the part of the period they would ideally like to make their purchase. One half the buyers would prefer to purchase during the first part of the period, one half during the second part. A buyer of type i is willing to pay $\overline{v}_i$ for a purchase during his or her preferred part of the period; and $\underline{v}_i$ for a purchase at another time.

Buyers of type 1, which constitute a fraction α of the population, are high-valuation, impatient buyers; that is, $\overline{v}_h$ is very high and $\underline{v}_h$ very low. High valuation implies that $\overline{v}_h$ is very high; impatience implies that $\underline{v}_h$ is very low: buyers of type 1 are not willing to buy at any time other than their preferred time. Buyers of type 2, by contrast, are very patient: $\overline{v}_l \approx \underline{v}_l$. Assume that α is relatively low; specifically, $\alpha < \overline{v}_l/\overline{v}_h$. To summarize: $\overline{v}_h > \overline{v}_l \approx \underline{v}_l > \alpha\overline{v}_h > \underline{v}_h \approx 0$.

a) Show that, under a constant-price strategy, the seller optimally sets $p = \overline{v}_l$.

b) Determine firm profits when it sets prices $p = \overline{v}_h$ and $p = \underline{v}_l$ in the first and second parts of the period, respectively.

c) Show that profits are greater under the "sales" strategy.

Solution: a) If $p > \overline{v_1}$, then there is no sale. If $\overline{v_2} < p < \overline{v_1}$, then the only purchasers are the impatient, high-valuation buyers, and the seller's profit is $\pi = \alpha p$, with maximum value $\alpha\overline{v_1}$. If $p < \overline{v_2}$, then all buyers make a purchase and the seller's profit is $\pi = p$, with maximum value $\overline{v_2}$. Since $\alpha\overline{v_1} < \overline{v_2}$, it is clear that the best constant-price strategy is to set $p = \overline{v_2}$.

b) Under this strategy the seller's profit is $\pi = \frac{1}{2}\alpha\overline{v_1} + (1-\alpha)\underline{v_2} + \frac{1}{2}\alpha\underline{v_2} = \underline{v_2} + \frac{1}{2}\alpha(\overline{v_1} - \underline{v_2}) > \overline{v_2}$, where the last inequality is based on the fact that $\underline{v_2} \approx \overline{v_2}$.

c) The proof is contained in part b.

■ **11.1*** Assume for the purposes of this problem that, contrary to their protestations, Microsoft has a monopoly in providing operating systems, called "Windows," for personal computers. Assume also that the marginal cost to Microsoft of supplying its operating system for one more computer is zero. Denote by w the price charged by Microsoft for its operating system. (Assume that Microsoft sets a single price per computer, i.e., does not employ two-part tariffs, quantity discounts, or other forms of price discrimination.)

Computer Original Equipment Manufacturers (OEMs) assemble computers. Suppose that the "bill of materials" for a computer, i.e., the cost to the OEM of all the parts necessary to build a computer, adds up to \$900 per machine, and that assembly costs another \$100 per machine. Finally, assume (contrary to the efforts of Dell and Compaq) that computers are a homogeneous good and the annual demand for computers is given by $Q = 50,000,000 - 10,000p$, where Q is quantity and p is price as usual.

Suppose that the OEM business is perfectly competitive.

(a) For any given price, w, of operating systems, what will be the price and sales of computers?

(b) What price w should Microsoft set for its operating system? How much money will Microsoft make? How much money will OEMs make? What will be the price of a computer?

Amusing if irrelevant note: Microsoft in fact charges in the \$50 to \$60 range per PC for Windows98. Microsoft argued in their antitrust trial that they must not really have a monopoly or else they would be charging a lot more.

(c) How much money would a vertically integrated firm controlling both the supply of Windows and the assembly of computers make? What price would such a firm charge for computers?

(d) Could Microsoft make more money by integrating downstream into computer assembly? Why or why not?

Suppose now (definitely contrary to reality) that a single firm, Compaq, has a monopoly over the assembly of computers.

(e) For a given price, w, for Windows, what price, p, would Compaq set for computers and how many computers would be sold?

(f) What price, w, should Microsoft set for its operating system? How much money will Microsoft make? How much money will Compaq make? What will be the price of a computer?

(g) Could Microsoft and Compaq make more money by merging? If so, how much? Would such a merger benefit or harm computer users? By how much?

Solution:

(a) Competition in the downstream computer market will drive prices in that market to the OEM's marginal cost. For the downstream computer makers, marginal cost is equal to w the price they pay Microsoft for its operating system plus the $900 + 100 = 1000$ per machine that they incur in materials and assembly. Therefore, the price in the downstream market will be $p = w + 1000$ and the total number of computers sold will be $Q = 50,000,000 - 10,000(w + 1,000) = 40,000,000 - 10,000w$.

(b) The demand curve calculated in part (a) is the derived demand for Microsoft's operating system since each computer sold has one copy of the Microsoft's operating system. Therefore, the correct price for Microsoft to charge is the price that maximizes its profits in this market, which is the monopoly price. Inverting this demand curve you get $w = 4000 - Q/10000$, which means that Microsoft's marginal revenue will be $MR_M = 4000 - Q/5000$. Since its marginal cost for the operating system is zero, the optimal quantity to sell will be $4000 - Q/5000 = 0$ or $Q = 20,000,000$ machines, or 20 million copies of its operating software. In order to sell this many copies, Microsoft needs to set its price, w such that $w = 4000 - 20,000,000/10000 = 2,000$. At this price, Microsoft earns $\$2000 \cdot 20,000,000 = \$40,000,000,000$ in profits. The OEMs earn zero profits since they price at marginal cost, which is equal to $\$2,000 + \$100 + 900 = \$3,000$.

(c) A vertically integrated firm has marginal costs of is \$0 for the operating system, \$100 for assembly, and \$900 for computer parts. Assuming that the vertically integrated firm only sells operating systems to its downstream computer subsidiary, this subsidiary would be a monopolist in the computer market. Therefore, its marginal revenue would be $MR = 5000 - Q/5000$. Setting this marginal revenue equal to marginal cost of \$1000 and solving for Q^* yields $Q^* = 20,000,000$ units. In order to sell this many units the firm would charge

$p = 5000 - 20,000,000/10000 = 5000 - 2000 = \$3,000$ per computer. Its profits would be $20,000,000(\$3000 - \$1000) = \$40,000,000,000$.

(d) Since all the market power in this industry is in the software business, Microsoft can make just as much money by staying in the upstream market as it could by entering the downstream market. You just demonstrated that fact in part (c).

(e) As a monopolist in the downstream market, Compaq faces marginal revenue equal to $MR = 5000 - Q/5000$. Equating this with its marginal cost of $w + 1000$ and solving for Q^* yields, $Q^* = 20,000,000 - 5000w$. This is the optimal amount for Compaq to sell, which requires a price of $p = 5000 - (20,000,000 - 5000w)/10000 = 3000 + w/2$.

(f) When setting its price, w, Microsoft knows that its demand will we $Q^* = 20,000,000, -5000w$ since this is the optimal amount for Compaq to sell when it pays Microsoft a price of w per unit of the operating system. For Microsoft, the optimal number of units of the operating system to sell are those that maximize its profits given this demand. Inverting Q^* and solving for w yields $w = 4000 - Q^*/5000$, which means that Microsoft's marginal revenue is $MR = 5000 - Q^*/2500$. Equating this with its marginal cost of zero, you find that the optimal amount for Microsoft to sell is $Q^* = 12,500,000$ units. In order to sell this many units, Microsoft's price to Compaq needs to be $w = 4000 - 12,500,000/5000 = \$1500$ per unit. Its profits will be $12,500,000\$1500 = \$18,750,000,000$.

