

ECN 594: Oligopoly Competition

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Welcome to Part 2

- Demand estimation and pricing
- **Models of competition and industry structure**
 - Oligopoly models (Cournot, Bertrand, Hotelling)
 - Entry and entry deterrence
 - Mergers
 - Vertical relationships
 - Collusion
- **HW2 released:** Merger simulation module

Plan

1. Cournot competition (refresher in IO notation)

2. Bertrand competition (refresher)

3. Cournot vs Bertrand: when does each apply?

4. Product differentiation: why it matters

5. Hotelling model

6. Connection to demand estimation

From ECN 532: Oligopoly models

- You covered Cournot and Bertrand in Hector's class
- Today: quick refresher in IO notation
- **New focus:** Market power measurement
 - Connecting oligopoly models to Lerner index
 - When does each model apply?

Cournot competition: setup

- n firms producing homogeneous goods
- Firms choose **quantities** simultaneously
- Inverse demand: $P = P(Q)$ where $Q = \sum_{i=1}^n q_i$
- Constant marginal cost: c
- Firm i profit: $\pi_i = P(Q) \cdot q_i - c \cdot q_i$

Cournot: first-order conditions

- Firm i maximizes profit taking q_{-i} as given:

$$\frac{\partial \pi_i}{\partial q_i} = P(Q) + P'(Q)q_i - c = 0$$

- Rearranging:

$$P(Q) - c = -P'(Q)q_i$$

- Divide by P :

$$\frac{P - c}{P} = \frac{-P'(Q)q_i}{P} = \frac{-P'(Q)Q}{P} \cdot \frac{q_i}{Q} = \frac{s_i}{|\varepsilon|}$$

- where $s_i = q_i / Q$ is firm i 's market share

Cournot: Lerner index

- **Key result:** In Cournot equilibrium,

$$L_i = \frac{P - MC}{P} = \frac{s_i}{|\varepsilon|}$$

- **Interpretation:**

- Markup depends on market share
- Larger firms have more market power
- More elastic demand → lower markup
- This connects to demand estimation from Part 1!

Worked example: Cournot with market power

- **Question:** Inverse demand is $P = 100 - Q$. Two symmetric firms with $MC = 10$.
- (a) Find equilibrium quantities and price.
- (b) Calculate the Lerner index for each firm.
- (c) Verify using the $L = s/|\varepsilon|$ formula.

Take 5 minutes.

Worked example: Cournot (solution)

Solution

- (a) FOC: $100 - 2q_i - q_j - 10 = 0$
- Symmetric: $q_1 = q_2 = q^*$, so $100 - 3q^* = 10 \Rightarrow q^* = 30$
- $Q = 60$, $P = 100 - 60 = 40$
- (b) $L = \frac{40-10}{40} = \frac{3}{4} = 0.75$
- (c) Market share: $s_i = 30/60 = 0.5$
- Elasticity: $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = (-1) \cdot \frac{40}{60} = -\frac{2}{3}$
- Check: $L = \frac{s_i}{|\varepsilon|} = \frac{0.5}{2/3} = 0.75 \checkmark$

Cournot with n symmetric firms

- General result for linear demand $P = a - bQ$:
- Symmetric equilibrium: $q^* = \frac{a-c}{b(n+1)}$
- Total quantity: $Q^* = \frac{n(a-c)}{b(n+1)}$
- Price: $P^* = \frac{a+nc}{n+1}$
- Lerner index: $L = \frac{1}{n|\varepsilon|}$ (symmetric case where $s_i = 1/n$)
- **Key insight:** As $n \rightarrow \infty$, $P \rightarrow MC$ (perfect competition)

Practice: Cournot comparative statics

- **True, False, or NEI:**
- (a) In Cournot equilibrium, a firm with a larger market share has a higher markup.
- (b) Adding a third firm to a Cournot duopoly always reduces industry profits.
- (c) In Cournot, all firms must have the same marginal cost.

Take 2 minutes.

