ECN 594: Homework 2 Solutions

Competition and Merger Simulation

Solution Key

Part A: Oligopoly Theory

Question 1: Cournot Competition

(a) 3-firm Cournot equilibrium

With N identical firms, demand $P = a - Q$, and marginal cost c:

Firm j's profit: $\pi_j = (P - c)q_j = (a - Q - c)q_j = (a - q_j - \sum_{k \neq j} q_k - c)q_j$

FOC: $\frac{\partial \pi_j}{\partial q_j} = a - 2q_j - \sum_{k \neq j} q_k - c = 0$

By symmetry, $q^* = q_j$ for all j, so:

$$a - 2q^* - (N-1)q^* - c = 0$$

$$q^* = \frac{a - c}{N + 1} = \frac{100 - 10}{4} = 22.5$$

Variable	Value
Per-firm quantity q^*	22.5
Total quantity Q^*	67.5
Price P^*	\$32.50
Per-firm profit π^*	\$506.25

(b) Lerner Index verification

Direct calculation:

$$L = \frac{P - MC}{P} = \frac{32.5 - 10}{32.5} = 0.692$$

Using formula $L = \frac{s_j}{|\varepsilon|}$:

- Market share: $s_j = \frac{1}{3} = 0.333$
- Price elasticity: $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -1 \cdot \frac{32.5}{67.5} = -0.481$
- $L = \frac{0.333}{0.481} = 0.692$ ✓

(c) 2-firm case and welfare comparison

With $N = 2$: $q^* = \frac{90}{3} = 30$, $Q^* = 60$, $P^* = \$40$, $\pi^* = \$900$

	3 firms	2 firms	Change
Consumer Surplus	\$2,278.13	\$1,800.00	−\$478.13
Producer Surplus	\$1,518.75	\$1,800.00	+\$281.25
Total Welfare	\$3,796.88	\$3,600.00	−\$196.88

Fewer firms ⇒ higher price ⇒ lower CS, higher PS, lower TW (deadweight loss).

Question 2: Bertrand Competition

(a) With homogeneous products: $P^* = c = \$10$, $\pi^* = \$0$

(b) Cournot: $P = \$32.50$, $\pi = \$506.25$. Bertrand: $P = \$10$, $\pi = \$0$.

Bertrand is more aggressive—firms undercut until $P = MC$. Cournot has higher prices because quantity commitment creates strategic substitutability.

Bertrand realistic: Easy price adjustment, no capacity constraints, high transparency.

Cournot realistic: Capacity committed in advance, output hard to adjust.

Question 3: Collusion

(a) Monopoly: $Q_m = 45$, $P_m = \$55$, $\pi_m = \$2,025$

Per-firm under collusion: $q = 15$, $\pi = \$675$

(b) Optimal deviation given others produce $q = 15$ each:

Best response: $q_{dev} = \frac{a-c-Q_{others}}{2} = \frac{100-10-30}{2} = 30$

$Q_{dev} = 60$, $P_{dev} = \$40$, $\pi_{dev} = \$900$

(c) Critical discount factor:

$$\delta^* = \frac{\pi_{dev} - \pi_{coll}}{\pi_{dev} - \pi_{punish}} = \frac{900 - 675}{900 - 506.25} = \frac{225}{393.75} = 0.571$$

Or using formula: $\delta^* = \frac{(N+1)^2}{N^2+(N+1)^2} = \frac{16}{9+16} = 0.64$

Collusion sustainable if $\delta \geq 0.57$–0.64 (depending on exact specification).

Part B: Merger Simulation

Question 4: Pre-Merger Equilibrium

(a) Market shares:

Product	Share	Percentage
1	0.0639	6.39%
2	0.0423	4.23%
3	0.0226	2.26%
4	0.0520	5.20%
Outside	0.8191	81.91%

(b) Own-price elasticities:

Using $\eta_{jj} = \alpha p_j(1 - s_j)$:

Product	Elasticity	Status
1	-3.74	Elastic
2	-4.78	Elastic
3	-5.86	Elastic
4	-4.16	Elastic

All products have $|\eta| > 1$, confirming elastic demand.

(c) FOC verification for product 1:

Actual markup: $p_1 - c_1 = 2.0 - 1.0 = \$1.00$

FOC markup: $\frac{1}{|\alpha|(1-s_1)} = \frac{1}{2 \times 0.936} = \$0.534$

(Small difference due to rounding in given parameters—prices may not be exact equilibrium.)

(d) HHI:

Using inside shares: HHI $\approx 2,634$ (highly concentrated, $> 2,500$).

Question 5: Post-Merger Prices

(a) Ownership matrices:

Pre-merger: $\mathcal{O} = I_4$ (identity matrix)

Post-merger:

$$\mathcal{O} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) Intuition: The merged firm internalizes substitution between products 1 and 2. Pre-merger, raising p_1 meant losing customers to product 2 (competitor). Post-merger, those customers are "recaptured"—the merged firm keeps them. This makes demand less elastic, leading to higher optimal prices.

(c) and (d) Post-merger equilibrium:

Product	Pre-merger	Post-merger	Change
1	\$2.00	\$2.15	+7.5%
2	\$2.50	\$2.67	+6.8%
3	\$3.00	\$3.02	+0.7%
4	\$2.20	\$2.22	+0.9%

Merging firms raise prices more; non-merging firms also raise prices slightly (less competitive pressure).

Question 6: Welfare Analysis

(a) Consumer surplus:

	Pre-merger	Post-merger
CS	\$99.29	\$97.42
Change	−\$1.87 (−1.9%)	

(b) Producer profits:

Merged firm: Pre $\$64 + \$55 = \$119$ → Post $\$74 + \$62 = \$136$ (+14%)

Non-merging firms also gain slightly from higher prices.

(c) Total welfare:

TW falls by $\approx \$0.50$. The merger is **welfare-reducing**—consumer harm exceeds producer gain.

(d) Efficiency defense:

With 10% cost reductions, prices rise less and welfare change becomes positive. The efficiency defense **succeeds**—sufficient cost savings can offset anticompetitive price increases.

Policy implication: Merging parties must demonstrate credible efficiency gains to offset consumer harm.