(g) If Microsoft and Compaq merged, their profits would be those calculated for the integrated computer maker calculated above (Part (c)), or \$40,000,000,000, which is an improvement of \$21,250,000,000. Consumers actually benefit as well since the total number of computers they buy will increase form 12,500,000 to 20,000,000 and the price they pay will fall. The total improvement for consumer is equal to the change in consumer surplus associated with the expansion of the number of computers sold and this price decline. In the vertically separated setting, the total cost of a computer is $p = 1500/2+3000 = 3750$ (from Part (e) above). Therefore, the consumer surplus is equal to $(12,500,000)(5000-3750)/2 = \$7,812,500,000$. In the vertically integrated setting, the price of a computer is \$3,000 (from Part (b) above) so the consumer surplus will be $(20,000,000)(5000-3000)/2 = \$20,000,000,000$. Therefore, the net improvement in consumer welfare will be \$12,187,500,000.

■ **11.2** Empirical evidence suggests that franchiser-owned McDonald's restaurants charge lower prices than independent ones. How can this difference be explained?

Solution: This is an example of double marginalization. If firms are vertically integrated (as is the case with franchise-owned McDonalds restaurants), then the retailer price is the monopoly price for the vertical structure. On the other hand, if the firms are not vertically integrated, then retailer's profit maximization leads to a second monopoly margin which takes as marginal cost the wholesale price. If the wholesale price is equal to the marginal cost of the upstream firm, then the two retail prices are the same. However, in such a

case the upstream firm makes zero profits. We would thus expect the wholesale price to be greater than marginal cost. It follows that the retail price for independent retailers is higher than for franchise-owned retailers.

■ **11.3** Suppose that a manufacturer sells to n retailers by means of a two-part tariff (f, w) including a fix fee f and a wholesale price w. Explain the intuition of the result that the greater the degree of retailer competition, the greater the optimal wholesale price.

Solution: See Section 11.2.

■ **11.4** The following industries are known to practice or have practiced resale price maintenance: fashion clothing, consumer electronics, fine fragrances. In each case, indicate the probable motivation for RPM and the likely welfare consequences.

Solution: In the case of consumer electronics, as pointed out in Section 11.3, there is an important positive externality from investing in sales effort. Retailers can free-ride on the investment efforts made by competing retailers, since one consumer can benefit from the point-of-sale services provided by a retailer (who invested in sales effort) and shop at a lower-price retailer (who did not invest). The result of this externality is that no retailer invests and the demand for the good is lower. An RPM policy induces investment in sales effort (which increases demand); instead of competing in price (which is now the minimum price required by the manufacturer), the retailers compete in investments in sales effort to attract customers. The final beneficiaries of this policy are, obviously, the retailers and the manufacturer. Nevertheless, consumers also benefit from better services at the point of sale.

in the fashion clothing and fine fragrance industries the degree of externality is likely to be much lower. Still, the incentives to invest may not high enough, since the retailer's benefit from investing in effort sales depends on the margin it receives. Specifically, if the margin is low, then the retailer will invest a small amount. Using an RPM policy, the manufacturer can create a larger margin for the retailer, thus inducing the optimal level of investment (see Section 11.4).

■ **11.5** Vermont Castings is a manufacturer of wood-burning stoves, a somewhat complex product. One of Vermont Castings's dealers once complained about the terms of the relations between the manufacturer and dealers, stating that "the worst disappointment is spending a great deal of time with a customer only to lose him to Applewood [a competing retailer] because of price." Specifically, the dealer lamented

"the loss of 3 sales of V.C. stoves ... to people whom we educated and spent long hours with."[29]

How do you think this problem can be resolved? How would you defend your solution in an antitrust/competition policy court?

Solution: Obviously, this is a case when one retailer makes an investment in sales effort while the other free-rides and gets the customer by charging a lower price. As we have seen in the discussions in sections 11.2 and 11.3, one possible solution to this problem is to use an RPM policy. In this way, the price would be "fixed" at the minimum level, while the retailers would compete in sales efforts to attract customers.

One would expect that the minimum price set in the RPM policy would be high, hence, an antitrust court would not agree with this policy. However, Vermont Casting may argue that, absent the RPM policy, the retailers will have no incentives to further invest in sales effort and to provide services to customers, making them (the customers) worse off. The price may be lower but the services may be poor. On the contrary, by using an RPM policy, the price may indeed increase, but the customer will now benefit from proper services provided at the point of sale. Obviously there is a trade-off between using and not using an RPM policy, with the crucial issue being the level at which the minimum price should be set.

■ **11.6** Should the European Union outlaw the practice of exclusive territories in car dealerships? Why or why not?

Solution: As we saw in Section 11.3 exclusive territories represent an instance of vertical restraints that helps in resolving the inter-retailer externality represented by underinvestment in sales effort. Thus, if awarded an exclusive territory, a car dealer has all the incentives to invest in advertising, educating customers, etc., while absent this policy it would, most likely, underprovide these services. The issue is to quantify the positive and negative effects of such a policy, that is, to observe how prices and service levels are set in areas where this policy is in use compared to areas where it is not.

■ **11.7** Beer producers are wont to impose an exclusive dealing clause on retailers. Discuss the efficiency and market power effects of this practice.

Solution: Exclusive dealing has, obviously, the effect of foreclosing upstream competition, that is, competition between manufacturers, which, a discussed in Sections 11.5 and 11.6, is likely to reduce welfare and increase market power. One possible defense of exclusive dealing

[29] Cf Judge R Posner's opinion, cited

Mathewson, Frank, and Ralph Winter (1998), "The Law and Economics of Resale Price Maintenance," *Review of Industrial Organization* **13**, 57–84..

is that there may be important investments to be made by the manufacturer at the retail store, so that, if there is competition between manufacturers, an externality may appear, leading to sub-optimal investments. In the case of car dealerships, such externality arise in the context of dealer training to be done by the manufactuer. In the case of beer, however, it is unlikely there are significant manufacturer externalities.

■ **11.8*** Two major music companies—Sony and Warner Music—have recently been subject to an antitrust inquiry by the FTC over allegations that they illegally discouraged retail discounting of compact disks. The investigation is centered on the practice of announcing suggested prices. Suggested prices are not illegal—only agreements among firms on such prices are illegal. But in practice retailers that advertise or promote CDs at a price below the suggested price are denied cash payments by the manufacturers, in effect "forcing" such suggested prices.[30]

How would you decide on this case?

Solution: De facto, this situation corresponds to one of RPM, even though it is not explicitly presented as such. The analysis of the costs and benefits from RPM should therefore be applied.

■ **11.9***** Consider the model presented in the beginning of Section **??**, but assume that retailers compete *à la* Cournot. Show that the optimal wholesale price is strictly between marginal cost and monopoly price.

Solution: As in the text, suppose that the upstream firm offers retailers a contract stipulating a fixed fee, f, as well as a wholesale price, w. From Chapter **??**, we know that the equilibrium price under Cournot competition is given by $p^N = \frac{1}{3}a + \frac{2}{3}w$, where w is the effective marginal cost paid by retailers. Output per firm is given by $q^N = \frac{1}{2}(a - p^N) = (a - w)/3$. Finally, equilibrium profit per firm is $\pi^R = (a - w)^2/3$. This implies that the upstream firm can ask for as much as $f = (a - w)^2/3$ as a fixed fee.