Practice: Cournot comparative statics (solution)

Answers

- **(a) TRUE.** From $L_i = s_i / |\varepsilon|$, larger s_i means higher L_i .
- **(b) TRUE.** More firms \Rightarrow lower price \Rightarrow lower industry profit. Each firm's profit falls, and total profit falls because P is closer to MC .
- **(c) FALSE.** Asymmetric costs work fine. Low-cost firms have higher q_i and s_i , hence higher margins.

Plan

1. Cournot competition (refresher in IO notation)

2. **Bertrand competition (refresher)**

3. Cournot vs Bertrand: when does each apply?

4. Product differentiation: why it matters

5. Hotelling model

6. Connection to demand estimation

Bertrand competition: setup

- n firms producing **homogeneous** goods
- Firms choose **prices** simultaneously
- Consumers buy from lowest-price firm
- If tie: split demand equally
- Constant marginal cost: c

Bertrand: the paradox

- **Nash equilibrium:** $p_1 = p_2 = c$ (marginal cost pricing!)
- **Why?**
 - If $p_i > p_j > c$: firm i can undercut and capture entire market
 - Undercutting continues until $p = c$
- **The “paradox”:**
 - Only 2 firms, but competitive outcome!
 - Zero profits with just 2 competitors
 - Seems unrealistic for most markets

Plan

1. Cournot competition (refresher in IO notation)
 2. Bertrand competition (refresher)
 3. **Cournot vs Bertrand: when does each apply?**
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Cournot vs Bertrand: summary

	Cournot	Bertrand
Strategic variable	Quantities	Prices
Equilibrium price	$P > MC$	$P = MC$
Profits	Positive	Zero
Lerner index	$L = s/ \varepsilon $	$L = 0$

- Which model is “right”?
- Answer: depends on the industry!

Escaping the Bertrand paradox

1. Capacity constraints (Kreps-Scheinkman)

- Can't serve entire market at low price
- Leads to Cournot outcome

2. Product differentiation (today's main topic!)

- Consumers have preferences
- Not all switch to lowest price

3. Repeated interaction (collusion, covered later)

- Firms can sustain $P > MC$ through punishment strategies

When does each model apply?

- **Cournot applies when:**

- Capacity constraints matter
- Firms commit to production before selling
- Quantities are hard to adjust quickly
- Examples: manufacturing, airlines (seat capacity)

- **Bertrand applies when:**

- Prices adjust quickly
- No capacity constraints
- Homogeneous products
- Examples: online retail, commodities

Industry examples: which model?

Industry	Model	Why?
Airlines	Cournot	Capacity (planes, gates) committed
Cement	Cournot	Production committed; transport costs
Gasoline stations	Bertrand	Prices adjust daily; commodity
Online retail	Bertrand	Instant price changes; no capacity
Smartphones	Differentiated	Product differentiation dominates

- In practice: most markets have differentiated products!

Practice: Cournot vs Bertrand

- **Question:** Two firms have $MC = 20$. Market demand is $P = 100 - Q$.
- (a) Find equilibrium price under Cournot.
- (b) Find equilibrium price under Bertrand.
- (c) Which model yields higher consumer surplus? Why?

Take 3 minutes.

Practice: Cournot vs Bertrand (solution)

Solution

- **(a) Cournot:** Using $P^* = \frac{a+nc}{n+1} = \frac{100+2(20)}{3} = 46.67$
- Or: $q^* = \frac{100-20}{3} = 26.67$, $Q = 53.33$, $P = 46.67$
- **(b) Bertrand:** $P = MC = 20$
- **(c)** Bertrand has higher CS:
 - Bertrand: $CS = \frac{1}{2}(100 - 20)(80) = 3200$
 - Cournot: $CS = \frac{1}{2}(100 - 46.67)(53.33) \approx 1422$
- Lower price \Rightarrow higher quantity \Rightarrow higher CS

Kreps-Scheinkman (1983): resolving the puzzle

- Two-stage game:
 1. Stage 1: Firms choose capacities (quantities)
 2. Stage 2: Firms compete in prices
- **Result:** Equilibrium outcome = Cournot!
- **Intuition:**
 - Capacity choice commits firms
 - Price competition is constrained by capacity
 - Undercutting is limited by what you can produce
- Key insight: commitment matters

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4. **Product differentiation: why it matters**

