there are five people bidding for a vintage baseball card and that the value v_i that each places on the card is, in increasing order: \$100, \$200, \$300, \$400, and \$500. Let's denote the bid of each player as b_i. If each player adopts the rule $b_{\mathrm{i}} = b(v_i) = v_i$, then in a second price auction the winning bid will be $b_i = \$500$, and the winner will pay the second-highest bid value of $b_i = \$400$. Likewise, in an English auction once the bid rises to an arbitrarily small amount above \$400, all bidders except the one with the highest valuation will have exited. Once again, if all bidders follow the rule $b_{\mathrm{i}} = b(v_i) = v_i$, that is, if all bidders bid their true value, the card will go to the bidder with the highest valuation at a price equal to second-highest willingness to pay, namely, \$400 in this case.

A question that immediately arises then is whether it makes sense for each bidder to bid a true valuation in both an English or second-price auction. Another way to put this is to ask whether the strategy $b_i = v_i$ for each player simultaneously constitutes a Nash equilibrium in these settings. To see that it does, it is helpful to recognize that in both the English and second-price sealed bid auctions, what one bids determines only whether or not one wins—not what one pays.

Returning to our example, suppose realistically that bidders know their own valuation of the baseball card but not the valuations of others. All a bidder knows in this latter regard is that the other bids are drawn randomly from a very large pool of potential bidders, each drawn from a uniform distribution with a range of \$0 to \$600. Now suppose again that all bidders are following the strategy of bidding their true valuation so that $b_j = v_j$ for each. Can any one bidder alter her behavior in a way that improves her expected outcome?

To see that the answer is no, think about a specific bidder, such as bidder 4. Suppose that she lowers her bid to \$390—\$10 below her valuation. Even without knowing the other valuations, she can foresee that this will lead to one of two outcomes. One of these is that all the other bidders had lower valuations and that the next highest bid was less than \$390. In this case, the change has no effect on her chances of winning or what she pays. The other possibility is that she had the highest valuation but that the next highest exceeded \$390, e.g., \$395. In that case, the fact that other bidders are bidding their true valuations will mean that bidding \$390 will result in bidder 4 losing the chance to buy a baseball card she values at \$400 for less than that amount. In short, bidder 4's decision to reduce her bid below \$400 will definitely not lead to any gain and may result in a possible loss.

The foregoing reasoning also applies to a decision by bidder 4 to raise her bid above \$400—say to \$410. If the winning bid was greater than this amount, e.g., \$500, then again bidder 4's increase to \$410 has no effect. However, if the winning bid was between \$400 and \$410, e.g., \$405, then bidder 4's decision to bid \$410 will mean she buys the baseball card at a price that exceeds her personal valuation. Once again, the choice to alter her bid—this time to raise it slightly—cannot improve her outcome but may lead to a loss.

We can repeat the foregoing analysis for each bidder. It then becomes clear that with all other bidders following the strategy $b_i(v_i) = v_i;\ i \neq j$, the best response for bidder j is to bid her valuation and set $b_j(v_j) = v_j$, as well. Thus, the symmetric Nash equilibrium in a second-price sealed bid auction is for each player to submit a bid equal to her true valuation. The winning bid will then pay an amount equal to the second-highest valuation. As noted, this is also exactly what would happen in an oral English auction. These two auctions are functionally equivalent.

Because the actual amount paid in English and second-price auctions is equal to the second-highest valuation, it is interesting to determine what the expected winning price will be. For this purpose, we need the concept of an order statistic. For a random sample of n from a given population, the kth order statistic is the value of the kth smallest value. For example, in our sample of $n = 5$ bidders with valuations \$100, \$200, \$300, \$400, and \$500, respectively, the first order statistic $v(1)$ is the smallest value, or \$100. The second order statistic is $v(2) = \$200$. Because we know that both a second-price sealed bid auction and an English auction both result in a winning payment equal to the second-highest valuation among the sample of bidders, we can work out the expected winning bid for any sample of size n by working out the n-1th order statistic, e.g., for a random sample of five bidders, the expected actual payment is the expected fourth-smallest valuation or the fourth order statistic $v(4)$. For the uniform distribution between 0 and V^{Max}, this is given by:

$$\text{Expected Value of } k\text{th Order Statistic with Sample Size } n = \mathrm{E}[v(\mathrm{k})] = \frac{k}{n+1}V^{Max} \tag{23.1}$$

Thus, for our five bidder case with bids drawn randomly from a uniform distribution with a maximum valuation of $V^{Max} = \$600$, we would expect that in a sample of five, the second-highest (fourth-smallest) value and therefore the price paid by the winning bidder would be $\mathrm{E}[v(4)] = \left(\frac{4}{6}\right)\$600 = \$400$. This is also the revenue that the seller of the baseball card should expect.

Note that as the number of bidders rises, the relevant order statistic also increases. If instead we had a sample of ten bidders drawn from this same uniform distribution, we would be interested in the ninth order statistic or the expected value of $v(9)$. This would be given by: $\mathrm{E}[v(9)] = \left(\frac{9}{11}\right)\$600 = \$490.91$. For the general case of n bidders drawn from a uniform distribution ranging from \$0 to V^{Max}, we have the expected winning price p as the n-1th order statistic or:

$$\mathrm{E}[v(n-1)] = \mathrm{E}(p) = \left(\frac{n-1}{n+1}\right)V^{Max}. \tag{23.2}$$

More bidders means that it is more likely that the pool will include buyers with very high valuations. As a result, the expected price or revenue to the seller rises closer and closer to the maximum valuation among all potential bidders.

Practice Problem 23.1

Show that a dominant bidding strategy in an English auction is to continue bidding as long as the price in the auction is less than your true value of the good.

23.2.2 Equilibrium Bidding Strategies in Dutch and First-Price Private Value Auctions

We now turn to the Dutch descending auction and the first-price sealed bid auction. A little thought quickly reveals that these two auction types are also equivalent because the essential strategic choice is the same in each. In each case, bidder i must decide on a strike price at which he or she will claim the item. The remaining question is what that strike price or bid should be. The answer is suggested by the central difference between our two broad auction types—the Dutch and first-price auctions on the one hand, and the English and second-price auctions discussed above, on the other. This difference is that unlike our earlier cases, the price the bidder pays in either a Dutch or first-price auction *is* determined by the bid he makes rather than the next highest bid. This immediately rules out ever bidding more than one's valuation. Instead, the fact that bidders in a Dutch or first-price auction pay precisely what they bid instead of the next-highest bid, suggests that they may want to shade their bids a bit relative to bidders in an English or second-price auction. The issue then becomes the extent of such shading. How far below their private valuation should bidders in a Dutch or first-price sealed bid auction bid?

Let's start by focusing on a sealed bid auction for the rare baseball card imagined above but also by simplifying that example by assuming just two random bidders. Bidder 1 personally values the card at \$200 while bidder 2 values it at \$400. However, the bidders only know their own valuation. As before, all bidder i knows about bidder j's valuation is that it is drawn randomly from a uniform distribution ranging from 0 to \$600. Because we are explicitly interested in bidding rules that may yield a bid lower than the bidder's true valuation, we will consider proportional rules of the form $b_i(v_i) = \lambda v_i$, where $0 \leq \lambda \leq 1$. When $\lambda = 1$, bidders bid their full valuation $b_i = v_i$. For smaller values of λ bids are only a fraction of that valuation.

Suppose that both bidders choose $\lambda = 0.9$, that is, each bids 90 percent of their true valuation. Because these bids are strictly proportional to the underlying valuations, the probability of having the highest bid is exactly the same as the probability of having the highest valuation. For bidder 1, given her value of \$200 and what she knows about the distribution of her rival's valuation, she can work out that she has a one-third chance of being the highest bidder. Similarly, bidder 2 can determine that he has a two-thirds chance of being the highest bidder. Is this a Nash equilibrium?

Consider bidder 1 first. Right now, she is bidding $0.9v_1 = 0.9 \times \$200 = \$180$. While she does not know her rival's valuation, she knows that her rival is also submitting a bid of $0.9v_2$. So, as noted, bidder 1's chance of submitting the winning bid is one-third. Because she only gains if she is the winner, bidder 1's expected outcome $\mathrm{E}[\mathrm{Y}_1]$ in the current setting is:

$$\mathrm{E}[\mathrm{Y}_1] = 0.3333 \times [\$200 - \$180] = \$6.67 \tag{23.3}$$

Suppose now that instead of continuing to bid 90 percent of her valuation, bidder 1 reduces her bid still further, say to 80 percent. This has the advantage of greatly increasing the gain if she is in fact the high bidder. However, she is now bidding sufficiently low that her chance of being the high bidder has also fallen. With a little effort, we can in fact determine how this tension works out.

