(b) The monopoly profit function is given by

$$\pi_m = Qp(Q) - C(Q_i) = Q(150 - 4Q) - 40Q.$$

The first order condition for profit maximization is given by:

$$150 - 8Q - 40 = 0. \tag{2}$$

Solving with respect to Q we get $Q = 13.75$, and then $p = 95$.

Under perfect competition the prevailing price would be given by marginal cost: $p = 40$; total quantity would be $Q = 27.5$ and welfare

$$W = CS = \frac{[p(0) - p]Q}{2} = 1512.5.$$

Under duopoly, total welfare is given by:

$$W_d = 2\pi + CS = 2q(p - c) + [p(0) - p]q = 1344.38.$$

Under monopoly, total welfare is given by

$$W_m = \pi + CS = (p - c)Q + \frac{[p(0) - p]Q}{2} = 1134.375.$$

Finally, the duopoly efficiency loss as a percentage of the monopoly efficiency loss is given by

$$EL = \frac{1512.5 - 1344.38}{1512.5 - 1134.375} = 44.5$$

■ **7.6**** Show analytically that equilibrium price under Cournot is greater than price under perfect competition but lower than monopoly price.

Solution: In a Cournot oligopoly, firm i's profit is given by $\pi_i = q_i P(Q) - C(q_i)$, where Q is total output. The first-order condition for profit maximization is given by

$$P(Q) + q_i \frac{dP}{dq_i} - MC = 0. \tag{3}$$

The first-order condition for a monopolist is given by

$$P(Q) + Q\frac{dP}{dQ} - MC = 0. \tag{4}$$

Finally, under perfect competition we have

$$P(Q) - MC = 0.$$

Notice that $\frac{dP}{dq_i} = \frac{dP}{dQ} < 0$. Consider the case of oligopoly and suppose that price is equal to monopoly price. Monopoly price is such that the (4) holds exactly. The only difference between (3) and (4)is that the latter has Q instead of q_i. Since $Q > q_i$, it follows that, for p equal to monopoly price, the left-hand side of (3) is positive. Finally, if it is positive, each firm has an incentive to increas output, which results in a lower price.

By a similar argument we can also show that price under Cournot competition is greater than marginal cost.

■ **7.7**** Consider a duopoly for a homogenous product with demand $Q = 10 - P/2$. Each firm's cost function is given by $C = 10 + q(q+1)$. Determine the values of the Cournot equilibrium.

Solution: Duopolist i's profit is given by $\pi_i = q_i p(Q) - C(q_i) = q_i[20 - 2(q_i + q_j)] - 10 - q_i(q_i + 1)$. The first order condition for profit maximization is given by:

$$20 - 2(q_i + q_j) - 2q_i - 2q_i - 1 = 0. \tag{5}$$

The problem of duopolist j is symmetric, therefore we have $q_i = q_j = 2.375$ and $p = 10.5$.

■ **8.1** Explain why collusive pricing is difficult in one-period competition and easier when firms interact over a number of periods.

Solution: In one-period competition each firm has a strong incentive to deviate from the pre-agreed collusive price, since the gains from deviating are higher than the losses. In terms of the example in Section 8.1, had the duopolists interacted in only one period, the gain would be given by one half of monopoly profits, while the loss from deviating would be 0. We would then be led to the usual Nash-Bertarand equilibrium when both firms price at marginal cost.

If, however, firms interact over a number of periods, history, in the form of past pricing behavior, becomes important. Deviation from the collusive price in one period can be met by punishment (deviation) in future periods. Hence, the original defector must weigh short-term gains against long-term losses, made possible exactly by multi-period interaction.

■ **8.2** After several years of severe price competition that damaged Boeing's and Airbus' profits, the two companies have recently pledged that they will not sink into another price war. However, in June 1999, Boeing made an unusual offer to sell 100 small aircraft to a leasing corporation at special discount prices. (Although customers never play list prices, it was felt that this deal was particularly attractive.) Boeing's move follows a similar one by Airbus.[14]

Based on the analysis of Section **??**, why do you think it is so difficult for aircraft manufacturers to collude and avoid price wars?

Solution: Aircraft manufacturers receive orders infrequently. Moreover, the terms of each sale are seldom made public. For these reasons, it is very difficult for them to collude. The incentive to cheat on a tacit or explicit agreement would be very high because: (a) the short run is very important with respect to the long run (low discount factor); (b) the probability that cheating would be detected is low.

■ **8.3*** In a market with annual demand $Q = 100 - p$, there are two firms, A and B, that make identical products. Because their products are identical, if one charges a lower price than the other, all consumers will want to buy from the lower-priced firm. If they charge the same price, consumers are indifferent and end up splitting their purchases about evenly between the firms. Marginal cost is constant and there are no capacity constraints.

(a) What are the single-period Nash equilibrium prices, p_A and p_B?

(b) What prices would maximize the two firms' joint profits?

Assume that one firm cannot observe the other's price until after it has set its own price for the year. Assume further that both firms know that if one undercuts the other, they will revert forever to the non-cooperative behavior you described in (a).

(c) If the interest rate is 10%, is one repeated-game Nash equilibrium for both firms to charge the price you found in part (b)? What if the interest rate is 110%? What is the highest interest rate at which the joint profit-maximizing price is sustainable?

(d) Describe qualitatively how your answer to (d) would change if neither firm was certain that it would be able to detect changes in its rival's price. In particular, what if a price change is detected with a probability of 0.7 each period after it occurs? Note: Do not try to calculate the new equilibria.

Return to the situation in part (c), with an interest rate of 10%. But now suppose that the market for this good is declining. The demand is $Q = A - p$ with $A = 100$ in the current period, but the value of A is expected to decline by 10% each year (i.e., to 90 next year, then 81 the following year, etc.).

(e) Now is it a repeated-game Nash Equilibrium for both firms to charge the monopoly price from part (b)?

Solution:

(a) Given that there is plenty of capacity to serve the entire market, each firm will be willing to undercut the other to make all the sales in the market so long as $p > 10$. The one-shot Nash equilibrium is for both firms to charge $p = 10$, the "Bertrand trap."

[14] *The Wall Street Journal Europe*, June 11–12, 1999.

(b) The greatest profits possible are found at the monopoly price. The capacity expenditures are sunk. A monopoly would set Q so that $MR = MC$. In this case, MC is 10. So the collusive outcome would split the market and price at 55. $p = 100 - Q \Rightarrow MR = 100 - 2Q$. $MR = MC \Rightarrow 100 - 2Q = 10 \Rightarrow Q = 45 \Rightarrow p = 55$.

Assume that each firm can monitor the other's price very closely and can respond instantly (before any consumers make a purchase decision) to a price change.

(c) Yes, one equilibrium is to stay at the monopoly price. If both firms are at the monopoly price, then each faces the following decision: "Assuming that the other firm will continue to charge the monopoly price, should I charge the monopoly price also, or should I charge slightly less today, knowing (believing) that we will then revert to $p = 10$ forever after?" Charging the monopoly price means getting half the monopoly profits forever, which is worth $PDV_{\text{cooperate}} = (1 + 1/r)(55 - 10)45/2 = 11137.5$ when the interest rate is 10%. Alternatively, $PDV_{\text{cheat}} = (54.99999 - 10)45 = 2025$. The logical conclusion is that it pays to cooperate indefinitely if you believe that the other firm will also. If, however, $PDV_{\text{cooperate}} < PDV_{\text{cheat}}$ then the monopoly price would not be sustainable. $PDV_{\text{cooperate}} < PDV_{\text{cheat}} \Rightarrow (1 + 1/r)(55 - 10)45/2 < 2025 \Rightarrow r > 100\%$. At any interest rate above 100%, the monopoly price would not be sustainable. Interest rates above 100% are rare, assuming that detection lags are on the order of weeks or months, so it looks like monopoly price could persist in this market.

