

ECN 594: Collusion

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Plan

1. Collusion refresher (from ECN 532)

2. Critical discount factor with N firms

3. Cournot vs Bertrand collusion

4. Detection and fines

5. Leniency programs

6. Antitrust enforcement

From ECN 532: Collusion basics

- **Collusion:** Firms coordinate to raise prices/restrict output
- Problem: each firm has incentive to deviate (undercut)
- **Solution:** Repeated game with punishment
- **Grim trigger strategy:**
 - Collude as long as everyone colludes
 - If anyone deviates → Nash forever
- You derived this in Hector's class

The collusion condition

- **Three profit levels:**

- π^C : Collusive profit (per period)
- π^D : Deviation profit (one-shot gain)
- π^{NE} : Nash equilibrium profit (punishment)

- **Collusion sustained if:**

$$\frac{\pi^C}{1 - \delta} \geq \pi^D + \frac{\delta \pi^{NE}}{1 - \delta}$$

- Rearranging:

$$\delta \geq \delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}}$$

Critical discount factor: intuition

- $\delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}}$
- **Numerator:** Gain from deviating ($\pi^D - \pi^C$)
- **Denominator:** Total loss from punishment ($\pi^D - \pi^{NE}$)
- **Higher δ^* means collusion is harder**
 - Need more patient firms
 - More frequent interaction helps (reduces effective δ)

T/F: Collusion theory

True, False, or Not Enough Information?

1. If $\delta = 0.8$ and $\delta^* = 0.6$, collusion can be sustained.
2. The critical discount factor δ^* decreases when the punishment becomes more severe.
3. Collusion is easier to sustain when firms interact infrequently (e.g., annual contracts).
4. In the grim trigger strategy, one deviation leads to Nash equilibrium forever.

Take 3 minutes.

T/F: Collusion theory (solutions)

Solutions

1. **TRUE.** $\delta = 0.8 > \delta^* = 0.6$, so condition satisfied.
2. **TRUE.** Harsher punishment \rightarrow lower π^{NE} \rightarrow larger denominator \rightarrow lower δ^* .
3. **FALSE.** Infrequent interaction means lower effective δ , making collusion *harder*.
4. **TRUE.** That's exactly what grim trigger means—permanent reversion to Nash.

Quick practice: Interpreting δ^*

- **Question:** Firm A has $\delta^* = 0.4$. Firm B has $\delta^* = 0.7$.
- Which firm is in an industry where collusion is easier to sustain?
- Explain your reasoning.

Take 2 minutes.

Quick practice: Interpreting δ^* (solution)

Solution

- **Answer:** Firm A's industry has easier collusion.
- **Reasoning:**
 - Lower δ^* = lower patience threshold
 - Firm A only needs $\delta \geq 0.4$ for collusion
 - Firm B needs $\delta \geq 0.7$ for collusion
 - Since most firms have δ around 0.9-0.95, both can collude...
 - ...but A's industry is “more robust” to low-patience entrants

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Cournot collusion with N firms

- Linear demand: $P = a - bQ$, symmetric firms with $MC = c$

- **Collusive profit per firm:**

$$\pi^C = \frac{\pi^M}{N} = \frac{(a - c)^2}{4bN}$$

- **Nash profit per firm:**

$$\pi^{NE} = \frac{(a - c)^2}{b(N + 1)^2}$$

- **Deviation profit:** Best response to $N - 1$ firms playing q^C

Critical discount factor: Cournot formula

- For symmetric linear Cournot with N firms:

$$\delta^* = \frac{(N+1)^2}{N^2 + (N+1)^2}$$

- Examples:

N	δ^*
2	$9/17 \approx 0.53$
3	$16/25 = 0.64$
4	$25/41 \approx 0.61$
10	$121/221 \approx 0.55$

- Key insight: Collusion harder with more firms

Worked example: Cournot collusion

- **Question:** 3 symmetric Cournot firms. $P = 100 - Q$, $MC = 10$.
- (a) Calculate π^C , π^{NE} , and π^D for each firm.
- (b) Find the minimum δ for collusion.

Take 7 minutes.