The upstream firm's total profit is therefore given by

$$\pi^M = 2\left((w - c)q^N + \pi^R\right) = 2\left(w\frac{a - w}{3} + \left(\frac{a - w}{3}\right)^2\right).$$

Maximizing with respect to w, we get the optimal value $w = \frac{1}{4}a + \frac{3}{4}c$. Notice that the optimal w is a convex combination of a and c, that is, the coefficients of a and c add up to 1. Moreover, from Chapter **??** we know that monopoly profit is given by $p^M = \frac{1}{2}a + \frac{1}{2}c$. Since the relative weight of w on c is greater than the weight of p^M on c (and $c < a$), it follows that w is less than monopoly price. By the same argument, it is also clear that w is greater than marginal cost.

[30] *The Wall Street Journal*, December 16, 1999.

■ **11.10***** Consider the following highly simplified picture of the personal computer industry.

There are many, price-taking firms that assemble computer systems. Call these firms "computer OEMs." ("OEMs" stands for "original equipment manufacturers.") Each of these firms must buy three inputs for each computer system that it sells: (1) a variety of components that are themselves supplied competitively and collectively cost the computer OEM \$500 per computer; (2) the Windows operating system, available only from Microsoft, at a price p_M, to be discussed below; and (3) a Pentium microprocessor, available only from Intel, at a price p_I, also to be discussed below. Since each computer system requires precisely one operating system and one microprocessor, the marginal cost of a computer to an OEM is $500+p_M+p_I$. Assume that competition among OEMs drives the price of a computer system down to marginal cost, so we have $p = 500 + p_M + p_I$, where p is the price of a computer system.

The demand for computer systems is given by $Q = 100,000,000 - 50,000p$.

Microsoft is the sole supplier of the Windows operating system for personal computers. The marginal cost to Microsoft of providing Windows for one more computer is zero.

Intel is the sole supplier of the Pentium microprocessors for personal computers. The marginal cost to Intel of a Pentium microprocessor for one more computer system is \$300.

(a) Suppose that Microsoft and Intel simultaneously and independently set the prices for Windows and Pentium chips, p_M and p_I. What are the Nash equilibrium prices, $\hat{p}_M$ and $\hat{p}_I$?

Now suppose that Microsoft and Intel sit down to negotiate an agreement to sell Windows and Pentium chips as a package to computer OEMs for a package price of p_{MI}.

(b) What package price would maximize Microsoft's and Intel's combined profits? By how much would an agreement between Microsoft and Intel boost their combined profits?

(c) Would final consumers benefit from such an agreement between Microsoft and Intel, or would they be harmed? What about computer OEMs? Relate your answer to your calculations in parts (a) and (b), and explain the economic principles underlying your answer.

Solution: [(a)] First consider Microsoft's best response to any given price p_I by Intel. Using the underlying demand for computers, the demand for Windows is given by $Q = 100,000,000 - 50,000(500 + p_M + p_I)$. For a given value of p_I, the demand for Windows is $Q = 75,000,000 - 50,000p_I - 50,000p_M$. The corresponding marginal revenue for Microsoft is $MR_M = 1500 - p_I - Q/25,000$. Setting this equal to Microsoft's marginal cost of zero gives $q^*_M = 37,500,000 - 25,000p_I$, and the corresponding optimal price of $p^*_M = 750 - p_I/2$. Next, repeat these steps to consider Intel's best response to any given price p_M by Microsoft. The only difference is that Intel has a marginal cost of \$300. These calculations imply that $MR_I = 1500 - p_M - Q/25,000$. Setting this equal to Intel's marginal cost of \$300 gives with the corresponding optimal price of $p^*_I = 900 - p_M/2$. Finally, solve these two equations together to get the Nash Equilibrium prices, which are $p^*_M = \$400$ and $p^*_I = \$700$. Note

that the resulting price of a computer is \$1600, so total computer sales are 20 million.

[(b)] This is a basic monopoly pricing problem for Microsoft and Intel collectively. If they set a package price of p_{MI}, the price of a computer system will be $500 + p_{MI}$. The number of computers sold will be $Q = 100,000,000 - 50,000(500 + p_{MI})$. The marginal revenue corresponding to this demand curve is $MR_{MI} = 1500 - Q/25,000$. Setting this equal to the (combined) marginal cost of \$300 gives a quantity of $Q^*_{MI} = 30,000,000$ and a corresponding package price of $p^*_{MI} = \$900$. At this price, the contribution to Microsoft's and Intel's combined profits is \$600 per computer times 30 million machines, or \$18 billion. In comparison, the Nash Equilibrium in part (a) involved a contribution of \$800 per computer times 20 million machines, or \$16 billion. Cutting a deal is worth \$2 billion to Microsoft and Intel together.

[(c)] Since Windows and Pentium are complements, Microsoft's profits are decreasing in the price of Pentium chips, and Intel's prices are decreasing in the price set by Microsoft. This implies that the two companies together would benefit from lower prices than they would set separately. Indeed, comparing parts (a) and (b) we see a lower price in part (b) than in part (a). Final consumers thus benefit from the cooperation between Microsoft and Intel. OEMs are indifferent, because their profits are driven to zero by competition, whatever the prices of components. (In practice, OEMs would benefit in the short run from the lower input prices, and OEMs able to differentiate themselves with their own brand names would benefit for a longer period of time.) The underlying principle is that cooperation among suppliers of complements tends to benefit consumers, just as cooperation among suppliers of substitutes (i.e., collusion) harms consumers. This is closely related to the theory of "double marginalization" that we discussed in this chapter; the only difference is that Microsoft and Intel stand in a "complements" relationship rather than a buyer/seller relationship.

■ **12.2** Empirical evidence suggests that, during the 1970s, a firm with an IBM 1400 was as likely as any other firm to purchase an IBM when making a new purchase, while a firm with an IBM 360 was more likely to purchase an IBM than a firm that did not own an IBM 360. Software for the IBM 1400 could not run on the succeeding generations of IBM models (360, 370, 3000, and 4300), while software for the IBM 360 could run on the 370, 3000 and 4300.[31]

How do you interpret these results?

Solution: These results suggest how backwards compatibility influences the degree of switching cost. Switching away from an IBM 1400 was less costly because there was no backwards compatibility between later models and the software developed for the IBM 1400. The same was not thue for the 360,370, 3000 and 4300 models. Consumers who bought one of these models had a higher opportunity cost of switching to a non-IBM computer. As

[31] See

Greenstein, Shane M. (1993), "Did Installed Base Give an Incumbent Any (Measurable) Advantages in Federal Computer Procurement?," *Rand Journal of Economics* **24**, 19–39..

we would expect, these consumers were more likely to buy IBM in the future than other consumers.

■ **12.3** Says a market analyst in Brussels:

> I think the euro [the new European single currency] will bring lower prices over all but that the price differences will be more or less the ones we have right now.

Do you agree? Why or why not?

Solution: As discussed in Section 12.4, there is significant price dispersion across European countries. Cross-country differences are partly due to price discrimination, partly to taxation and regulation, and partly to search costs (and possibly other factors). Search costs may be reduced because consumers are no longer confused by transforming prices from one currency to another. Therefore, the reduction in search costs should reduce the market power of firms (perhaps not to a great extent, though), resulting in lower prices. Overall differences will, however, persist, due to the above mentioned price discrimination, taxes, regulations, etc.

■ **12.4** A study on retail price for books and CDs finds that price dispersion (weighted by market shares) is lower for internet retailers than for conventional retailers.[32] Discuss.

Solution: Lower price dispersion may result from two factors. First, it is easier to obtain information about online store prices than it is about conventional retailers. Second, online stores have one less dimension of differntiation with respect to traditional stores: geographical location.

