ECN 453: Bertrand Competition

Nicholas Vreugdenhil

Static Models of Oligopoly

- We will start the second part of the course today.
    - (So, the material we will talk about today will not be on the first mid-term exam)

- In this part of the course, we will study different **static models of oligopoly**

Static Models of Oligopoly

- What are **static models of oligopoly**?
  - 'Static': this means that the game is only played once
    - Note that there might be a sequential element to it e.g. players take turns to choose like in the entry deterrence example we saw in previous lectures, but ultimately the game is played only once
    - (In the third part of the course, we will contrast 'static' games with 'dynamic' or 'repeated' games which are played again and again)
  - 'Oligopoly': this means when we have (potentially) more than one firm competing in the market
- These models (arguably) the central building blocks of industrial organization, and the models will correspond to different types of competition we observe in real-world markets.

Plan

1. Bertrand competition
2. Bertrand competition with capacity constraints

Plan

1. **Bertrand competition**
2. Bertrand competition with capacity constraints

Bertrand Competition

- Named after Joseph Bertrand
- 11 March 1822 – 5 April 1900
- He was a mathematician

Bertrand Competition

- **Players**: two firms (firm 1 and firm 2)
- **Strategies**
  - Firms choose prices
  - Prices are set simultaneously
- **Payoffs**
  - Each firm has a constant marginal cost (for now, assume they have the *same* marginal cost)
  - Consumers buy from the firm which sets the lower price
  - If the prices are the same, consumers split their demand equally between the firms
  - So: total demand is $Q = D(p)$ where $p = min\{p_1, p_2\}$

Bertrand Competition

- Before getting into the implications of Bertrand competition, first consider: what kind of real-world competitive situations does Bertrand competition correspond to?

- It is a model of **price competition** (the decision is 'what price do I choose'?)

- Best suited to markets where firms offer a **homogeneous** (identical) product
  - In other words, the products are **perfect substitutes**
  - E.g. gas stations next to each other offer the same gas
  - If the price is just a few cents lower, all demand will go to the station with the lowest price
  - If the products were *not* homogeneous, then the strong assumption that 'consumers buy from the firm which sets the lower price' would probably be violated and we would need to use a different model.

Bertrand Competition: Continuous Strategies

- Unlike in the previous lectures, where strategies were limited to just a few discrete alternatives (e.g. choose a high price/low price), here firms can choose *any* price ('continuous strategies').
- All of our definitions about best responses, Nash equilibrium, etc still work!
- I will first show you the mathematical definitions (which are pretty abstract), but they will hopefully make more sense when applied to the Bertrand competition example.

Bertrand Competition: Continuous Strategies - Best Responses

- Here, the **best responses** are prices (where $p_1^*(p_2)$ is just another way of writing $BR_1(p_2)$ and $p_2^*(p_1)$ is just another way of writing $BR_2(p_1)$):

$$p_1^*(p_2) = BR_1(p_2) \in argmax_{p_1} \Pi_1(p_1, p_2)$$
$$p_2^*(p_1) = BR_2(p_1) \in argmax_{p_2} \Pi_2(p_1, p_2)$$

- Notation:
  - $BR_1(p_2)$: the best response of firm 1 given that firm 2 chooses the price p_2
  - $BR_2(p_1)$: the best response of firm 2 given that firm 1 chooses the price p_1
  - *argmax*: the argument(s) (i.e. prices) that maximize profit
  - $\in$: 'in' the set of best responses
  - p_1, p_2: prices that firm 1 and firm 2 set, respectively
  - $\Pi_1(p_1, p_2)$: profit of firm 1 given that firm 1 and firm 2 set the prices p_1, p_2.

Bertrand Competition: Continuous Strategies - Nash Equilibrium

- The idea behind the Nash equilibrium is the same as before: 'a Nash equilibrium is two prices (p_1, p_2) where each firm has no incentive to unilaterally deviate by choosing a different price'

- Equivalently: a Nash equilibrium is a price for each firm (p_1, p_2) that is the best response to the price of the other firm.

$$p_1 \in p_1^*(p_2)$$
$$p_2 \in p_2^*(p_1)$$

Bertrand Competition: Best Responses - Firm 1

- Let's apply the definitions on the previous two slides to Bertrand competition, starting with firm 1.