Worked example: Cournot collusion (solution)

Solution

- (a) Profit calculations:
 - $\pi^M = (90)^2/4 = 2025$, so $\pi^C = 2025/3 = 675$
 - $q^C = 45/3 = 15$ per firm (monopoly quantity split)
 - Nash: $q^{NE} = 90/4 = 22.5$, $\pi^{NE} = 90^2/16 = 506.25$
 - Deviation: BR to $2 \times 15 = 30$ is $q^D = (90 - 30)/2 = 30$
 - $P = 100 - 60 = 40$, $\pi^D = (40 - 10) \times 30 = 900$

Worked example: Cournot collusion (solution cont.)

Solution

- (b) Critical discount factor:

$$\delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}} = \frac{900 - 675}{900 - 506.25} = \frac{225}{393.75} = 0.571$$

- Or use formula: $\delta^* = \frac{(3+1)^2}{3^2 + (3+1)^2} = \frac{16}{9+16} = \frac{16}{25} = 0.64$
- (Small difference due to rounding in worked example)
- **Interpretation:** Firms must value future at 64% of present

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Bertrand collusion with N firms

- Homogeneous Bertrand: $\pi^{NE} = 0$ (price = cost)
- Collusion: split monopoly profits
- **Key difference:** Punishment is more severe ($\pi^{NE} = 0$)
- **Critical discount factor for Bertrand:**

$$\delta^* = \frac{\pi^D - \pi^C}{\pi^D - 0} = \frac{\pi^M - \pi^M/N}{\pi^M} = \frac{N-1}{N}$$

- **Examples:**

- $N = 2$: $\delta^* = 0.5$
- $N = 4$: $\delta^* = 0.75$

Cournot vs Bertrand collusion

N	δ^* (Cournot)	δ^* (Bertrand)
2	0.53	0.50
3	0.64	0.67
4	0.61	0.75

- At $N = 2$: Bertrand collusion **easier**
- **Why?** Bertrand punishment is harsher ($\pi^{NE} = 0$)
- At higher N : Bertrand collusion harder
- **Why?** Deviation captures entire market (bigger temptation)

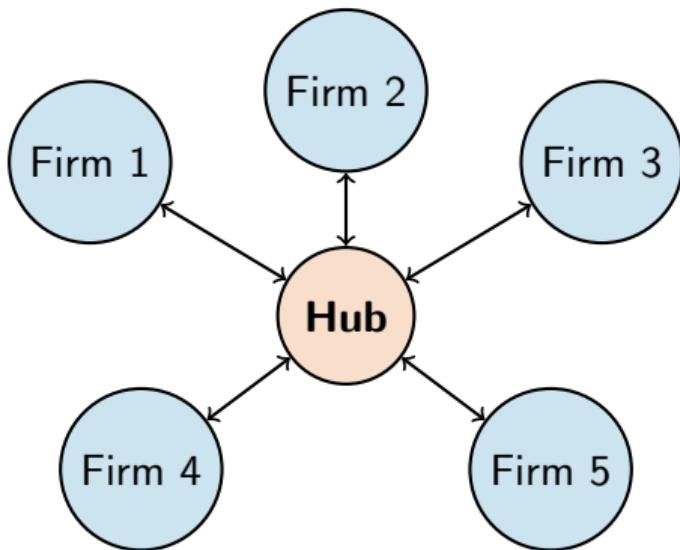
Tacit vs explicit collusion

	Explicit Collusion	Tacit Collusion
Definition	Direct communication and agreement	Parallel behavior without communication
Legal status	Illegal (per se violation)	Generally legal (hard to prove intent)
Examples	Lysine cartel meetings, price-fixing calls	Gasoline price matching, oligopoly pricing
Evidence	Documents, recordings, testimony	Statistical patterns only

- Same economic effect, very different legal treatment

Hub-and-spoke arrangements

- **Structure:** Competitors (spokes) coordinate through common partner (hub)
- **Example:** Retailers share pricing info through common supplier



- **Legal issue:** Is hub liable? Are spokes colluding?

