imperfect competition, a number of its predictions are borne out by empirical evidence.

It is important to remember though that, as pointed out in Chapter 4, market structure is endogenous. Strategies that generate above normal profits for existing firms will induce new firms to enter. At the same time, incumbent firms may be able to take actions that deter such entry. We need to extend our analysis in ways that allow us to examine these issues.

Further, the Cournot model studied in this chapter has firms interacting only once. The reality, of course, is that firms are involved in strategic interactions repeatedly. In such a setting, issues such as learning, establishing a reputation, and credibility can become quite important. We turn to a consideration of how the nature of strategic interaction over time affects market structure in Chapters 11–13.

Problems

1. Harrison and Tyler are two students who met by chance the last day of exams before the end of the spring semester and the beginning of summer. Fortunately, they liked each other very much. Unfortunately, they forgot to exchange addresses. Fortunately, each remembers that they spoke of attending a campus party that night. Unfortunately, there are two such parties. One party is small. If each attends this party, they will certainly meet. The other party is huge. If each attends this one, there is a chance they will not meet because of the crowd. Of course, they will certainly not meet if they attend separate parties. Payoffs to each depending on the combined choice of parties are shown below, with Tyler's payoffs listed first.

		Harrison	
		Go to Small Party	Go to Large Party
Tyler	Go to Small Party	(1,000, 1,000)	(0, 0)
	Go to Large Party	(0, 0)	(500, 500)

   a. Identify the Nash equilibria for this problem.
   b. Identify the Pareto optimal outcome for this "two party" system.

2. Suppose that the small party of Problem 1 is hosted by the "Outcasts," twenty men and women students trying to organize alternatives to the existing campus party establishment. All twenty Outcasts will attend the party. But many other students—not unlike Harrison and Tyler—only go to a party to which others (no one in particular, just people in general) are expected to come. As a result, total attendance A at the small party depends on just how many people X everyone *expects* to show up. Let the relationship between A and X be given by: $A = 20 + 0.6X$
   a. Explain this equation. Why is the intercept 20? Why is the relation between A and X positive?
   b. If the equilibrium requires that partygoers' expectations be correct, what is the equilibrium attendance at the Outcasts' party?

3. A game known well to both academics and teenage boys is "Chicken." Two players each drive their car down the center of a road in opposite directions. Each chooses either STAY or SWERVE. Staying wins adolescent admiration and a big payoff *if* the other player chooses SWERVE. Swerving loses face and has a low payoff when the other player stays. Bad as that is, it is still better than the payoff when both players choose STAY in which case they each are badly hurt. These outcomes are described below with Player A's payoffs listed first.

		Player B	
		Stay	Swerve
Player A	Stay	(−6, −6)	(2, −2)
	Swerve	(−2, 2)	(1, 1)

   a. Find the Nash equilibria in this game.
   b. This is a good game to introduce mixed strategies. If Player A adopts the strategy STAY one-fifth of the time, and SWERVE four-fifths of the time, show

that Player B will then be indifferent between either strategy, STAY or SWERVE.

c. If *both* players use this probability mix, what is the chance that they will both be badly hurt?

4. You are a manager of a small "widget"-producing firm. There are only two firms, including yours, that produce "widgets." Moreover your company and your competitor's are identical. You produce the same good and face the same costs of production described by the following total cost function: Total Cost $= 1500 + 8q$ where q is the output of an individual firm. The market-clearing price, at which you can sell your widgets to the public, depends on how many widgets both you and your rival choose to produce. A market research company has found that market demand for widgets can be described as: $P = 200 - 2Q$ where $Q = q_1 + q_2$, where q_1 is your output and q_2 is your rivals. The Board of Directors has directed you to choose an output level that will *maximize* the firm's profit. How many widgets should your firm produce in order to achieve the profit-maximizing goal? Moreover you must present your strategy to the Board of Directors and explain to them why producing this amount of widgets is the profit-maximizing strategy.

5. You are still a manager of a small widget-producing firm. Now however there are fourteen such firms (including yours) in the industry. Each firm is identical; each one produces the same product and has the same costs of production. Your firm, as well as each one of the other firms, has the same total cost function, namely: Total Cost $= 200 + 50q$ where q is the output of an individual firm. The price at which you can sell your widgets is determined by market demand, which has been estimated as: $P = 290 - \frac{1}{3}Q$ where Q is the sum of all the individual firms producing in this industry. So, for example, if 120 widgets are produced in the industry then the market-clearing price will be 250, whereas if 300 widgets are produced then the market-clearing price will be 190. The Board of Directors has directed you to choose an output level that maximizes the firm's profit. You have an incentive to maximize profits because your job and salary depend upon the profit performance of this company. Moreover you should also be able to present your profit-maximizing strategy to the Board of Directors and explain to them why producing this amount maximizes the firm's profit.

6. The inverse market demand for fax paper is given by $P = 400 - 2Q$. There are two firms who produce fax paper. Each firm has a unit cost of production equal to 40, and they compete in the market in quantities. That is, they can choose any quantity to produce, and they make their quantity choices simultaneously.

   a. Show how to derive the Cournot-Nash equilibrium to this game. What are firms' profits in equilibrium?

   b. What is the monopoly output, i.e., the one that maximizes total industry profit? Why isn't producing one-half the monopoly output a Nash equilibrium outcome?

7. Return to Problem 6, but suppose now that firm 1 has a cost advantage. Its unit cost is constant and equal to 25 whereas firm 2 still has a unit cost of 40. What is the Cournot outcome now? What are the profits for each firm?

8. We can use the Cournot model to derive an equilibrium industry structure. For this purpose, we will define an equilibrium as that structure in which no firm has an incentive to leave or enter the industry. If a firm leaves the industry, it enters an alternative competitive market in which case it earns zero (economic) profit. If an additional firm enters the industry when there are already n firms in it, the new firm's profit is determined by the Cournot equilibrium with $n + 1$ firms. For this problem, assume that each firm has the cost function: $C(q) = 256 + 20q$. Assume further that market demand is described by: $P = 100 - Q$.

   a. Find the long-run equilibrium number of firms in this industry.

   b. What industry output, price, and firm profit levels will characterize the long-run equilibrium?

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10

Oligopolistic Price Competition

In 2007, Amazon introduced its first *Kindle* e-reader at a price of $399. The product was a quick success. However, in the face of an announcement by Barnes & Noble that it would soon launch its own *Nook* e-reader, the price was reduced in 2008 to $259. In 2009, when the *Nook* was launched, the competition intensified and Amazon dropped the price to $199. In March of 2010, Apple began taking orders for its new *iPad* product, which also included e-reading capabilities. As this text is being written, both the *Kindle Touch* and the *Nook Simple Touch* are selling for $99, while the latest top-of-the-line *Kindle Fire* that also serves as a viewer and game platform is being offered at a sale price of $139. This rapid fall of e-reader prices has been all the more remarkable in that it has been accompanied by an almost equally rapid rise in their quality. Relative to the initial products, the current *Kindle* and *Nook* are more than 50 percent lighter, have a battery life more than twice as long, and a storage capacity as much as 100 times greater.

The digital and mobile technology markets are similar to many markets, including restaurants, electricians, moving companies, consulting firms, and financial services, in which consumers favor those products that best match their preferences at the lowest price. In these markets, it is in fact the firm's price choice that largely determines its demand and its profit, and each firm may set a different price. While this may seem eminently plausible, it is quite different from the way competition works in the Cournot model. There each firm independently chooses a level of production. It is only afterwards that the market price adjusts so that consumers will buy the combined output of all the firms at a price that is the same for each company.

In a monopolized market, it would of course make no difference whether the firm initially set a price and then produced whatever amount consumers demanded at that price or, instead, first chose its production and let the price settle at whatever level was necessary to sell that output. When a profit-maximizing monopolist optimally sets price, that choice will imply, via the demand curve, a specific output level. That same optimal price is precisely the price that would have emerged if instead the monopolist had initially chosen its profit-maximizing production.

Once we leave the world of monopoly, however, the equivalence of price and output strategies vanishes. In oligopolistic markets, it matters very much whether firms compete in terms of quantities, as in Cournot, or in prices like the e-reader firms above. To understand these differences, we begin as we did with the Cournot model by examining the simple duopoly case in which the two firms sell an identical good but now compete by first setting prices instead of production levels. This is known as the Bertrand model. Later in the chapter we allow the products to be less than perfect substitutes, or to be differentiated. As in Chapter 9, we also focus on static or simultaneous models of price competition limited to a single market period.