As shown in Section 12.4, imperfect information leads to higher prices and possible price dispersion. In Section 12.2, we argued that product differentiation leads to higher prices. And, although this was not formally shown, product differentiation may also lead to price dispersion.

In summary, lower price dispersion by online sellers may result both from imperfect information and product differentiation. In fact, one of the points of this chapter is that the effects of imperfect information and product differentiation are often similar.

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Brynjolfsson, E., and M. Smith (1999), "Frictionless Commerce? A Comparison of Internet and Conventional Retailers," Working Paper, MIT..

■ **12.5** "Price dispersion is a manifestation — and indeed it is a measure — of ignorance in the market."[33] Do you agree? Compare with possible alternative explanations for price dispersion.

Solution: If we consider search costs as being a measure of market ignorance, then indeed the above claim holds. As in note f) in section 12.4, quotation of prices in different currencies makes comparison shopping more difficult by increasing "search costs". The fact that buyers do not know or bother to learn how to transform prices from one currency into another is a sign of ignorance, which supports price dispersion.

Other alternative explanations for price dispersion may be: price discrimination, different regulatory or taxation regimes (geographical price dispersion) or different shopping experience (see the example for CDs bought in a small music shop or in a supermarket).

■ **12.6** Consider the model of price dispersion sketched in Section **??**. Show that there can be at most two different prices in equilibrium.

Solution: See S. Salop and J. Stiglitz, "Bargains and Ripoffs," Review of Economic Studies 44 (1976), 493–510; or the summary discussion in H. Varian, Microeconomic Analysis, New York: Norton (1978), Chapter 8.

■ **12.7*** Two firms are engaged in Bertrand competition. There are 10,000 people in the population, each of whom is willing to pay at most 10 for at most one unit of the good. Both firms have a constant marginal cost of 5. Currently, each firm is allocated half the market. It costs a customer s to switch from one firm to the other. Customers know what prices are being charged. Law or custom restricts the firms to charging whole-dollar amounts (e.g., they can charge 6, but not 6.50).

(a) Suppose that $s = 0$. What are the Nash equilibria of this model? Why does discrete (whole-dollar) pricing result in more equilibria than continuous pricing?

(b) Suppose that $s = 2$. What is (are) the Nash equilibrium (equilibria) of this model?

(c) Suppose that $s = 4$. What is (are) the Nash equilibrium (equilibria) of this model?

(d) Comparing the expected profits in (b) to those in (c), what is the value of raising customers' switching costs from 2 to 4?

Solution:

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[33] Stigler, George (1961), "The Economics of Information," *Journal of Political Economy* **69**, 213–225..

(a) There are three Nash equilibria: (1) both firms charge $p = 5$, (2) both firms charge $p = 6$, and (3) both firms charge $p = 7$. The reason whole-dollar pricing results in multiple Nash equilibria is that one has to undercut by a discrete amount, not by just a fraction of a cent.

(b) Now to undercut your rival, you must drop price by at least 3 to get the whole market. (If you undercut by 2, you get half the other's customers). There is only one Nash equilibrium: both firms charge $p = 10$.

(c) Same as in part (b). One Nash equilibrium: both firms charge $p = 10$.

(d) There is no advantage to further increasing switching costs once $s = 2$.

■ **12.8*** Twenty five different stores sell the same product in a given area to a population of two thousand consumers. Consumers are equally likely to first visit any of the twenty five stores. Half of the consumers have no search costs and purchase at the lowest price. The other half is willing to buy one unit of the product up to a maximum of \$70 and must incur a cost of \$44 in order to find out about the prices charged by other stores. Each store can sell up to 50 units and has a unit cost of \$25.

(a) Show that, in equilibrium, there exist at most two different prices.

(b) Show that, if there exist two different equilibrium prices, then the higher price must be 70.

(c) Show that the following is an equilibrium: 5 firms set a price of 70 and the remaining 20 firms set a price of 45.

Solution: **a.** As in Exercise 12.6;

b. If the high price is lower than 70, a firm that deviates by slightly increasing price does not lose market share since consumers are not willing to pay the search cost. Therefore, the firm is strictly better off. Hence, all firms would want to deviate upwards, so that the high price must be 70.

c. [There are two typos in this problem: each store's capacity is given by 90 units, not 50. Moreover, consumers with zero search cost have willingness to pay of 45.]

First notice that, given the search costs for first type of consumers, we can safely assume that these consumers will not search, rather will compare price to their willingness to pay (70).

At the proposed prices, profits are as follows: for a firm setting $p = 70$, demand is given by 1,000/25=40 and total profit is $40(70 - 25) = 1,800$. For a firm setting $p = 45$, total demand is $1,000/25 + 1,000/20 = 90$ and total profit is $90(45 - 25) = 1,800$.

A $p = 45$ firm could deviate by setting a lower price. It would get more demand but, since it is selling at capacity, profit would be lower. It could set a higher price but would then only keep the high valuation consumers. It could at most make a profit equal to the profit currently earned by the $p = 70$ firms, which in turn is equal to its current profit. We thus conclude that such firm would not want to charge a different price.

A $p = 70$ firm could deviate by setting a lower price. Any price below 70 and above 45 leads to the same demand but a lower margin. By setting a price equal or lower than 45, the firm would get less than what $p = 45$ currently get, which in turn is the same as a $p = 70$ firm currently gets.

■ **13.1** Explain how advertising expenditures with no direct informational content can increase market efficiency.

Solution: As discussed in Section 13.1, advertising expenditures may signal product quality. In the presence of repeat purchases, a firm that produces a high-quality good and sells the good not only in the present but also in the future, will have more to gain from getting customers to try its product than a firm that produces a low-quality good. This is because once a good is purchased, consumers become aware of its quality; in the future they will buy the high-quality good. If, however, a consumer does not get to try the good in the present, in the future he or she will still be uncertain about the good's quality. Therefore, high-quality goods producers will try to lure customers in the present since their gain is higher. They thus have an incentive to differentiate themselves from low-quality goods producers.

Although advertising has no direct informational content, the equilibrium with advertising may be more efficient than the equilibrium without advertising. Absent advertising, high-quality firms have no incentive to produce, since they cannot differentiate themselves; their products are ex-ante identical to the ones produced by low-quality firms. Therefore, if consumers value high-quality goods, even if there are savings in advertising expenditures, the overall efficiency effect may be negative, due to the loss in the availability of high-quality goods.

■ **13.2*** Empirical evidence suggests that the probability of a household switching to a different brand of breakfast cereal is increasing in the advertising intensity of that brand. However, the effect of advertising is significantly lower for households who have previously tried that brand.[34] What does this suggest about the nature of advertising expenditures (persuasion vs information)?

Solution: To answer this question one can simply parallel the explanation provided in Box 13.1. The effect on the probability of switching is high when the consumer did not try the product before and low if the consumer has already tried the product. This is consistent with the hypothesis that advertising has an informative effect.

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Shum, Matthew (1999), "Advertising and Switching Behavior in the Breakfast Cereals Market," University of Toronto..

■ **13.3** Consider the following industries: pharmaceuticals, cement, perfumes, fast food, compact cars. How would you expect them to be ordered by advertising intensity? Why?

Solution: According to the Dorfman-Steiner formula, advertising intensity is proportional to the demand elasticity of advertising expenditures and inversely proportional to the price elasticity of demand. Price elasticity of demand is lowest for pharmaceuticals and perfumes, highest for cement. Advertising elasticity is lowest for cement, highest for perfumes (and some pharmaceuticals). We would expect advertising intensity to be highest for pharmaceuticals and perfumes, lowest for cement, intermediate for fast food and compact cars.

