Industrial Organization
Answer Key to Assignment # 2

1. *Consider the following games in normal form. For each game:*

   i. *Determine whether the game can be solved by dominant strategies*

   ii. *Determine all Nash equilibria in pure strategies.*

   (a)

	L	R
T	2, 5	3, 2
B	1, 2	4, 3

       i. No, the game cannot be solved by dominant strategies.

       ii. There are two Nash equilibria: (B,R), (T,L)

   (b)

	L	R
T	−10, −10	5, −5
B	−5, 5	1, 1

       i. No, the game cannot be solved by dominant strategies.

       ii. There are two Nash equilibria: (B,L), (T,R)

   (c)

	L	C	R
T	100, 100	0, 0	50, 200
M	0, 0	1, 1	0, 0
B	0, 300	0, 0	200, 200

       i. No, the game cannot be solved by dominant strategies.
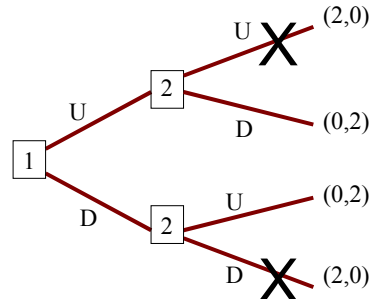
       ii. There is one Nash equilibrium: (M,C)

   (d)

	L	C	R
T	10, 4	1, 5	98, 4
M	9, 9	0, 3	99, 8
B	1, 98	0, 100	100, 98

       i. No, the game cannot be solved by dominant strategies.
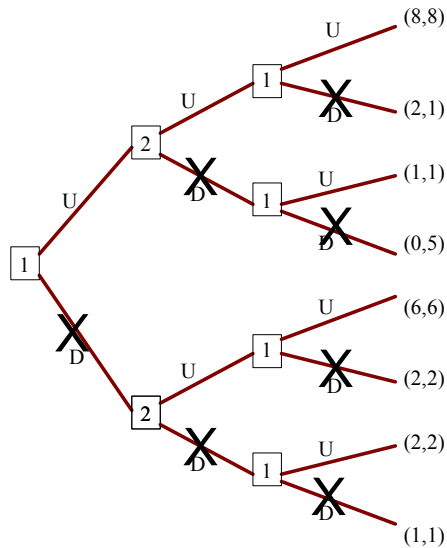
       ii. There is one Nash equilibrium: (T,C)

2. Find the Subgame Perfect Nash Equilibrium outcomes and payoffs of the following games:

(a)

There are two subgame perfect Nash equilibia (SPNE)

(b)

The unique SPNE of the game calls for player 1 to play Up, then player 2 plays Up and finally player 1 one plays Up. Both players get a payoff of 8.

3. *Cabral, problem 4.2.: The UK Office of Fair Trading has recently unveiled a plan that will offer immunity from prosecution to firms who blow the whistle on their co-cartel conspirators. In the U.S., this tactic has proven extremely successful: since its introduction in 1993, the total amount of fines for anti-competitive behavior has increased twentyfold. Show how the tactic initiated by the U.S. Department of Justice and soon to be followed by the Office of Fair Trading changes the rules of the game played between firms in a secret cartel.*

Prior to the introduction of the plan, each cartel firm would have two options: (a) to stick by the agreement or (b) to deviate and set lower prices. With the introduction of the plan, the firm has

a third option: (c) to blow the whistle. Let α be the probability that the DOJ discovers the price conspiracy. High values of α imply a low expected value from (a). The same is true of (b), though probably to a lesser extent. Finally, (c) is invariant to the value of α. We would thus expect that, for high values of α, (c) is the best strategy.

With the introduction of the plan, the firms now play a second prisoner's dilemma type of game. Before, it was whether to price high or price low. Now, it's whether to blow the whistle or not. Firm would be better off if neither of them blew the whistle. However, if α is high, blowing the whistle is a dominat strategy.