(d) If the probability of being detected is less than one, then a company that cheated would have a chance of getting the high profits of cheating for more than one period before it got caught. This would raise the incentive to cheat and lower the interest rate at which the monopoly price is sustainable. In fact, one can think of a detection probability of 70% as corresponding to an interest rate of 30% (added on top of whatever interest rate applies based on the time value of money).

(d) Declining demand generally makes cooperative pricing more difficult to support. The rate of decline acts much like a discount rate on future earnings, since the cost to a firm of "cheating" in the current period, namely the loss of its share of future profits, is less in a declining market. However, a rate of decline of "only" 10% acts much like raising the interest rate by 10% (from 10% to 20% here), which is still safely below the interest rate at which cooperative pricing breaks down (assuming perfect detection and continuing to assume that these "grim trigger" strategies are credible punishments for cheating).

■ **8.4*** You compete against three major rivals in a market where the products are only slightly differentiated. The "Big Four" have historically controlled about 80% of the market, with a fringe of smaller firms accounting for the rest. Recently, prices have been rather stable, but your market share has been eroding slowly, from 25% just a few years ago to just over 15% now. You are considering adopting an aggressive discounting strategy to gain back market share.

Discuss how each of the following factors would enter into your decision.

(a) You have strong brand identity and attribute your declining share to discounting by your rivals among the Big Four.

(b) The Big Four have all been losing share gradually to the fringe, as the product category becomes more well known and customers become more and more willing to turn to smaller suppliers to meet their needs.

(c) Your believe your rivals are producing at close to their capacity, and capacity takes a year or two to expand.

(d) You can offer discounts selectively, in which case it will take one or two quarters before your rivals are likely to figure out that you have become more aggressive on pricing.

(e) Your industry involves high fixed costs and low marginal costs, as applies for most information goods.

(f) The entire market is in rapid decline due to technological shifts unfavorable to this product.

Solution:

(a) Discounting can cheapen your brand image and identity, but may be worthwhile if you still have relatively large margins and thus find it profitable to halt your slide in market share. Since the discounting is from other members of the Big Four, an aggressive response on your part, perhaps followed by an exploratory price increase, might signal that you will fight to avoid losing market share but are willing to accept today's shares if your rivals raise prices somewhat.

(b) There is little you can do about this problem, since the fringe is hard to control in any way, and entry of new fringe players is not likely to be very difficult. This is the situation to emphasize your brand and to try to segment the market to retain your share of those customers willing to pay a premium for a well-known brand (yours!).

(c) Generally, you can be more confident pushing prices up if rivals are at or near capacity. You will lose some sales, assuming that industry demand is not perfectly inelastic, but you will lose little or no customers to your rivals in the short run (a year or two) if they cannot expand production. Of course, if fringe firms are viewed as offering close substitutes, and do not face capacity constraints, then the capacity limitations faced by the other major players don't help you much at all.

(d) Such "detection lags" always make discounting look more attractive, simply because any competitive responses will be delayed. Indeed, it seems that this is exactly how you lost market share, to rivals who were discounting before you realized what was going on.

(e) Now discounting is more attractive because marginal cost is low, so setting marginal cost to the marginal revenue (associated with your residual demand curve) involves a lower price. Plus, even if you can engage in "cooperative pricing," the resulting price is lower, the lower are marginal costs.

(f) In a declining market, the future is relatively less important commercially relative to the present. In terms of our theories of "cooperative pricing," declining demand is much like a higher interest rate: the scale tips more towards maximizing current profits and away from a "patient" approach of sacrificing short-run profits to support or sustain long-run cooperation. So, discounting now to avoid a further loss of market share (or to gain market share back) looks more attractive in a declining market, even if this will trigger or inflame a price war.

■ **8.5** "Price wars imply losses for all of the firms involved. The empirical observation of price wars is therefore a proof that firms do not behave rationally." True or false?

Solution: False. As Section 8.2 shows, price wars may be part of the equilibrium of a game played between rational firms.

■ **8.6** Empirical evidence from the U.S. airline industry suggests that fare wars are more likely when carriers have excess capacity, caused by GDP growth falling short of its predicted trend. Fare wars are also more likely during the Spring and Summer quarters, when more discretionary travel takes place.[15] Explain how these two observations are consistent with the theories presented in Section **??**.

Solution: The first model in Section 8.2 (secret price cuts) predicts that price wars start in periods of unexpected low demand. This is consistent with the first observation above. However, the effect of unexpected low demand is also consistent with a theory of price wars caused by financial distress (see the end of Section 8.2). The observation that prices fare wars take place during periods of higher demand is consistent with the second model in Section 8.2 (demand fluctuations).

■ **8.7** A 1998 news article reported that

> Delta Air Lines and American Airlines tried to raise leisure air fares 4% in most domestic markets, but the move failed Monday when lone-holdout Northwest Airlines refused to match the higher prices.
>
> The aborted price boost illustrates the impact Northwest's woes already are having on the industry. Months of labor unrest ... are prompting passengers to book away from the fourth largest carrier.[16]

[15]

Morrison, Steven A., and Clifford Winston (1996), "Causes and Consequences of Airline Fare Wars," *Brookings Papers on Economic Activity* (Microeconomics), 205–276..

[16] *The Wall Street Journal Europe*, August 12, 1998.

What does this say about the nature of price dynamics in the airline industry?

Solution: The event seems consistent with the view, presented at the end of Section 8.2, that price wars are asymmetric in nature. In this case, they are caused by firms, like Northwest Airlines, that are in financial distress.

■ **8.8** In the third quarter of 1999, most North American paper and forest-products companies experienced an improvement in their results. The industry, analysts said, was in a cyclical upswing: not only was demand increasing at a moderate pace; more importantly, the industry practiced restraint in keeping low production levels, thus providing support for higher prices.[17]

How do you interpret these events in light of the models presented in Section **??**?

Solution: The analysis of Section 8.1 predicts that collusion is easier in growing industries (the promise of future profits under collusion is worth more). This is consistent with the fact that "restraint in keeping low production levels" took place during the "cyclical upswing."

■ **8.9** In 1918, the U.S. Congress passed a low allowing American firms to form export cartels. Empirical evidence suggests that cartels were more likely to be formed in industries where American exporters had a large market share, in capital-intensive industries, in industries selling standardized goods, and in industries that enjoyed strong export growth.[18] Discuss.

Solution: The effect of export growth seems consistent with the analysis in Section 8.1. The effect of standardization may correspond to the fact that it is easier to monitor collusion with a standardized product (however, the effect of product differentiation on collusion is a controversial issue). The effect of market share is consistent with the analysis in Section 8.3 (concentration facilitates collusion).