10.1 THE BERTRAND DUOPOLY MODEL

The standard duopoly model, recast in terms of price strategies rather than quantity strategies, is typically referred to as the Bertrand model. Joseph Bertrand was a French mathematician who in 1883 reviewed and critiqued Cournot's work nearly fifty years after its publication in the *Journal des Savants*. Bertrand was critical of mathematical modeling in economics, and to prove his point he analyzed the Cournot model in terms of prices rather than quantities. The legacy of Bertrand is not, however, his criticism of what he termed "pseudo-mathematics" in economics. Instead, Bertrand's contribution was the recognition that outcome of the strategic interaction depends critically on the nature of the strategic variable—in this case, quantity versus price. This is just a way of expressing the point made at the start of the previous chapter. The rules of the game matter ... a lot.

Therefore, consider the simple duopoly model of Chapter 9, but now let each firm choose the price it will charge rather than the quantity it will produce. All other assumptions, however, remain unchanged. Each firm produces the same good at the same constant marginal cost c, and the firms choose their strategies simultaneously. Each firm knows the structure of market demand and its rival's cost, as well as the fact that total demand is: $P = A - BQ$. In this case, it is more convenient to write the demand function with output as the dependent variable.[1] Hence:

$$Q = a - bP; \quad \text{where} \quad a = \frac{A}{B} \quad \text{and} \quad b = \frac{1}{B} \tag{10.1}$$

Consider the pricing problem first from firm 2's perspective. In order to determine its best price response to its rival, firm 2 must first work out the demand for its product *conditional* on both its own price, denoted by p_2, and firm 1's price, denoted by p_1. Rationally speaking, firm 2's reasoning would go as follows. If $p_2 > p_1$, firm 2 will sell no output. The product is homogenous so that consumers always buy from the cheapest source. Setting a price above that of firm 1 therefore means that firm 2 will serve no customers. The opposite is true if $p_2 < p_1$. When firm 2 sets the lower price, it will supply the entire market, and firm 1will sell nothing. Finally, if $p_2 = p_1$, the two firms will split the market evenly. When both firms charge identical prices, we will assume that each firm's demand is one half the total market demand at that price.

The foregoing reasoning tells us that demand for firm 2's output, q_2, may be described as follows:

$$\begin{aligned} q_2 &= 0 && \textit{if} \ \ p_2 > p_1 \\ q_2 &= \frac{a - bp_2}{2} && \textit{if} \ \ p_2 = p_1 \\ q_2 &= a - bp_2 && \textit{if} \ \ p_2 < p_1 \end{aligned} \tag{10.2}$$

[1] When firms choose quantities (as in Cournot's model) it is often easier to work with the inverse demand curve and treat price as the dependent variable. When firms select prices, as in Bertrand's analysis, it is often best to let quantity be the dependent variable.

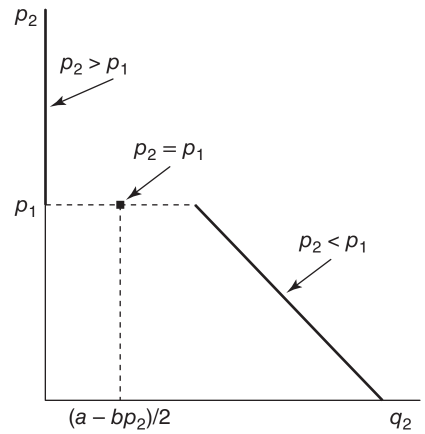

Figure 10.1 Firm 2's demand curve in the Bertrand model
Industry demand equal to $a - bp_2$ is the same as firm 2's demand for all p_2 less than p_1. If $p_2 = p_1$, then the two firms share equally the total demand. For $p_2 > p_1$, firm 2's demand falls to zero.

As Figure 10.1 shows, this demand function is *not* continuous. For any p_2 greater than p_1, demand for q_2 is zero. But when p_2 falls and becomes exactly equal to p_1, demand jumps from zero to $\frac{a - bp_2}{2}$. When p_2 then falls still further so that it is below p_1, demand then jumps again to $a - bp_2$.

This discontinuity in firm 2's demand curve was not present in the Cournot model of quantity competition, and it turns out to make a crucial difference in terms of firms' strategies. The discontinuity in demand carries over into a discontinuity in profits. Firm 2's profit, Π_2, as a function of p_1 and p_2 is:

$$\begin{aligned} &\Pi_2(p_1, p_2) = 0 && \textit{if} \;\; p_2 > p_1 \\ &\Pi_2(p_1, p_2) = (p_2 - c)\frac{a - bp_2}{2} && \textit{if} \;\; p_2 = p_1 \\ &\Pi_2(p_1, p_2) = (p_2 - c)(a - bp_2) && \textit{if} \;\; p_2 < p_1 \end{aligned} \tag{10.3}$$

To find firm 2's *best response* function, we need to find the price p_2 that maximizes firm 2's profits $\Pi_2(p_1, p_2)$ for any given choice of p_1. For example, suppose firm 1 chooses a very high price—higher even than the pure monopoly price, which in this case is $p^M = \frac{a + bc}{2b}$.[2] Because firm 2 could capture the entire market by selecting any price lower than p_1, its best response would be to choose the pure monopoly price p^M, and thereby earn the pure monopoly profits.

Conversely, what if firm 1 set a very low price, say one below its unit cost c? This would be an unusual choice, but if we wish to construct a complete *best response* function for firm 2, we must determine its value for *all* the possible values p_1 can take. If $p_1 < c$, then firm 2 is best setting its price a level above p_1. This will mean that firm 2 will sell nothing

[2] This is, of course, the same monopoly price as we showed in Chapter 8 for the quantity version of the model with the notational change that $a = A/B$, and $b = 1/B$.

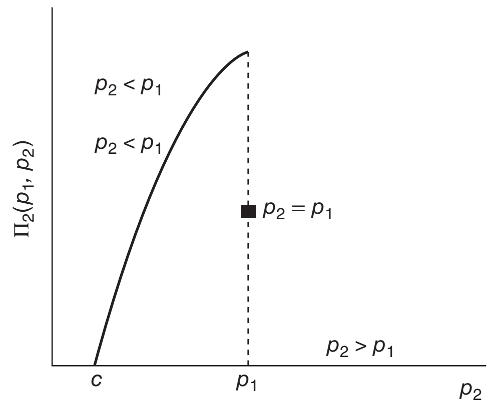

Figure 10.2 Firm 2's profits as a function of p_2 when firm 1 prices above cost but below the pure monopoly price

Firm 2's profits rise continuously as its price rises from the level of marginal cost, c, to just below firm 1's price. When p_2 equals p_1, firm 2's profits fall relative to those earned when p_2 is just below p_1. For p_2 greater than p_1, firm 2 earns zero profits.

and earn zero profits. The alternative of setting $p_2 \leq p_1 < c$ will lead to *negative* profits: firm 2 sells a positive amount of output, but at a price below unit cost so that it loses money on each unit sold.

What about the more likely case in which firm 1 sets its price above marginal cost c but either equal to or below the pure monopoly price p^M? How should firm 2 optimally respond in these circumstances? The simple answer is that it should set a price *just a bit less than* p_1. The intuition behind this strategy is illustrated in Figure 10.2, which shows firm 2's profit given a price p_1, satisfying the relationship $\frac{a + bc}{2b} \geq p_1 > c$.

Note that firm 2's profits rise continuously as p_2 rises from c to just below p_1. Whenever p_2 is less than $p_{1,}$, firm 2 is the only company that sells anything. However, when p_1 is less than or equal to p^M, the monopoly power that firm 2 obtains from undercutting p_1 is constrained. In particular, the firm cannot sell at the pure monopoly price, p^M, and earn the associated profit because at that price, firm 2 would lose all its customers. Still, the firm will wish to get as close to that result as possible. It could, of course, just match firm 1's price exactly. But whenever it does so, it shares the market equally with its rival. If, instead, firm 2 just *slightly* reduces its price below p_1 level, it will double its sales while incurring only an infinitesimal decline in its profit margin per unit sold. This is a trade well worth the making, as Figure 10.2 makes clear. In turn, the implication is that for any p_1 such that $p^M \geq p_1 > c$, firm 2's best response is to set $p_2^* = p_1 - \varepsilon$, where ε is an arbitrarily small amount.