4. *Cabral, problem 4.5.: Hernan Cortez, the Spanish navigator and explorer, is said to have burnt his ships upon arrival to Mexico. By so doing, he effectively eliminated the option of him and his soldiers returning to their homeland. Discuss the strategic value of this action knowing the Spanish colonists were faced with potential resistance from the Mexican natives.*

By eliminating the option of turning back, Hernan Cortez established a credible commitment regarding his future actions, that is, to fight the Mexican natives should they attack. Had Cortez not made this move, natives could have found it better to attack, knowing that instead of bearing losses the Spaniards would prefer to withdraw.

5. *Cabral, problem 4.6.: Consider the following game depicting the process of standard setting in high-definition television (HDTV). The U.S. and Japan must simultaneously decide whether to invest a high or a low value into HDTV research. Each country's payoffo are summarized in Figure 3.*

		Japan			
		Low		High	
U.S.	Low	4	3	2	4
	High	3	2	1	1

Figure 3: The HDTV game: each country chooses a high or a low level of R&D on HDTV.

(a) *Are there any dominant strategies in this game? What is the Nash equilibrium of the game? What are the rationality assumptions implicit in this equilibrium?*

For the United States investing, a low value in HDTV research is a dominant strategy. The Nash equilibrium of the game is given by the U.S. choosing Low and Japan choosing High. The rationality assumptions implicit in this solution are that both players are rational and, moreover, Japan belives the U.S. acts rationally.

(b) *Suppose now the U.S. has the option of committing to a strategy ahead of Japan's decision. How would you model this new situation? What are the Nash equilibria of this new game?*

See Figure 3. (See also Section 4.2.) By solving backwards, with get the following Nash equilibrium: U.S. chooses High, Japan chooses Low.

(c) *Comparing the answers to (a) and (b), what can you say about the value of commitment for the U.S.?*

Comparing the answers from (a). and (b). we can see that the value of commitment to the U.S. is 1 that is, 3 minus 2.

(d) "When pre-commitment has a strategic value, the player that makes that commitment ends up 'regretting' its actions, in the sense that, given the rivals' choices, it could achieve a higher payoff by choosing a di®erent action." In light of your answer to (b), how would you comment this statement?

Given that Japan chooses Low, the U.S. would be better off by choosing Low as well. However, it must be the case that the cost of switching from High to Low is so high that the U.S. won't do it (ex post). Otherwise, the commitment to stick to High would not be credible.

6. *Cabral, problem 7.1.: According to Bertrand's theory, price competition drives firms' profits down to zero even if there are only two competitors in the market. Why don't we observe this in practice very often?*

Section 7.2 suggests three possible explanations: (a) product differentiation, (b) dynamic competition, (c) capacity constraints.

7. *Cabral, problem 7.2.: Three criticisms are frequently raised against the use of the Cournot oligopoly model: (i) firms normally choose prices, not quantities; (ii) firms don't normally take their decisions simultaneously; (iii) firms are frequently ignorant of their rivals' costs; in fact, they do not use the notion of Nash equilibrium when making their strategic decisions. How would you respond to these criticisms?*

(i) If firms are capacity constraint, then price competition \looks like" like quatitiy competition. See Section 7.2.

(ii) If there are significant information lags, then sequential decisions "look like" simultaneous decisions. See Chapter 4 (\first section).

(iii) The last section of Chapter 7 presents an argument for the relevance of Nash equilibrium which only requires each firm to know its own profit function.

8. *Suppose there are two firms. IT&T and Horizon, that are the only two cellular providers in Fargo, ND. Both firms act as Cournot competitors and face the (inverse) demand function $P(Q) = A - BQ$, where $Q = q_1 + q_2$. Assume that each firm has constant marginal costs equal to c.*
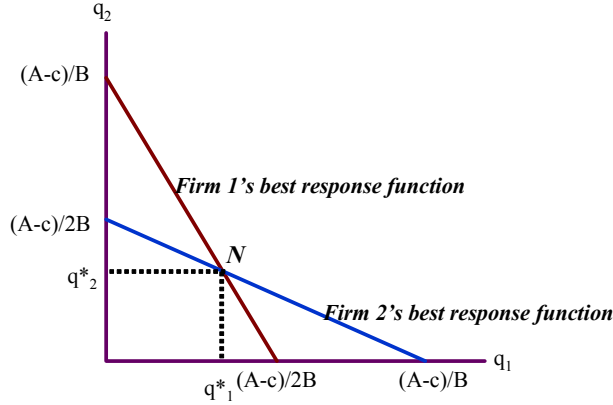
(a) *Find an expression for IT&T's output as a best response function to the output of firm 2. Find the analogous expression for Horizon. Graph the two reaction functions.*