■ **8.10** The endowments of the Ivy League universities have increased significantly in recent years. Princeton, the richest of all, boosted its endowment from $400,000 per student in 1990 to more than $750,000 in 1997. In the same period, both Harvard and Yale more than doubled their endowments. Notwithstanding these riches, the universities have restrained from using financial incentives as a means to compete for

[17] *The Wall Street Journal*, October 11, 1999.
[18]

DICK, ANDREW (1997), "If Cartels Were Legal, Would Firms Fix Prices?," Antitrust Division, U.S. Department of Justice..

students. For many years, the manual of the council of Ivy League Presidents stated that the schools should "neutralize the effect of financial aid so that a student may choose among Ivy Group institutions for non-financial reasons." In 1991, the Justice Department argued that this amounted to price collusion and forced the agreement to end. However, no significant price competition took place until 1998, when Princeton University started offering full scholarships for students with incomes below $40,000. Stanford, MIT, Dartmouth and Cornell followed suit. Allegedly, Harvard sent a letter to accepted 1998 applicants stating that "we expect that some of our students will have particularly attractive offers from the institutions with new aid programs, and those students should not assume that we will not respond."[19]

How do you interpret these events in light of the theories discussed in this chapter?

Solution: If the Department of Justice was right in assuming the council manual's clause was an explicit form of price collusion, then what happened after 1991 is that collusion ceased to be explicitly supported by the clause and turned into tacit collusion. In fact, the analysis in Chapter 8 suggests that explicit, contractual arrangements are not necessary to sustain a collusive agreement. The chapter also states that, under tacit collusion, each firm balances the short-run benefits from deviation against the long-term cost of entering into non-cooperative play. The fact that endowments have increased so much (especially Princeton's) may be what has tipped the balance in the direction of giving away full scholarships.

■ **8.11** Based on data from the Spanish hotel industry, it was estimated that the rate set by hotel i in market k is positively influenced by a variable that measures the intensity of multimarket competition between hotel i and its competitors in market k: the more markets $m \neq k$ in which firm i and its competitors meet, the greater the measure of multimarket contact. It was also observed that the measure of multimarket contact is highly correlated with hotel chain size, that is, the larger hotel i's chain, the greater the measure of multimarket contact for firm i.[20]

Provide two interpretations for the positive coefficient of multimarket contact on hotel rates, one based on collusion, one based on a different effect.

Solution: When interaction between oligopolists takes place over a number of periods, it is easier to sustain collusion: long-term losses weigh more compared to short-term gains from deviation. Multimarket contact adds another "dimension" to the balance between gains and losses. A firm's gain from deviating in one market may be punished by its competitors in all the markets they meet, making the potential cost from deviation higher. However, the optimal behavior of the deviating firm would call for deviation in all markets. Thus, we have higher losses from deviating but also higher gains. As discussed in Section 8.3, if everything

[19] *The Economist*, December 5, 1998.

[20]

Fernández, Nerea, and Pedro Marín (1998), "Market Power and Multimarket Contact: Some Evidence from the Spanish Hotel Industry," *Journal of Industrial Economics* **46**, 301–315..

is identical (firms, markets) then, multimarket contact does not increase the likelihood of collusion because the potential gains from deviation increase in the same proportion as the losses. However, asymmetries between firms or markets can make losses weigh more than gains, thus increasing the likelihood of collusion. This justifies the positive correlation beteween multimarket contact and average rates.

There is, however, an alternative interpretation. Maybe rates are higher in hotels of greater size. This could happen either because consumers attach a greater value to hotels that have larger chains or because bigger hotel chains command greater (unilateral) market power. Given the empirical correlation between hotel size and multimarket contact this would also imply a correlation between multimarket contact and rates, even if there is no implicit or explicit collusion between hotel chains.

■ **8.12** Consider the following excerpt from a 1998 news item.[21]

> LONG-STALLED SHIPPING REFORM BILL TAKEN UP BY SENATE. Washington — The Senate has formally begun consideration of a shipping reform bill that, if passed, would create changes for all countries shipping manufactured goods to and from the United States ...
>
> Until now U.S. shipping law has been founded on the principle of common carriage — "Everybody pays the same tariff (rate) to go from Oakland to Yokohama," said the Department of Transportation (DOT) official, who asked not to be identified. Under this system, groups of liners called conferences — legal cartels with immunity from antitrust law — set the rates for their members and make those rates public through registration with the federal government. If the shipping bill passes, however, liners could make private, confidential deals with exporters-importers outside of conferences at market-set rates.
>
> "This is going to bring marketplace economics into ocean shipping like we've never seen before," the official said. "It's going to really change the influence of ocean shipping conferences in the marketplace." ...
>
> The Transportation Department official said the Clinton administration has generally supported legislation for shipping reform in line with its promotion of deregulation in airlines and trucking, but has stated concerns about specific provisions of the Senate bill. Probably the administration's biggest concern is a provision of the bill allowing conferences also to engage in confidential contracting, he said. "In the administration view that conveys too much market power to the conferences," the official said.

Do you agree with the Clinton administration's view? Why or why not?

Solution: The example in Box 8.6 shows that making information public is not a panacea to the collusion problem. Although the market becomes more transparent, and collusive

[21] USIA *EPF513 04/03/98, written by USIA Staff Writer Bruce Odessey.

agreements are easier to monitor, this may come at a cost: It gives firms the opportunity to coordinate on a collusive equilibrium.

The approach taken by the U.S. Senate in its shipping reform bill is to switch from a public information exchange to the possibility of secretly priced individual/group contracts. The idea is that, although we may end up with a collusive equilibrium that is difficult to detect, this equilibrium is likely to feature price wars in order to be sustainable (cf Section 8.2). In the shared information approach, collusion is probably easier to detect but firms may (tacitly) coordinate on a higher price collusive equilibrium.

■ **8.13** In 1986, the U.S. Congress enacted a regulation (PL99-509) requiring railroads to disclose contractual terms with grain shippers. Following the passing of the legislation, rates increased on corridors with no direct competition from barge traffic, while rates decreased on corridors with substantial direct competition.[22] How do you interpret these events?

Solution: One possible interpretation for these results is that, when there is no competition to railroad shipping, there is potential for collusion among railroad operators, whereas the opposite is true when there is direct competition from barge traffic. In this context, increased information about railroad contracts has the effect of

1. improving collusion among railroad operators when the latter have no competition. This is consistent with the idea that when price cuts are difficult to observe collusion is more difficult to sustain.
2. increasing competition in markets where railroad operators compete with barge operators. This is consistent with the idea that, in a competitive environment, better information about prices increases demand elasticity (consumer are more aware of price differences) and thus decreases margins.

■ **8.14*** Consider an n firm homogeneous-good oligopoly with constant marginal cost, the same for all firms. Let $\bar{\delta}$ be the minimum value of the discount factor such that it is possible to sustain monopoly prices in a collusive agreement. Show that $\bar{\delta}$ is decreasing in n. Interpret the result.

Solution: Let π^M be total industry profits. Under the collusive agreement, each firm receives π^M/n. If one of the firms undercuts its rivals, then it gets approximately π^M. Finally, if firms revert to a (perpetual) price war each firm gets zero. It follows that the

[22] See

Schmitz, John, and Stephen W. Fuller (1995), "Effect of Contract Disclosure on Railroad Grain Rates: An Analysis of Corn Belt Corridors," *The Logistics and Transportation Review* **31**, 97–124..

condition such that it is an equilibrium for firms to price at the monopoly level is given by

$$\frac{1}{1-\delta}\frac{\pi^M}{n} \geq \pi^M.$$

Solving with respect to δ we get

$$\delta \geq \frac{n-1}{n}.$$

It follows that collusion is stable if and only if $\delta > \bar{\delta} \equiv \frac{n-1}{n}$. (Note that the condition is independent of the value of π^M, so the same condition would apply for any level of collusion.)