By bidding only 80 percent of her valuation, bidder 1 reducers her bid to \$160. Hence, if she wins the auction, she now gains \$40 instead of only \$20. What is the probability with which this happens?

To answer this question first note that if bidder 1 were to subit a bid of \$160 when *all* bidders including bidder 1 bid 90 percent of their valuation, it would imply that bidder 1 had a true valuation of $v_1 = \$160/0.9 = \$177.78$. In reality, this is not her true value. But this thought experiment is nevertheless instructive. In a world in which all other bidders are bidding 90 percent of their true value, bidder 1's decision to submit a bid equal to 80 percent of her true \$200 value gives her the same chance of winning as if she had continued to bid 90 percent of her value but had a value of only \$177.78. Specifically, her chance of winning now given her rival's strategy is \$177.78/\$600 = 0.296. As expected, this is a lower chance than before, but this fall in the likelihood of winning is more than offset by the increased gain of winning. Bidder 1's expected outcome now is:

$$\mathrm{E}[\mathrm{Y}_1] = 0.296 \times [\$200 - \$160] = \$11.84 \tag{23.4}$$

From this it is clear that the strategy combination of each bidder shading his or her bid to 90 percent of its true value is *not* a Nash equilibrium. If bidder 2 were to do this, bidder 1 would do better by unilaterally reducing her bid to 80 percent of its true value. Moreover, this result is symmetric. If bidder 1 were submitting a bid equal to 90 percent of her true value, bidder 2 could likewise do better by shading his bid more deeply as well.

We have just seen that $\lambda = 0.9$ is too high a proportional bid to be consistent with a Nash equilibrium. It is pretty clear however that some choices of λ would be too small. For example if both bidders set λ arbitrarily close to zero, either one could virtually assure herself of winning the auction by increasing its bid a little while still enjoying a very sizable gain after the auction is won. The issue is then whether we can find a λ value that is neither too small nor too large—one that if used by both bidders implies that neither can gain by raising or lowering his or her bid a small amount. As it turns out, we can and for this simple two-bidder case, the Nash equilibrium is for each bidder to submit a bid that is one half of the true value, i.e., $b(v_i) = 0.5v_i$. In this case, the expected gain for each bidder is

$$\begin{aligned} \mathrm{E}[\mathrm{Y}_1] &= 0.3333 \times [\$200 - \$100] = \$33.33 \\ \mathrm{E}[\mathrm{Y}_2] &= 0.6667 \times [\$400 - \$200] = \$133.33 \end{aligned} \tag{23.5}$$

To illustrate that $b(v_i) = 0.5v_i$ constitutes a Nash equilibrium for both bidders, we work through the logic of our earlier example and ask whether a small rise in the bid of either bidder would yield a gain given that the rival bidder continues to set $b(v_i) = 0.5v_i$.

Consider bidder 1. With $b(v_i) = 0.5v_i$, she bids \$100 so that \$33.33 is her expected win. If she raises her bid to \$110, her chances of winning rise to $\left(\frac{110/0.5}{600}\right) = 0.367$. Her gain if she wins is then (\$200–\$110) = \$90. So, her expected win if she increases her bid to \$110 while bidder 2 still plays $b(v_i) = 0.5v_i$ is

$$\mathrm{E}[\mathrm{Y}_1] = 0.367 \times [\$200\text{–}\$110) = \$33 \tag{23.6}$$

This is less than the expected win if she continues with the strategy $b(v_i) = 0.5v_i$. Accordingly, it does not pay to raise her bid.

Now consider what happens if bidder 1 lowers her bid, say to \$90. In this case, her chances of winning the auction fall to $\left(\frac{90/0.5}{600}\right) = 0.30$. Her expected win as a result becomes:

$$\mathrm{E}[\mathrm{Y}_1] = 0.3 \times [\$200\text{–}\$90) = \$33 \tag{23.7}$$

Just as she has no reason to raise her bid, bidder 1 has no reason to lower it either.

What about bidder 2? The same calculations show that he too has no reason to deviate from the strategy $b(v_i) = 0.5v_i$, given that bidder 1 has not done so. With both bidders setting $b(v_i) = 0.5v_i$, he has a 0.67 chance of winning \$200 for an expected payoff of \$134. If he raises his bid to \$220, he will increase his chance of winning to $\left(\frac{220/0.5}{600}\right) = 0.733$, but lower the gain from winning to \$400–\$220 = \$180. His expected payoff now given that bidder 1 still plays $b_i(v_i) = 0.5v_i$ declines to

$$E[Y_2] = 0.733 \times [\$400 - \$220) = \$132 \tag{23.8}$$

Similarly, if bidder 2 lowers his bid to \$180, his gain if he wins rises to \$220 but his chances of winning fall to $\left(\frac{180/0.5}{600}\right) = 0.60$. Hence, his expected payoff in this case is again:

$$E[Y_2] = 0.60 \times [\$400 - \$180) = \$132 \tag{23.9}$$

As \$132 again implies a fall in bidder 2's expected gain, it appears that with two bidders submitting sealed bids in a first-price auction, there is a symmetric Nash equilibrium in which each bidder submits a bid equal to one-half of his or her private evaluation given that the valuation of the rival bidder is drawn from a uniform distribution. We have not proven this formally here (the proof is in the appendix), yet the heuristic analysis is compelling. We therefore assert the result here. In particular, we assert that:

If two bidders, each with a private valuation drawn randomly from an identical uniform distribution, compete in a first-price, sealed-bid auction the symmetric Nash equilibrium bidding strategy is for each bidder to bid exactly one half of his or her true value, v_i, *i.e.*, $b(v_i) = 0.5v_i$.

What happens if there are more than two bidders? We already have some intuition regarding the answer. In our discussion of English and second-price auctions we learned that as the number of bidders increased, the bidding became more competitive and the winning bid rose higher toward its maximum possible value. It is reasonable to assume that that same competitive pressure also holds in Dutch and first-price auctions. It follows that as the number n of bidders rises above two, that we should expect to see bidders offering to pay a higher fraction—choose a higher λ value—as part of their bidding strategies.

As it turns out, the foregoing intuition is exactly right. As we show in the Appendix to this chapter, for bidders in a first-price or Dutch auction, the optimal value of λ for the general case of n bidders with independent private value drawn from a common uniform distribution is $\lambda = \frac{n-1}{n}$. Again, the proof is given in the appendix but the insight is reflected in the following result:

In a Dutch or first-price auction with n bidders with independent private values v_i drawn from a common uniform distribution, the symmetric equilibrium bidding is for each bidder to submit a bid given by: $b(v_i) = \frac{(n-1)}{n}v_i$.

Once again, it is useful to determine the expected winning bid or equilibrium price that will be paid for the auctioned item. For this purpose, we recall the order statistics for the uniform distribution. Because both the winning bid and the actual payment are equal to the highest bid submitted, the expected price is just equal to the bid generated by the above equilibrium bidding strategy when applied to the nth highest expected value v_i from a random sample of n bidders whose values are drawn independently from a uniform distribution ranging from 0 to V^{Max}. That nth smallest value is given by $\left(\frac{n}{n+1}\right)V^{Max}$ so that the expected winning bid or price is:

$$\mathrm{E}(p) = \left(\frac{n-1}{n}\right)\left(\frac{n}{n+1}\right)V^{Max} = \left(\frac{n-1}{n+1}\right)V^{Max} \tag{23.10}$$

Practice Problem 23.2

You are bidding for an original John Lennon hat in a sealed bid first-price auction. You are one of eight bidders in this auction and the most you would be willing to pay for this hat is \$200. Show that your optimal strategy is to submit a bid of \$175.

23.2.3 The Revenue Equivalence Theorem

We are now in a position to state what may be the most famous result in all of auction theory. A comparison of equations (23.2) and (23.10) reveals that they are identical. That is, the expected price ultimately paid by the winning bid in an English or second-price auction is exactly the same as the expected price paid in either a Dutch or first-price auction. That is, from the standpoint of the seller, all auction designs yield the same expected revenue. This result, originally due to Vickrey (1961), is known as the Revenue Equivalence Theorem. We have shown it here for the specific case of the uniform distribution. However, it is a quite general result that holds across different distributions of individual private values. We state this theorem below:

Revenue Equivalence Theorem: *Any auction in which the item goes to the highest bidder and in which risk neutral bidders have private values drawn independently from an identical and continuous distribution and in which the expected payment from a bidder with value 0 is 0, will yield the same expected revenue to the seller*.

The Revenue Equivalence Theorem is a remarkable result.[5] It also applies when more than one unit is being auctioned so long as each bidder wants only one unit and the auction gives the m units being sold to the m highest bidders. There are, of course, important situations in which Revenue Equivalence will not hold. In particular, it will not hold when any of the assumptions on which it relies such as risk-neutrality are not satisfied.[6] Yet in such cases it is precisely the ability of the theorem to identify the reason that revenue equivalence fails that makes the theorem so useful.