- We want to find the best response $p_1^*(p_2)$
  - i.e. the optimal price for firm 1 given firm 2 plays p_2.

- There are three main cases to consider, corresponding to different potential p_2
  - **Case 1**: Firm 2 plays $p_2 > p^M$ (where p^M is the monopoly price)

  - **Case 2**: Firm 2 plays $MC < p_2 \leq p^M$

  - **Case 3**: Firm 2 plays $p_2 \leq MC$

Bertrand Competition: Best Responses - Firm 1

- **Case 1**: Firm 2 plays $p_2 > p^M$ (where p^M is the monopoly price)
- The best response: $p_1^*(p_2) = p^M$.
- Why?
  - Since $p_2 > p^M$, for any $p_1 \leq p^M$ firm 1 gets all of the total demand.
  - So, firm 1 can just set prices like it is a monopoly for the entire market and can set the monopoly price.

Bertrand Competition: Best Responses - Firm 1

- **Case 2**: Firm 2 plays $MC < p_2 \leq p^M$
- The best response: $p_1^*(p_2) = p_2 - \epsilon$
  - Define ϵ: a tiny change in the price
  - Idea: undercut firm 2 by a tiny amount.
- Why? (Intuition)
  - Firm 1 should undercut firm 2 by a tiny amount, and receive the entirety of total demand, rather than setting a price equal to firm 2 (and splitting the market), or setting a price higher than firm 2 (and receiving 0 demand).

Bertrand Competition: Best Responses - Firm 1

- **Case 2**: Firm 2 plays $MC < p_2 \leq p^M$
- The best response: $p_1^*(p_2) = p_2 - \epsilon$
  - Define ϵ: a tiny change in the price
  - Idea: undercut firm 2 by a tiny amount.
- Why? (Math)
  - Note that the profit at $p_1 = p_2 - \epsilon$ is:

$$\Pi_1(p_1 = p_2 - \epsilon, p_2) = D(p_2 - \epsilon)(p_2 - \epsilon - MC)$$

  - Since firm 1 already gets all of total demand at $p_1 = p_2 - \epsilon$, any price lower than this will just reduce profit.
  - If firm 1 were to set a price that exactly matched p_2, the profit would be $0.5D(p_2)(p_2 - MC) < D(p_2 - \epsilon)(p_2 - \epsilon - MC)$ if ϵ is sufficiently small
  - If firm 1 were to set a price $p_1 > p_2$ then firm 1's profits would be $= 0$ since noone would buy from them, but firm 1's profits would be positive if ϵ is small enough because $p_2 > MC$.

Bertrand Competition: Best Responses - Firm 1

- **Case 3**: Firm 2 plays $MC \geq p_2$
- The best response: $p_1^*(p_2) = MC$
- Why?
  - Any price $p_1 < p_2$ would give firm 1 negative profit, since $p_2 < MC$.
  - So, set $p_1 = MC$: here you get 0 profit.
  - (Note: technically any $p_1 > p_2$ is a best response if $MC > p_2$ and any $p_1 \geq MC$ is a best response if $p_2 = MC$, since for all these p_1 prices profits are zero for firm 1. But, dealing with these situations just complicates the proof and does not change the equilibrium; the textbook just ignores them and so will I).

