

# ECN 453: Homework 3 - solutions

## 1. New technology and market structure (30 points)

Consider an industry with market demand  $Q = a - p$  and an infinite number of potential entrants with access to the same technology. Initially, the technology is given by  $C = F + cq$ . A new technology allows for a lower marginal cost  $c' < c$  at the expense of a higher fixed cost  $F' > F$ .

Given  $a = 10, F = 2, F' = 3, c = 2, c' = 1$ .

- Use the following formula (from the lecture slides) for the number of firms  $n$ :

$$n = \left[ (a - c) \sqrt{\frac{S}{F}} - 1 \right]$$

Then, use the following formula (from the lecture slides) for the price, given the above number of firms:

$$p = \frac{a + nc}{n + 1}$$

Under the old technology:

Here,  $a = 10, c = 2, S = 1, F = 2$ . So:  $n = \left[ (10 - 2) \sqrt{1/2} - 1 \right] = 4$ .

Then, equilibrium price under old technology:

$$p = \frac{10 + 4 \times 2}{4 + 1} = 3.6$$

Under the new technology:

Here,  $a = 10, c = 1, S = 1, F = 3$ . So:  $n = \left[ (10 - 1) \sqrt{1/3} - 1 \right] = 4$ .

Then, equilibrium price under new technology:

$$p = \frac{10 + 4 \times 1}{4 + 1} = 2.8$$

## 2. Repeated games (50 points)

Consider the following game and suppose that it is repeated an infinite number of times. Players have a discount value of  $\delta$ .

		Player 2	
		L	R
		10	12
Player 1	T	10	0
	B	0	1
		12	1

a. Equilibrium payoff:

$$\Pi = 10 + \delta 10 + \delta^2 10 + \delta^3 10 + \dots = \frac{10}{1 - \delta}$$

Deviation payoff:

$$\Pi' = 12 + \delta + \delta^2 + \delta^3 + \dots = 12 + \frac{\delta}{1 - \delta}$$

Therefore, for collusion to sustain, we need:

$$\begin{aligned} \Pi &\geq \Pi' \\ \frac{10}{1 - \delta} &\geq 12 + \frac{\delta}{1 - \delta} \\ 10 - \delta &\geq 12(1 - \delta) \\ \delta &\geq \frac{2}{11} \end{aligned}$$

b. Equilibrium payoff is the same.

Deviation payoff:

$$\Pi' = 12 + 0\delta + 0\delta^2 + 0\delta^3 + \dots = 12$$

Therefore, for collusion to sustain, we need:

$$\begin{aligned} \Pi &\geq \Pi' \\ \frac{10}{1 - \delta} &\geq 12 \\ 10 &\geq 12(1 - \delta) \\ \delta &\geq \frac{2}{12} \end{aligned}$$

- c. The players sustain collusion on  $(T,L)$  using the grim trigger punishment of playing  $(B,R)$  in all future periods, and the discount factor  $\delta$  indexes how much agents care about this future punishment. Since in Part b, the punishment is harsher than in Part a, (since the players get  $(0,0)$  for all future periods rather than  $(1,1)$ ), collusion can be sustained for lower discount factors.