5 The initial observations behind the theorem are due to Vickrey (1961). The formalization of the theorem itself is typically credited to both Riley and Samuelson (1981) and Myerson (1981).

6 Recall that bidders in the first-price auction take some risk of being outbid by a lower-valuation bidder in that each bidder shades his bid below his personal valuation. If bidders are risk averse, they will shade their bids less (buy insurance with a higher bid), in such cases leading to a higher overall revenue in first-price auctions.

23.3 COMMON VALUE AUCTIONS AND THE WINNER'S CURSE

In the baseball card example above, we assumed that each bidder had his or her own private value of the card, independent of what other bidder's valued it. This may well be the case for certain heirloom items, memorabilia, or art objects and similar items. Yet for many items, the ultimate worth to the buyer depends on how much the item can be sold for later, i.e., on its worth to others. Thus, when two companies bid for drilling rights on a particular tract of land, or the legal rights to a specific patent, and when two bidders bid for a jar with an undisclosed amount of quarters in it or perhaps even for a vintage baseball card, what they truly care about is the item's true market value or the price at which it can be sold—a common value for which each bidder may only have a private estimate.

For example, suppose that we again have n bidders interested in buying a vintage baseball card, not as a personal prize but as an investment. As a result, each is interested in the card's true market value V^*, which is a random variable. After doing some research, the bidders obtain an estimate v_i of V^* that is related to their estimate by an error ε_I as follows:

$$v_i = V^* + \varepsilon_i \tag{23.11}$$

where the ε_i term is distributed uniformly from $-\$100$ to $+\$100$. This means that for any bidder, v_i is an unbiased estimate of the true value $V^* : \mathrm{E}(v_i) = V^* + \mathrm{E}(\varepsilon_i) = V^*$. Each bidder of course only sees v_i—not its decomposition into V^* and ε_i. Yet given that the expected value of ε_i is 0, a bidder who receives information that the baseball card is worth \$400, i.e., one for whom $v_i = \$400$, can reasonably take this value as an unbiased estimate of the true value V^*. As a result, each bidder may well be tempted to bid up to the bidder's observed v_i value in order to procure the card.

Unfortunately, bidding v_i is likely to lead to overpaying in these situations. This is because the winner will be the bidder for whom ε_i was greatest. In other words, the auction winner will likely be one whose value estimate v_i included a large positive value of ε_i. When the true market value V^* is revealed, this winner will discover that she has overpaid. This phenomenon is known as "the winner's curse." The problem is that the bidder is interested in more than just an unbiased expectation of V^*, which in fact is provided by her observed v_i. What the bidder is really interested in is the expected value of V^* *conditional on her having the winning bid*. It is clear in this regard that the winning bidder will be the one whose individual estimate v_i included the highest value of the error term ε_i. That is, each bidder can foresee that if she wins, it will be because her v_i estimate was the most optimistic and likely overstates the true market value V^*. Rational bidders need to take this into account and again shade their bids to minimize the winner's curse.

Once again we can work out the optimal amount of shading using the order statistics. Effectively, the v_i are random observations drawn from a uniform distribution ranging from V^*–\$100 to $V^* + \$100$. We know that if there are n bidders, the expected value of the highest bidder's observed v_i lies a fraction $n/(n+1)$ of this distance from the lower bound. That is:

$$\begin{aligned}
&\mathrm{E}[n\text{th order statistic}] = V^* - \$100 + \left(\frac{n}{n+1}\right)\$200 \\
&= V^* - \left(\frac{n+1}{n+1}\right)\ \$100 + \left(\frac{n}{n+1}\right)\$200 = V^* + \left(\frac{n-1}{n+1}\right)\$100.
\end{aligned} \tag{23.12}$$

Equation (23.12) says that, conditional on winning, each bidder can infer that he was the highest of the n bidders in which case his observed value v_i is $\left(\frac{n-1}{n+1}\right)$ \$100 above the true value V^*. Thus each bidder should shade his bid below v_i by this amount. If there are just two bidders, this shading amounts to bidding \$33.33. If there are three bidders, the optimal shading rises to \$50. It continues to rise with the number of bidders and asymptotically reaches \$100.

The reason that the optimal shading increases with the number of bidders is that the winner's curse does as well. When there are many bidders, the bidder who finds himself with the top estimate v_i has a very good reason to believe his estimate is an outlier. By analogy, consider a student in a classroom who wants to ask a question. If there are just a few other students, the student might not be too worried about raising his hand. However, if there are many students, the student might rationally infer that if it were a good question, someone else would have asked it. By the same logic, it is natural for any bidder to take a very high estimate of the baseball card's true value with several grains of salt if there are a lot of other bidders interested in it as well.[7]

Note that in terms of the impact of the number of bidders n, the winner's curse phenomenon in common value auctions cuts against the insight of equation (23.10) that large numbers lead to higher bids. Economists are naturally drawn to the idea that more bidders means more competition and that this will bid prices up. When there is a "winner's curse" possibility, though, as there is in common value auctions, having many bidders can work in the opposite direction to lower the equilibrium price.[8]

23.3

Practice Problem

Suppose your local town is auctioning off a franchise to sell hot dogs at the July 4th celebration. You and your partner decide to bid for the franchise. Including your partnership there are eight groups bidding in the auction. Your market research on expected attendance, hot dog consumption, and costs suggests that the franchise is worth \$20, 000. You know that this value is an unbiased estimate of the true value but that it includes an error distributed uniformly between −\$3000 and +\$3000. What is your expected monetary "curse" if you bid \$20, 000 for the concession?

23.4 AFFILIATED VALUES

The private value and common value cases represent the two polar auction cases. The intermediate case involves elements of both. Thus, to return to our baseball card analogy, it is possible that each bidder has his or her own private value for the card but that this private value is nevertheless influenced by the values assigned to the card by other bidders. This seems the most realistic assumption for many cases. Even devoted art collectors who know and love specific artists or styles will likely care about the resale value of their acquisitions notwithstanding their passionate views on an item's worth. Conversely, while homebuyers

[7] Revenue Equivalence can still hold in a common value auction if buyers' signals are completely independent.

[8] For examples in which the winner's curse hedging is the dominant force, see Bulow and Klemperer (2002).

clearly care about the market value of their property, each may differ on the utility or psychic income that a specific parcel yields depending on how much it "speaks" to them.

When the values of each bidder are affiliated, it means that it is a little unclear what we mean by the bidder's privately observed value because for any given signal, the true value is known to be affiliated with others' observed signals. As a result, there is some ambiguity regarding what is meant by a bidding rule that says the bidder should bid her own valuation, i.e., $b_i = v_i$. A good rule of thumb however is that bidders should bid their valuation as if the next highest bidder had observed the same signal. This will still lead to differential bids because each bidder observes her own individual signal or value v_i. Moreover, the fact that the v_i are affiliated means that if one bidder observes a fairly high value or v_i, others are more likely to observe high values as well. The winner's curse effect may thus be mitigated in affiliated auctions but it is not in general eliminated.

The selection bias that leads to the winner's curse phenomenon means that whenever players' signals are not independent, revenue equivalence will not hold. It follows that if some auction designs are better than others in reducing the winner's curse, such auctions will also lead to less shading and higher equilibrium prices. In this respect, both an open English auction and a second-price auction are likely to lead to higher equilibrium prices than either a Dutch of first-price auction. Further, an English auction is likely to yield a higher price than a second-price auction. The intuition behind these results is straightforward. The open English auction limits the winner's curse because bidders can see the bids and therefore infer the signals received by other bidders. This gives more probabilistic support to their own estimated value and reduces the possibility that they will greatly overbid. Likewise, the second-price or Vickrey auction helps to reduce the winner's curse because as noted earlier, the bid determines only whether one wins. What one pays is decided by the next highest bid. So, some protection against the winner's curse is already built into the auction process. The reason that an English auction dominates or, at least, can never yield a lower price than the second price auction is that it naturally includes the second-price mechanism in its final stage. That is, after $n - 2$ bidders have dropped and there are just two bidders remaining, the open English auction is equivalent to the second-price auction with the winner paying a price equal to the value at which the next-to-last bidder quits.