The last case to consider is the case in which firm 1 prices at cost so that $p_1 = c$. Clearly, firm 2 has no incentive to undercut this value of p_1. To do so would only lead to losses for firm 2. Instead, firm 2 will do best to set p_2 either equal to or above p_1. If it prices above p_1, firm 2 will sell nothing and earn zero profits. If it matches p_1, it will enjoy positive sales but break even on every unit sold. Accordingly, firm 2 will earn zero profits in this latter case, too. Thus, when $p_1 = c$, firm 2's *best response* is to set p_2 either greater than or equal to p_1.

Our preceding discussion may be summarized with the following description of firm 2's best price response:

$$\begin{aligned}
p_2^* &= \frac{a+bc}{2b} && \text{if } \; p_1 > \frac{a+bc}{2b} \\
p_2^* &= p_1 - \varepsilon && \text{if } \; c < p_1 \le \frac{a+bc}{2b} \\
p_2^* &\ge p_1 && \text{if } \; c = p_1 \\
p_2^* &> p_1 && \text{if } \; c > p_1 \ge 0
\end{aligned} \tag{10.4}$$

By similar reasoning, firm 1's best response p_1^* for any given value of p_2 would be given by:

$$\begin{aligned}
p_1^* &= \frac{a+bc}{2b} && \text{if } \; p_2 > \frac{a+bc}{2b} \\
p_1^* &= p_2 - \varepsilon && \text{if } \; c < p_1 \le \frac{a+bc}{2b} \\
p_1^* &\ge p_2 && \text{if } \; c = p_2 \\
p_1^* &> p_2 && \text{if } \; c > p_2 \ge 0
\end{aligned} \tag{10.5}$$

We may now determine the Nash equilibrium for the duopoly game when played in prices. We know that a Nash equilibrium is one in which neither firm has an incentive to change its strategy. For example, the strategy combination $\left[p_1 = \frac{a+bc}{2b}, p_2 = \frac{a+bc}{2b} - \varepsilon\right]$ *cannot* be an equilibrium. This is because in that combination, firm 2 undercuts firm 1's price and sells at a price just below the monopoly level. However, in such a case, firm 1 would have no customers and earn zero profit. Because firm 1 could earn a substantial profit by lowering its price to just below that set by firm 2, it would wish to do so. Accordingly, this strategy cannot be a Nash equilibrium. To put it another way, firm 2 could never expect firm 1 to set the monopoly price of $p_1 = (a+c)/2b$ precisely because firm 1 would know that so doing would lead to zero profit as firm 2 would undercut that price by a small amount ε and steal all of firm 1's customers.

As it turns out, there is one and only one Nash equilibrium for the Bertrand duopoly game described above. It is the price pair, $(p_1^* = c, p_2^* = c)$.[3] If firm 1 sets this price in the expectation that firm 2 will do so, and if firm 2 acts in precisely the same manner, neither will have an incentive to change. Hence, the outcome of the Bertrand duopoly game is that the market price equals marginal cost. This is, of course, exactly what occurs under perfect competition. The only difference is that here, instead of many small firms, we have just two firms, each of which is large relative to the market.

It is no wonder that Bertrand made note of the different outcome obtained when price replaces quantity as the strategic variable. Far from being a cosmetic or minor change, this alternative specification has a dramatic impact. It is useful, therefore, to explore the nature and the source of this powerful effect more closely.

[3] This outcome assumes that prices can be varied continuously.

10.1

Practice Problem

Let the market demand for carbonated water be given by $Q^D = 100 - 5P$. Let there be two firms producing carbonated water, each with a constant marginal cost of 2.

a. What is the market equilibrium price and quantity when each firm behaves as a Cournot duopolist choosing quantities? What profit does each firm earn?
b. What is the market equilibrium price and quantity when each firm behaves as a Bertrand duopolist choosing price? What firm profit does each firm earn now?

10.2 BERTRAND RECONSIDERED

Like its Cournot cousin, the Bertrand analysis of a duopoly market is not without its critics. One chief source of criticism with the Bertrand model is its assumption that *any* price deviation between the two firms leads to an immediate and complete loss of demand for the firm charging the higher price. It is, of course, this assumption that gives rise to the discontinuity in both firms' demand and profit functions. It is also this assumption that underlies our derivation of each firm's best response function and their intersection at $P = MC$.

There are two very sound reasons that a firm's decision to charge a price higher than its rival may not result in the complete loss of all its customers. One reason is that typically the rival firm does not have the capacity to serve all of the customers who demand the product or service at its low price.[4] The second is that consumers may not view the two products as perfect substitutes.

To see the importance of capacity constraints, consider the fictional case of a small New England area with two ski resorts, Pepall Ridge and Snow Richards, each located on different sides of Mount Norman. Skiers regard the services at these resorts to be the same and will choose when possible to ski at the resort that quotes the lowest lift ticket price. Pepall Ridge and Snow Richards are roughly the same size and each can handle 1,800 skiers a day. We will further assume that the overall demand for skiing at Mount Norman is: $Q = 6000 - 60P$, where P is the price of a daily lift ticket and Q is number of skiers per day.

The two resorts compete in price. Suppose that the marginal cost of providing lift services is the same at each resort and is equal to \$10 per skier. However, the outcome in which each resort sets a price equal to marginal cost *cannot* be a Nash equilibrium. Demand when the price of a lift ticket is equal to \$10 would be equal to 5,400 skiers, far exceeding the total capacity of the two resorts. To be sure, if each resort had understood the extent of demand, each might have built additional lifts, ski runs, and parking facilities in order to increase capacity. Nevertheless, it is still not likely that the Nash equilibrium will end up with each resort charging a lift ticket price equal to the marginal cost of \$10 per skier. Why? Think of it this way. If Pepall Ridge sets a price of \$11, Snow Richards will only find it profitable to undercut this price—say, charge \$10.90 for a lift ticket—if it can serve all the skiers who would come at this lower price. Yet because total demand at $P = \$10.90$ is $Q = 5346$ it clearly cannot do so with its current capacity of 1,800.

[4] Edgeworth (1925) was one of the first economists to investigate the impact of capacity constraints on Bertrand analysis.

We might then ask how each resort ended up with a capacity of just 1,800. Wouldn't it have been profitable for each to expand their capacity precisely because this would then enable either to undercut the other's price and gain more profit? The answer to this question, however, is no. Consider an initial capacity choice by each firm of 5,400. Clearly, such a choice is required for each firm to make a credible threat of cutting its price to \$10. Yet if each firm did cut its price to this marginal cost, each would only serve half the market, i.e., 2,700 skiers per day. Given that building capacity is expensive, neither firm will choose an initial capacity of 5,400 because this would imply an equilibrium in which each firm sets a price equal to marginal cost = \$10 and in which each would serve only 2,700 skiers and have an equal amount of costly but unused capacity.

We can see then that when capacity constraints are binding, a firm charging a higher price than its rival only loses all its customers if its rival has the capacity to serve them. We can also see that such a capacity choice for both firms is not likely in the range of $P =$ marginal cost = \$10. Hence, once capacity constraints are considered, Bertrand's proposed solution that $p_1 = p_2 =$ marginal cost is no longer a Nash equilibrium. This leaves open, though, the question of precisely what capacity choice for each firm does make sense, i.e., what is the Nash equilibrium capacity choice?

When capacity constraints come into play, the game between the two firms really becomes a two-stage one. In the first stage, the two firms choose capacity levels. In the second, they compete in price. Viewed in this light, it is straightforward to show that the outcome is likely to be close to the Cournot solution despite the fact that competition is in prices as Bertrand proposed.[5] To see this point more clearly, let's return to the ski resort competition between Pepall Ridge and Snow Richards. We assume that at any price at which a resort has demand beyond its maximum capacity, the skiers that the resort serves are those skiers who are the most eager and who have the highest willingness to pay.[6] Suppose, for instance, that each resort sets a lift price of \$30. Then total market demand is 4,200, well beyond the total capacity of 3,600 across both firms. Therefore, each resort will need somehow to ration or choose which skiers will actually ski. Our assumption, sometimes called the efficient rationing assumption, is that the resorts will do this by serving customers in order of their willingness to pay. Pepall Ridge, for example, will serve the 1,800 potential skiers with the 1,800 highest willingesses to pay. In turn, the assumption of efficient rationing allows us to derive the residual demand curve facing Snow Richards at any price.