5. Hotelling model
6. Connection to demand estimation

Why differentiation matters

- Bertrand paradox: $P = MC$ with homogeneous products
- **Solution:** Product differentiation!
- If products are different, consumers don't all buy from lowest-price firm
- Firms have some pricing power
- This is exactly what we modeled in Part 1 (logit demand)
- Now: a classic spatial model of differentiation

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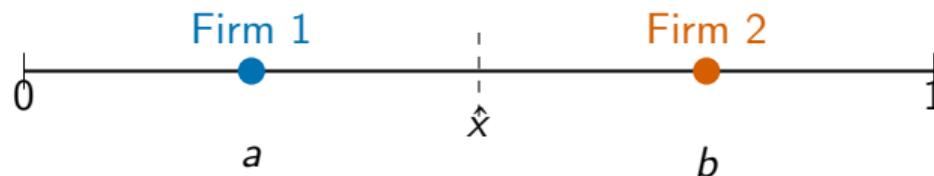
Hotelling model: setup

- Consumers uniformly distributed on $[0, 1]$ ("Main Street")
- Two firms located at positions a and b on $[0, 1]$
- Consumer at location x has utility:

$$u_j = v - p_j - t|x - \ell_j|$$

- v : base value of product
- p_j : price of firm j
- t : transport cost per unit distance
- $|x - \ell_j|$: distance to firm j

Hotelling: graphical intuition



- Consumers to the left of \hat{x} buy from Firm 1
- Consumers to the right of \hat{x} buy from Firm 2
- \hat{x} is the “indifferent consumer”

Finding the indifferent consumer

- Consumer at \hat{x} is indifferent between firms:

$$v - p_1 - t|\hat{x} - a| = v - p_2 - t|b - \hat{x}|$$

- With $a = 0$ and $b = 1$ (firms at endpoints):

$$v - p_1 - t\hat{x} = v - p_2 - t(1 - \hat{x})$$

$$p_2 - p_1 = t(1 - 2\hat{x})$$

$$\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

- Demand for firm 1: $D_1 = \hat{x}$

- Demand for firm 2: $D_2 = 1 - \hat{x}$

Hotelling: equilibrium prices

- Firm 1 maximizes: $\pi_1 = (p_1 - c) \cdot \hat{x}(p_1, p_2)$

- FOC: $\frac{\partial \pi_1}{\partial p_1} = \hat{x} + (p_1 - c) \frac{\partial \hat{x}}{\partial p_1} = 0$

- With $\frac{\partial \hat{x}}{\partial p_1} = -\frac{1}{2t}$:

$$\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{p_1 - c}{2t} = 0$$

- Symmetric equilibrium ($p_1 = p_2 = p^*$):

$$p^* = c + t$$

- **Markup = transport cost!**

Hotelling: interpretation

- $p^* = c + t$: Firms charge above marginal cost
- **Transport cost t measures differentiation**
 - High t : products very different \rightarrow high markup
 - Low t : products similar \rightarrow low markup
 - $t \rightarrow 0$: products identical \rightarrow Bertrand ($p \rightarrow c$)
- **No Bertrand paradox:** Differentiation creates pricing power
- Each firm gets half the market: $D_1 = D_2 = 1/2$
- Profit: $\pi = (p^* - c) \cdot \frac{1}{2} = \frac{t}{2}$

Worked example: Hotelling

- **Question:** Two ice cream vendors on a beach of length 1 mile. Transport cost $t = 2$ dollars per mile. Marginal cost $c = 1$.
- (a) Find the equilibrium price.
- (b) If firm 1 raises price to $p_1 = 4$, what is its market share?
- (c) Calculate firm 1's demand elasticity at the equilibrium.

Take 4 minutes.