Multi-market contact

- **Multi-market contact:** Same firms compete in multiple markets
- **Effect on collusion:**
 - More opportunities to punish deviators
 - Deviation in market A triggers punishment in markets B, C, ...
 - Increases total punishment → lowers δ^*
- **Example:** Airlines compete on multiple routes
 - American and United both fly NYC-LA and NYC-Chicago
 - Price war on one route can spread to others
- **Bernheim & Whinston (1990):** Formalized this theory

Price wars and demand uncertainty

- **Problem:** Firms can't observe rivals' prices directly
- Low demand could be:
 - Bad luck (demand shock)
 - Or a rival secretly cutting prices
- **Green & Porter (1984):** Price wars as equilibrium
 - When demand is low, trigger “punishment phase”
 - Not because anyone cheated—just to maintain incentives
 - Punishment is temporary (return to collusion after T periods)
- **Implication:** Price wars can occur without anyone deviating
- This explains why we see cyclical pricing in some industries

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Detection probability and fines

- In reality: cartels may be detected and punished

- **Each period:**

- Detection probability: ρ
 - Fine if detected: F

- **Modified collusion condition:**

$$\delta^* = \frac{\pi^D - \pi^C + \rho F}{\pi^D - \pi^{NE} + \rho F}$$

- Higher ρ or higher $F \rightarrow$ higher $\delta^* \rightarrow$ harder to collude

Worked example: Detection and fines

- **Question:**
- Cartel earns $\pi^C = 100$ per period
- $\pi^{NE} = 25$, $\pi^D = 150$
- Detection probability $\rho = 0.1$, fine $F = 500$
- Find the minimum δ for collusion.

Take 3 minutes.

Worked example: Detection (solution)

Solution

- Expected fine per period: $\rho F = 0.1 \times 500 = 50$
- Apply formula:

$$\delta^* = \frac{\pi^D - \pi^C + \rho F}{\pi^D - \pi^{NE} + \rho F} = \frac{150 - 100 + 50}{150 - 25 + 50} = \frac{100}{175} = 0.571$$

- **Compare to no detection:**

$$\delta_{\text{no detection}}^* = \frac{150 - 100}{150 - 25} = \frac{50}{125} = 0.4$$

- Detection and fines make collusion harder ($0.4 \rightarrow 0.57$)

Criminal vs civil penalties

- In the US, price-fixing is a felony
- Civil penalties:
 - Corporate fines (up to 10% of affected commerce)
 - Treble damages in private lawsuits
- Criminal penalties:
 - Individual prison sentences (up to 10 years)
 - Individual fines (up to \$1 million)
- Why criminal penalties matter:
 - Executives can't pass prison time to shareholders
 - Creates personal incentive not to collude
 - Lysine: executives actually went to prison

Optimal deterrence: Becker (1968)

- **Expected cost of crime:** $\rho \times F$
- To deter, need: Expected cost > Expected gain
- **Question:** How should we set ρ and F ?
- **Becker's insight:**
 - Detection is costly (investigations, lawyers)
 - Fines are cheap to impose
 - → Set F very high, ρ low
- **Problem:** Firms may not have enough to pay huge fines
- **Solution:** Criminal penalties fill the “gap”

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Leniency programs

- **Leniency:** First firm to report cartel gets reduced/zero fine
- **US Corporate Leniency Program (1993):**
 - First to report: automatic immunity
 - Second: significant reduction possible
- **Effect on incentives:**
 - Creates “race to report”
 - Each firm fears others will report first
 - Destabilizes existing cartels

Why leniency works

- **Without leniency:**
 - If detected, everyone pays fine
 - No incentive to report
- **With leniency:**
 - First to report gets immunity
 - Creates Prisoner's Dilemma within cartel
 - Each firm thinks: "Better report before they do"
- **Result:** Cartel detection increased dramatically after 1993
- **Exam question:** "Explain why leniency programs help detect cartels."

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Factors facilitating collusion

1. **Few firms:** Easier to coordinate and monitor
2. **Frequent interaction:** Higher effective δ
3. **Similar costs:** Easier to agree on price
4. **Stable demand:** Easier to detect deviations
5. **Homogeneous products:** Easier to monitor prices
6. **Industry associations:** Facilitate communication

Bid-rigging in auctions

- **Bid-rigging:** Collusion in procurement auctions
- **Common schemes:**
 - **Bid rotation:** Firms take turns “winning”
 - **Bid suppression:** Some firms don’t bid at all
 - **Complementary bidding:** Losers submit high “cover” bids
 - **Subcontracting:** Winner pays off losers
- **Example:** Construction bid-rigging in NYC (1980s)
- Often easier to detect than retail price-fixing (paper trail)

T/F: Detection and policy

True, False, or Not Enough Information?