Taking the derivative of $\bar{\delta}$ with respect to n, we get

$$\frac{d\,\bar{\delta}}{d\,n} = \frac{n-(n-1)}{n^2} = 1/n^2 > 0.$$

It follows that $\bar{\delta}$ is increasing in n. In words, the more firms there are, the more difficult it is to sustain a collusive agreement. The idea is that the relative gain from cheating is greater the greater the number of firm (the profit from cheating is always the same, but the profit from collusion is lower the greater n is).

■ **8.15**** Consider the model of multimarket contact presented in Subsection **??**. Determine the minimum value of the discount factor such that the optimal collusive solution is stable.

Solution: The setting of the problem consists of Firms 1 and 2, and Markets A and B. Firm 1 has cost $\underline{c}$ in Market A, while Firm 2 has a cost of $\overline{c}$. The situation is reversed in Market B. Demand is the same both markets. It is assumed that $\underline{c} < \overline{c} < p^M$.

As discussed in section 8.3 the efficient collusive agreement is the following: In each market, the firm with a cost advantage sets the monopoly price, while the other sets a higher price and sells 0. Let us use the following notation: π^M represents the monopoly profit of the firm with cost advantage, $\pi^{M'} = \pi(p^M - \varepsilon, \overline{c})$ is the profit of the firm with high marginal cost when it charges (slightly less than) the monopoly price and $\pi^C = \pi(\overline{c}, \underline{c})$ is the profit of the firm with low cost when it charges a price equal to the other firm's costs.

In the efficient collusive agreement each firm gets:

$$\pi^M + \ \delta\pi^M + \ \delta^2\pi^M + \ ... = \pi^M(1 + \delta + \delta^2 + ...) = \frac{1}{1-\delta}\pi^M \ 1 \qquad (6)$$

If a firm decides to deviate, it will do so only in the market where it has a cost disadvantage, since in the other market it already earns monopoly profits. Suppose that the punishment for deviation is be for both firms to engage in a price war so that the prevailing price in each market is $\overline{c}$. If Firm 2 deviates in Market A, then it gets π^{MC} in that market in the first period, plus 0 from then on; and $\pi^M + \delta\pi^C + \delta^2\pi^C + \ldots$ in the other market. The situation is symmetric. Therefore, the deviating firm's total profits are given by:

$$\pi^{MC} + \pi^M + \delta\pi^C + \delta^2\pi^C + \ldots = \pi^{MC} + \pi^M + \frac{\delta}{1-\delta}\pi^C.$$

The stability condition requires:

$$\frac{1}{1-\delta}\pi^M \geq \pi^{MC} + \pi^M + \frac{\delta}{1-\delta}\pi^C.$$

This gives the minimum value for the discount factor:

$$\underline{\delta} = \frac{\pi^{MC}}{\pi^{MC} + \pi^M - \pi^C}.$$

■ **9.2** Based on data from local cement markets in the U.S., a series of regressions were estimated for seven years in the period 1948–1980. Each regression has the form price = $\beta \cdot C_4$ + (other variables). The coefficient β was estimated to be positive in five of the seven years considered, negative in the remaining two. How can these results be explained?

Solution: It was shown in Section 9.1 that, under the assumption of Cournot competition, the higher the number of firms in the market, the lower price and the lower allocative inefficiency. Moreover, the more concentrated an industry (the smaller the number of firms), the easier is to sustain collusion. These arguments suggest that when market concentration (as measured by C_4, for example) is greater equilibrium price is further apart from the competitive price (the structure-performance hypothesis).

However, as pointed out in Section 9.2, reverse causation is an important problem. If one assumes that market structure is endogenously determined (i.e., entry is possible) and market price is exogenous, then we obtain that a high price would induce entry from other firms and consequently decrease in concentration—a negative relation between the two variables.

■ **9.3**** Based on monthly data for Portuguese commercial banks, the following relation was estimated:

$$r_t = 0.098 + 0.814\, m_t,$$

where r_t is the interest rate charged by commercial banks and m_t is the money market rete, that is, the interest rate that banks must pay to borrow in the short term. The standard deviation of the second coefficient estimate is .0878. Knowing that the money market interested rate is highly correlated with the marginal cost of giving out loans, and knowing that H is approximately .125, what can you say about market power in this sector?

Solution: Applying the equation on p. 161, we get

$$\theta = \frac{1 - 0.814}{0.814} \frac{1}{.125} = 1.828$$

This value is higher than Cournot (1) but lower that perfect collusion ($1/H = 8$). Another way of evaluating the result is to consider what the result would be under Cournot. From Table 9.2, we get that, under Cournot,

$$\xi = \frac{H}{1 + H} = \frac{1}{1.125} = .888$$

The statistical test that the estimated coefficient is greater than the Cournot value would correspond to the value (.888-.814)/.0878=.84. Although we don't have complete information about sample size, etc, this is a relatively low value. The conclusion is that behavior is between Cournot and collusive behavior but not statistically different from Cournot.

Finally, note that the above analysis is only valid under the assumption that demand and costs are linear.

■ **9.4**** Consider the following criteria for a good measure of market concentration:

1. Non-ambiguity. Given any two different industries, it must be possible to rank concentration between the two.
2. Invariance to scale. A concentration measure ought not to depend on measurement units.
3. Transfers. Concentration should increase when a large firm's market share increases at the expense of a small firm's market share.
4. Monotonicity. Given n identical firms, concentration should be decreasing in n.
5. Cardinality. If we divide each firm into k smaller firms of the same size, then concentration should decrease in the same proportion.

verify whether the indeces C_n and H satisfy these requirements.

Solution: Recall that $C_m \equiv \sum_{i=1}^{m} s_i$ and $H \equiv \sum_{i=1}^{n} s_i^2$, where n is the total number of firms and $m \leq n$. [Note: there is a typo on p. 164. It should be C_m, not C_n.]

(a) We can compute both C_m and H for any two industries with the result being a rational number. Since rational numbers form an ordered set we can rank any two industries based on the two measures.

(b) This condition is satisfied since when computing the share of each firm the measure becomes units-free. For example, if we consider the share of firm i as the proportion of the firm's sales in the total industry sales, it is irrelevant whether we measure sales in billions or millions or thousands of dollars.

(c) Let firm k's market share increase at the expense of firm j's, so that $s_k^t = s_k + \alpha$ and $s_j^t = s_j - \alpha$, where the superscript indicates post-transfer values. Moreover, as required by the condition, let $s_k > s_j$.

$$H^t = \sum_{i=1, i\neq j,k}^{n} s_i^2 + (s_j^t)^2 + (s_k^t)^2 = \sum_{i=1, i\neq j,k}^{n} s_i^2 + (s_j - \alpha)^2 + (s_k + \alpha)^2 = H + 2\alpha(\alpha + s_k - s_j) > H.$$

We conclude that H satisfies the transfer condition. Now suppose that $j, k < m$. Then we have

$$C_m^t = \sum_{i=1, i\neq j,k}^{m} s_i + s_j^t + s_k^t = \sum_{i=1, i\neq j,k}^{m} s_i + s_j - \alpha + s_k + \alpha = C_m.$$

We conclude that C_m does not satisfy the transfer condition.