A price of particular interest is \$40. Suppose then that both resorts have set $p_1 = p_2 =$ \$40. At these prices, total demand is equal to 3,600, which is just equal to the total capacity of the two resorts. Is this a Nash equilibrium? We can answer this question by using the logic above to determine the demand function facing Snow Richards when Pepall Ridge sets a price equal to \$40. Under our assumption of efficient rationing, this is shown in Figure 10.3. It is the original demand curve shifted to the left by 1,800 units, i.e. it is $Q = 4200 - 60P$ (or, in inverse form, $P = 70 - Q/60$). The associated marginal revenue curve— $MR = 70 - Q/30$—is also shown.

Note though that while changes in its price also change its quantity demanded, Snow Richards is always constrained to serve no more than its capacity of 1,800. This is why Figure 10.3 shows the residual demand and marginal revenue curves truncated at that output level. More importantly, note that when Pepall Ridge sets a price of \$40, Snow Richards

[5] This result is formally modeled in a two-stage game in Kreps and Scheinkman (1983).

[6] We also implicitly assume that in the first stage, installing capacity of any amount is costless but that, once installed, that capacity cannot be expanded in the second stage.

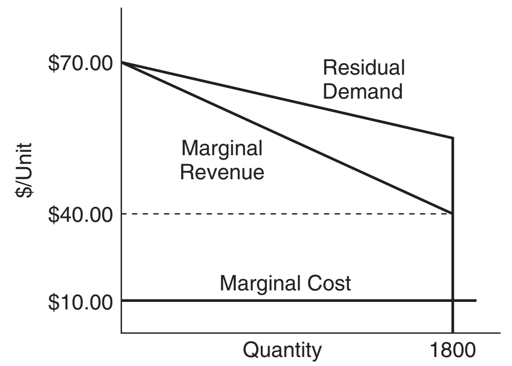

Figure 10.3 Snow Richards residual demand curve

finds that $MR = MC$ at an output exactly equal to its capacity of 1,800, implying that this is indeed the best response for Snow Richards. It also implies a price of \$40 for this resort as well. In turn, both firms charging a price of \$40 also leads to the full utilization of the 1,800 skiers per day capacity at Pepall Ridge. There can be no doubt then that price = \$40 for both firms is a Nash equilibrium if both choose capacity of 1,800. But is this the right capacity level? Given Pepall Ridge's choice, would Snow Richards have preferred to have a greater or lesser capacity?

Suppose for instance that Snow Richards initially chose a capacity of 1,700 while, as noted, Pepall Ridge had chosen 1,800. In this case, \$40 could no longer be the price at both firms because then the total demand (3,600) would exceed the total capacity (3,500). What would the new price equilibrium be? Start by noting what happens if the price set by both firms is \$41.67, i.e., the price point at which total demand would equal the capacity of 3,500. If each firm charges \$41.67 per lift ticket, Pepall Ridge will serve 1,800 skiers per day, Snow Richards will serve 1,700, and the market will clear. If just one firm raises its price, it loses no customers to its rival (which is working at capacity) but it does lose marginal consumers who are not willing to pay any more for a lift ticket. That is, for each firm, we can again work out the residual demand curve and associated marginal revenue curve given the price and capacity choice of its rival. These are:

$$
\begin{array}{rll}
\text{Snow Richards} & \text{—Residual Demand :} & P = 70 - Q/60 \\
 & \text{—Residual Marginal Revenue :} & MR = 70 - Q/30 \\
\text{Pepall Ridge} & \text{—Residual Demand :} & P = 71.67 - Q/60 \\
 & \text{—Residual Marginal Revenue :} & MR = 71.67 - Q/30
\end{array}
$$

Profit maximization implies that each firm will want to expand its output and lower its price so long as marginal revenue exceeds marginal cost and so long as there is spare capacity. It is straightforward to show, though, that at the capacity limit of each firm (1,700 and 1,800, respectively), the marginal revenue for each firm exceeds or equals its marginal cost so that each will want to produce at least to capacity. To put it another way, with each firm setting a price of \$41.67 and each working to capacity, neither will want to raise its price because to do so would lose more in revenue than it would save in costs. Of course, neither firm wants to lower the price either because they are both working at capacity and cannot serve any additional customers. Thus, if Snow Richards chooses instead the lower capacity of 1,700 in response to a Pepall Ridge capacity choice of 1,800, each firm charging \$41.67 per lift ticket will indeed be the Nash equilibrium in prices. From the perspective of Snow Richards, however, this Nash equilibrium is not as profitable as the equilibrium that we earlier derived when Snow Richards responds to Pepall Ridge's capacity choice of 1,800 by selecting 1,800 for its capacity as well. As can be readily verified, Snow Richards earns \$53,833.33 when it sets capacity at 1,700 but \$54,000 when it sets it at 1,800. In short, if Pepall Ridge has a capacity of 1,800, Snow Richards will not want any less.

It will also not want any more. Suppose, for example, that Snow Richards had a 1,900 skier capacity. In this case, at least one firm would need to lower its price to \$38.33 in order to clear the market. Pepall Ridge, though, has no interest in making such a price cut. If it keeps its price at \$40, it will serve its capacity of 1,800 skiers regardless of the Snow Richards price because that firm can only serve 1,900 skiers. Thus, there is no reason for Pepall Ridge to cut its price if Snow Richards expands its capacity by 100 units. This means that Snow Richards faces exactly the same situation as in the original Nash equilibrium, except that now its residual marginal revenue curve, $MR = 70 - Q/30$, extends to an output of 1,900 before it is truncated. Yet that marginal revenue falls below the marginal cost of \$10 as soon as the firm's output exceeds 1,800 units. If Snow Richards had the additional capacity, it would not want to use it. Hence, there is no point in installing the extra 100 units of capacity in the first place.

Obviously, the foregoing argument is symmetric so that just as Snow Richards wants neither to expand nor contract its capacity from 1,800, neither does Pepall Ridge. It follows that each firm choosing a capacity of 1,800 and setting a price of \$40 is the Nash equilibrium for this price competition game. More importantly, this outcome is exactly the same as the Cournot outcome in a market without capacity constraints but in which each firm has a constant marginal cost of 10 and market demand is described by $Q = 6000 - 60P$. That is, once capacity constraints are considered and assuming efficient rationing, Bertrand price competition yields Cournot quantity competition outcomes. This is in fact a fairly general result, again, conditional on the key assumptions. Of course, in reality, capacity may be more expandable than assumed here and rationing may not be efficient. Nevertheless, the analysis provides a strong counter argument to the dramatic marginal-cost pricing implication of the simple Bertrand model.

10.2 **Practice Problem**

Suppose now that market demand for skiing increases to $Q^D = 9000 - 60P$. However, because of environmental regulations neither Pepall Ridge nor Snow Richards can increase their capacities and serve more skiers beyond their current level of 1,800. What is the Nash equilibrium price outcome for this case?

10.3 BERTRAND IN A SPATIAL SETTING

There is a second reason that the simple Bertrand efficient outcome of price equal to marginal cost may not occur. This is that the two firms often do not, as Bertrand assumed, produce identical products. Think of hair salons, for example. No two hair stylists cut and style hair in exactly the same way. Nor will the salons have exactly the same sort of equipment or furnishings. Also, as long as the two firms are not side by side, they will differ in their locations. This is often sufficient by itself to generate a preference by some consumers for one salon or the other even when different prices are charged. In short, differences in locations, furnishings, or cutting styles can each be sufficient to permit one salon to price somewhat higher than its rival without immediately losing all of its customers.

We presented a spatial model of product differentiation in Chapter 7. There our aim was to understand the use of such differentiation by a monopoly firm to extract additional surplus from its customers. The same model, however, may also be used to understand the nature of price competition when competing firms market differentiated products. Let's review the basic setup presented earlier. There is a line of unit length (say one mile) along which consumers are uniformly distributed. This market is supplied by two stores. This time though, the same company does not operate the two stores. Rival firms operate them. One firm—located at the west end of town—has the address $x = 0$. The other—located at the east end of town—has the location, $x = 1$. Each of the firms has the same constant unit cost of production c.

