

ECN 594: Homework 2 Solutions

Competition and Merger Simulation

Solution Key

Part A: Oligopoly Theory

Question 1: Cournot Competition

(a) 3-firm Cournot equilibrium

With N identical firms, demand $P = a - Q$, and marginal cost c :

Firm j 's profit: $\pi_j = (P - c)q_j = (a - Q - c)q_j = (a - q_j - \sum_{k \neq j} q_k - c)q_j$

FOC: $\frac{\partial \pi_j}{\partial q_j} = a - 2q_j - \sum_{k \neq j} q_k - c = 0$

By symmetry, $q^* = q_j$ for all j , so:

$$a - 2q^* - (N - 1)q^* - c = 0$$

$$q^* = \frac{a - c}{N + 1} = \frac{100 - 10}{4} = 22.5$$

Variable	Value
Per-firm quantity q^*	22.5
Total quantity Q^*	67.5
Price P^*	\$32.50
Per-firm profit π^*	\$506.25

(b) Lerner Index verification

Direct calculation:

$$L = \frac{P - MC}{P} = \frac{32.5 - 10}{32.5} = 0.692$$

Using formula $L = \frac{s_j}{|\varepsilon|}$:

- Market share: $s_j = \frac{1}{3} = 0.333$
- Price elasticity: $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -1 \cdot \frac{32.5}{67.5} = -0.481$
- $L = \frac{0.333}{0.481} = 0.692 \checkmark$

(c) 2-firm case and welfare comparison

With $N = 2$: $q^* = \frac{90}{3} = 30$, $Q^* = 60$, $P^* = \$40$, $\pi^* = \$900$

	3 firms	2 firms	Change
Consumer Surplus	\$2,278.13	\$1,800.00	−\$478.13
Producer Surplus	\$1,518.75	\$1,800.00	+\$281.25
Total Welfare	\$3,796.88	\$3,600.00	−\$196.88

Fewer firms \Rightarrow higher price \Rightarrow lower CS, higher PS, lower TW (deadweight loss).

Question 2: Bertrand Competition

(a) With homogeneous products: $P^* = c = \$10$, $\pi^* = \$0$

(b) Cournot: $P = \$32.50$, $\pi = \$506.25$. Bertrand: $P = \$10$, $\pi = \$0$.

Bertrand is more aggressive—firms undercut until $P = MC$. Cournot has higher prices because quantity commitment creates strategic substitutability.

Bertrand realistic: Easy price adjustment, no capacity constraints, high transparency.

Cournot realistic: Capacity committed in advance, output hard to adjust.

Question 3: Collusion

(a) Monopoly: $Q_m = 45$, $P_m = \$55$, $\pi_m = \$2,025$

Per-firm under collusion: $q = 15$, $\pi = \$675$

(b) Optimal deviation given others produce $q = 15$ each:

Best response: $q_{dev} = \frac{a-c-Q_{others}}{2} = \frac{100-10-30}{2} = 30$

$Q_{dev} = 60$, $P_{dev} = \$40$, $\pi_{dev} = \$900$

(c) Critical discount factor:

$$\delta^* = \frac{\pi_{dev} - \pi_{coll}}{\pi_{dev} - \pi_{punish}} = \frac{900 - 675}{900 - 506.25} = \frac{225}{393.75} = 0.571$$

Or using formula: $\delta^* = \frac{(N+1)^2}{N^2 + (N+1)^2} = \frac{16}{9+16} = 0.64$

Collusion sustainable if $\delta \geq 0.57$ –0.64 (depending on exact specification).

Part B: Merger Simulation

Question 4: Pre-Merger Equilibrium

(a) Pre-merger equilibrium prices:

Solving the FOC system $s_j + (p_j - c_j) \frac{\partial s_j}{\partial p_j} = 0$ with initial guess $p = c + 0.5$:

Product	Equilibrium Price	Markup
1	\$1.51	\$0.51
2	\$1.71	\$0.51
3	\$2.01	\$0.51
4	\$1.61	\$0.51

(b) Market shares:

Product	Share	Percentage
1	0.0268	2.68%
2	0.0221	2.21%
3	0.0135	1.35%
4	0.0243	2.43%
Outside	0.9133	91.33%

(c) Own-price elasticities:

Using $\eta_{jj} = \alpha p_j(1 - s_j)$:

Product	Elasticity	Status
1	-2.95	Elastic
2	-3.35	Elastic
3	-3.96	Elastic
4	-3.15	Elastic

All products have $|\eta| > 1$, confirming elastic demand.

(d) FOC verification for product 1:

Actual markup: $p_1 - c_1 = 1.51 - 1.0 = \0.51

FOC markup: $\frac{1}{|\alpha|(1-s_1)} = \frac{1}{2 \times 0.973} = \0.51

The FOC is satisfied (difference $< \$0.001$).

(e) HHI:

Using inside shares: $\text{HHI} \approx 2,634$ (highly concentrated, $> 2,500$).

Question 5: Post-Merger Prices

(a) Ownership matrices:

Pre-merger: $\mathcal{O} = I_4$ (identity matrix)

Post-merger:

$$\mathcal{O} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) Intuition: The merged firm internalizes substitution between products 1 and 2. Pre-merger, raising p_1 meant losing customers to product 2 (competitor). Post-merger, those customers are “recaptured”—the merged firm keeps them. This makes demand less elastic, leading to higher optimal prices.

(c) and (d) Post-merger equilibrium:

Product	Pre-merger	Post-merger	Change
1	\$1.51	\$1.53	+0.75%
2	\$1.71	\$1.73	+0.81%
3	\$2.01	\$2.01	+0.00%
4	\$1.61	\$1.61	+0.00%

Merging firms raise prices; non-merging firms see negligible changes (logit with small shares leads to limited strategic interaction).

Question 6: Welfare Analysis

(a) Consumer surplus:

	Pre-merger	Post-merger
CS	\$45.37	\$44.77
Change	−\$0.60 (−1.3%)	

(b) Producer profits:

Merged firm: Pre $\$13.78 + \$11.29 = \$25.07 \rightarrow$ Post $\$25.08$ (+0.04%)

Non-merging firms see negligible changes (Firm 3: +\$0.01, Firm 4: +\$0.01).

Total PS: Pre $\$44.39 \rightarrow$ Post $\$44.42$ (+\$0.03)

(c) Total welfare:

TW falls by $\$0.57$ (−0.63%). The merger is **welfare-reducing**—consumer harm exceeds producer gain.

(d) Efficiency defense:

With 10% cost reductions on products 1 and 2, welfare change becomes +\$10.44 (+11.6%). The efficiency defense **succeeds**—sufficient cost savings offset anticompetitive price increases.

Policy implication: Merging parties must demonstrate credible efficiency gains to offset consumer harm.