$P(Q) = A - B(q_1 + q_2)$, then $MR_1 = A - Bq_2 - 2Bq_1$. By equating $MR_1 = c$, we get

$$q_1 = \frac{A - Bq_2 - c}{2B}$$

Analogously,

$$q_2 = \frac{A - Bq_1 - c}{2B}$$

(b) *Find an expression for each firm's equilibrium output and the equilibrium price.*

Using the best response functions, we substitute one into the other to get the equilibrium outputs. For firm 2,

$$
\begin{aligned}
q_2 &= \frac{A - Bq_1 - c}{2B} \\
&= \frac{A - B\left(\frac{A - Bq_2 - c}{2B}\right) - c}{2B} \\
\Rightarrow q_2^* &= \frac{A - c}{3B}
\end{aligned}
$$

Likewise,

$$
q_1^* = \frac{A - c}{3B}
$$

Recalling that $P(Q) = A - B(q_1 + q_2)$, then

$$
P^* = A - B\left(2\left(\frac{A - c}{3B}\right)\right) = \frac{A + 2c}{3}
$$

9. *Consider a market for a homogeneous product with demand given by $Q = 37.5 - \frac{P}{4}$. There are two firms, A and B, each with constant marginal cost equal to 54.*

(a) *Determine output and price under a Cournot equilibrium*

$P(Q) = 150 - 4(q_1 + q_2)$, then $MR_a = 150 - 4q_b - 8q_a$. By equating $MR_a = MC = 54$, we get

$$
q_a = 12 - \frac{1}{2}q_b
$$

Analogously,

$$
q_b = 12 - \frac{1}{2}q_a
$$

Using both reaction curves, we substitute one into the other to get the equilibrium outputs. For firm A,

$$
\begin{aligned}
q_a &= 12 - \frac{1}{2}q_b \\
&= 12 - \frac{1}{2}\left(12 - \frac{1}{2}q_a\right) \\
\Rightarrow \quad q_a^* &= 8
\end{aligned}
$$

Likewise,

$$
q_b^* = 8
$$

Recalling that $P(Q) = 150 - 4(q_a + q_b)$, then

$$
P^* = 150 - 4\,(8 + 8) = 86
$$

(b) *Suppose that firm A has a first-mover advantage. Determine output and price under a Stackelberg equilibrium. Explain your results.*

Solving using backward induction, firm B's reaction curve is again:

$$
q_b = 12 - \frac{1}{2}q_a
$$

Firm A internalizes firm B's reaction curve into its demand, to find its (inverse) effective demand, $q_a^e = Q - q_b$. Notice that $Q = 37.5 - \frac{1}{4}P$. Then

$$
\begin{aligned}
q_a^e &= \left(37.5 - \frac{1}{4}P\right) - \left(12 - \frac{1}{2}q_a^e\right) \\
q_a^e &= 25.5 - \frac{1}{4}P + \frac{1}{2}q_a^e \\
\frac{1}{4}P &= 25.5 - \frac{1}{2}q_a^e \\
P &= 102 - 2q_1
\end{aligned}
$$

Then, the effective marginal revenue for firm 1 is $MR_1 = 102 - 4q_1$. By equating $MR_1 = MC$, we get:

$$
\begin{aligned}
102 - 4q_1 &= 54 \\
\Rightarrow \quad q_1^* &= 12
\end{aligned}
$$

Replacing $q_1^* = 12$ into firm 2's reaction curve:

$$
q_2^* = 12 - \frac{1}{2}\,(12) = 6
$$

Recalling that $P(Q) = 150 - 4(q_1 + q_2)$, then

$$
P^* = 150 - 4\,(12 + 6) = 78
$$

10. *The following table contains sales figures for three hypothetical industries in the year 2002.*