Incidentally, this seems like a good moment to make clear a vital point. The winner's curse is an outcome that *can* happen—the winner will regret how much she paid—if bidders are irrational and do not shade their bids properly. If bidders are rational however, they will make the appropriate adjustments such that the winning bid will not be too high, on average. The winner's curse should not be a commonly observed event in real auctions. To the extent that it is observed (recall that Didius Julianus totally lost his head in winning the Roman Empire), it calls into question bidder rationality—not the rationality of the bidding strategies analyzed here.[9]

23.5 AUCTIONS AND INDUSTRIAL ORGANIZATION

What are the implications of auction theory for industrial organization? The study of auctions is a relatively new field. Despite Vickrey's (1961) path-breaking early work, it is only in the last twenty years that the applications of auction theory have begun to be widely

[9] Thaler's (1992) well-known piece renewed interest in the winner's curse by providing evidence that it was frequently real.

Reality Checkpoint

You're Watched and Wanted

Log on to a web page. Chances are you will see in the margin or elsewhere on the site one or more product advertisements. What may be more surprising is that if you then go to a different website, you may seem many of the same ads. This is no accident. Your time and attention and, ultimately, your money, are wanted.

Digital advertising space—what the industry calls an impression—is valuable. For any would-be advertiser, however, just how valuable an impression is depends critically on the type of consumer seeing it—their age, their gender, their income, their location, what other sites they visit, and so forth. Cookies and other tracking mechanisms allow this information to travel with you each time you visit a site. In a manner of milliseconds, this information can be recorded and decoded so that each potential advertiser can very quickly know who is visiting a particular site.

This is where modern advertising exchanges such as Yahoo's *Right Media*, Google's *AdEx*, and the *Rubicon Project* come into the picture. These firms act as auctioneers gathering all the information on each particular impression visitor and then auctioning off that particular digital spot to the highest bidder. The bidders themselves are either firms trying to sell a product or their marketing agents. These firms in turn rely on complicated algorithms to respond with a bid to the various attributes the exchange reports. Again, all this happens at a speed so fast that you, the Internet user, will never know it is happening.

The real time technology allows advertisers to reach highly targeted audiences rather than simply buy a space on a website based on the general characteristics of its visitors. The auction process for allocating these impression rights should further serve to insure that those firms that place the highest value on getting access to your digital persona will be the ones whose ads you see. Your cyber self is being sold almost every second you are on the web.

Source: N. Singer, "Your Online Attention, Bought in an Instant," *New York Times*, November 18, 2012, p. B1.

understood. While the insights from auction theory for economic analysis are numerous and growing, we focus here on two that are particularly relevant to industrial organization. The first of these has to do with oligopoly pricing. The second has to do with market asymmetries, perhaps most notably, those between an entrant and an incumbent.

23.5.1 Auctions and Oligopoly Pricing

Consider the simple Bertrand model, in which two firms sell an identical product. Let us start with a simple case. There is a buyer willing to pay at most V for exactly one unit of the good and each firm has a constant and identical marginal cost c_i with $0 \leq c_i < V$. To make matters specific, let us assume that $V = \$110$, $c_1 = \$10$, and $c_2 = \$15$. In this initial case, we will assume that all of the foregoing is common information known to all participants. The market interaction then proceeds as follows. Each firm is asked to post a price at which it will sell and the buyer chooses the firm that offers the lowest price. In this case, it is easy to see that the Nash equilibrium requires that each firm will quote a price equal to $c_2 = \$15$. If either firm ever expected the other to quote a price above \$15, say \$16, the other would win the competition by offering a price of \$15.99. Yet this would mean that the high-price firm would want to change its price quote, i.e., it would not meet the Nash equilibrium criterion. Yet firm 2 will also never set a price below \$15. If it does, it may win the competition but it will lose money on the sale. The only possible Nash equilibrium for this game is $p = c_2 = \$15$, with firm 1 winning the bid. As is usual with Bertrand pricing, we get price equal to the second-lowest marginal cost even though there are only two firms. Indeed, adding a third or fourth with costs $c_3 = \$20$ and $c_4 = \$25$ would not change the outcome.

Let us now, however, change the model slightly by altering the information structure. In particular, let us assume that there are just the original two firms and that each firm knows its own marginal cost c_i, but does not know the rival's marginal cost. Instead, it knows only that its rival's marginal cost is distributed uniformly between 0 and \$100. What price should a firm post in this market setting?

The answer to this question comes from auction theory. The difference is that unlike the analysis in Section 23.2, we are now considering a selling auction rather than a buying auction. Nevertheless, the underlying principles are the same. Each firm will adopt a pricing rule $p_i = p(c_i)$ that determines the price that will maximize its expected profit from the sale of the one unit to be bought given the firm's observed cost c_i. We show below that the pricing rule that achieves this outcome is:

$$p(c_i) = c_i + \frac{100 - c_i}{2} = \frac{100 + c_i}{2} \tag{23.13}$$

Equation (23.13) says that in this duopoly case, each firm sets its price equal to its own cost c_i plus an amount equal to half the difference between this cost and the maximum cost (\$100) possible. This result is just the mirror image of our earlier work on bidding in a second-price auction. The underlying logic is that this rule still implies that the firm with the lowest cost will post the lowest price. It follows that the probability that firm i wins the sale is exactly the same as the probability that it has the lowest cost c_i.

In other words, if both firms follow the pricing rule, then

$$prob(b_i < b_j) = prob(c_i < c_j);\ i = 1, 2;\ i \neq j.$$

Given the uniform distribution of c_i between 0 and \$100, this probability is equal to:

$$prob(c_i < c_j) = \frac{100 - c_i}{100} = 1 - \frac{c_i}{100};\ i = 1, 2;\ i \neq j.$$

The cumulative distribution of c_i at some value c_1 is just the probability of the random variable c_i being less than or equal to c_1, and for the case at hand, this is given by $c_1/100$. Therefore the probability of some randomly chosen c_i variable being greater than c_1 is just 1 minus the cumulative distribution at c_1 or $1–c_1/100$.

For example, if firm 1 has a marginal cost of $c_1 = \$10$, it would know that 90 percent of the time its rival will be a firm with a higher marginal cost given that the marginal cost is distributed between 0 and \$100. If both firms follow the pricing rule of equation (23.13), this will also then be the probability with which firm 1 wins the sale. Recall that pricing rule calls for firm 1 to set a price $p_1 = \$55$, that is $(\$100–\$10)/2 = \$45$ above cost. Firm 1 will then have an expected profit $E(\pi)$ of

$$E(\pi) = prob(p_1 < p_2)(p_1 - c_1) = 0.9(\$45) = \$40.50 \tag{23.14}$$

Suppose instead that firm 1 set a price of \$65. This will obviously increase its profit margin but also decrease its chance of winning if firm 2 continues to follow the pricing rule specified by equation (23.13). Specifically, by working that equation backward, we can determine that if firm 1 sets a price of \$65, it is acting as if its cost is \$30. Firm 1's chance of winning the sale therefore becomes equivalent to the probability that its rival has a cost greater than \$30, namely, 70 percent. Hence, firm 1's expected profit under this alternative strategy becomes:

$$\mathrm{E}(\pi) = prob(\$30 < c_2)(p_1 - c_1) = 0.7(\$65 - \$10) = \$38.50 \tag{23.15}$$

Similarly, if firm 1 decreased its bid to \$50, it would be pricing as if it had a marginal cost of $c_1 = \$0$. It would raise its chance of winning the sale to 100 percent. Yet while it would win with certainty, it would only earn \$50–\$10 = \$40, which is still less than the \$40.50 earned using the optimal bidding strategy. In short, given that firm 2 is pricing according to the rule specified in equation (23.13), the profit-maximizing choice for firm 1 is to follow this rule as well. Obviously, the same is true in terms of firm 2's best response if firm 1 follows that pricing rule. Thus, with each firm pricing as indicated by equation (23.13), each will be making its best response to the other, i.e., the market will be in a Nash equilibrium. While we have demonstrated this result here only by example, a general proof is provided in the appendix.

There are a number of points worth noting about the Bertrand pricing equilibrium just described. First, it generalizes to the case of $n = 3$, 4, or more competitors. That is, for the general case of n firms the Nash equilibrium pricing rule is:

$$p_i(c_i, n) = c_i + \frac{100 - c_i}{n} = \frac{100}{n} + \frac{(n-1)}{n}c_i \tag{23.16}$$

Thus, as the number of firms n rises, the Bertrand pricing outcome now gets closer and closer to the competitive outcome of $p = c =$ marginal cost. Firms optimally reduce the margin of their posted price over cost as they face more competitors.