1. Leniency programs reduce the total number of cartels that form.
2. If detection probability doubles, collusion becomes twice as hard.
3. Criminal penalties are more effective than fines because they can't be passed to shareholders.
4. Tacit collusion is legal because firms don't explicitly communicate.

Take 3 minutes.

T/F: Detection and policy (solutions)

Solutions

1. **NEI.** Leniency destabilizes existing cartels but may reduce fear of detection, potentially encouraging cartel formation. Empirically ambiguous.
2. **FALSE.** From formula, ρ appears in both numerator and denominator—effect is not linear.
3. **TRUE.** Prison time is “non-transferable”—executives face personal consequences.
4. **TRUE.** (Generally) Without evidence of explicit agreement, parallel pricing is hard to prosecute.

Famous cartel cases

- **Lysine cartel (1990s):**
 - Price-fixing among feed additive producers
 - FBI surveillance, recorded meetings
- **LCD screen cartel (2000s):**
 - Samsung, LG, Sharp, others
 - \$1.4 billion in fines
- **LIBOR scandal (2012):**
 - Banks manipulated interest rate benchmark
 - \$9 billion in fines

Detecting collusion: what regulators look for

- **Pricing patterns:**

- Parallel price changes
- Price rigidity despite cost changes
- Similar prices despite different costs

- **Market characteristics:**

- High concentration
- Frequent meetings/communication
- History of antitrust violations

- **Whistleblowers:** Leniency program tips

Plus-factors in antitrust law

- **Problem:** Parallel pricing is not illegal by itself
- **Courts require “plus-factors”:** Evidence beyond parallel conduct
- **Common plus-factors:**
 - Actions against individual self-interest
 - Secret meetings or communications
 - Artificial standardization of products
 - Exchange of price information
 - Price increases during demand slumps
- **Economic testimony:** Experts analyze whether behavior is consistent with competition

Algorithmic collusion: a new frontier

- **Question:** Can algorithms collude without human instruction?
- **How it might work:**
 - Firms use pricing algorithms (AI/ML)
 - Algorithms learn to “cooperate” through repeated interaction
 - No explicit programming to collude
- **Legal challenge:**
 - Traditional antitrust requires “agreement”
 - If algorithms learn collusion independently, is there agreement?
- **Active research area:** Assad et al. (2024) on German retail gasoline
- Policy is still catching up

Collusion summary table

Concept	Key Formula/Idea	Policy Implication
Critical discount factor	$\delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}}$	Lower δ^* = easier collusion
Detection & fines	Increase ρF in formula	Higher penalties deter
Leniency	First to report = immunity	Creates “race to confess”
Multi-market contact	More markets = more punishment	Monitor multi-market firms
Plus-factors	Evidence beyond parallel pricing	Required for prosecution

Course connection: collusion and mergers

- **Merger review considers coordinated effects:**
 - Will the merger make collusion easier?
 - Fewer firms → lower δ^* → easier collusion
- **What makes post-merger collusion more likely?**
 - Merger creates symmetric cost structures
 - Merger eliminates a “maverick” (aggressive competitor)
 - Merger increases multi-market contact
- **HW2 connection:** Your merger simulation focuses on unilateral effects, but DOJ also considers coordinated effects

Key Points

1. **Critical discount factor:** $\delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}}$
2. **Cournot with N firms:** $\delta^* = \frac{(N+1)^2}{N^2 + (N+1)^2}$
3. **Bertrand with N firms:** $\delta^* = \frac{N-1}{N}$
4. More firms → generally harder to collude
5. **Detection and fines** raise δ^* : $\delta^* = \frac{\pi^D - \pi^C + \rho F}{\pi^D - \pi^{NE} + \rho F}$
6. **Leniency programs:** Create “race to report,” destabilize cartels
7. Collusion easier with: few firms, frequent interaction, similar costs

Next time

- **Lecture 13:** Final Review
 - Comprehensive review of Part 1 and Part 2
 - Practice problems for final exam
- **HW2 due before Lecture 13**