(d) If all firms are identical, then $H = \frac{1}{n}$ and $C_m = m/n$. Clearly, both indeces satisfy the condition.

(e) It is easy to find examples where C_m violates cardinality. For example, suppose that $s_1 = .4, s_2 = .2$. In this case, $C_2 = .6$. Suppose all firms are divided by $k = 2$. The new value of C_2 is $.2 + .2 = .4$, which is different from $.6/2$.

We now show that H does satisfy cardinality. Let H' be the new value of H when each of the initial n firms is divided by k.

$$H' = \sum_{i=1}^{n} \left(\sum_{j=1}^{k} \left(\frac{s_i}{k} \right)^2 \right) = \sum_{i=1}^{n} k \left(\frac{s_i}{k} \right)^2 = \sum_{i=1}^{n} \frac{1}{k} s_i^2 = \frac{H}{k}.$$

■ **9.5**** Suppose you only know the value of the market shares for the largest m firms in a given industry. While you do not possess sufficient information to compute the Herfindahl index, you can find a lower and an upper bound for its values. How?

Solution: A lower bound would result from an industry where, in addition to the top m firms, there is a very large number of firms with a very small market share. In the limit of infinitesimal shares, the value of H would be $H = \sum_{i=1}^{m} s_i^2$. An upper bound would result from an industry where all the remaining firms have the same market share as the m-th firm. The value of H would then be $H = \sum_{i=1}^{m} s_i^2 + \left(1 - \sum_{i=1}^{m} s_i\right) s_m$. (Notice that the remaining firms would be $\left(1 - \sum_{i=1}^{m} s_i\right) / s_m$ in number.

The above lower and upper bounds are frequently very close, so a fairly good approximation if often possible.

■ **10.1** First-time subscribers to the *Economist* pay a lower rate than repeat subscribers. Is this price discrimination? Of what type?

Solution: This is an example of third-degree price discrimination. The market is segmented into new subscribers and repeat subscribers. New subscribers, know the product less well and are thus likely to be more price sensistive. Moreover, the fact that they have not subscribed in the past indicates that they are likely to be willing to pay less than current subscribers. It is therefore optimal to set a lower price for new subscribers.

■ **10.2** Many firms set a price for the export market which is lower than the price for the domestic market. How can you explain this policy?

Solution: A possible explanation is that there is a "domestic product bias" that makes consumers less price sensitive to domestic products (see Box 10.1). It is then rational to set higher prices in the domestic market.

■ **10.3** Cement in Belgium is sold at a uniform delivered price throughout the country, that is, the same price is set for each customer, including transportation costs, regardless of where the customer is located. The same is practice is also found in the sale of plasterboard in the United Kingdom.[23] Are these cases of price discrimination?

Solution: Yes, these are cases of price discrimination. Consider the total price being paid by each customer, P, as being composed of the price actually charged and the transportation cost; $P = p_i + t_i$. Since locations are different, transportation costs are different, thus, each consumer is charged a price p_i that depends on his or her location. This is a clear example of geographic price discrimination.

■ **10.4** A restaurant in London has recently removed prices from its menu: each consumer is asked to pay what he or she thinks the meal was worth. Is this a case of price discrimination?

Solution: It is likely that each consumer will pay a price that reflects his or her willingness to pay. In that sense, this is a situation of close to perfect price discrimination.

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[23]

Phlips, Louis (1983), *The Economics of Price Discrimination*, Cambridge: Cambridge University Press., pp. 23–30.

■ **10.5** In the New York Fulton fish market, the average price paid for whiting by Asian buyers is significantly lower than the price paid by White buyers.[24] What type of price discrimination does this correspond to, if any? What additional information would you need in order to answer the question?

Solution: This appears to be a case of third-degree price discrimination, whereby a group of buyers (a market segment) pays a different price than another group. Theory predicts that in a non-competitive market (monopoly, oligopoly) buyers with higher price elasticity should be charged a lower price; as a result, we can conclude that Asian buyers have higher price elasticity than white buyers.

In order to have a more accurate picture, however, more information is needed. Different prices could could simply result from quantity discounts and the possible fact that different quantities are bought by the different groups. It could also be the case that different groups use different types of payment type (cash or credit), so that different prices reflect different costs. Also, the time of purchase (e.g., before 5am or after 5am) could be correlated with race, so that it is not race that determines the price difference. The same reasoning applies to the type of establishment does the buyer represents (store, fry shop, etc.).

For a more complete discussion, see the cited reference.

■ **10.6** Supermarkets frequently issue coupons that entitle consumers to a discount in selected products. Is this a promotional strategy, or simply a form of price discrimination? Empirical evidence suggests that paper towels are significantly more expensive in markets offering coupons than in markets without coupons.[25] Is this consistent with your interpretation?

Solution: This may be interpreted as a case of price discrimination. By offering coupons (hence a lower price), supermarkets can serve the buyers with a higher price elasticity at a different price. In order for this strategy to improve revenues with respect to single price, supermarkets should then set a higher regular price. Hence, empirical evidence is consistent with the explanation that this is a form of price discrimination.

■ **10.7**** A market consists of two population segments, A and B. An individual in segment A has demand for your product $q = 50 - p$. An individual in segment B has

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24

Graddy, Kathryn (1995), "Testing for Imperfect Competition at the Fulton Fish Market," *Rand Journal of Economics* **26**, 75–92..

25

Levedahl, J W (1984), "Marketing, Price Discrimination, and Welfare: Comment," *Southern Economic Journal* **3**, 886–891..

demand for your product $q = 120 - 2p$. Segment A has 1000 people in it. Segment B has 1200 people in it. Total cost of producing q units is $C = 5000 + 20q$.

(a) What is total market demand for your product?

(b) Assume that you must charge the same price to both segments. What is the profit-maximizing price? What are your profits?

(c) Imagine now that members of segment A all wear a scarlet "A" on their shirts or blouses and that you can legally charge different prices to these people. What price do you change to the scarlet "A" people? What price do you change to those without the scarlet "A"? What are your profits now?

Solution:

(a) Segment A people buy zero at or above $p = 50$. At $p < 50$, the total demand from segment A types is $Q_A = 1000(50 - p) = 50000 - 1000p$. Segment B people buy zero at or above $p = 60$. At $p < 60$, the total demand from segment B types is $Q_B = 1200(120 - 2p) = 144000 - 2400p$. At $p \geq 60$, quantity demanded is zero. At $50 < P < 60$, total demand is just the demand from B, $Q = 144000 - 2400p$. At $P < 50$ total demand is from both types $Q = (144000 - 2400p) + (50000 - 1000p) = 194000 - 3400p$.