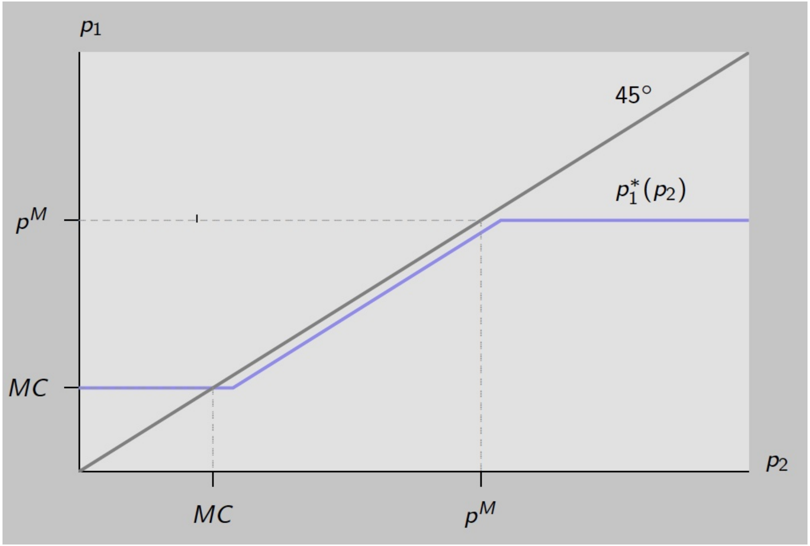
Bertrand Competition: Best Responses - Firm 1, summary

- $p_1^*(p_2) = MC$ if $p_2 \leq MC$
- $p_1^*(p_2) = p_2 - \epsilon$ if $MC < p_2 \leq p^M$
- $p_1^*(p_2) = p^M$ if $p_2 > p^M$

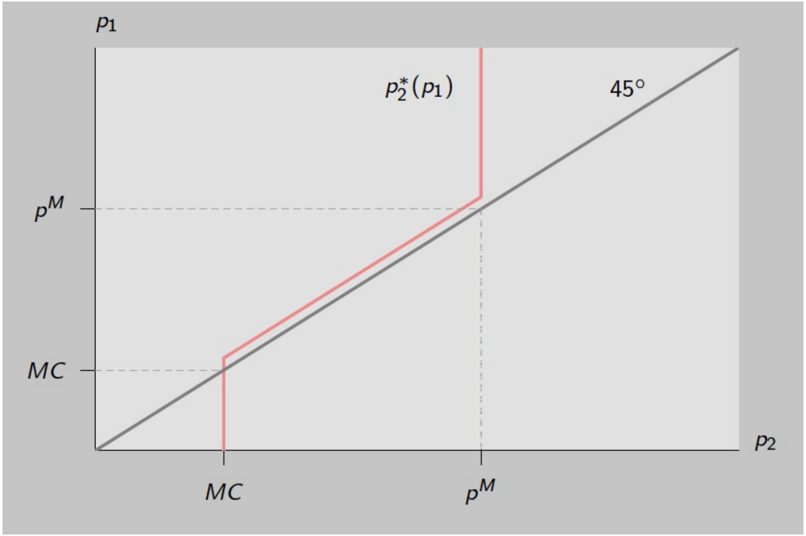
Bertrand Competition: Best Responses - Firm 2, summary

- By exactly the same arguments, we can find the best responses for firm 2:
- $p_2^*(p_1) = MC$ if $p_1 \leq MC$
- $p_2^*(p_1) = p_1 - \epsilon$ if $MC < p_1 \leq p^M$
- $p_2^*(p_1) = p^M$ if $p_1 > p^M$

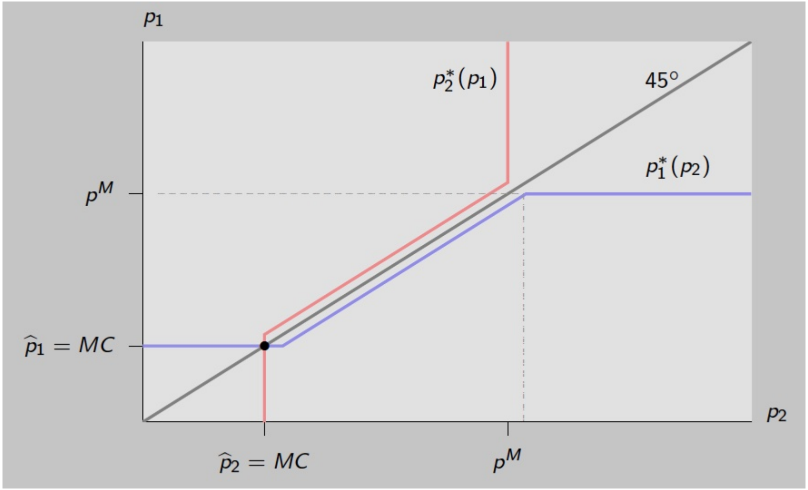
Bertrand Competition: Best Responses - Firm 1, graph