■ **13.4** In Section **??**, it was argued that advertising intensity under duopoly should be greater than under monopoly. DeBeers, the dominant firm in the diamond industry (a cartel that in many respects is like a monopoly), spends vast resources on advertising. More recently, DeBeers has also started to advertise diamonds *and* the name DeBeers. Is this consistent with the analysis of Section **??**? What aspects of the diamond industry are not reflected in the analysis of Section **??**?

Solution: The value of diamonds is, to a great extent, a consequence of the perception of scarcity. Advertising has played a very important role in the diamond industry, both by increasing demand and by inducing a perception of scarcity. In this sense, there is a strong "public good" element in the advertising of diamonds. By controlling the distribution of diamonds, DeBeers is able to internalize this extrenality. Recent events in the industry (the cartel defection of the Australian mines and the emergence of non-cartel mines in Canada) is likely to lead to a more fragmented market structure. The "public good" effect would then imply lower levels of advertising. However, with DeBeers controlling a smaller market share, the market-share-shifting effect of advertising is now more important, leading possibly to higher levels of advertising. Finally, in addition to changes in the *level* of advertising we are also likely to observe a shift in the nature of the advertising expenditures, with a greater emphasis on branding and less on generic characteristics of diamonds.

■ **13.5** Which of the two cars, BMW series 5 and Nissan Sentra, would you expect to have a greater price elasticity? Based on this, which car would you expect to have a greater advertising to sales ratio? Is the empirical evidence consistent with this?

Solution: One would expect the price elasticity of demand to be higher for compact cars, both because branding effects are likely to be smaller in this price range and because the number of competing models is greater. See Box 12.1 for data from the US car market.

Table 1: Advertising, income and price elasticities in specific industries.

Industry	Income	Price	Advertising Short-run	Advertising Long-run
Bakery products	.757	−.263	.223	.265
Books	2.205	−.774	.250	.348
Canning	.359	−.820	.614	.963
Cereals and grain mill products	.177	−1.469	.224	.320
Cigars and cigarettes	.001	−1.809	.408	.575
Costume jewelry	−1.407	−3.007	.282	.307
Distilled liquor	.179	−.253	.641	.745
Drugs	.719	−1.079	.663	1.042
Jewelry (precious metal)	1.792	.661	.147	.201
Malt liquor	−.184	−.562	.004	.010
Soaps	1.684	−.758	.284	.294
Soft drinks	2.008	−1.478	.567	.591
Wines	.407	−.680	.972	1.202

■ **13.6** Consider the values in Table 1. In which industries do you expect advertising intensity to be higher?

Solution: We know from Equation 13.1 that $\frac{a}{R} = \frac{\eta}{\varepsilon}$, that is, advertising intensity is proportional to advertising elasticity and inversely proportional to price elasticity. Therefore, for the table in the exercise we have:

- Bakery products: ;
- Books: $\frac{a}{R} = \frac{.223}{.263} = 0.85$;
- Canning: $\frac{a}{R} = \frac{.250}{.774} = 0.32$;
- Cereals and grain mill products: $\frac{a}{R} = \frac{.614}{.820} = 0.75$;
- Cigars and cigarettes: $\frac{a}{R} = \frac{.224}{1.469} = 0.15$;
- Costume jewelry: $\frac{a}{R} = \frac{.408}{1.809} = 0.225$;
- Distilled liquor: $\frac{a}{R} = \frac{.282}{3.007} = 0.09$;
- Drugs: $\frac{a}{R} = \frac{.663}{1.079} = 0.61$;
- Jewelry (precious metal): $\frac{a}{R} = \frac{.147}{.661} = 0.22$;
- Malt liquor: $\frac{a}{R} = \frac{.004}{.562} \approx 0$;
- Soaps: $\frac{a}{R} = \frac{.284}{.758} = 0.37$;
- Soft drinks: $\frac{a}{R} = \frac{.567}{1.478} = 0.38$;

- Wines: $\frac{a}{R} = \frac{.972}{.680} = 1.43$.

All of the above are computed for the short-run advertising elasticity of demand.

■ **13.7**** Your company sells expensive, branded fountain pens. Currently, there are 100,000 people aware of your pens. Each of these 100,000 people has his or her own willingness to pay for your pens. These willingness-to-pay numbers are uniformly distributed between \$0 and \$500. So, your demand curve is given by $Q = 100000(1-p/500)$. Your marginal cost per pen is \$100. Well-versed in economics, you are pricing your pens at \$300 each, and selling 40,000 pens, generating a contribution of \$8 million.

You have just become brand manager for these fountain pens. The previous brand manager engaged in very little advertising, but you are considering running a major promotional campaign to build your brand image and visibility. Your are considering two possible advertising campaigns, call them "Build Value," "Expand Reach." (You will either run one of these campaigns or none at all; you cannot run both.)

The "Build Value" campaign will not reach any new potential customers, but will increase the willingness to pay of each of your current 100,000 existing customers by 25%. This campaign costs \$2.5 million to run.

The "Expand Reach" campaign will expand the set of potential customers by 25%, from 100,000 to 125,000. The 25,000 new customers reached will have the same distribution of willingness-to-pay as the pre-existing 100,000 potential customers (namely, uniformly distributed between \$0 and \$500). This campaign costs \$1.8 million to run.

(a) If your choice were between running the "Build Value" campaign and running no campaign at all, would you choose to run the "Build Value" campaign?

(b) If your choice were between running the "Expand Reach" campaign and running no campaign at all, would you choose to run the "Expand Reach" campaign? Show your calculations.

(c) What choice would you make in this situation: run the "Build Value" campaign, run the "Expand Reach" campaign, or run neither?

Solution:

(a) If you run the "Build Value" campaign, the willingness-to-pay of your 100,000 potential customers will be uniformly distributed between \$0 and \$625, since \$625 is 25% higher than \$500. Thus, your demand will shift from $Q = 100,000(1 - p/500)$ to $Q = 100,000(1 - p/625)$. Put differently, demand will shift from $p = 500(1 - Q/100,000)$ to $p = 625(1 - Q/100,000)$. With this new demand curve, the corresponding marginal revenue curve is $MR = 625(1-Q/50,000)$. Setting MR equal to the marginal cost of \$100 and solving for Q gives $Q^* = 42,000$. The corresponding price is $p^* = \$362.50$. This generates a contribution of \$11,025,000, or \$3,025,000 higher than without the campaign. Since this exceeds the \$2.5 million cost of the campaign, the "Build Value" campaign is worth running, rather than no campaign at all.

(b) If you run the "Expand Reach" campaign, you will now face 125,000 customers with willingness-to-pay uniformly distributed between \$0 and \$500. Thus, your demand will shift from $Q = 100,000(1 - p/500)$ to $Q = 125,000(1 - p/500)$. Solving for p gives $p = 500(1 - Q/125,000)$, with corresponding marginal revenue of $MR = 500(1 - Q/62,500)$. Setting this equal to the marginal cost of \$100 and solving for Q gives $Q^* = 50,000$. The corresponding price is $p^* = \$300$. This generates a contribution of \$10,000,000, or \$2,000,000 higher than without the campaign. Since this exceeds the \$1.8 million cost of the campaign, the "Expand Reach" campaign is worth running, rather than no campaign at all.

(c) In comparison with running no campaign, the "Build Value" campaign adds \$525,000 to profits. In comparison with running no campaign, the "Expand Reach" campaign adds \$200,000 to profits. Since you can only pick one, you should pick the "Build Value" campaign.