We define a consumer's "location" in this market to be that consumer's most preferred product, or style. Thus "consumer x" is located distance x from the left-hand end of the market, where distance may be geographic in a spatial model or measured in terms of characteristics in a more general product differentiation sense. While consumers differ regarding which variant or location of the good they consider to be the best, they are identical in their reservation price V for their most preferred product. We further assume that the reservation price V is substantially greater than the unit cost of production c. Each consumer buys at most one unit of the product. If consumer x purchases a good that is not an ideal product, the consumer incurs a utility loss. Specifically, consumer x incurs the cost tx if good 1 (located at $x = 0$) is consumed, and the cost $t(1 - x)$ if good 2 (located at $x = 1$) is consumed. If the consumer buys good 1 at price p_1, the consumer enjoys consumer surplus $V - p_1 - tx$; if good 2 at price p_2 is purchased, there will be a consumer surplus of $V - p_2 - t(1 - x_2)$. Of course, the consumer will purchase the good that offers the greater consumer surplus provided that this is greater than zero. Figure 10.4 describes this market setting.

It bears reemphasizing that the concept of location that we have introduced here serves as a metaphor for all manner of qualitative differences between products. Instead of having two stores geographically separated, we can think of two products marketed by two different firms that are differentiated by some characteristic, such as sugar content in the case of soft drinks, fat content in the case of fast food, or fuel efficiency in the case of automobiles. Our unit line in each case represents the spectrum of products differentiated by this characteristic and each consumer has a most preferred product specification on this line. For the case of soft drinks our two firms could be Pepsi and Coca-Cola. In the case of fast food, our two firms could be McDonald's and Burger King, whereas for automobiles, our two firms could be Ford and GM.

As in the simple Bertrand model, the two firms compete for customers by setting prices p_1 and p_2, respectively. These are chosen simultaneously, and we want to solve for a Nash

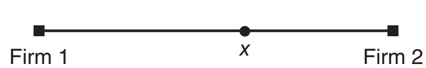

Figure 10.4 The "main street" spatial model once again

equilibrium solution to the game. If $V > c$ then in equilibrium it must be the case that both firms have a positive market share. Because any equilibrium will require that firms charge at least c per unit and because customers always prefer the closer shop at any common price, there is no way that either firm can steal all the customers from its rival. Here, we assume not only that $V > c$, but also that V is sufficiently large that the entire market is served. That is, the market outcome is such that every consumer buys one unit of the product from either firm 1 or firm 2.[7] When V is large, each customer can be charged a price sufficiently high to make each such sale profitable.

When the entire market is served, then it must be the case that there is some consumer, called the marginal consumer x^m, who is indifferent between buying from either firm 1 or firm 2. That is, this consumer enjoys the same consumer surplus either way. Algebraically, this means that for consumer x^m:

$$V - p_1 - tx^m = V - p_2 - t(1 - x^m) \tag{10.6}$$

Equation 10.6 may be solved to find the address or the location of the marginal consumer, x^m. This is:

$$x^m(p_1, p_2) = \frac{(p_2 - p_1 + t)}{2t} \tag{10.7}$$

At any set of prices p_1 and p_2, all consumers to the left of x^m buy from firm 1. All those to the right of x^m buy from firm 2. In other words, x^m is the fraction of the market that buys from firm 1 and $(1 - x^m)$ is the fraction that buys from firm 2. If the total number of consumers is N and they are uniformly distributed over the market space, the demand function facing firm 1 at any price combination (p_1, p_2) in which the entire market is served is:[8]

$$D^1(p_1, p_2) = x^m(p_1, p_2) = \frac{(p_2 - p_1 + t)}{2t}N \tag{10.8}$$

Similarly, firm 2's demand function is:

$$D^2(p_1, p_2) = (1 - x^m(p_1, p_2)) = \frac{(p_1 - p_2 + t)}{2t}N \tag{10.9}$$

These demand functions make sense in that each firm's demand is decreasing in its own price but increasing in its competitor's price. Notice also that, unlike the simple Bertrand duopoly model in Section 10.1, the demand function facing either firm here is continuous in both p_1 and p_2. This is because when goods are differentiated, a decision by say firm 1 to set p_1 a little higher than its rival's price p_2 does not cause firm 1 to lose all of its customers. Some of its customers still prefer to buy good 1 even at the higher price simply because they prefer that version of the good to the style (or location) marketed by firm 2.[9]

7 Refer to Figure 7.3 in Chapter 7 for a discussion of this point.

8 We are using N here to refer to the *number of consumers* in the market.

9 Our assumption that the equilibrium is one in which the entire market is served is critical to the continuity result.

The continuity in demand functions carries over into the profit functions. Firm 1's profit function is:

$$\Pi^1(p_1, p_2) = (p_1 - c)\frac{(p_2 - p_1 + t)}{2t}N \tag{10.10}$$

Similarly, firm 2's profits are given by:

$$\Pi^2(p_1, p_2) = (p_2 - c)\frac{(p_1 - p_2 + t)}{2t}N \tag{10.11}$$

In order to work out firm 1's best response pricing strategy we need to work out how firm 1's profit changes as the firm varies price p_1 in response to a given price p_2 set by firm 2. The most straightforward way to do this is to take the derivative of the profit function (10.10) with respect to p_1. When we set the derivative equal to zero we can then solve for the firm's best response price p_1^* to a given price p_2 set by firm 2.[10]

However, careful application of the alternative solution method of converting firm 1's demand curve into its inverse form and solving for the point at which marginal revenue equals marginal cost will also work. From (10.7), we can write firm 1's inverse demand curve for a given value of firm 2's price p_2 as $p_1 = p_2 + t - \frac{2t}{N}q_1$. Hence firm 1's marginal revenue curve is $MR_1 = p_2 + t - \frac{4t}{N}q_1$. Equating firm 1's marginal revenue with its marginal cost gives the first-order condition for profit maximization, $p_2 + t - \frac{4t}{N}q_1 = c$. Solving for the optimal value of firm 1's output, again given the price chosen by firm 2, we then obtain:

$$q_1^* = \frac{N}{4t}(p_2 + t - c) \tag{10.12}$$

When we substitute the value of q_1^* from equation (10.12) into firm 1's inverse demand curve, we find the optimal price for firm 1 to set given the value of the price set by firm 2. This is by definition firm 1's best response function:

$$p_1^* = \frac{p_2 + c + t}{2} \tag{10.13}$$

where t is the per unit distance transportation or utility cost incurred by a consumer. Of course, we can replicate this procedure for firm 2. Because the firms are symmetric, the best response function of each firm is the mirror image of that of its rival. Hence, firm 2's best price response function is:

$$p_2^* = \frac{p_1 + c + t}{2} \tag{10.14}$$

The best response functions described in (10.13) and (10.14) for the two firms are illustrated in Figure 10.5. They are upward sloping. The Nash equilibrium set of prices is, of course, where these best response functions intersect. In other words, the Nash equilibrium

10 Setting $\partial \Pi^1(p_1, p_2)/\partial p_1 = 0$ in equation (10.10) yields immediately: $p_1^* = (p_2 + c + t)/2$.

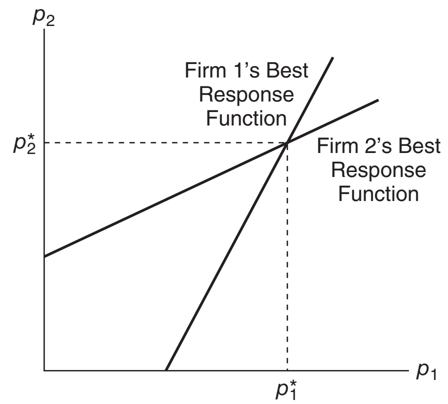

Figure 10.5 Best response functions for price competition with imperfect substitutes

is a pair of prices (p_1^*, p_2^*) such that p_1^* is firm 1's best response to p_2^* and p_2^* is firm 2's best response to p_1^*. Thus, we may replace p_1 and p_2 on the right-hand-side of the equations in (10.13) and (10.14) with p_1^* and p_2^*, respectively. Solving jointly for the Nash equilibrium pair (p_1^*, p_2^*) yields:

$$p_1^* = p_2^* = c + t \tag{10.15}$$

In equilibrium, each firm charges a price that is equal to the unit production cost *plus* an amount t, the utility cost per unit of distance a consumer incurs in buying a good that is at some distance from the preferred good. At these prices, the firms split the market. The marginal consumer is located at the address $x = 1/2$. The profit earned by each firm is the same, and equal to $(p_i^* - c)N/2 = tN/2$.