(b) First note that $MC = 20$ at all output levels. For $p > 50$, the only consumers in the market are segment B consumers so $TR = Q(60 - Q/2400) = 60Q - Q2/2400$. Using calculus, one can then take the derivative and find $MR = 60 - Q/1200$ in this range. But note that at the break point $p = 50$, where segment A customer begin to enter the market, $Q = 24000$, and $MR = 40$, which is still greater than MC. Therefore, the firm would keep lowering its price to sell more units. This would induce segment A consumers to buy so the demand function should we consider at $p < 50$, is now a combination of A and B segment customers so $TR = 57.06Q - Q2/3400$, $MR = 57.06 - Q/1700$. Taking the derivative of this total revenue function and setting it equal to MC we have $57.06 - Q/1700 = 20$ which yields an optimal output of $Q = 63002$, which yields $p = 38.53$. To avoid doing the calculus, one could set up a spreadsheet with every possible quantity and find the profit maximizing Q.

(c) The problem can now be solve as two separate markets. In each, you pick the profit maximizing quantity to sell to the segment by setting marginal cost equal to the marginal revenue for that segement..

$Q_A = 50000 - 1000p \Rightarrow p = 50 - Q_A/1000 \Rightarrow TR = 50Q_A - Q_A^2/1000 \Rightarrow MR = 50 - Q_A/500. MR = MC \Rightarrow 50 - Q_A/500 = 20 \Rightarrow Q_A = 15000 \Rightarrow p_A = 35$.

$Q_B = 144000 - 2400p \Rightarrow p = 60 - Q_B/2400 \Rightarrow TR = 60Q_B - Q_B^2/2400 \Rightarrow MR = 60 - Q_B/1200. MR = MC \Rightarrow 60 - Q_B/1200 = 20 \Rightarrow Q_B = 48000 \Rightarrow p_B = 40$.

■ **10.8*** Coca-Cola recently announced that it is developing a "smart" vending machine. Such machines are able to change prices according to the outside temperature.[26]

[26] *Financial Times*, October 28, 1999.

Suppose for the purposes of this problem that the temperature can be either "High" or "Low." On days of "High" temperature, demand is given by $Q = 280 - 2p$, where Q is number of cans of Coke sold during the day and p is the price per can measured in cents. On days of "Low" temperature, demand is only $Q = 160 - 2p$. There is an equal number days with "High" and "Low" temperature. The marginal cost of a can of Coke is 20 cents.

(a) Suppose that Coca-Cola indeed installs a "smart" vending machine, and thus is able to charge different prices for Coke on "Hot" and "Cold" days. What price should Coca-Cola charge on a "Hot" day? What price should Coca-Cola charge on a "Cold" day?

(b) Alternatively, suppose that Coca-Cola continues to use its normal vending machines, which must be programmed with a fixed price, independent of the weather. Assuming that Coca-Cola is risk neutral, what is the optimal price for a can of Coke?

(c) What are Coca-Cola's profits under constant and weather-variable prices? How much would Coca-Cola be willing to pay to enable its vending machine to vary prices with the weather, i.e., to have a "smart" vending machine?

Solution:

(a) On a Hot day, $Q = 280 - 2p$, or $p = 140 - Q/2$. Marginal revenue is $MR = 140 - Q$. Equating to marginal cost (20) and solving, we get $Q^* = 120$ and $p^* = 80$. On a Cold day, $Q = 160 - 2p$, or $p = 80 - Q/2$. Marginal revenue is $MR = 80 - Q$. Equating to marginal cost (20) and solving, we get $Q^* = 60$ and $p* = 50$.

[(b)] Observe from part (a) that even on a Hot day the optimal price is no greater than 80 cents. So, we can restrict our attention to prices of 80 cents or less. In this price range, the expected demand is given by $Q = .5(280 - 2p) + .5(160 - 2p) = 220 - 2p$. Solving for p gives $p = 110 - Q/2$. The marginal revenue associated with this expected demand curve is given by $MR = 110 - Q$. Equating this marginal revenue to marginal cost, we get $Q^* = 90$ and $p^* = 65$.

[(c)] Under price discrimination, from part (a), profits on a Hot day are $(80 - 20)120 =$ \$72, and profits on a Cold day are $(50 - 20)60 =$ \$18. Expected profits per day are therefore (\$72 + \$18)/2 = \$45. Under uniform pricing, expected profits per day are $(65 - 20)90 =$ \$40.50. It follows that Coca-Cola should be willing to pay up to an extra \$4.50 per day for a "smart" vending machine.

■ **10.9*** Suppose the California Memorial Stadium has a capacity of 50,000 and is used for exactly seven football games a year. Three of these are OK games, with a demand for tickets given by $D = 150,000 - 3p$ per game, where p is ticket price. (For simplicity, assume there is only one type of ticket.) Three of the season games are not so important, the demand being $D = 90,000 - 3p$ per game. Finally, one of the games is really big, the demand being $D = 240,000 - 3p$. The costs of operating the Stadium are essentially independent of the number of tickets sold.

(a) Determine the optimal ticket price for each game, assuming the objective of profit maximization.

Given that the Stadium is frequently full, the idea of expanding the Stadium has arisen. A preliminary study suggests that the cost of capacity expansion would be \$100 per seat per year.

(b) Would you recommend that the University of California go ahead with the project of capacity expansion?

Solution:

(a) Demand for OK games is given by $D = 150 - 3p$, where number of tickets is measured in thousands. Inverse demand is $p = 50 - Q/3$. Marginal revenue is $MR = 50 - 2/3 * Q$. Marginal cost is zero, since costs do not depend on the number of tickets sold. Equating marginal cost to marginal revenue, we get $Q = 75$. This is greater than capacity. Therefore, the optimal solution is simply to set price such that demand equals capacity: $150 - 3p = 50$, which implies $p = \$33.3$

Demand for not-so-important games is given by $D = 90 - 3p$. Inverse demand is $p = 30 - Q/3$. Marginal revenue is $MR = 50 - 2/3 \cdot Q$. Equating marginal revenue to marginal cost, we get $Q = 45$. Substituting back in the inverse demand curve we get $p = \$15$.

Since demand for the Big Game is greater than for the OK games, it will surely be the case that $MR = MC$ implies a demand level greater than capacity. The optimal price is therefore determined by equating demand to capacity: $240 - 3p = 50$, or simply $p = \$63.3$

(b) The marginal revenue of an additional seat is the sum of the difference between marginal revenue and marginal cost for all games where capacity was a constraint. For OK games, marginal revenue is given by $MR = 50 - 2/3 \cdot 50 = 16.7$. For the Big Game, $MR = 80 - 2/3 \cdot 50 = 46.7$. Adding these up (three times the first plus the second) we get \$96.7. Since this is less than the marginal cost of capacity exapansion, it is not worth it to pursue the project.

■ **10.10**** Your software company has just completed the first version of SpokenWord, a voice-activated word processor. As marketing manager, you have to decide on the pricing of the new software. You commissioned a study to determine the potential demand for SpokenWord. From this study, you know that there are essentially two market segments of equal size, professionals and students (one million each). Professionals would be willing to pay up to \$400 and students up to \$100 for the full version of the software. A substantially scaled-down version of the software would be worth \$50 to consumers and worthless to professionals. It is equally costly to sell any version. In fact, other than the initial development costs, production costs are zero.

(a) What are the optimal prices for each version of the software?

Suppose that, instead of the scaled-down version, the firm sells an intermediate version that is valued at \$200 by professionals and \$75 by students.

(b) What are the optimal prices for each version of the software? Is the firm better off by selling the intermediate version instead of the scaled-down version?