Worked example: Hotelling (solution)

Solution

- (a) $p^* = c + t = 1 + 2 = 3$

- (b) At $p_1 = 4, p_2 = 3$:

$$\hat{x} = \frac{1}{2} + \frac{3 - 4}{2(2)} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

- Firm 1's market share falls to 25%

- (c) At equilibrium: $D_1 = 1/2, \frac{\partial D_1}{\partial p_1} = -\frac{1}{2t} = -\frac{1}{4}$

$$\varepsilon_1 = \frac{\partial D_1}{\partial p_1} \cdot \frac{p_1}{D_1} = -\frac{1}{4} \cdot \frac{3}{1/2} = -1.5$$

Hotelling: why transport cost matters

- **High t :** Strong differentiation
 - Consumers have strong location preferences
 - Firm can raise price without losing many customers
 - Example: specialty restaurants vs fast food
- **Low t :** Weak differentiation
 - Consumers nearly indifferent across locations
 - Small price cut steals many customers
 - Approaches Bertrand as $t \rightarrow 0$
- In demand estimation: $1/\alpha$ plays similar role to t

Practice: T/F on Hotelling

- **True, False, or NEI:**
- (a) In Hotelling equilibrium, firms always split the market equally.
- (b) A monopolist in Hotelling should locate at the center of the line.
- (c) Higher transport costs benefit consumers because products are more differentiated.

Take 2 minutes.

Practice: T/F on Hotelling (solution)

Answers

- **(a) TRUE** (with caveat). If firms are at endpoints and have same costs, yes. With asymmetric locations or costs, shares differ.
- **(b) TRUE.** Monopolist minimizes total transport costs by locating at center. This maximizes consumer value and thus WTP.
- **(c) FALSE.** Higher t means higher prices ($p^* = c + t$) and higher transport costs. Both hurt consumers.

Welfare in Hotelling

- **Total welfare** = Consumer surplus + Profits
- Transport costs are deadweight loss
- **Socially optimal locations:** $a = 1/4$, $b = 3/4$
 - Minimizes total transport costs
- **Equilibrium locations:** Both firms at $1/2$ (minimum differentiation)
 - Firms want to capture more customers
 - But this increases total transport costs
- “Principle of minimum differentiation” (but fragile)

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Connection to demand estimation

- Hotelling is a specific **differentiated Bertrand** model
- Location \leftrightarrow product characteristics
- Transport cost \leftrightarrow preference heterogeneity
- **Logit demand** generalizes this idea:
 - Products differ in characteristics space
 - Consumers have heterogeneous preferences
 - Price competition with differentiated products
- Next lectures: how to use demand estimates for merger simulation

From Hotelling to logit: the connection

Hotelling	Logit
Location on line	Product characteristics
Transport cost t	Sensitivity $1/ \alpha $
Consumer position	Consumer preferences
Linear utility	Logit choice probabilities
2 products	J products

- Logit demand lets us do empirical Hotelling-style analysis
- Estimate α from data → calibrate “differentiation”

Oligopoly models: summary

- **Homogeneous products:**

- Cournot (quantity): $L = s_i / |\varepsilon|$
- Bertrand (price): $P = MC$

- **Differentiated products:**

- Hotelling/Differentiated Bertrand: $P > MC$
- Markup depends on differentiation (transport cost)

- **Empirical approach:**

- Estimate demand (logit)
- Assume pricing behavior (usually differentiated Bertrand)
- Back out marginal costs

Preview: merger simulation

- **Key question:** If two firms merge, what happens to prices?
- **Approach:**
 1. Estimate demand → get elasticities
 2. Assume Bertrand pricing → back out MC
 3. Change ownership structure
 4. Solve new Bertrand equilibrium
 5. Compare prices and welfare
- This is exactly what HW2 will do!
- Will cover in detail in Lecture 10

Key Points

1. **Cournot:** Firms choose quantities; $L = s_i / |\varepsilon|$
2. **Bertrand (homogeneous):** $P = MC$, zero profits
3. **Bertrand paradox:** Only 2 firms but competitive outcome
4. Cournot applies with capacity constraints; Bertrand with flexible prices
5. **Product differentiation** creates pricing power
6. **Hotelling:** $p^* = c + t$ (markup = transport cost)
7. Higher t (more differentiation) \rightarrow higher markup
8. Hotelling connects to logit demand from Part 1

Next time

- **Lecture 9:** Entry and Market Structure

- Free entry condition
- Entry deterrence: limit pricing, excess capacity
- Strategic entry barriers