■ **13.8**** The effect of advertising expenditures can be decomposed into (a) effect on total market demand and (b) effect on market shares. Accordingly, the following cases can be distinguished, where q_i is firm i's demand and a_i its advertising expenditure:[35]

Cooperative advertising:	$\partial q_j / \partial a_i > 0$
Predatory advertising:	$\partial q_j / \partial a_i < 0$
Perfectly cooperative advertising:	$\partial q_i / \partial a_i = \partial q_j / \partial a_i$
Completely predatory advertising:	$\partial q_i / \partial a_i + \partial q_j / \partial a_i = 0$

Empirical studies suggest the following values of demand elasticity with respect to advertising levels.[36]

	Advertising Elasticity	
Product	Own	Cross**
Coca Cola	.25	−.06
Pepsi Cola	.32	−.62
Saltine crackers*	.16	−.05
High-tar cigarettes	.005***	−.001***

* Long-run elasticity for major brands.

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Friedman, James (1983), "Advertising and Oligopolistic Equilibrium," *Bell Journal of Economics* **14**, 464–373..

36 Source:

Roberts, Mark, and Larry Samuelson (1988), "An Empirical Analysis of Dynamic, Nonprice Competition in an Oligopolistic Industry," *Rand Journal of Economics* **19**, 200–220..

Gasmi, F, Jean-Jacques Laffont, and Quang Vuong (1992), "Econometric Analysis of Collusive Behavior in a Soft-Drink Market," *Journal of Economics, Management and Strategy* **1**, 277–312..

Slade, Margaret E (1995), "Product Rivalry with Multiple Strategic Weapons: An Analysis of Price and Advertising Competition," *Journal of Economics, Management and Strategy* **4**, 445–476..

** Cross elasticity is the elasticity of q_i with respect to a_j.
*** NB: these are derivatives of market share with respect to advertising level.

Based on the above classification, how do you characterize advertising expenditures on cola drinks, saltine crackers and cigarettes?

Solution: These are all instances of predatory advertising since the cross-elasticities are negative, that is, increasing advertisisng decreases the market share of competitors.

■ **14.1** Explain in words why the number of firms in a free-entry equilibrium may be less than proportional to market size.

Solution: The explanation lies in the fact that as the number of firms increases, so does competition. As a result, prices will fall, reducing the margin, $p - c$. Therefore, variable profit per unit of market size decreases, making the number of firms the market can sustain increase less than proportionally to market size.

■ **14.2*** Suppose that two countries, initially in autarchy, decide to create a single market. For simplicity, assume that, in both economies, there is only one product. Demand for this product is given by $D_i = S_i(a - p_i)$, $(i = 1, 2)$, where S_i is a measure of country i's size. Upon the creation of a single market, total demand is given by the horizontal sum of the two initial demands.

Assuming there is free entry and that firms compete *à la* Cournot, determine the equilibrium number of firms in autarchy and after the completion of the single market. Interpret the results.

Solution: In autarchy we have $p_i = a - \frac{D_i}{S_i}$. Assuming that the cost function takes the form $c(q_{ik}) = F + cq_{ik}$ (where i indexes the country and k indexes the firm), each firm solves the problem $\max[(p - c)q_{ik} - F]$ which is equivalent to $\max[(a - c - \frac{\sum_k q_{ik}}{S_i})q_{ik} - F]$. The first order condition is given by $a - c - \frac{(n+1)q_{ik}}{S_i} = 0$ (due to the symmetry assumption), therefore, we have the solution for each firm's quantity $q_{ik} = \frac{(a-c)S_i}{n+1}$. The profits for each firm will be $\pi_i(n_i) = \left(\frac{a-c}{n_i+1}\right)^2 S_i - F$. In a free entry equilibrium these profits should be 0. Therefore we have the solution: $n_i = \left[(a - c)\sqrt{\frac{S_i}{F}} - 1\right]$.

After the completion of the single market the size of the market increases, and, as it is assumed, demand becomes $D_{1+2} = D_1 + D_2 = (S_1 + S_2)(a - p)$. Using the same general formula that we derived for the autarchy case we obtain that $n_{1+2} = \left[(a - c)\sqrt{\frac{S_1+S_2}{F}} - 1\right]$. This tells us that some firms will exit, the explanation for this being the same as in Exercise 14.1.

■ **14.3*** The number of imported automobiles in California is four times higher than in Montana, in per capita terms. The population of Californian is mainly urban, whereas the population of Montana is mainly rural. How do demographic differences and the model presented in Section **??** explain the differences in consumption patterns?[37]

Solution: The model predicts that smaller markets will have fewer firms and higher margins. The fact that the population of Montana is mainly rural implies that the typical market for a car dealer is smaller than in California.

■ **14.4*** Retail in Switzerland is mostly dominated by highly profitable cartels. The Swiss authorities anticipate the gradual collapse of these cartels as the country becomes better integrated with the rest of Europe. OECD, by contrast, hold a more sceptical view, claiming that the collapse of cartels does not necessarily lead to more competitive markets; rather, they add, cartel breakdowns are frequently associated with an increase in concentration. Which prediction seems more reasonable? Are the two views inconsistent?

Solution: Integration is likely to imply greater competition from foreign suppliers. Lower margins will then imply that the Swiss market cannot hold the same number of firms as currently. It is therefore possible that the two predictions hold true: that prices go down and that the industry becomes more concentrated.

■ **14.5** "Barriers to entry may be welfare improving." What particular industry characteristics might make this statement valid?

Solution: Following the discussion in Section 14.3, free entry may decrease welfare when the business stealing effect dominates. For this to happen, as in the example of retail banking, the product or service should be relatively homogenous (so that product differentiation is unimportant) and price competition should be soft. In this case, paying a fee for setting up a branch represents a barrier to entry and may act as an efficient means of blocking excessive entry.

■ **14.6**** Show that the coefficient of scale economies, AC/MC, is greater than one if and only if average cost is decreasing.

[37] Adapted from an exercise written by T. Bresnahan.

Solution: Average Cost is given by the ratio Cost / Output. Taking the derivative with respect to Output q, we get

$$\frac{d\,AC}{d\,q} = \frac{d}{d\,q}\frac{C}{q} = \frac{\frac{d\,C}{q}q - C}{q^2} = (MC - AC)/q,$$

It follows that AC is greater than MC if and only if $\frac{d\,AC}{d\,q} < 0$, that is, average cost is decreasing.

■ **14.7***** Consider the model presented in Section **??**. Suppose that firms can choose one of two possible technologies, with cost functions $C_i = F_i + c_i q$.

a) Derive the conditions for a free-entry equilibrium.

b) Show, by means of numerical example, that there can be more than one equilibrium, with different numbers of large and small firms.