Bertrand Competition: Best Responses - Firm 2, graph



Bertrand Competition: Nash Equilibrium, graph



- Nash equilibrium is where the two best response curves cross.
- This is at $p_1 = p_2 = MC$.

Bertrand Competition: recipe for how to solve it

1. Find the best responses for firm 1 (i.e. find the optimal prices p_1 for all prices p_2 that firm 2 could set)

2. Find the best responses for firm 2 (i.e. find the optimal prices p_2 for all prices p_1 that firm 1 could set)

3. Find where the two best responses cross: this is a Nash equilibrium!

- Note: This recipe is exactly the same as in the simultaneous games we saw before, the 'trick' is splitting the best responses into different cases.

Bertrand Competition: alternative way to think about it

- Essentially, **Bertrand competition is a model of a price war**.
- Suppose that firm 1 chooses a price $p^M > p_1 > MC$.
  - Firm 2 will then slightly undercut it by a tiny amount...
  - Firm 1 then responds by undercutting by a tiny amount...
  - Firm 2 then responds by undercutting by a tiny amount...
  - ...this continues until each firm is setting price = MC.

Bertrand Competition: example 1

- **Question:** Assume we have two firms with the same marginal cost (=2) and these firms produce homogenous products. These two firms compete under Bertrand competition and the total demand curve: $Q = 100 - p$.
- 1. What are the best response functions?
- 2. Are the prices $p_1 = p_2 = 4$ a Nash equilibrium?
- 3. What is the Nash equilibrium?
- 4. What are the profits of the firms?
- 5. What is consumer surplus?

Bertrand Competition: example 1

- **Question:** Assume we have two firms with the same marginal cost (=2) and these firms produce homogenous products. These two firms compete under Bertrand competition and the total demand curve: $Q = 100 - p$.

- 1. What are the best response functions? (Write down the three cases as before with the monopoly price $q^M = 49, p^M = 51$ and $MC = 2$)

- 2. Are the prices $p_1 = p_2 = 4$ a Nash equilibrium? No, firms will undercut each other.

- 3. What is the Nash equilibrium? (set $p = MC = 2$)

- 4. What are the profits of the firms? (0)

- 5. What is consumer surplus? ($CS = 0.5 * 98 * 98 = 4802$)

Bertrand Competition: example 2

- **Question:** Assume we have two firms with the different marginal costs $p_1^M > p_2^M > c_1 > c_2$ and these firms produce homogenous products. These two firms compete under Bertrand competition with total demand curve denoted $D(p)$.
- 1. What are the best response functions?
- 2. What is the Nash equilibrium?
- 3. What are the firm profits?
- (Note: the textbook does this example but with $c_2 > c_1$ on p 191)

Bertrand Competition: example 2

- **Question:** Assume we have two firms with the different marginal costs $p_1^M > p_2^M > c_1 > c_2$ and these firms produce homogeneous products. These two firms compete under Bertrand competition with total demand curve denoted $D(p)$.

- 1. What are the best response functions? Similar to before, except the same 'MC' from before is now replaced with each firm's specific marginal cost (either c_1 or c_2 for firm 1 and firm 2 respectively)

- 2. What is the Nash equilibrium? $p_1 = c_1, p_2 = c_1 - \epsilon$ (note that if ϵ is really small then $p_2 \approx p_1 = c_1$)

- 3. What are the firm profits? Firm 1: 0. Firm 2: $D(p_2)(p_2 - c_2) = D(c_1 - \epsilon)(c_1 - \epsilon - c_2) \approx D(c_1)(c_1 - c_2) > 0$ since $c_1 > c_2$. I.e. firm 2 now competes firm 1 down to its marginal cost and makes a profit on the rest of demand.