By way of example, consider the two hair salons located one mile apart on Main Street. All the potential customers live along this stretch of Main Street and are uniformly spread out. Each consumer is willing to pay at most $50 for a haircut done at the consumer's home. However, if a consumer has to travel to get a haircut a round-trip travel cost of $5 per mile will be incurred. Each of the hair salons can cut hair at a constant unit cost of $10 per cut, and each wants to set a price per haircut that maximizes the salon's profit. Our model predicts that the equilibrium price of a haircut in this town will be $15, a price that is greater than the marginal cost of a haircut.

Two points are worth making in connection with these results. First, note the role that the parameter t plays. It is a measure of the value each consumer places on obtaining the most preferred version of the product. The greater t is, the less willing the consumer is to buy a product "far away" from the favorite location, product, or style. That is, a high t value indicates consumers have strong preferences for their most desired product and incur a high utility loss from having to consume a product that is less than ideal. The result is that neither firm has much to worry about when charging a high price because consumers prefer to pay that price rather than buy a low-price alternative that is "far away" from their preferred style. When t is large, the price competition between the two firms is softened. In other words, a large value of t means that product differentiation makes price competition much less intense.

Reality Checkpoint

Unfriendly Skies: Price Wars in Airline Markets

Following the euphoria that accompanied deregulation in 1977, the major airlines have been in for a bumpy ride with generally lower and more volatile profits. One source of this downward trend has been the persistent recurrence of price wars. Morrison and Winston (1996) define such conflicts as any city-pair route market in which the average airfare declines by 20 percent or more within a single quarter. Based on this definition, they estimate that over 8l percent of airline city-pair routes experienced such wars in the 1979–95 time period. In the wars so identified, the average fare in fact typically falls by over 37 percent and sometimes falls by as much as 79 percent. While these conflicts abated in the last part of the 1990s, new wars had emerged by the early 2000s and continued through the decade leading to more than six bankruptcies and a number of mergers. The continued price pressure largely stemmed from two forces. First, the growth of low-cost airlines such as Southwest and JetBlue placed continued pressure on fares. Berry and Jia (2010) document that this pressure was increasingly strong as consumers grew to view the low-cost carriers as close substitutes for the more established ones. Second, in the wake of the recessions associated with the dot-com bubble and, later, the sub-prime mortgage collapses, business demand became substantially more price sensitive. These factors push prices closer to marginal cost and raise consumer surplus, but this is not much comfort to airline executives.

Source: S. Morrison and C. Winston, 1996. "Causes and Consequences of Airline Fare Wars," *Brookings Papers on Economic Activity*, Microeconomics: 85–124.; M. Maynard, "Yes, It Was a Dismal Year for Airlines; Now for the Bad News," *New York Times*, December 16, 2002, p. C2.; S. Berry and P. Jia, 2010. "Tracing the Woes: An Empirical Analysis of the Airline Industry," *American Economic Journal* 2 (August): 1–43.

However, as t falls consumers place less value on obtaining their most preferred styles, and become more sensitive to lower prices. This intensifies price competition. In the limit, when $t = 0$, product differentiation is of no value to consumers. They treat all goods as essentially identical. Price competition becomes fierce and, in the limit, drives prices to marginal cost just as in the original Bertrand model.

The second point concerns the location of the firms. We simply assumed that the two firms were located at either end of town. However, the location or product design of the firm is also part of a firm's strategy. Allowing the two firms to choose simultaneously *both* their price and their location strategies makes the model too complicated to solve here. Still, the intuition behind location choice is instructive. There are two opposing forces affecting the choice of price and location. On the one hand, the two firms will wish to avoid locating at the same point because to do so eliminates all differences between the two products. Price competition in this case will be fierce, as in the original Bertrand model. On the other hand, each firm also has some incentive to be located near the center of town. This enables a firm to reach as large a market as possible. Evaluating the balance of these two forces is what makes the solution of the equilibrium outcome so difficult.[11]

[11] There is a wealth of literature on this topic with the outcome often depending on the precise functional forms assumed. See, for example, Eaton (1976); D'Aspremont, Gabszewicz, and Thisse (1979); Novshek (1980); and Economides (1989).

10.3 **Practice Problem**

Imagine that the two hair salons located on Main Street no longer have the same unit cost. In particular, one salon has a constant unit cost of \$10 whereas the other salon has a constant unit cost of \$20. The low-cost salon, we'll call it Cheap-Cuts, is located at the west end of town, $x = 0$. The high-cost salon, The Ritz, is located at the east end, $x = 1$. There are 100 potential customers who live along the mile stretch, and they are uniformly spread out along the mile. Consumers are willing to pay \$50 for a haircut done at their home. If a consumer has to travel to get a haircut, then a travel cost of \$5 per mile is incurred. Each salon wants to set a price for a haircut that maximizes the salon's profit.

a. The demand functions facing the two salons are not affected by the fact that now one salon is high cost and the other is low cost. However the salons' best response functions are affected. Compute the best response function for each salon. How does an increase the unit cost of one salon affect the other salon's best response?
b. Work out the Nash equilibrium in prices for this model. Compare these prices to the ones derived in the text for the case when the two salons had the same unit cost equal to \$10. Explain why prices changed in the way they did. It may be helpful in your explanation to draw the best response functions when the salons are identical and compare them to those when the salons have different costs.

10.4 STRATEGIC COMPLEMENTS AND SUBSTITUTES

Best response functions in simultaneous-move games are extremely useful tools for understanding what we mean by a Nash equilibrium outcome. But an analysis of such functions also serves other useful purposes. In particular, examining the properties of best response functions can aid our understanding of how strategic interaction works and how that interaction can be made "more" or "less" competitive.

Figure 10.6 shows both the best response functions for the standard Cournot duopoly model and the best response functions for the Bertrand duopoly model with differentiated

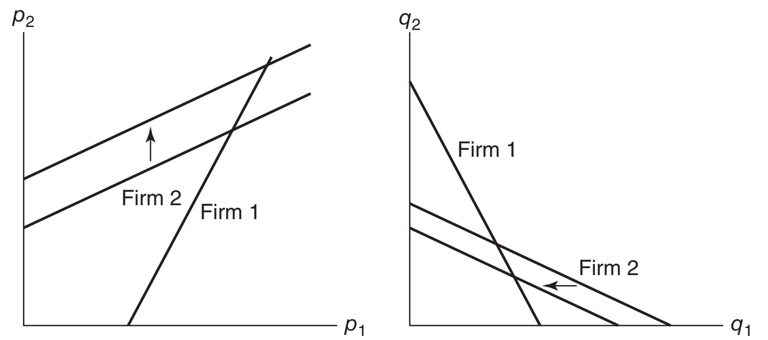

Figure 10.6 Best response functions for the Cournot (quantity) case and the Bertrand (price) case
A rise in firm 2's cost shifts its response function inwards in the Cournot model but outwards in the Bertrand model. Firm 1 reacts aggressively to increase its market share in the Cournot case. It reacts mildly in the Bertrand price by *raising* its price.

products. One feature in the diagram is immediately apparent. The best response functions for the Cournot quantity model are *negatively* sloped—firm 1's best response to an increase in q_2 is to *decrease* q_1. But the best response functions in the Bertrand price model are *positively* sloped. Firm 1's best response to an increase in p_2 is to increase p_1, as well.

Whether the best response functions are negatively or positively sloped is quite important. The slope reveals much about the nature of competition in the product market. To see this, consider the impact of an increase in firm 2's unit cost c_2. Our analysis of the Cournot model indicated that the effect of a rise in c_2 would be to shift *inward* firm 2's best response curve. As Figure 10.6 indicates, this leads to a new Nash equilibrium in which firm 2 produces less and firm 1 produces more than each did before c_2 rose. That is, in the Cournot quantity model, firm 1's response to firm 2's bad luck is a rather aggressive one in which it seizes the opportunity to expand its market share at the expense of firm 2.

Consider now the impact of a rise in c_2 in the context of the differentiated goods Bertrand model. The rise in this case shifts firm 2's best response function *upwards*. Given the rise in its cost, firm 2 will choose to set a higher p_2 than it did previously in response to any given value of p_1. How does firm 1 respond? Unlike the Cournot case, firm 1's reaction is less aggressive. Firm 1—seeing that firm 2 is now less able to set a low price—realizes that the price competition from firm 2 is now less intense. Hence, firm 1 now reacts by raising p_1.

