likely -0.5.

- Is this consistent with endogeneity bias? Why?

Worked example: Bias direction

- **Question:** You estimate a logit demand model using OLS and get $\hat{\alpha} = -0.3$. A colleague says the true α is likely -0.5.
- Is this consistent with endogeneity bias? Why?
- **Answer:** Yes!
  - OLS overstates how much consumers like expensive products
  - So OLS finds a smaller (less negative) price coefficient
  - $-0.3 > -0.5$, so this is exactly what we'd expect

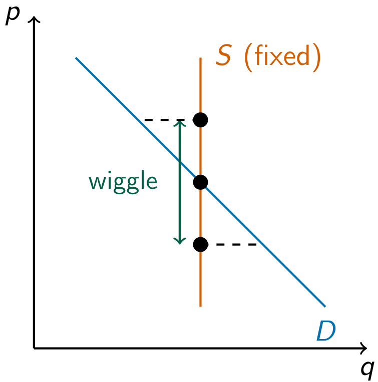
Case study: Uber surge pricing

- Uber's surge pricing: prices rise when demand is high
- **Why is surge pricing endogenous?**
- Surge responds to BOTH:
    - **Supply shocks:** Fewer drivers available $\rightarrow$ surge up
    - **Demand shocks:** Concert ends, rain starts $\rightarrow$ surge up
- Can't just regress rides on surge multiplier!
- Same problem as before: observing equilibrium prices

The solution: Uber's price "wiggle" experiment

- Uber's insight: **randomize** prices!
- **Cohen et al. (2016):** Uber experimentally varied surge multipliers
- Same time, same location $\rightarrow$ different riders see different prices
- Key: Some riders randomly see 1.9x, others see 2.1x
- Demand conditions are identical, only price differs

Tracing out the demand curve



- Randomization holds supply fixed
- Price "wiggles" up/down randomly
- Traces out points ON the demand curve
- This identifies demand!

Uber experiment: Results

- Result: demand elasticity ≈ -0.5
- **Inelastic!** A 10% price increase $\rightarrow$ only 5% fewer rides
- Why so inelastic?
  - When you need a ride, you *need* a ride
  - Limited substitutes late at night
- **Key lesson:** Experiments give clean identification
- But most industries can't randomize prices...

Why experiments are powerful

- No confounding: price variation is independent of demand shocks
- Clean identification of the demand curve
- **But:**
  - Tech companies can run experiments
  - Traditional industries can't randomize prices
  - Most IO settings require **instrumental variables**

Instrumental variables: the solution

- Need variables z that are:
  1. **Relevant:** Correlated with price ($\text{Cov}(z, p) \neq 0$)
  2. **Exogenous:** Uncorrelated with ξ ($\text{Cov}(z, \xi) = 0$)
- These are cost shifters or other supply-side variables

Common IVs in demand estimation

1. **Hausman IVs:** Prices in other markets
   - Same product in different cities has similar costs
   - But demand shocks may differ across markets
2. **BLP IVs:** Characteristics of competing products
   - More/different competitors $\rightarrow$ lower prices
   - Competitors' characteristics don't affect YOUR ξ
3. **Cost shifters:** Input prices, exchange rates
   - Affect production costs, hence prices
   - No direct effect on demand

Worked example: IV intuition

- **Question:** Why do competitor characteristics work as IVs?

Worked example: IV intuition

- **Question:** Why do competitor characteristics work as IVs?
- **Answer:**
- **Relevance:** More competitors nearby $\rightarrow$ more competition $\rightarrow$ lower price ✓
- **Exogeneity:** Competitor characteristics don't affect YOUR unobserved quality ξ_j ✓
- Example: If Toyota enters with a new Camry, this affects Civic's price but not Civic's unobserved quality

Worked example: Evaluating an IV

- **Question:** A researcher proposes using gasoline prices as an IV for car prices. Is this valid?

Worked example: Evaluating an IV

- **Question:** A researcher proposes using gasoline prices as an IV for car prices. Is this valid?
- **Relevance:** Higher gas prices $\rightarrow$ higher operating costs $\rightarrow$ might affect car prices? Maybe weakly.
- **Exogeneity:** Do gas prices affect car quality ξ_j?
  - Gas prices affect *demand* for fuel-efficient cars
  - This might shift which cars look "good" to consumers
  - Potentially problematic!
- **Verdict:** Probably not a great IV

Summary: IV conditions

- For z to be a valid IV:
  1. **Relevant:** z must predict prices
     - Can test this! Run first-stage regression
  2. **Exogenous:** z must not affect demand directly
     - Cannot test this directly (requires economic reasoning)
- This is the standard IV framework from econometrics
- Applied to demand estimation: use supply-side variation

What makes a good IV? Summary

Instrument	Relevance	Exogeneity
Cost shifters (input prices)	Strong	Usually OK
Hausman IVs (prices elsewhere)	Strong	Debatable
BLP IVs (competitor chars)	Medium	Usually OK
Experiments (Uber wiggle)	Perfect	Perfect

- No perfect instrument in observational data
- **Best practice:** Use multiple IVs, check robustness
- **Experiments** are gold standard but often infeasible

From estimation to policy

- We've seen how to estimate demand with logit
- **Why do we care?**
- Demand estimates let us:
    1. Compute **elasticities** (how sensitive are consumers?)
    2. Compute **consumer welfare** (how much do consumers benefit?)
    3. Simulate **counterfactuals** (what if a product is removed? what if firms merge?)
- Next: demographics to relax IIA, then these applications

Key Points

1. **Logit model**: ε_{ij} is Type I Extreme Value $\rightarrow$ closed-form shares
2. **Share equation**: $s_j = \frac{\exp(\delta_j)}{1+\sum_k \exp(\delta_k)}$
3. **Berry inversion**: $\ln(s_j) - \ln(s_0) = \delta_j$ turns demand estimation into a regression
4. **Elasticities**: Own $= -\alpha p_j(1 - s_j)$; Cross $= \alpha p_k s_k$
5. **IIA**: Cross-elasticities don't depend on product similarity (a limitation)
6. **Price is endogenous**: Firms observe ξ_j when pricing $\rightarrow$ $\text{Cov}(p, \xi) > 0$
7. **Bias direction**: OLS gives $\hat{\alpha}$ biased toward zero
8. **Solution**: Instrumental variables (cost shifters, BLP IVs, Hausman IVs)

Next time

- **Lecture 3:** Demographic Interactions and pyblp
  - Extending logit to allow preference heterogeneity
  - Estimation using pyblp package
  - Worked example with car data