Bertrand Competition

- We just showed that the equilibrium of the Bertrand model is for both firms to price at marginal cost (for the case where they have the same marginal cost) and make zero profit.

- This is really surprising! Particularly since the assumptions behind the Bertrand model seemed (at least at first glance) quite reasonable.

- The result is so surprising that economists have names for the predictions of the model:
  - The **Bertrand trap**: when firms get caught in a fierce price war where they compete prices down to marginal cost.

  - The **Bertrand paradox**: the predictions of the Bertrand model imply that as we move from monopoly (1 firm) to duopoly (2 firms), price will change from the monopoly price to the perfect com petition price.
    - If this is the case, no role for competition policy in markets with > 1 firm!

Bertrand Competition: the Bertrand trap

- Example of the Bertrand trap: the case of encyclopedias
  - Encyclopedia Britannica: Until the 1990s, 32 volume hardback sold for $1600
  - Entry by Microsoft Encarta in the 1990s, sold on CD for less than $100
  - In 2000: both Encarta and Britannica sold for $89.99
- Example of the Bertrand trap: airline industry
  - American Airlines: in 1992 introduce a ‘value pricing’ plan that cut fares
  - Competitors announced even bigger cuts, American Airlines undercut these further, rest of the industry also cut prices
  - Total cost (to the airlines) of the price war: 4 billion dollars.

Bertrand Competition

- How do the predictions of the Bertrand model hold up in real-world settings?

Bertrand Competition

- How do the predictions of the Bertrand model hold up in real-world settings?
  - The answer is - for the vast majority of markets - not very well.
  - We typically see firms making positive profits.
  - Therefore, *something* about the assumptions of the Bertrand model must be wrong.

Potential solutions to the Bertrand paradox/ways out of the Bertrand trap

- How could we change the assumptions of the Bertrand model to get a more realistic model with positive profits (i.e. firms pricing about MC)?

Potential solutions to the Bertrand paradox/ways out of the Bertrand trap

- How could we change the assumptions of the Bertrand model to get a more realistic model with positive profits (i.e. firms pricing about MC)?
- **Asymmetric (i.e. different) costs**
  - Like 'cost leadership'
- **Product differentiation/branding**
  - Undercutting the price by a small amount may no longer deliver all of total demand
- **Dynamic competition**
  - What if rivals can retaliate? E.g. you set a high price but threaten to retaliate next time if your rival undercuts you?
  - We will see this case in Part 3 of the course when we study 'dynamic models of competition'
- **Capacity constraints**
  - What good is undercutting your rival to get total demand if you cannot actually supply all this demand due to capacity constraints?

Bertrand Competition: example 3

- **Question:** Suppose that total demand for golf balls is $Q = 90 - 3P$ and Q is measured in kilos of balls. There are two firms that supply the market. Firm 1 can produce a kilo of balls at a constant unit cost of \$15 whereas firm 2 has a constant unit cost equal to \$10.

- 1. Suppose the firms compete in price. How much does each firm sell in a Bertrand equilibrium? What is the market price and what are firms profits?

- 2. How would your answer to 1. change if there were three firms, one with unit cost $=$ \$20 and two with unit cost $=$ \$10?

- 3. How would your answer to 2b change if firm 1's golf balls were green and endorsed by a famous golfer, but firm 2's were plain and white?

Plan

1. Bertrand competition
2. **Bertrand competition with capacity constraints**

Bertrand competition with capacity constraints

- **Setup:**
- Same assumptions as before
  - Firms set prices simultaneously; constant MC (set $= 0$ for simplicity), homogeneous product
  - Denote the inverse demand curve by $P(Q)$.
- But: each firm is constrained to not be able to sell more than k_i (where i is either 1 or 2 depending on the firm)
- **Question:** What is the Nash equilibrium?
- **Answer**(solution on the next slide): $p_1 = p_2 = P(k_1 + k_2)$.