Solution: a) Suppose that demand is given by $Q = a - p$. There are two types of firms, Firm i's profit is given by $(a-Q)q_i - C_i$. The first-order condition for profit maximization is $q_i = a - c_i - Q$. Suppose that in equilibrium each of the n_i firms with technology i product output q_i. Then $Q = n_1 q_1 + n_2 q_2$. Solving the system of first-order conditions, we get

$$q_i^*(n_1, n_2) = \frac{a - c_j - n_i(c_j - c_i)}{1 + n_1 + n_2},$$

for $i, j = 1, 2$ and $i \neq j$. From these equations, we can get $Q^*(n_1, n_2)$, the equilibrium total output when there are n_i firms of each type:

$$Q^*(n_1, n_2) = n_1 \frac{a - c_2 - n_1(c_2 - c_1)}{1 + n_1 + n_2} + n_2 \frac{a - c_1 - n_2(c_1 - c_2)}{1 + n_1 + n_2}.$$

The equilibrium conditions are then given by

$$\begin{aligned}
\Big(a - Q^*(n_1, n_2)\Big)\, q_i^*(n_1, n_2) &\geq F_i + c_i q_i^*(n_1, n_2) \\
\Big(a - Q^*(n_i + 1, n_j)\Big)\, q_i^*(n_i + 1, n_j) &\leq F_i + c_i q_i^*(n_i + 1, n_j) \\
\Big(a - Q^*(n_i + 1, n_j - 1)\Big)\, q_i^*(n_i + 1, n_j - 1) - & \\
\Big(F_i + c_i q_i^*(n_i + 1, n_j - 1)\Big) &\leq \Big(a - Q^*(n_1, n_2)\Big)\, q_j^*(n_1, n_2) \\
& \quad - \Big(F_j + c_j q_j^*(n_1, n_2)\Big)
\end{aligned}$$

The first of these conditions implies that incumbent firms make positive profits. The second condition implies that a potential entrant would make negative profits. The third condition implies that an incumbent would not gain from switching technologies. Note that all conditions apply for $i = 1, 2$. We thus have a total of six equilibrium conditions.

b) The following values satisfy the equations above: $a = 1000, F_1 = 173, c_1 = 0, F_2 = 10.3, c_2 = 10, n_1 = 60, n_2 = 60$.

■ **14.8*** Consider the monopolistic competition model, presented in Chapter **??**. What is, according to this model, the relation between the degree of product differentiation and market structure?

Solution: Refer to Figure 6.3 the solution to Exercise 6.2. The greater the degree of product differentiation, the steeper the demand curve d faced by each firm. In the long run, price equal average cost. Therefore, the steeper d is the lower each firm's output is in the long run equilibrium. We would therefore expect a more fragmented market structure when the degree of product differentiation is higher.

■ **14.9**** T. Bresnahan and P. Reiss collected data for small, geographically isolated U.S. towns, on population as well as on the number of doctors, dentists, plumbers, etc., in each town. Based on these data, they estimated that the minimum town size that justifies the entry of a second doctor is approximately 3.96 times the required size for the first doctor to enter. For plumbers, the number is 2.12. How can these numbers be interpreted?

Solution: The higher number for doctors has two interpretations. The first one is that competition between two doctors is very intense, so that it would take a much larger market before the second doctor could recoup entry costs. The second interpretation is that there are specific barriers to entry by a second doctor which are not present in the case of a plumber.

■ **14.10**** Derive Equation (**??**).

Solution: $\pi_i = (p - c)q_i - F$ and $p = a - \frac{Q}{S}$, therefore, $\pi_i = (a - c - \frac{\sum q_j}{S})q_i - F$. The first-order condition for profit maximization is $a - c - \frac{\sum q_j}{S} - \frac{q_i}{S} = 0$. Using the symmetry assumption, we get $q_i = \frac{(a-c)S}{n+1}$. Plugging this into the profit function we obtain $\pi_i = (\frac{a-c}{n+1})^2 S - F$.

■ **14.11***** Consider the following model of entry into an advertising-intensive industry. To simplify the analysis, and to concentrate on the effects of advertising, suppose that there is no price competition. Specifically, the value of the market, in total sales, is given by S. (One can think of a demand curve $D(p)$ and an exogenously given price, whereby $S = pD(p)$.) S is therefore a measure of market size .

Each firm must decide whether or not to enter the industry. Entry cost is given by F. If a firm decides to enter, then it must also choose how much to invest in advertising; let a_i be the amount chosen by firm i. Finally, firm i's market share, s_i, is assumed to be equal to its share of the industry total advertising effort:

$$s_i = \frac{a_i}{\sum_{i=1}^{n}} = \frac{a_i}{A},$$

where n is the number of firms in the industry and $A \equiv \sum_{i=1}^{n}$ is total industry advertising.

(a) Show that each firm i's optimal level of advertising solves

$$\frac{A - a_i}{A^2} S - 1 = 0.$$

(b) Show that, in a symmetric equilibrium,

$$a = \frac{n-1}{n^2} S.$$

where a is each firm's level of advertising.

(c) Show that equilibrium profit is given by

$$\pi = \frac{S}{n^2}.$$

(d) Show that the equilibrium number of entrants is given by

$$\hat{n} = \left[\sqrt{\frac{S}{F}} \right],$$

where $[x]$ means the highest integer lower than x.

(e) Interpret this result in light of the previous discussion on the effects of endogenous entry costs.

Solution: a) The profit of each firm is given by $\pi_i = pq_i - a_i - F = pQ\frac{q_i}{Q} - a_i - F = Ss_i - a_i - F$. Therefore, each firm is solving $\max[S\frac{a_i}{\sum a_j} - a_i - F]$. The first-order condition is given by $\frac{S}{A} - \frac{Sa_i}{A^2} - 1 = 0$, which is equivalent to $\frac{S(A-a_i)}{A^2} - 1 = 0$.

b) In a symmetric equilibrium we have $a_i = \frac{A}{n}$ and using the result from a) we obtain $a = \frac{S(n-1)}{n^2}$.

c) $\pi = \frac{S}{n} - \frac{S(n-1)}{n^2} - F = \frac{S}{n^2} - F$.

d) The equilibrium requires profits to be 0, hence we have $\pi = \frac{S}{n^2} - F = 0$ so that $n = [\sqrt{\frac{S}{F}}]$.

e) With this specification of the model we have, from b), that advertising expenditures increase with market size. This is an instance of endougenous entry costs, where because of this costly investment in advertising, the net industry profit grows by less than the market size (as can also be observed form c)). As a result, even if price is exogenously given, as it is in our model, the number of firms increases by less than the market size, as the result in d) shows.

■ **15.1*** LC Burgers is currently the sole fast-food chain in Linear city, a city that is one mile long and consists of one street, with one thousand consumers distributed uniformly along the street. The price for the BigLC, the only product sold by the LC Burger chain, is set nationally at \$4, so that the local Linear city manager's decision is limited to choosing the number and location of its stores.

Each store costs \$600,000 to open and lasts indefinitely. Each consumer buys one burger per week at the current price of \$4. However, no consumer will walk for more than a quarter of a mile to buy a burger. Operating costs are \$1 per burger. The interest rates is 0.1% per week. The market conditions are unchanging, so present discounted profits can be regarded as level perpetuities.

(a) Suppose that LC Burgers faces no competition and no threat of entry. How many stores should LC Burgers open, and at what locations?

CS Burgers is contemplating entering Linear city. CS Burgers' costs and price are the same as those of LC Burgers. Moreover, consumers regard the products at both chains as equally good, so, if both brands are in town, each consumers buys from the closest store.

(b) At what locations should CS Burgers open stores, given that LC Burgers has opened the locations found to be optimal in part (a)?

(c) Recognizing the threat of entry by CS Burgers, at what locations should LC Burgers open stores?

(d) Would your analysis of these product-location decisions be affected if you also considered the possibility of pricing competition, i.e., if prices were then set independently given the locations of the stores (rather than taking prices as fixed, as was done above)?

(e) Moving beyond this particular model, does product positioning involve a first-mover advantage, a second-move advantage, or does this depend upon particular aspects of the market in question?