Second, it is worthwhile to determine the expected price that will be paid in this Bertrand market. For this purpose, it is helpful to recall the first two order statistics for the uniform distribution between 0 and 100—the expected value of the lowest and second-lowest of n random draws. These are:

$$C^1 = \frac{100}{n+1} \text{ and } C^2 = \frac{200}{n+1} \tag{23.17}$$

Thus, with $n = 2$ firms the lowest expected cost in our example is $C^1 = \$100/3 = \$33.33$ while the second-lowest expected cost is $C^2 = \$200/3 = \$66.67$. It follows that when $n = 2$, as in our example, we would expect *ex ante* that the lowest cost firm would have $c_i = \$33.33$. The pricing rule in (23.16) then implies that such firm would set a price equal to $(\$100 + \$33.33)/2 = \$66.67$. Note that this expected price in the duopoly case is exactly equal to C^2 or the expected value of the second-lowest cost. This is no accident. It follows directly from the Revenue Equivalence Theorem. If we were to hold a Dutch auction and keep lowering the price until just one firm remained, we would expect that on average the price at which one firm dropped out would be $C^2 = \$66.67$.

Moreover, this result generalizes to all values of n. For example, when n rises to 3, the first order statistic falls to $C^1 = \$25$. In this case, the pricing formula of equation (23.16) says that the expected market price is then:

$$E(p, n) = \frac{\$100}{n} + \frac{(n-1)}{n}C^1 = \frac{\$100}{3} + \frac{2}{3}\$25 = \$50 \text{ when } n = 3 \tag{23.18}$$

From equation (23.17) we know that when $n = 3$, the second order statistic C^2 and the expected winning bid $= \$200/4 = \$50$, as well. We can repeat this for all values of n. However, simple algebra makes clear that with $C^1 = \$100/(n+1)$, the pricing formula of equation (23.16) will always yield an expected winning price of C^2. Again, this is really just an example of the Revenue Equivalence Theorem. The more important point here is that even though we have Bertrand competition and identical products, we nevertheless get a result in which the expected price generally exceeds the lowest marginal cost but moves asymptotically closer to it as the number of firms n rises. Market structure matters.

23.5.2 Auctions, Asymmetries, and Firm Rivalry

Thus far we have assumed that the bidders in auctions are symmetric. They may each realize a different draw from a random distribution but each draws from the same distribution with the same probabilistic parameters, and the price or payoff, conditional on that expectation, is the same. Yet we know that in many real strategic settings, the players may not be equal. For example, the gain to an incumbent firm from a new patent that partially replaces the profit from its existing intellectual property may be less than the gain that the new patent would yield to a new firm. Auction theory offers a powerful insight into the analysis of these econometric cases as well.[10]

To understand the role of asymmetry, consider the following somewhat contrived but revealing example. An incumbent, firm 1, dominates a local market but does face some competition from a small, high-cost rival R with a loyal following. More threatening to firm 1 is the potential entry of a highly efficient rival, firm 2. However, given the setup costs of entering and establishing its own brand, firm 2 can only enter by buying the existing rival R and transforming R's operations with firm 2's significantly more cost-effective technology and management. The dominant incumbent can stop this, however, if it beats firm 2 to the punch and acquires R for itself. Hence, firm 1 and firm 2 will each be interested in buying R—the latter with a view to entering the market and the former with a view to blocking such entry.

In bidding for R each firm makes use of the information that it has on the benefits and costs of successful entry. This information is structured as follows. Based on its years of experience of actual market operations, firm 1 knows that it will lose at least G_1 in profit if firm 2 enters. However, it may lose an additional but uncertain amount G_2 depending on

[10] See Klemperer (1998) and Maskin and Riley (2000) for formal analysis of the role of asymmetry in auctions.

firm 2's cost-effectiveness and marketing skills. From firm 1's perspective, G_2 is distributed continuously between 0 and a large number. Conversely, firm 2 knows its cost-efficiency and marketing skills and therefore knows that it will gain at least G_2 if it enters. It may also gain the (to firm 2) unknown amount G_1 depending on key features of the market known to firm 1. Here again, firm 2 knows only that G_1 is distributed continuously between 0 and a much larger value.

In short, both firms face an expected gain of $G_1 + G_2$ from buying R. For firm 1, this total includes a known part G_1 and an unknown component G_2, and reflects the profit gain from keeping firm 2 out of the market. For firm 2 the total is comprised of an uncertain amount G_1 and a certain component G_2. This total reflects the entrant's potential profit gains from successful entry. The firms bid for R in an English auction and the bidding stops when one firm drops out.

How much should each firm bid? It is straightforward to show that firm 1 should bid up to $2G_1$ while firm 2 should spend up to $2G_2$. That this is a Nash equilibrium can be seen as follows. Given that both firms are following this strategy, firm 2 will drop out (forego entry) as soon as the required expenditure reaches $2G_2$. If this happens, firm 1 will claim R and the total gain $G_1 + G_2$ for an expense of $2G_2$. Given the hypothesized bidding rule however, this only happens when $G_1 > G_2$. As a result, firm 1 will know that with this strategy combination, any time that it wins the auction, it gains $G_1 + G_2 – 2G_2 = G_1 – G_2 > 0$ precisely because $G_1 > G_2$ whenever firm 1 wins. Analogously, firm 2 will know that if it wins it does so at a commitment of $2G_1$. Yet for this to be a winning bid means that $G_2 > G_1$ which, in turn, means that when firm 2 wins, it gains $G_1 + G_2 – 2G_1 = G_2 – G_1 > 0$. Clearly, there is no sense in either firm i committing to a higher expenditure strategy as this will not increase its chances of winning when $G_i > G_j$ but will result in firm i enjoying a smaller gain when it does win. Likewise, there is no sense in either firm adopting a lower expense strategy. This will only result in a lost opportunity for a net gain, on average. In this setting, we should therefore envision an equilibrium in which each firm i will bid up to $2G_i$ to buy the target firm R as a means of either invading or defending the market. The winning bid will therefore be equal to $2 \times \text{Minimum}[G_1, G_2]$.

As structured, the entry game just described is a common value auction in which each player has specialized information about a common value $G_1 + G_2$ but in which the bidders are symmetric in that there is no reason to believe that either one will consistently face an essentially different payoff. Because it is a common value auction, we know that any bidding strategy combination that is sensible must somehow involve a degree of optimal shading to avoid the winner's curse. The shading here is reflected in the fact that once the bid reaches say $2G_1$ firm 1 will accede to firm 2's entry even though firm 1 then knows, by virtue of the very fact that firm 2 has pushed the bidding up to $2G_{1,}$ that G_2 must be at least as large as G_1 and may well be larger. Nevertheless, firm 1 does not increase its bid any higher than $2G_1$ because, again, again, firm 1 (and firm 2 as well) is not interested in the expected value of the total gain but that expected value conditional on its winning the bid.

Now consider one small change to the above scenario. Specifically, let us make use of what we know to be generally true in entry games, namely that the gain to the incumbent of keeping the entrant out exceeds the gain to the entrant of successfully coming into the market. This extra gain does not need to be large. Indeed, the main point of the exercise is to show that even a small asymmetry can have very large consequences.

A numerical example may help. Let us imagine that G_1 and G_2 both vary between 0 and \$25 (million), and that when firm 2 wins the bidding for R it gains as before the sum, $G_1 + G_2$. However, due to the asymmetry, if firm 1 outspends firm 2, it gains $G_1 + G_2 + \$1$ (million). How does this change the outcome?

Suppose that firm 2 continues to bid up to the value $2G_2$. In that case, the incumbent firm 1 will find it advantageous to bid up to the value $2(G_1 + 1)$ (in millions). If firm 1 wins, it must be because $G_1 + 1 > G_2$. In this case, firm 1 buys R at a price of $2G_2$ and gains $G_1 + 1 + G_2$ for a net gain of $G_1 + 1–G_2 > 0$. For example, if $G_2 = \$20$ million and $G_1 = \$19.05$ million, firm 1 will bid up to $2 \times (\$19.05 + 1) = \$40.1$ million. Because $G_2 = \$20$ million, firm 1 will win the bidding at a price of \$40 million and gain \$0.1 million.

If, however, firm 1 adopts the bidding rule of bidding up to $2(G1 + 1)$ for R, firm 2 will quickly find that bidding $2G_2$ is no longer optimal. The more aggressive bidding by firm 1 now increases the winner's curse facing firm 2. Suppose again that firm 1 knows that $G_1 = \$19.05$ million and therefore bids up to \$40.1 million for R. This time though, let firm 2 know that $G_2 = \$20.051$ million and therefore bid up to \$40.102 million. It will find it gains only $\$19.05 + \$20.051 = \$40.101$ million, implying a loss of \$0.001 million or \$1, 000. Firm 2 can avoid this loss and the enlarged winner's curse by shading its bid further. In particular, firm 2 should set its top bid equal to $2(G_2–1)$. In that case, it will only win if $G_2–1 > G_1 + 1$ or $G_2 > G_1 + 2$, given that firm 1's top bid is $2(G_1 + 1)$. Hence, this means that firm 2 will now pay $2(G_1 + 1)$ for a gain of $G_1 + G_2$ or a net gain of $G_2–G_1–2 > 0$ if $G_2 > G_1 + 2$.