When the best response functions are upward sloping, we say that the strategies (prices in the Bertrand case) are *strategic complements*. When we have the alternative case of downward sloping best response functions, we say that the strategies (quantities in the Cournot case) are *strategic substitutes*.[12] What should be clear at this point is that the choice of modeling firm competition in terms of variables that are strategic substitutes or complements is a crucial one. If we are interested in understanding the competitive dynamics of a particular industry, we need to know the underlying features of that industry that will determine that strategic variable choice. For example, in those industries in which firms set their production schedules far in advance of putting the goods on the market for sale, there is a good case to assume that firms compete in quantities. Examples include aircraft producers, coffee-growers, and automobile manufacturers. In many service industries—such as banking, insurance, and air travel—in which production levels can be quickly adjusted, it is much more natural to think in terms of price competition.

10.5 EMPIRICAL APPLICATION: BRAND COMPETITION AND CONSUMER PREFERENCES—EVIDENCE FROM THE CALIFORNIA RETAIL GASOLINE MARKET

Gasoline is typically produced by refiners and then shipped to a central distribution point. The gasoline is then bought either by an unbranded independent retailer such as RaceTrac, or by service stations selling a branded product such as an Exxon or a Chevron station. In the latter case, a special additive unique to the brand has to be added. That is, to sell "Chevron" gasoline, a station has to have added *Techronas*™ to the fuel. Thus, each specific brand is differentiated by the use of its own additive. Independent stations, however, simply sell the basic gasoline without any additive. Here, we briefly describe a paper by Justine

[12] This terminology comes from Bulow, Geanakopolos, and Klemperer (1985) and reflects similar terminology in consumer demand theory.

Hastings (2004) that examines the nature of price competition in the retail gasoline market in Southern California.

The background to the study is as follows. In June of 1997, the Atlantic Richfield Company (ARCO), a well-known refiner and retail brand, acquired control of about 260 gasoline stations that formerly had been operated by the independent retailer, Thrifty, in and around Los Angeles and San Diego. ARCO then converted these to ARCO stations—a process that was essentially completed by September of that same year. Thus, the ARCO-Thrifty acquisition resulted in the exit of a large number of independent service stations in Southern California, as these were replaced in part by ARCO sellers.

Hastings (2004) asks what effect the ARCO-Thrifty deal had on retail gasoline prices. In principle, the effect could be either positive or negative, depending on consumer preferences. If consumers identify brands with higher quality and independents with lower quality, then conversion of the unbranded (low-quality) stations to the ARCO brand would mean that these stations now sell a closer substitute to the other branded products. This would intensify price competition and *lower* branded gasoline prices. However, if a large pool of consumers is unresponsive to brand labels because their willingness to pay for higher quality is limited and they only want to buy gasoline as cheaply as possible, then the loss of the Thrifty stations removes this low-cost alternative and *raises* gasoline prices.

To isolate the effect of the ARCO-Thrifty merger, Hastings (2004) looked at how prices charged by gasoline stations in the Los Angeles and San Diego areas differed depending on whether they competed with a Thrifty or not. Her data cover the prices charged by 699 stations measured at four different times: February 1997, June 1997, October 1997, and December 1997. Notice that the first two dates are for prices before the conversion while the last two dates are for prices after the conversion. She then defines submarkets in which each station's competitors are all the other stations within one mile's driving distance. A simple regression that might capture the effect of the merger would be:

$$p_{it} = Constant + \alpha_i + \beta_1 X_{it} + \beta_2 Z_{it} + e_{it} \tag{10.16}$$

where p_{it} is the price charged by station i at time t; α_i is a firm-specific dummy that lets the intercept be different for each service station; X_{it} is a dummy variable that has the value 1 if station i competes with an independent (Thrifty) at time t and 0 otherwise; likewise Z_{it} is 1 if a competitor of station i has become a station that is owned by a major brand as opposed to a station that operates as a franchisee or lessee of a major brand, and 0 otherwise. This last variable, Z_{it} is meant to capture the impact of any differential effects depending on the contractual relationship between a major brand and the station that sells that brand. The key variable of interest however is X_{it}. We want to know whether the estimated coefficient β_1 is negative, which would indicate that having independent rivals generally leads to lower prices—or is positive, which would indicate that the presence of independents softens competition and raises prices.

However, there is a potentially serious problem with estimating equation (10.16). The problem is that over the course of 1997, gasoline prices were rising generally throughout Southern California. Equation (10.16) does not allow for this general rising trend. Consider our key variable X_{it}. In the data, this will be 1 for a lot more stations before the merger in February and June than it will be in October and December. As a result, the coefficient β_1 will likely be negative because prices were lower in February and June (when there were a lot more independents) than in September and December (after the merger removed the Thrifty stations). That is, β_1 will be biased because it will pick up time effects as well as the effects of independents. The solution to this problem is to include variables that explicitly isolate this time effect so that what is left after this effect is removed is a pure measure of the impact of having an independent gasoline retailers on market prices. Accordingly, Hastings (2004) puts in location specific time dummies for February, June, and September. (The effect of December is of course captured in the regression constant.) That is, she estimates an equation something like:

$$p_{it} = Constant + \alpha_i + \beta_1 X_{it} + \beta_2 Z_{it} + \beta_3 T_i + e_{it} \tag{10.17}$$

where T_i or time is captured not as a continuous variable but, again, by dummy variables capturing the sequence of specific times in each location. Her results, both with and without the time dummies (but suppressing the firm specific intercepts) are shown in Table 10.1, below.

Consider first the column of results for the equation that includes the location time dummies. Here, the estimate of β_1, the coefficient on having a Thrifty or independent rival in a station's local market, implies that this led the station to lower its price by about five cents per gallon. The standard error on this estimate is very small, so we can be very confident of this measure. Note too how this contrasts with the effect measured in the regression results shown in the first column that leaves out the time effects. That estimate suggests a much larger effect a ten cents per gallon decline when a station has independent rivals. Again, this is because in leaving out the time effects, the regression erroneously attributes the general rise in gasoline prices throughout the region to the merger, when in fact prices were clearly rising for other reasons as well. We should also note that the coefficient estimate for β_2 is not significant in either equation. So, the type of ownership by a major brand does not seem to be important for retail gasoline prices.

One picture is often worth a large number of words. The graph below in Figure 10.7 illustrates the behavior of Southern California gasoline prices over the period covered by Hastings's data for each of two groups: 1) the treatment group that competed with a Thrifty station; and 2) the control group of stations that did not.

Notice the general rise in prices in both groups through October. Clearly, this is a phenomenon common to the gasoline market in general and not the result of the merger per

Table 10.1 Brand competition and gasoline prices

Variable	*Without location-time dummies Coefficient (Standard Error)*	*With location-time dummies Coefficient (Standard Error)*
Constant	1.3465 (0.0415)	1.3617 (0.0287)
X_{it}	−0.1013 (0.0178)	−0.0500 (0.0122)
Z_{it}	−0.0033 (0.0143)	−0.0033 (0.0101)
LA*February		0.0180 (0.0065)
LA*June		0.0243 (0.0065)
LA*December		0.1390 (0.0064)
SD*February		−0.0851 (0.0036)
SD*June		−0.0304 (0.0036)
SD*December		0.0545 (0.0545)
R^2	0.3953	0.7181

Dependent Variable = price per gallon of regular unleaded

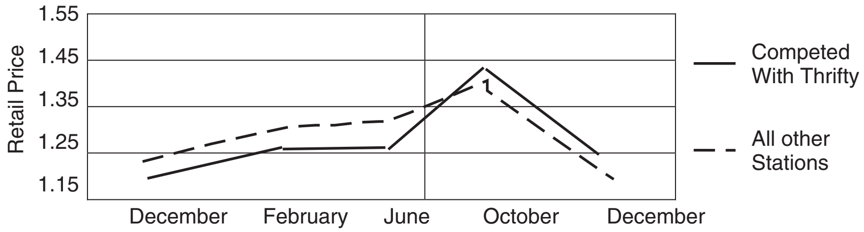

Figure 10.7 "Thrifty" competition and gasoline prices in Southern California

se. However, a close look at the data does reveal that the merger did have some impact. In the months before the merger, stations that competed with a Thrifty had prices that were two to three cents *lower* than those in the control group. Starting about the time of the merger in June, however, and continuing afterwards, these same stations had prices two to three cents *higher* than those in the control group. It is this roughly five-cent effect that is being picked up in the final column of the preceding table.[13] For both groups, those that initially competed with a Thrifty prior to the merger and those that did not, prices differ between the beginning of 1997 and the end. To isolate the effects of the merger, we need to look at how these differences over time were different between the two groups. If we recall that for the treatment group stations $X_{it} = 1$ at first but 0 after the merger, while it is always zero for firms in the control group, the price behavior for the two groups is:

	Before Merger	*After Merger*	*Difference*
Treatment group:	$\alpha_i + \beta_1$	$\alpha_i + \textit{time effects}$	$-\beta_1 + \textit{time effects}$
Control Group:	α_j	$\alpha_j + \textit{time effects}$	*time effects*

Thus, β_1 in our regression reflects the difference between the difference over time in the treatment group and that in the control group. For this reason, β_1 is often referred to as a *difference-in-differences* estimator.