Suppose that professionals are willing to pay up to \$800$(a - .5)$, and students up to \$100a, for a given version of the software, where a is the software's degree of functionality: $a = 1$ denotes a fully functional version, whereas a value $a < 1$ means that only $100a\%$ features of the software are functional. It is equally costly to produce any level of a. In fact, other than the initial development costs, production costs are zero.

(c) How many versions of the software should the firm sell? Which versions? What are the optimal prices of each version?

Solution:

(a) It is optimal to price the full version at 400 and the scaled-down version at 50. Total profits are 450.

(b) One first possibility would be to price the intermediate version at 75 and the full version at 400. However, this would lead professionals to choose the intermediate version since the difference between willingness to pay and price is greater for the intermediate version. In order to induce professionals to by the full version, the full version's price would need to be 75+(400-200)=275, where the value in parentheses is the professionals' difference in willingness to pay between the two versions. This would lead to a total profit of 275 + 75=350, which is lower than initially. Still another possibility would be to price the full version at 400 and the intermediate version at 400 - (400-200) = 200. In this case, professionals would buy the full version but students would not buy the intermediate version. Profits would then be 400: better than 350 but still less than the 450 the firm would get with the truly scaled-down version.

(c) There are two candidates for optimal price: \$400 and \$100. Profits are given by \$400m in the first case and \$200m in the second case (recall that there are one million professionals and one million students). It follows that $a = 1, p = 400$ is the optimal solution.

Since there are only two types of consumers, it will not be necessary to offer more than two different versions. Since it is equally costly to produce any version and willingness to pay is increasing in a, it follows that one of the versions should have have full functionality ($a = 1$), the other one $a \leq 1$. Since professionals value at zero any version with $a \leq .5$, we conclude that the "damaged" version has $.5 \leq a \leq 1$.

At the margin, professionals are willing to pay more for greater functionality than students. Therefore, if there is to be self-selection between two different versions, it will be the case that professionals choose the fully functional version and students the other one. If professionals prefer the fully functional version, it must be that $p_1 - p_a \leq 800(1-.5) - 800(a-.5)$, that is, the price difference must be smaller than the difference in willingness to pay (p_1 and p_a are the prices of the fully functional and damaged versions, respectively). Moreover, it must be $p_1 \leq 400$. By the same token, if students prefer to purchase the "damaged" version, it must be that $p_1 - p_a \geq 100 - 100a$ and $p_a \leq 100a$.

Suppose the first and fourth inequalities are binding. Profits as a function of a are then given by $p_1 + p_a$, which is equal to $200a + 800(1 - .5) - 800(a - .5)$. This is decreasing in a, implying that the optimal value would be $a = .5$. But this would lead to $p_1 = 450$, which violates the second inequality. It follows that the optimal solution is to choose the minimum value of a such that the second constraint is just satisfied, that is, $100a + 800(1 - .5) - 800(a - .5) = 400$, or simply $a = 4/7$. (Notice that the third constraint is satisfied for these values.) Optimal prices are therefore given by $p_1 = 400$ and $p_a = 100(4/7) \approx 57.14$.

Profits under one version are \$400m. Under two versions, the firm gets \$457.14m, an increase of \$57.14m. Basically, the increase corresponds to student sales.

■ **10.11*** One of the arguments used in Microsoft's defense against allegations of monopoly behavior is that it "cannot charge a monopoly price because it faces competition from ... its own installed base." Based on the above discussion on durable goods, how would you qualify/extend Microsoft's defense?

Solution: In Section 10.4, we discussed the problem faced by a monopolist selling a durable good. If the monopolist can set different prices over time (inter-temporal price discrimination), then its profits may be lower that they would be if the monopolist could not set different prices over time. Rational buyers know that, once high-valuation buyers have purchased the good the seller has an incentive to lower price and capture lower-valuation consumers who would otherwise not purchase the product.

In order for this to take place, it is important that potential buyers have some flexibility regarding the time of purchase (as is usual with durable goods). Operating systems seem a good candidate for this: typically, consumers are already using a give operating system when they buy a new one, and thus delay is a reasonable option. However, many computer purchases are bundled with the latest operating system, in which case buyers don't really make a decision of when to purchase the operating system. In summary, it's unclear how important the durable-good constraint is in this case.

■ **10.12** In 1998, the European Commission fined Volkswagen more than \$100m for preventing its dealers in Italy from selling to foreign buyers. Is this consistent with the European Commission's policy regarding price discrimination? Is this the right decision from a social welfare point of view?

Solution: Section 10.5 presents several cases concerning the European Union's policy towards price discrimination. The E.U. appears concerned with price discrimination within the union but less so between the E.U. and the rest of the world. Since both Italy and Germany (home to Volkswagen) are part of the E.U., the decision is consistent with the E.U. policy goal of creating a single market.

From a social welfare point of view, as Section 10.5 suggests, things are not straightforward. Price discrimination may be more efficient if total welfare is increased. However, price discrimination may be considered unfair by consumers: German buyers may not like the idea of paying more for the same car as Italian buyers.

■ **10.13*** Can coupons be used to price discriminate? How? Empirical evidence suggests that, in U.S. cities where coupons are used more often, breakfast cereals are sold at a lower price.[27] Is this consistent with the interpretation that coupons are used for price discrimination? If not, how can the empirical observation be explained?

Solution: Paralleling the explanation in Exercise 10.6, one could argue that coupons can be used for price discrimination. However, the empirical evidence from the breakfast cereal market is not consistent with this explanation (as was the example with the market for paper towels in Exercise 10.6). The interpretation of coupons as a promotion strategy is probably a better explanation.

For more information, see the cited reference.

■ **10.14**** In September 1997, the New York state's attorney general pressed charges against Procter & Gamble over the fact that P&G eliminated the use of coupons. The argument was that P&G was colluding with rivals to eliminate coupons, for doing so "only works if everybody goes along with it."[28] What does this suggest about the practice of price discrimination in the context of oligopoly? (In the end, P&G, while not admitting any wrongdoing, agreed on a \$4.2m settlement of the charges.)

Solution: Price discrimination may be viewed as a "prisoners' Dilemma:" If oligopolists can commit not to use coupons (and price discriminate) then everybody is better off (as are both prisoners in the case they do not defect). However, using coupons may be a dominant strategy, implying that every player would use it. The equilibrium where no player uses coupons can then only be achieved through collusion / cooperation.

■ **10.15*** Suppose that perfect price discrimination implies a transaction cost T, incurred by the seller. Show that perfect price discrimination may be optimal for the seller but welfare decreasing for society as a whole.

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[27]

Nevo, Aviv, and Catherine Wolfram (1999), "Prices and Coupons for Breakfast Cereals," University of California, Berkeley, and Harvard University..

[28] *The Economist*, August 1st, 1997.

Solution: Refer to Figure 10.1. By going from no price discrimination to (perfect) price discrimination, the seller's gross profits increase by $B + C$, whereas consumer surplus decreases by B. The net social gain is C. Suppose however that the seller must incur a cost T in order to implement perfect price discrimation. If $C < T < B + C$, then perfect price discrimination is profitable but not socially desirable.

■ **10.16*** Consider the model of a monopolist with two markets presented in Section ??. Suppose that the seller has a limited capacity and zero marginal cost up to capacity (or very low marginal cost). An example of this would be an airline with two types of passengers or a football stadium with two types of attendees.

Derive the conditions for optimal pricing. How do they relate to the case when there are no capacity constraints?