Unfortunately, the revised bidding strategy on firm 2's part is not the end of the story. This is because firm 2's decision to bid less aggressively lowers the potential winner's curse for firm 1 and therefore encourages firm 1 to bid even higher. Specifically, if firm 2's top bid is $2(G_2–1)$, then firm 1 will now find it optimal to set a top bid of $2(G_1 + 2)$. Firm 1 will then find that it wins the bid whenever $G_1 + 2 > G_2–1$, i.e., whenever $G_1 > G_2–3$. When it wins, firm 1 will gain $G_1 + G_2 + 1$ for a payment of $2(G_2–1)$, and therefore a net gain of $G_1–G_2 + 2 + 1$, which of course is positive whenever the condition $G_1 > G_2–3$ is satisfied. In turn though, this more aggressive bidding by firm 1 will induce firm 2 to reduce its bid for R still further. Where will the process end?

It should be clear that firm 2 will never bid less than G_2 to acquire R. No matter what firm 1's bidding strategy is, firm 2 cannot lose with a winning bid of G_2 because it knows that the entry access that ownership of R confers is always worth at least this much. Yet the surprising logic of our analysis above is that in equilibrium, the small asymmetry that we have assumed also means that firm 2 will never bid *more* than G_2. Any higher bid will put it at risk. Thus, in the usual case in which the incumbent's gain G_1 asymmetrically exceeds the entrant's gain G_2, the incumbent will have a strong advantage in bidding for R.

There are two lessons from the foregoing example. First, the fact that asymmetries transform common value auctions into almost-common value auctions, means that auction design again becomes important in such settings. In particular, the Revenue Equivalence Theorem will no longer hold necessarily.

The second insight, though, is the one more important one from an industrial organization perspective. This is the fact that even small advantages can have major implications for the outcomes in imperfectly competitive markets. Thus, the advantages of say incumbency—even if they are small—may have a major effect not just for entry battles but also for patent races, advertising rights, and a host of other non-cooperative games.

23.6 EMPIRICAL APPLICATION: SCHOOL MILK AUCTIONS, COMPETITION, AND COLLUSION

Each summer, public school districts around the country run an auction to solicit bids for providing school lunch milk. Ohio is no exception to this rule. Unfortunately, Ohio is also not an exception to the rule that school lunch auction bids often reflect collusion instead of competition. The predictability and relatively inelastic demand for school milk coupled with the homogeneity of the product, and therefore, the similarity of cost structures across producers, facilitates collusive bidding. In addition, the fact that the districts announce the identity of the winning bidder and the amount of the wining bid along with that fact that the game is repeated each year so that any firm that does not cooperate can be punished soon also make it easier to monitor and enforce collusive agreements. Consequently, it is not surprising that collusion in school milk markets is relatively common, especially because milk transport costs mean that the number of firms that could potentially serve a given school district is typically small. Over the past twenty-five years, more than two dozen states have launched investigations of price fixing in school milk auctions and guilty pleas have been entered in over half of these cases.

Definitive legal proof of price-fixing is, however, difficult. Somehow, overt collusion must be distinguished from the normal rivalry, which includes imperfect competition. This is particularly true when conspirators engage in what is commonly referred to as complementary or courtesy bidding. In such cases, the colluding firms submit bids to a wide variety of buyer auctions in an effort to give the appearance of competition. In reality though, the bids are deliberately set too high to provide any real competitive pressure and thus allow the designated conspirator to win a particular auction at a high price. Nevertheless, a sensible use of auction theory may help provide such compelling evidence. Economists Robert Porter and J. Douglas Zona (1999) attempted to do just that and later reviewed their findings in the paper "Ohio School Milk Markets: An Analysis of Bidding."

The case investigated by Porter and Zona (1999) stemmed from state investigations that led in 1993 to confessions by two Cincinnati dairies, Meyer and Coors Brothers, to participation in bid-rigging schemes. Executives from both companies described a collusive bidding ring that included a third Cincinnati dairy, Louis Trauth, in which different school districts were allocated to each specific dairy with the other two dairies agreeing to submit bids so as to give the appearance of competition but bids that, in actuality, were excessively high bids so that the chosen dairy would still win the bidding with a profitable price. In other words, the Myers and Coors executives described a standard case of complementary bidding. Despite the confessions of Myers and Coors, Trauth maintained its innocence. The case then proceeded as the state of Ohio pursued collusion charges against a number of dairies.

Porter and Zona (1999) review bids from the roughly sixty different firms in 509 Ohio school districts over the eleven years, 1980–90. With these data, they construct a control group of the vast majority of dairy distributors and processors presumed innocent of any bid rigging. They then attempt to identify collusive bidding by testing for any differences between the actual bidding practices of Meyer, Coors, and Trauth and the bidding practices that these firms would have exhibited if they behaved like the competitive control group firms.

Porter and Zona (1999) break the bidding process into two steps. First, there is the decision simply to submit a bid. That is, each supplier has to determine whether or not to submit a bid to any particular district. Second, conditional on submitting a bid to a particular district, the firm has to determine how much to bid, i.e., what price it should ask for should it indeed be the winning bidder.

The authors therefore start by using data from the control group to estimate a model of the probability that a specific district receives a bid from a specific firm in a particular year. Their findings may be briefly summarized as follows. First, processing firms that actually transform raw milk into a finished product are more likely to submit bids to any given district than are distributor firms that simply distribute processed milk. Second, all milk sellers are more likely to submit bids in one direction from their base rather than in multiple directions. Third, all sellers are more likely to submit bids in a nearby districts rather than ones far away. Finally, fewer bids are submitted to those districts requiring that complementary goods such as straws be provided along with the milk.

Turning to the price quoted in the bid given that a bid is submitted, the most important finding is that the level of the bid rises as the distance between the bidder and the district increases. However, Porter and Zona (1999) also find that distributors tend to submit higher bids than do processing firms and that bids that have an escalator clause allowing for the price to increase over the life of the contract, are also lower.

All of the foregoing results are quite plausible. Because processors often serve many milk demands, they have more routes and therefore a greater likelihood of serving a route that includes a school. Similarly, it is easier to serve schools that lie along an existing route structure that runs, say, from east to west, rather than to serve a school along a new route running from, say, south to north. Because transport costs are very significant, it makes sense that firms will tend to submit bids in closer districts. Likewise, because providing straws and other services (e.g., coolers) is expensive, districts that require these items should also expect to get fewer bids.

The bid level results are also consistent with expectations. Again, the presence of significant transport costs implies that a dairy firm's bid should rise the further the dairy has to transport the milk to delivery. Because distributors buy their milk from processors, one would again expect that distributors would ask for a higher price, all else equal. Finally, if a firm is protected against cost increases over time by means of an escalator clause, we would expect that the firm can submit a lower bid free from any need for a premium to protect against such risk.

In sum, there is ample reason to suspect that the behavior of the control group captures the normal workings of Ohio's imperfectly competitive school milk markets. In light of these results, Porter and Zona (1999) then seek to determine whether and in what ways the behavior of the three main alleged conspirators—Meyer, Coors, and Trauth—is different from that of the control group.

Table 23.1 demonstrates one clear difference. Relative to the typical dairies in the control group, the three accused firms submit bids much more frequently in nearby markets—markets not far from any of the firms. Thus, the third row of the table indicates that considering districts twenty to thirty miles from the dairy's office, Coors submits 22.9 percent more bids, Meyer submits 18.5 percent more bids, and Louis Trauth submits 20.6 percent more bids than would the typical competitive firm draws from the control group analysis. Moreover, many of these differences are statistically significant.

The true importance of the foregoing finding though only emerges when Porter and Zona (1999) investigate the actual prices that the accused firms submit when they do bid. In contrast to the behavior of the control group firms, the delivered milk prices offered by the three Cincinnati firms *fall* with distance. That is, these firms tend to offer a lower milk price to school districts farther away even though shipping milk this distance incurs much

Table 23.1 Accused conspirators' difference in bidding propensity from control group by distance

Distance in Miles	*Coors Brothers*	*Meyer*	*Louis Trauth*
0–10	24.1% >	5.6% >	7.0% >
10–20	42.9% >	8.2%	15.2% >
20–30	22.9% >	18.5% >	20.6% >
30–40	−17.1% <	18.6% >	0.1%
40–50	−9.5% <	−2.2%	−4.3%
50–60	−6.0%	−5.5%	6.9%
60–70	−6.0%	−18.6% <	47.1% >
70–80	−4.9% <	−25.0% <	10.0% >
80–90	−2.4% <	−17.5% <	−2.5% <
90–100	−1.7%	−7.7% <	11.8% <
100–110	−1.3%	30.7% >	8.7% >
110–120	−0.6%	0.5%	−4.2% <
120–130	−0.5%	−0.9%	−3.6% <
130–140	−0.2%	−0.3%	−2.0%
140–150	−0.2%	−0.1%	−1.2%

> Indicates statistical significance

higher transportation costs. This pattern is especially evident for Meyer and Trauth who, as Table 23.1 shows, were the two of the three firms who submitted a lot of bids far away. Taken together then, these findings reveal a pattern in which the Coors, Myer, and Trauth submitted lots of bids in school districts close to their home town of Cincinnati where they were in most direct competition. These bids were relatively high while the prices offered to more distant school districts were noticeably lower. This of course is very consistent with the practice of complementary bidding to which the Meyer and Coors executives had confessed.