Summary

In the Bertrand model, firms compete in prices. In the simple model of Bertrand competition, prices are pushed to marginal cost even if there are just two firms. This result stands in sharp contrast with the outcome under quantity or Cournot competition in which prices remain substantially above marginal cost so long as the number of firms is not large. In addition, high-cost firms can survive in Cournot competition whereas they will be forced out of business in the simple Bertrand model. In short, Bertrand's initial analysis predicts competitive and efficient market outcomes even when the number of firms is quite small.

However, the efficient outcomes predicted by the simple Bertrand model depend upon two key assumptions. The first is that firms have extensive capacity so that it is possible to serve all of a rival's customers after undercutting the rival's price. The second key assumption is that the firms produce identical products so that relative price

13 More recently, C. Taylor, N. Kreisle, and P. Zimmerman, (2010) use an alternative data set that suggests the impact of the merger was to raise prices by much less than the effect found by Hastings (2004).

is all that matters to consumers when choosing between brands. If either of these assumptions is relaxed, the efficiency outcomes of the simple Bertrand model can no longer be obtained. If firms must choose production capacities in advance, the outcome with Bertrand price competition becomes closer to what occurs in the Cournot model. If products are differentiated, prices are again likely to remain above marginal cost. Indeed, given the fierceness of price competition, firms have a real incentive to differentiate their products.

A useful model of product differentiation is the Hotelling (1929) spatial model, which we first introduced in Chapter 4. This model uses geographic location as a metaphor for more general distinctions between different versions of the same product. It thereby makes it possible to consider price competition between firms selling differentiated products. The model makes it clear that Bertrand competition with differentiated products does not result in efficient marginal cost pricing. It also makes clear that the deviation from such pricing depends on how much consumers value variety. The greater value that the typical consumer places on getting his or her most preferred brand or version of the product, the higher prices will rise above marginal cost.

Ultimately, the differences between Cournot and Bertrand competition reflect underlying differences between quantities and prices as strategic variables. The quantities chosen by Cournot firms are strategic substitutes—increases in one firm's production lead to decreases in the rival's output. In contrast, the prices chosen by Bertrand competitors are strategic complements. A rise in one firm's price permits its rival to raise price, too. To be accurate, analysis of any industry requires familiarity with those industry features that determine whether the competitive rivalry is played out in a setting of strategic substitutes or strategic complements.

Models of price competition based on consumer preferences and perceived quality have provided an extremely useful framework for academicians and policy makers alike. It is important in this regard to identify the precise mechanism in which consumers' preferences for specific brands and designs are modeled. For instance, depending on the nature of consumer preferences, a merger between a high-quality and a low-quality firm that results in the transformation of the low-quality firm outlets to high-quality ones could either weaken competition because it removes a low-quality competitor or intensify competition because it adds to the high-quality supply. In the case of retail gasoline markets in Southern California, work by Hastings (2004) finds that a merger leading to removal of a low-quality firm raised prices, although subsequent work by Taylor, Kreisle, and Zimmerman (2010) offers some contrasting evidence.

Problems

1. Suppose firm 1 and firm 2 each produce the same product and face a market demand curve described by $Q = 5000 - 200P$. Firm 1 has a unit cost of production c_1 equal to 6 whereas firm 2 has a higher unit cost of production c_2 equal to 10.
   a. What is the Bertrand-Nash equilibrium outcome?
   b. What are the profits of each firm?
   c. Is this outcome efficient?
2. Suppose that market demand for golf balls is described by $Q = 90 - 3P$, where Q is measured in kilos of balls. There are two firms that supply the market. Firm 1 can produce a kilo of balls at a constant unit cost of \$15 whereas firm 2 has a constant unit cost equal to \$10.
   a. Suppose the firms compete in quantities. How much does each firm sell in a Cournot equilibrium? What is the market price and what are the firms' profits?
   b. Suppose the firms compete in price. How much does each firm sell in a Bertrand equilibrium? What is market price and what are the firms' profits?
3. Refer again to the golf ball market described in problem 2.
   a. Would your answer in 2b change if there were three firms, one with unit cost = \$20 and two with unit cost = \$10? Explain why or why not.

b. Would your answer in 2b change if firm 1's golf balls were green and endorsed by Tiger Woods, whereas firm 2' s were plain and white? Explain why or why not.

4. In Tuftsville, everyone lives along Main Street, which is 10 miles long. There are 1,000 people uniformly spread up and down Main Street, and every day they each buy a fruit smoothie from one of the two stores located at either end of Main Street. Customers ride their motor scooters to and from the store using $0.50 worth of gas per mile. Customers buy their smoothies from the store offering the lowest price, which is the store's price plus the customer's travel expenses getting to and from the store. Ben owns the store at the west end of Main Street and Will owns the store at the east end of Main Street.
   a. If both Ben and Will charge $1 per smoothie, how many will each of them sell in a day? If Ben charges $1 per smoothie and Will charges $1.40, how many smoothies will each sell in a day?
   b. If Ben charges $3 per smoothie, what price would enable Will to sell 250 smoothies per day? 500 smoothies per day? 750 smoothies per day? 1,000 smoothies per day?
   c. If Ben charges p_1 and Will charges p_2, what is the location of the customer who is indifferent between going to Ben's or Will's shop? How many customers go to Will's store and how many go to Ben's store? What are the demand functions that Ben and Will face?
   d. Rewrite Ben's demand function with p_1 on the left-hand side. What is Ben's marginal revenue function?
   e. Assume that the marginal cost of a smoothie is constant and equal to $1 for both Ben and Will. In addition, each of them pays Tuftsville $250 per day for the right to sell smoothies. Find the equilibrium prices, quantities sold, and profits net of the $250 license fee.

5. Return to Main Street in Tuftsville. Now suppose that George would like to open another store at the midpoint of Main Street. He must pay Tuftsville an addional $250 per day to operate this new store.
   a. If Ben and Will do not change their prices, what is the best price for George to charge? How much profit would he earn?
   b. What do you think would happen if George did open another store in the middle of Main Street? Would Ben and Will have an incentive to change their prices? Their locations? Would one or both leave the market?

6. Suppose that there are two firms, firm B and firm N, that produce complementary goods, say bolts and nuts. The demand curve for each firm is described as follows:

$$Q_B = Z - P_B - P_N \text{ and } Q_N = Z - P_N - P_B$$

For simplicity, assume further that each firm faces a constant unit cost of production, $c = 0$.
   a. Show that the profits of each firm may be expressed as $\Pi^B = P_B = (P_B)(Z - P_B - P_N)$ and $\Pi^N = P_N(Z - P_B - P_N)$.
   b. Show that each firm's optimal price depends on the price chosen by the other as given by the optimal response functions: $P_B^* = (Z - P_N)/2$ and $P_N^* = (Z - P_B)/2$.
   c. Graph these functions. Show that the Nash equilibrium prices are: $P_B = P_N = Z/3$.
   d. Are the prices set by each of the two firms either strategic complements or strategic substitutes?

7. Assume that two firms sell differentiated products and face the following demand curves:

$$q_1 = 15 - p_1 + 0.5p_2 \text{ and } q_2 = 15 - p_2 + 0.5p_1;$$

   a. Derive the best response function for each firm. Do these indicate that prices are strategic substitutes or strategic complements?
   b. What is the equilibrium set of prices in this market? What profits are earned at those prices?

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