Firm	Industry 1	Industry 2	Industry 3
A	$300	$1000	$400
B	$300	$100	$300
C	$300	$60	$300
D	$300	$40	$300
E	$300	$30	$300
All Others	$10 \times 50	$22 \times 35	$4 \times 100

Figures in millions of current dollars

(a) *Compute the 4-firm concentration ratio (C_4) for each industry. According to this index, which industry is most concentrated?*

The 4-firm concentration ratio is the sum of the top four firms' market shares. The market share is given by dividing the firm's sales by the total industry sales. In the above table, each industry has total sales of $2000. The following table gives the market shares:

Firm	Industry 1	Industry 2	Industry 3
A	15%	50%	20%
B	15%	5%	15%
C	15%	3%	15%
D	15%	2%	15%
E	15%	1.5%	15%
All Others	$10 \times 2.5\%$ each	$22 \times 1.75\%$ each	$4 \times 5\%$ each

Thus, we get the following results for C_4:

	Industry 1	Industry 2	Industry 3
Sum of top four market shares	$1200	$1200	$1300
C_4	60%	60%	65%

According to the C_4, industry 3 would be the most concentrated.

(b) *Compute the Herfindahl-Hirschman Index (HHI) for each industry. According to this index, which industry is most concentrated?*

To compute the HHI, we sum the squared market share of every firm in a given industry.

For industry 1: HHI$=5*(15)^2 + 10*(2.5)^2 = 1187.5$

For industry 2: HHI$=1*(50)^2 + 1*(5)^2 + 1*(3)^2 + 1*(2)^2 + 1*(1.5)^2 + 22*(175)^2 = 2607.6$

For industry 3: HHI$=1*(20)^2 + 4*(15)^2 + 4*(5)^2 = 1004.0$

Here, industry 2 would be the most concentrated.

(c) *Why do you think HHI does a better job than C_4 of measuring industry concentration? Give two reasons for your answer and make sure you refer to the above table.*

    i. HHI does a better job because it can describe the concentration of an industry within the top four firms, whereas CR$_4$ cannot make this distinction. For example, from the above table, both industry 1 and industry 2 have the same 4-firm concentration ration, yet industry 2 has one firm with half of the industry's total sales. In this sense, HHI does a better job showing that industry 2 is more concentrated that industry 1.

ii. On a similar note, HHI does a better job because it can describe the industry beyond just the top four firms. The HHI index gives a stronger prediction that industry 3 is more concentrated than industry 1 than CR_4 does. Notice that the HHI for industry 3 is over 15% greater than the HHI for industry 1, whereas the CR_4 for industry 3 is less than 10% greater than industry 1's ratio.

(d) *Which industry would you consider most concentrated? Why?*

Industry 2 would appear to be most concentrated since it has one very large firm and all other firms are small.

11. *Cabral, problem 12.1: Consider a duopoly where horizontal product differentiation is important. Firms first simultaneously choose their product locations, then simultaneously set prices in an infinite series of periods. Suppose that firms collude in prices in the second stage and anticipate this at the product-positioning stage. What do you expect this implies in terms of the degree of product differentiation?*

Assuming that both firms are identical, and that there is no punishment regarding the choice of location, then both firms will locate right in the middle, so that there is no product differentiation between them.

12. *In the Bertrand-Hotelling duopoly, if firms have the same costs but occupy different locations, the Nash equilibrium prices are directly related to how differentiated are the products of the two firms. Explain*

True: With fixed locations on a Hotelling line, our index of differentiation is the transportation cost. If the transportation cost is very large, then it would take a huge price difference for customers "located" near one product to go for the other product (i.e. the products are very differentiated). On the other hand, if the transportation cost is very small, it only takes a small price difference for customers to get higher net utility from a product located very far away (i.e. the products are undifferentiated). We have found that in the our linear model, the mark-up over cost was exactly the transportation cost. Thus, the more the products are differentiated, the higher equilibrium prices are.