In addition to the above finding, Porter and Zona (1999) also offered further evidence of bid rigging. First, the bids of the indicted conspirators were significantly lower in 1983 and 1989, the two years in which the Meyer and Coors executives testified that there was a breakdown in the cartel. More formally, Porter and Zona (1999) examine two correlation measures among the accused firms. Briefly put, if the three firms are truly submitting bids in an independent fashion, then the fact that, say, Meyer bid unexpectedly in a particular district should be totally uncorrelated with whether either Coors or Trauth also bid unexpectedly in that district. However, complementary bidding would imply that these unexpected bids would be positively correlated. In agreement with Meyer and Trauth, Coors would submit bids in the same districts these firms did more often than not. Similarly, independence would imply that in a district in which say, Coors submitted an unexpectedly high price, we would not expect the bids of either Meyer or Trauth to show any pattern. In contrast, a complementary bidding scheme would suggest that in such districts, Meyer and Trauth would, if they bid at all, also submit bids that are unexpectedly high.

Porter and Zona (1999) therefore also consider the correlation between: 1) the unexpected bid propensity measured as the unexplained residuals for the three firms from the regression predicting their bidding submission; and 2) the unexpected level of the bid again measured

Table 23.2 Pair-wise correlation coefficients on unexplained propensity to bid and unexplained bid price

	Coors & Myer	*Meyer & Trauth*	*Trauth & Coors*
Propensity to Bid	0.58	0.54	0.43
Bid Price	0.66	0.67	0.54

as the unexplained residuals now taken from the pricing equation estimated for the three firms. Table 23.2 displays the pairwise correlations for each case.

As Table 23.2 indicates, the correlations are all positive and in each case, the estimated correlation is significantly different from zero. In districts where one of the three firms unexpectedly submitted a bid, the other two were statistically very likely to do the same. Further, when bidding, if one of the three firms submitted an unexpectedly high bid, the others were very likely to do so as well. Coupled with the earlier evidence, these findings strongly reject the hypothesis of independent bidding. They are instead very consistent with the complementary bidding scheme that was alleged.

Overall, Porter and Zona (1999) estimate that on average, the collusion raised prices by 6.5 percent over what they otherwise would have been. For some school districts—particularly those in which one of the three alleged conspirators already had a contract and in which therefore the other two were strongly encouraged not to undercut that incumbent's prices—the bid-rigging is estimated to have raised school milk prices by as much as 49 percent.

Summary

The common use of auctions for all sorts of market transactions raises many important questions. One of the most important is the impact of auction design. That is, are the market outcomes different for English or Dutch or first-price or second-price sealed-bid auctions? Vickrey's (1961) path-breaking work provides a key answer. His main result, typically referred to as the Revenue Equivalence Theorem, is that the expected price or revenue is the same under each auction design *provided* that the bidders' values are independent.

Apart from auction design, auctions also differ depending on whether the item(s) being sold has only a purely private value or, instead, a common value that will be revealed after the auction is completed. A common value auction—in which the item(s) being sold ultimately have a true market value common to all bidders but unknown to any bidder prior to the bidding—has the potential for a "winner's curse." Bidders in auctions for real estate, oil tract rights, radio spectra, and so on know that their signal or guess as to what the item is truly worth is based on the information that they happened to observe. Therefore, any bidder who wins a common value auction can infer that he or she must have had the most optimistic signal or estimate of the item's true market value. Unless bidders shade their bids below the value implied by the information they have they will find that they have paid more than the item's true value. If bidders shade their bids optimally, they will avoid the "winner's curse" of paying too much in a common value auction.

When the signals of bidders are affiliated, the potential for a "winner's curse" and other features imply that auction design does matter. In general, the ranking is that an open English or ascending auction leads to a higher price than does a second-price auction. In turn, a second-price auction yields a higher selling price than does a first-price, sealed bid auction.

Auction theory offers many insights for industrial organization analysis. It provides an alternative interpretation of the Bertrand price competition model in which the number of competitors does matter. It also makes clear how market asymmetries can have a lasting impact on market outcomes.

Equally important, auctions markets are subject to the same industrial organization influences as are other markets. In such markets, there still remains the temptation for firm imperfect competitors to collude. Examination of the auction markets for school milk in Ohio using standard microeconomic concepts to determine non-collusive bidding strongly implies that some firms in Ohio did collude. They bid too much, too high, and offered bids that declined with distance despite the fact of very high transport costs for processed milk. In short, auction theory has become an important part of both the theory and practice of industrial organization.

Problems

1. Suppose that there are six bidders in an English auction for an antique 1950s Vaporizer. Each has his or her own private value for the vintage machine. In ascending order, these values are \$50, \$60, \$70, \$80, \$90, and \$100. What will be the winning bid?
   a. What will be the winning bid if the highest three bidders collude?
   b. What will be the winning bid if the middle three bidders collude?

2. Imagine that you are an educational consultant and that you have been asked to submit a bid for a month-long project by a small liberal arts college. You know that the college's willingness to pay for your service is uniformly distributed between \$5,000 and \$15,000. Assuming that you have other earning opportunities for that month for which you would be paid \$5,000, what should you bid?

3. In a "war of attrition" two bidders with valuations drawn independently from the same uniform distribution ranging from 0 to \$10, bid for an object but in this case, *both* pay the losing bid. Derive the equilibrium for this game. [Hint: Use the Revenue Equivalence Theorem.]

4. The wallet game is a well-known common value auction. In this game, two players are each given a chance to bid for a total amount equal to the sum of the money they are carrying in their wallets. For example, if the amount in player 1's wallet w_1 is \$20 and the amount in player 2's wallet w_2 is \$45, the total prize is \$65. Of course, each bidder only knows the amount in his or her own wallet. Player 1 knows w_1 and player 2 knows w_2. Show that each player following the bidding rule $b(w_i) = 2w_i$ is a (Nash) equilibrium.

5. Up until the 1990s, the US Treasury has auctioned off its debt instruments of Treasury bills and notes using a discriminatory format. Under this procedure, the supply of securities would be auctioned off in lots at different prices until the available supply was purchased. That is, bidders submit bids indicating both the price they will pay and how many securities they wish to buy at that price. In response, the Treasury fills the demand of the highest priced bidder first. It then moves on to the demand of the second-highest bidder, and so on until all the supply of Treasury securities is sold. However, in the 1990s the Treasury moved to a uniform price auction in which roughly the above procedure was followed except that now, all bidders paid the same price—namely the lowest price at which the supply cleared. Why might the uniform price auction have encouraged more bidders to participate in Treasury auctions?

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Appendix

OPTIMAL BIDDING IN FIRST-PRICE AUCTIONS

We show that optimal bidding in a sealed-bid first price auction with n bidders with valuations v_i drawn from a uniform distribution ranging from 0 to V^{Max} is for each bidder i to bid $b_i = \dfrac{(n-1)}{n} v_i$. We assume that any equilibrium bidding rule $b(v_i)$ adopted by all bidders must imply $b(v_i) > b(v_j)$ if $v_i > v_j$, i.e., the bidder with the highest value v_i will have the highest bid. Bidder i's expected gain is then:

$$\text{Prob}\ (v_i > v_j \text{ for } j = 1,\ n \text{ and } j \neq i) \times [v_i – b(v_i)] \tag{23.A1}$$

Uniformity implies that the probability that any randomly drawn valuation will have a value less than v_i is v_i / V^{Max}. In a sample of n bidders with independent valuations the probability that all of the $n-1$ valuations are below v_i is $\left(\dfrac{v_i}{V^{Max}}\right)^{n-1}$. Hence, any admissible bidding rule $b(v)$ in (23.A1) implies:

$$\text{Expected Gain for Bidder } i = \left(\frac{v}{V^{Max}}\right)^{n-1} [v_i - b(v)] \tag{23.A2}$$

Taking the derivative with respect to v and setting it to zero at $v = v_i$ we have:

$$[v_i - b(v_i)](n-1)\frac{(v_i{}^{n-2})}{(V^{Max})^{n-1}} - \left(\frac{v_i}{V^{Max}}\right)^{n-1} b'(v_i) = 0 \tag{23.A3}$$

Simplifying, we than obtain the differential equation:

$$b'(v_i) = (n-1)\left[1 - \frac{b\left(v_i\right)}{v_i}\right] \tag{23.A4}$$

For which the solution is readily confirmed to be:

$$b(v_i) = \left(\frac{n-1}{n}\right) v_i \tag{23.A5}$$

When $n = 2$, each bidder bids an amount equal to $0.5v_i$. As n grows ever larger, bidding becomes more competitive. In the limit, $b_i \Rightarrow v_i$ as $n \Rightarrow \infty$.