Solution: With two stores, one at .25 and the other at .75 (miles from the left end of the street), LC Burgers is able to cover the entire market. Any additional store would not increase demand and would thus be sub-optimal. By opening two stores, LC Burgers makes a discounted profit of $1000(\$4-\$1)/.1\%-2\$600,000 = \$1.8$m. If LC Burgers were to open one store only, the maximum it could possibly get is $500*(\$4-\$3)/.1\%-\$600,000 = \$900,000$.

CS Burgers is contemplating entering Linear city. CS Burgers' costs and price are the same as those of LC Burgers. Moreover, consumers regard the products at both chains as equally good, so, if both brands are in town, each consumers buys from the closest store.

(b) CS Burger should open four stores, to the immediate left and right of LC Burger's stores, thus stealing all of the market demand. Given these locations, CS Burger would receive a demand of 1,000 and a discounted profit (net of entry costs) of $1,000(\$4 - \$3)/.1\% - 4\$600,000 = \$600,000$. Notice that, under this outcome, LC Burger's profit is $-\$1.2$m (two stores, no revenues).

(c) LC Burger should open three stores and locate them at .1666, .5 and .8333. Given these locations, the maximum an entrant can get is one sixth of the market (check). Given this demand, discounted profits are $166(\$4 - \$3)/.1\% - \$600,000 = -\$100,000$. Under this location strategy, LC Burgers gets a total profit of $1,000(\$4 - \$3)/.1\% - 3\$600,000 = \$1.2$m. This is substantially less than (unchallenged) monopoly profits (as in (a)). However, it is more than LC Burgers would get by choosing the same locations as in (a) ($-\$1.2$m).

Notice that, while these locations are optimal, they are not the only optimal solution. The important thing is that an entrant cannot achieve a market share of 20% or more, where 20% is the minimum market share necessary to recover entry costs (check). Therefore, any solution with a store between .1 and .2, one at .5, and a third one between .8 and .9 would also be optimal. The solution proposed above, however, is the only three-store solution that deters entry when entry costs are as low as \$500,000.

(d) If there were price competition, then we would expect firms not to locate their stores very close to each other. In particular, CS Burger's entry strategy in (b). would unlikely take place as firms would then compete as in the Bertrand model, yielding zero profits for incumbent and entrant.

(e) In the case considered above, there is clearly a first-mover advantage: the first-mover makes positive profits, whereas the second mover stays out of the market and makes zero profits. Suppose however that each firm has limited resources and can open on store only. Then it can be seen that, whichever location the first firm chooses, the second firm can choose a location that gives it profits at least as large.

■ **15.2** In less than one year after the deregulation of the German telecommunications market at the start of 1998, domestic long-distance rates have fallen by more than 70%. Deutsche Telekom, the former monopolist, accompanied some of these rate drops by increases in monthly fees and local calls. MobilCom, one of the main competitors, fears it may be unable to match the price reductions. Following the announcement of a price reduction by Deutche Telekom at the end of 1998, shares of MobilCom fell by 7%. Two other competitors, O.tel.o and Mannesmann Arcor, said they would match the price cuts. VIAG Interkom, however, accused Telekom of "competition-distorting behavior," claiming the company is exploiting its (still remaining) monopoly power in the local market to subsidize its long-distance business.[38]

[38] *International Herald Tribune*, December 29, 1998.

Is this a case of predatory pricing? Present arguments in favor and against such assertion.

Solution: One culd indeed argue that this is a case of predatory pricing. If Deutsche Telekon has monopoly in local markets, it likely has financial resouces strong enough to afford losing money in the long distance market by pricing below marginal cost. However, since there are two other competitors that matched Deutsche Telekom's prices, one can argue that there exists technology with marginal cost less than the low-price charged. Evidently, other explanations can also invoked, namely low-cost signaling and reputation for toughness. (See the discussion in the chapter.)

■ **15.3** "The combined output of two merging firms decreases as a result of the merger." True or false?

Solution: If the merger implies little or no cost efficiencies (namely at the level of marginal cost), we would expect the combined output of the merging firms to decline. If however the merger reduces the marginal cost of the combined firm significantly, then it is possible that the combined output increases as a result of the merger.

■ **15.4*** One of the efficiencies created by mergers in the paper industry results from reorganization of production. A machine is more efficient the narrower the range of products it produces, among other reasons because the length of each production run can be made longer.

The paper industry underwent a wave of mergers in the 1980s. Of the firms that merged, about two thirds increased their market share as a result of the merger. Assuming that (i) firms compete by setting production capacity and (ii) paper products are relatively homogeneous across firms, explain how the previous paragraph explains the pattern of changes in market shares. Which firms would you expect to increase their market share?[39]

Solution: According to the paragraph, there are increased cost efficiencies from mergers. Applying the analysis from Section 15.3, it seems that for two thirds of the merging firms the cost efficiencies were so big that the merging firms increased their output and market share, while for the rest the efficiencies were not big enough, resulting in a decreased market share.

39

Pesendorfer, Martin (1998), "Horizontal Mergers in the Paper Industry," Department of Economics, Yale University, September..

■ **15.5** "The renewed prospect of a link-up between British Aerospace PLC and the Marconi defense arm of General Electric Co. PLC of the U.K. as led to revived talks between the top defense companies in Germany and France."[40] Discuss.

Solution: Refer to the discussion on merger waves in this chapter.

■ **15.6** Consider a homogeneous product industry with inverse demand given by $p = 100 - 2Q$. Variable cost is given by $C = 10q$. There is currently one incumbent firm and one potential competitor. Entry into the industry implies a sunk cost of F.

(a) Determine the incumbent's optimal output in the absence of potential competition.

(b) Suppose the entrant takes the incumbent's output choice as given. Show that the entrant's equilibrium profit is decreasing in the incumbent's output.

(c) What output should the incumbent firm set in order to deter entry?

(d) Assuming that the incumbent firm decides to deter entry, determine the Lerner index as a function of F. Discuss the result.

(e) Determine the lowest value of E such that the incumbent firm prefers to deter entry.

Solution: a) The incumbent solves $\max[pq - c(q)] = \max[(100 - 2q)q - 10q]$. The first order condition is $90 - 4q = 0$ and the solution is $q = 22.5$.

b) Taking the incumbent's output choice as given, the potential entrant solves the following problem: $\max[100 - 2(q_i + q_e)]q_e - 10q_e - F$. The first-order condition is given by $90 - 2q_i - 4q_e = 0$, and the solution is $q_e = 22.5 - \frac{q_i}{2}$. Plugging this result into the entrant's profit function we obtain $\pi_e = 2(22.5 - \frac{q_i}{2})^2 - F$. As one can see, the bigger is q_i, the lower the entrant's profits are.

c) Knowing that $\pi_e = 2(22.5 - \frac{q_i}{2})^2 - F$, in order to deter entry the incumbent has to set q_i such that $\pi_e = 0$. Therefore we have $q_i = 2(22.5 - \sqrt{\frac{F}{2}})$.

d) Since there is only one firm in the market (entry is deterred) the market share is equal to 1, therefore, the lerner index is $L = \frac{p - MC}{p}$. In our case, $p = 100 - 2q_i = 100 - 4(22.4 - \sqrt{\frac{F}{2}}) = 10 + \sqrt{8F}$, $MC = 10$, hence $L = 1 - \frac{10}{10 + \sqrt{8F}}$. This basically says that the higher the sunk costs, the higher the concentration index. In order to deter entry, the incumbent deviates from its optimal monopoly output choice. However, sunk cost act as a barrier to entry. Therefore, the higher the sunk costs, the smaller the incumbent's deviation from the monopoly output choice and the higher the concentration.

[40] *The Wall Street Journal Europe*, January 15–16, 1999.