Solution: Let K denote capacity and $p_1(q_1)$, $p_2(q_2)$ denote the inverse demand functions.

The monopolist's problem becomes:

$$\max_{q_1,q_2} q_1 p_1(q_1) + q_2 p_2(q_2) - c(q_1 + q_2)$$

subject to

$$q_1 + q_2 \leq K.$$

The Lagrangean for this problem is

$$\mathcal{L} = q_1 p_1(q_1) + q_2 p_2(q_2) - c(q_1 + q_2) + \lambda \Big(K - q_1 - q_2 \Big) .$$

The first-order conditions are:

$$\begin{aligned} MR_1 &= MC + \lambda \\ MR_2 &= MC + \lambda, \end{aligned}$$

or simply

$$\begin{aligned} MR_1 &= \lambda \\ MR_2 &= \lambda, \end{aligned}$$

since marginal cost is zero up to capacity. Depending on whether capacity constraints are binding or not, we will have λ positive or zero. Whichever is the case, the above equations show that optimality implies that *marginal revenue* be equated across markets. Notice that, if demand elasticity differs across markets, then this implies different prices for the different markets.

The same result can be obtained intuitively. Suppose that the seller is capacity constrained. Is the current set of prices optimal? One alternative is to take one unit from one market and sell it the other market, changing prices accordingly. Whould the seller want to do this? By taking one unit away from Market 1, the seller would lose MR_1. By selling it in Market 2, the seller would get MR_2. Optimality then requires that $MR_1 = MR_2$.

■ **10.17***** Consider the model of non-linear pricing introduced in Section **??**. Suppose there are two types of consumers, in equal number. Type 1 have demand $D_1(p) = 1 - p$, and type 2 $D_2(p) = 2(1 - p)$. Marginal cost is zero.

(a) Show that if the seller is precluded from using non-linear pricing, then the optimal price is $p = \frac{1}{2}$ and profit (per consumer) $\frac{3}{8}$.

(b) Show that if the seller must set a single two-part tariff, then the optimal values are $f = \frac{9}{32}$ and $p = \frac{1}{4}$, for a profit of $\frac{9}{16}$.

(c) Show that if the seller can set multiple two-part tariffs, then the optimal values are $f_1 = \frac{1}{8}$, $p_1 = \frac{1}{2}$, $f_2 = \frac{7}{8}$, $p_2 = 0$, for a profit of $\frac{5}{8}$.

d) Show that, like profits, total surplus increases from (a) to (b) and from (b) to (c).

Solution: (a) Total demand from a consumer of Type 1 and a consumer of Type 2 is given by $D(p) = D_1(p) + D_2(p) = 1 - p + 2(1 - p) = 3(1 - p)$. The monopolist's problem is:

$$\max_p \; 3p(1 - p) \tag{7}$$

The solution to this problem is given by the first order condition, $1 - 2p = 0$, so that we get $p = \frac{1}{2}$ and the profit is $\frac{3}{4}$. Social welfare is given by the sum of the firm's profit and the consumer surplus and is equal to: $W_a = 3p(1 - p) + (1 - p)^2 = 1$.

(b) In this case the monopolist's demand is the same. However, the monopolist now can also charge a fixed fee, f, from both consumers. The problem becomes:

$$\max_p \; 3p(1 - p) + 2f$$
$$\text{s.t. } \frac{(1 - p)^2}{2} \geq f,$$

where the constraint comes from the fact that the consumer of Type 1 must have a positive surplus, otherwise it will not buy. Once the constraint for the Type 1 consumer is satisfied, the constraint for Type 2 is also satisfied; we can therefore ignore it. The monopolist is better off when it extracts as much surplus as possible from consumers. Thus, its optimal policy requires that the fixed fee be equal to the Type 1 consumer surplus, that is, the constraint should be binding. The monopolist's problem becomes:

$$\max_p \; 3p(1 - p) + (1 - p)^2,$$

and the solution is given by the first order condition, $3 - 6p - 2 + 2p = 0$, so that we get $p = \frac{1}{4}$, $f = \frac{9}{32}$ and the profit is $\frac{9}{8}$. Welfare is given by $W_b = 3p(1-p) + (1-p)^2 + 0 + \frac{(1-p)^2}{2} = \frac{45}{32} > W_a$.

(c) In this case the monopolist's problem is more complex:

$$\begin{aligned} \max_{p_1,p_2} \quad & p_1(1-p_1)+f_1+2p_2(1-p_2)+f_2 & \\ s.t. \quad & CS_1(p_1) \geq f_1 & (PC1) \\ & CS_2(p_2) \geq f_2 & (PC2) \\ & CS_1(p_2)-f_2 \leq CS_1(p_1)-f_1 & (IC1) \\ & CS_2(p_1)-f_1 \leq CS_2(p_2)-f_2, & (IC2) \end{aligned}$$

where the participation constraints assure that the consumer will prefer to consume and the incentive compatibility constraints assure that each plan is chosen by the targeted type of consumers, that is, Type 1 consumers will prefer plan 1 to plan 2 while Type 2 consumers will prefer plan 2 to plan 1.

One can show that PC1 and IC2 are binding, while PC2 and IC1 are not. Suppose that PC1 and IC2 are satisfied. We have: $CS_2(p_2)-f_2 \geq CS_2(p_1)-f_1 \geq CS_2(p_1)-CS_1(p_1) \geq 0$, where the last inequality comes from the fact that, at any price, the surplus of the Type 2 consumers is higher, since they consume more. Therefore, PC2 is automatically satisfied. PC2 will not be binding unless consumers of Type 1 are not served. To see this, suppose PC2 is binding. From IC2 and PC1 we get $CS_2(p_1) \leq f_1 \leq CS_1(p_1)$ which is obviously impossible. In contrast, PC1 must be binding: if PC1 and PC2 would not bind the monopolist could increase its profits by increasing both f_1 and f_2 with the same small amount without violating the ICs. If IC2 is not binding the monopolist could increase f_2 with a small amount and keep all other constraints satisfied, while increasing her profits.

Therefore, we have $f_1 = CS_1(p_1) = \frac{(1-p_1)^2}{2}$ and $f_2 = CS_2(p_2) - CS_2(p_1) + f_1 = (1-p_2)^2 - \frac{(1-p_1)^2}{2}$. The monopolist's problem becomes:

$$\max_{p_1,p_2} \quad p_1(1-p_1)+2p_2(1-p_2)+(1-p_2)^2$$

The first order conditions are: $1-2p_1=0$ and $2-4p_2-2+2p_2=0$, and the solutions are: $p_1=\frac{1}{2}$, $f_1=\frac{1}{8}$, $p_2=0$, $f_2=\frac{7}{8}$ and the profit is $\frac{5}{4}$. The welfare is given by $W_c = p_1(1-p_1)+\frac{(1-p_1)^2}{2}+2p_2(1-p_2)+(1-p_2)^2 = \frac{11}{8} < W_b$.

(d) The proof is already contained in the previous points.

■ **10.18***** Many retail stores set lower-than-usual prices during a fraction of the time (sale). One interpretation of this practice is that it allows for price discrimination between patient and impatient buyers.

Suppose that each buyer wants to purchase one unit per period. Each period is divided into two subperiods, the first and the second part of the period. Suppose there are two types of buyers, $i=1,2$. Each type of buyer is subdivided according to