OPTIMAL BIDDING IN OLIGOPOLISTIC BERTRAND COMPETITION WITH INCOMPLETE INFORMATION

We assume n firms competing in price to sell a good for whom the consumer has valuation V and each firm has cost c_i drawn from a uniform distribution ranging from 0 to C^{Max}. The probability that c_i is the lowest cost is $\left(1 - \frac{c_i}{C^{Max}}\right)^{n-1}$. Given a pricing function $p(c_i)$ that preserves this ranking, firm i's expected net gain is:

$$\mathrm{E}(\mathrm{Gain}_i) = \left(1 - \frac{c}{C^{Max}}\right)^{n-1} [p(c) - c_i] \tag{23.A6}$$

Therefore, the optimal pricing function must satisfy:

$$\left(1 - \frac{c}{C^{Max}}\right)^{n-1} p'(c_i) = \frac{(n-1)}{C^{Max}}\left(1 - \frac{c_i}{C^{Max}}\right)^{n-2} [p(c_i) - c_i] \tag{23.A7}$$

Simplifying equation (23.A7) then yields:

$$(n-1)[p(c_i) - c_i] = p'(c_i)[C^{Max} - c_i] \tag{23.A8}$$

The pricing function that satisfies this differential equation for all permissible values of c_i is:

$$p(c_i) = \frac{C^{Max}}{n} + \left(\frac{n-1}{n}\right) c_i \tag{23.A9}$$

24

Strategic Commitments and International Trade

The first US Secretary of the Treasury was the brilliant but prideful Alexander Hamilton whose picture still adorns the US ten-dollar bill. Hamilton came to his position just as the new country was struggling with a host of financial issues accumulated from its long war for independence and the somewhat chaotic financing that had thereafter characterized the eight years under the Articles of Confederation. Of crucial importance to Hamilton was the young country's international profile. He was convinced that America had to establish its identity in international markets. In particular, Hamilton argued forcefully that America had to raise sufficient taxes to pay off its accumulated foreign debt and eliminate any fear of default. Yet Hamilton's vision did not stop there. He also had a very clear idea about the sort of taxes that would best serve the United States in its quest for a respected place in the international community. In Hamilton's view, the United States had to have an internationally competitive manufacturing sector. Therefore, in his *Report on Manufactures* (1791), Hamilton argued strongly for tariffs on manufacturing imports. This would help raise the revenue necessary for debt service and, by discouraging imports, encourage the development of US manufacturing that Hamilton viewed so central to the country's future economic success. In fact, he also recommended the establishment of a Society for Useful Manufactures to subsidize certain key industries that he saw as critical to a vibrant manufacturing sector.

The issues raised by Hamilton's analysis have carried forward to this day. As this text is going to press, the countries of the Euro zone are struggling with the issue of credible debt reduction. Simultaneously, countries around the globe are concerned about the trade policies that will best insure the health of their manufacturing sector, especially in light of the rapid emergence of manufacturing bases in China and other newly industrialized nations.

As it turns out, these issues have a significant industrial organization component. The gains from strategic trade policy depend critically on both the element of commitment that they introduce into trade models and also on the nature of competition in those trade sectors. In this chapter, we use the tools developed earlier to explore industrial competition in an international context.[1]

[1] Many authors have contributed to the strategic trade literature. Central contributions include Spencer and Brander (1983), Brander and Spencer (1985), Eaton and Grossman (1986), and Krugman (1986). Fudenberg and Tirole (1984) offer a classic analysis of strategic commitment.

24.1 STRATEGIC COMMITMENTS IN INTERNATIONAL MARKETS

We begin with a simple numerical illustration. Assume that Boeing and Airbus are the only two international producers of large passenger aircraft. Assume that each is considering a major investment in the development of a new super jumbo jet capable of carrying over 500 passengers. Both recognize that the size of the jet may make it economical but also limit its market to routes connecting airports that have both the demand for and the facilities needed to handle such a large passenger load. As a result, there is really only room for one firm to develop the aircraft successfully. If a firm sinks the development costs and is not the firm to survive it will suffer a major loss. This is illustrated in Table 24.1 below in which all payoffs are in millions of Euros.

As can be seen, this simple game has two Nash equilibria. In one of these, Airbus develops the new superjumbo jet. In the other, Boeing does. Absent some explicit coordination or other device, there is no way to determine which of these two equilibria will prevail. We could imagine a probabilistic equilibrium where each firm develops the new plane with a probability of, say, one-third, but does not with a probability of two-thirds. Instead, however, let's assume that the European Union commits to financing Airbus's R&D in the amount of €3.5 billion. With this change, the payoff matrix is now that of Table 24.2.

Now Airbus has a dominant strategy, namely, to develop the new aircraft. The Nash equilibrium therefore becomes one in which Airbus develops the plane and Boeing stays out of the race. As a result, Airbus earns a surplus of €9500 *million*. Assuming these funds accrue to Europeans, the Union as a whole has gained from its commitment to Airbus. The subsidy of €3500 million has been worthwhile.

24.1.1 Strategic Subsidies in an International Cournot Model

To investigate the role of R&D subsidies more formally, suppose that there are two countries, A and B, in each of which there is a domestic monopoly firm, *a* and *b*, respectively. However, while firms *a* and *b* do not compete with each other in their home markets, they do compete in other markets which we shall simply designate as the

Table 24.1 The strategic R&D game without subsidies

		Boeing	
		Don't Develop	*Develop*
Airbus	Don't Develop	0,0	0,€6000
	Develop	€6000, 0	−€3000, −€3000

Table 24.2 The strategic R&D game with a subsidy for airbus

		Boeing	
		Don't Develop	*Develop*
Airbus	Don't Develop	0,0	0, €6000
	Develop	€9500, 0	€500, −€3000

international market. Demand in this market is given by $P = A - Q$, and each firm has a constant marginal cost $c > 0$. Hence, from Chapter 9 we know that the equilibrium quantity and profit of each firm are: $q_1 = q_2 = (A - c)/3$; and $\pi_a = \pi_b = (A - c)^2/9$.

Now imagine that firm a persuades its government to subsidize its costs to the extent of s per unit. As a result, firm a now faces a reduced constant marginal cost of $c - s$. Its profit function will therefore be:

$$\pi_a = (A - q_a - q_b - c + s)q_a \tag{24.1}$$

As we saw in Chapter 9, firm a's best response function is now:

$$q_a = \frac{(A - c + s)}{2} - \frac{q_b}{2} \tag{24.2}$$

Of course, because it receives no subsidy, firm b's best response function is given by equation (24.2) with $s = 0$. Combining these two equations, we then have the new equilibrium outputs in the international market:

$$q_a = \frac{(A - c + 2s)}{3};\ q_b = \frac{(A - c - s)}{3};\ Q = \frac{2(A - c) + s}{3};\ P = \frac{A + 2c - s}{3} \tag{24.3}$$

In turn, this implies that firm a earns profit:

$$\pi_a = \frac{(A - c + 2s)^2}{9} \tag{24.4}$$

In the absence of any subsidy, $s = 0$ and firm a earns the standard Cournot duopoly profit $\pi_a = \dfrac{(A - c)^2}{9}$. Therefore, the profit increase $\Delta\pi$ for firm a that the subsidy generates is:

$$\Delta\pi = \frac{(A - c + 2s)^2}{9} - \frac{(A - c)^2}{9} = \frac{4(A - c)s + 4s^2}{9} \tag{24.5}$$

The total cost $TC(s)$ of the subsidy is s times the number of units firm a produces. Hence:

$$TC(s) = s\frac{(A - c + 2s)}{3} = \frac{3(A - c)s + 6s^2}{9} \tag{24.6}$$

Hence, the net benefit of the subsidy $NB(s)$ is:

$$NB(s) = \Delta\pi - TC(s) = \frac{(A - c)s - 2s^2}{9} \tag{24.7}$$

The subsidy should of course be chosen so as to maximize the net benefit shown in equation (24.7). A straightforward application of calculus then yields the optimum per unit subsidy s^*:

$$s^* = \frac{A - c}{4} \tag{24.8}$$