Bertrand competition with capacity constraints

- **Question:** What is the Nash equilibrium?

- **Answer**: $p_1 = p_2 = P(k_1 + k_2)$.

- **Why is this a Nash equilibrium?**
  - Suppose that Firm 1 is setting $p_1 = P(k_1 + k_2)$. Consider Firm 2's decision:

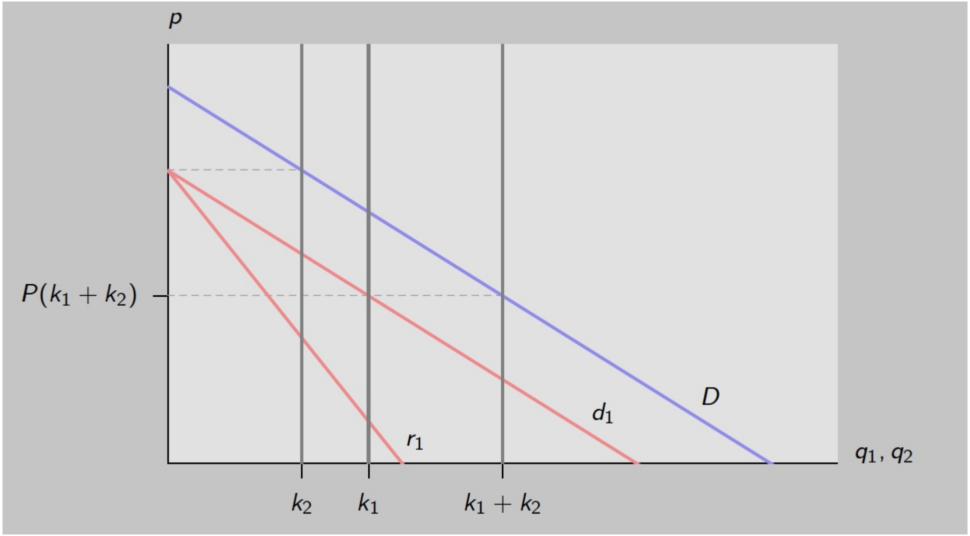
  - Can Firm 2 do better than $p_1 = p_2$ by deviating and setting $p_2 < P(k_1 + k_2)$?
    - No. Although Firm 2 now gets *all* demand, this price actually lowers its profits: it can still only sell k_2 units but it now does this at a lower price.

  - Can Firm 2 do better than $p_1 = p_2$ by deviating and setting $p_2 > P(k_1 + k_2)$?
    - Firm 2 now receives positive demand even though it prices above Firm 1, since Firm 1 is capacity constrained.

    - Specifically, Firm 2 gets the 'residual demand' $d_1 = D(p_2) - k_1$, with the corresponding marginal revenue curve r_1.

    - But, looking at the diagram on the next slide, $MR > MC = 0$ for all the quantities below its capacity k_2. Hence, Firm 2 is not setting the optimal price but should in fact lower its price.

Bertrand competition with capacity constraints



Bertrand competition with capacity constraints

- **Question:** What is the Nash equilibrium?

- **Answer**: $p_1 = p_2 = P(k_1 + k_2)$.

- **Why is this a Nash equilibrium?** (continued)
  - A similar argument holds for Firm 1's decision.

  - So, neither firm has an incentive to unilaterally deviate from $p_1 = p_2 = P(k_1 + k_2)$ and so it is a Nash equilibrium.

- **Note:** this only works if capacity levels are low. If capacity levels are high, it may be optimal to undercut the rival's price.

Summary of key points*

- Know the assumptions behind Bertrand competition
- Know that Bertrand competition is a model of a price war that ends in firms charging marginal cost
- Know what is the Bertrand paradox/Bertrand trap
- Understand how to construct and read best responses/Nash equilibria with continuous strategies (including graphing them)
- Understand how to 'solve' the Bertrand paradox with capacity constraints

*To clarify, all the material in the slides, problem sets, etc is assessable unless stated otherwise, but I hope this summary might be a useful place to start when studying the material.