

ECN 594: Consumer Surplus, IIA, and Price Discrimination

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Plan for today

1. Consumer surplus: the log-sum formula
 2. The IIA problem: Red Bus / Blue Bus
 3. From demand to supply
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4. Types of price discrimination
 5. Selection by indicators
 6. Worked example: optimal pricing across markets

Part 1: Consumer Surplus and IIA

Why do we care about consumer surplus?

- Policy analysis requires measuring welfare
- Questions we want to answer:
 - How much do consumers gain from a new product?
 - How much do consumers lose from a merger?
 - What is the welfare cost of a price increase?
- Need a way to compute consumer surplus from our demand model

Consumer surplus in logit: the log-sum formula

- For consumer i , expected utility from choosing among J products:

$$E[\max_j u_{ij}] = \ln \left[\sum_{j=0}^J \exp(\delta_j + \mu_{ij}) \right] + \text{constant}$$

- This is the “log-sum” or “inclusive value”
- Consumer surplus (in dollars):

$$CS_i = \frac{1}{\alpha} \ln \left[\sum_{j=0}^J \exp(\delta_j + \mu_{ij}) \right]$$

- Divide by α (price coefficient) to convert to dollars

Worked example: CS change from removing a product

- **Question:** Market has 2 products with $\delta_1 = 1$, $\delta_2 = 0.5$. Outside option has $\delta_0 = 0$. Suppose $\alpha = 0.5$.
- What is the consumer surplus loss if product 1 is removed?

Take 3 minutes to solve this.

Worked example: CS change (solution)

- **Before removal:**

$$CS^{\text{before}} = \frac{1}{0.5} \ln(e^0 + e^1 + e^{0.5}) = 2 \ln(1 + 2.72 + 1.65) = 2 \ln(5.37) = 3.36$$

- **After removal:**

$$CS^{\text{after}} = \frac{1}{0.5} \ln(e^0 + e^{0.5}) = 2 \ln(1 + 1.65) = 2 \ln(2.65) = 1.95$$

- **Loss:** $3.36 - 1.95 = 1.41$ dollars per consumer
- This is on HW1!

The IIA problem: Red Bus / Blue Bus

- **Setup:** Consumers choose how to commute
- Choices: Car, Red Bus
- Suppose: half choose Car, half choose Red Bus
- So: $\delta_{\text{car}} = \delta_{\text{red bus}} = 0$

Red Bus / Blue Bus: introducing a new option

- Now introduce a **Blue Bus**
- But consumers are color-blind!
- Blue Bus is identical to Red Bus in every way
- **Reality:** Welfare should NOT change
 - It's the same bus, just different color
 - No real new option

What does logit predict?

- **Before Blue Bus:**

$$\text{Inclusive value} = \ln(e^0 + e^0) = \ln(2)$$

- **After Blue Bus:** $\delta_{\text{blue bus}} = \delta_{\text{red bus}} = 0$

$$\text{Inclusive value} = \ln(e^0 + e^0 + e^0) = \ln(3)$$

- Logit says welfare **increased!**
- But nothing real changed...

The IIA problem: what went wrong?

- Logit gives an extra “lottery ticket” for each product
- It doesn't know that buses are close substitutes
- **IIA:** The ratio s_j/s_k doesn't depend on other options

$$\frac{s_{\text{car}}}{s_{\text{red bus}}} = \frac{e^0}{e^0} = 1 \quad (\text{before and after!})$$

- Adding Blue Bus steals equally from Car and Red Bus
- But Car and Red Bus are NOT equally similar to Blue Bus

Why IIA matters

- IIA affects:
 1. **Valuing new products:** May overstate welfare gains
 2. **Merger analysis:** May mispredict substitution patterns
 3. **Cross-elasticities:** All products same cross-elasticity with any given product
- **When is logit “good enough”?**
 - Products are genuinely similar (e.g., brands of cereal)
 - You're not analyzing entry/exit of close substitutes

How demographics help (partial solution)

- With demographics: different consumer types have different substitution patterns
- Bus riders vs car commuters substitute differently
- Aggregate substitution is richer
- But IIA still holds *within* each consumer type
- **Mixed logit** (random coefficients) fully relaxes IIA
 - Beyond our scope, but important to know

From demand to supply

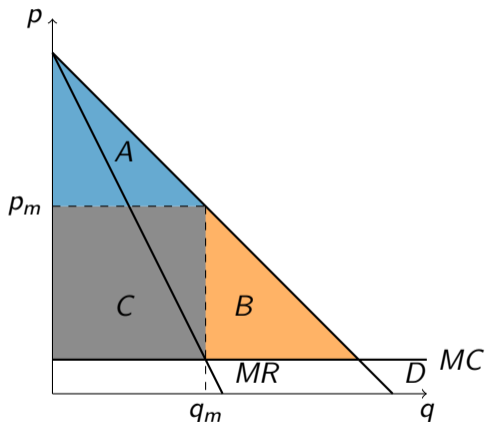
- We've focused on demand estimation
- **Key insight:** Demand gives us the hard part
 - Elasticities
 - Substitution patterns
 - Consumer welfare
- Costs can often be *backed out* from pricing behavior
- Using the Lerner index: $mc = p - p/|\epsilon|$

Part 2: Price Discrimination

Price Discrimination

- Price discrimination: **setting different prices for the same good**
- Examples: airline tickets, software, pharmaceuticals, student discounts
- We will look at different ways firms price discriminate

Why price discriminate?



- **Area A:** Consumers WTP $> p_m$
 - Could charge them more!
- **Area B:** Consumers WTP between MC and p_m
 - Could sell to them at lower price
- **Area C:** Current profit

Types of price discrimination (Cabral terminology)

1. Perfect price discrimination

- Charge each consumer their exact WTP
- Extracts all surplus; unrealistic benchmark

2. Selection by indicators

- Divide buyers into groups by observable characteristics
- Set different price for each group

3. Self-selection

- Cannot observe type directly
- Design menu to induce consumers to reveal type
- (Covered next lecture)

Selection by indicators

- Divide buyers into groups based on **observable characteristics**
- Set different price for each group
- Examples:
 - Student discounts (show student ID)
 - Senior discounts
 - Geographic pricing (different prices in different countries)
 - Time-based pricing (matinees vs evening shows)

Selection by indicators: setup

- Two markets: market 1 and market 2
- Demand: $q_1 = D_1(p_1)$ and $q_2 = D_2(p_2)$
- Cost: $C(q_1 + q_2)$, with constant MC
- **Goal:** Find optimal price in each market

Selection by indicators: solution

- Apply optimal pricing rule in each market:

$$MR_1 = MC \quad \text{and} \quad MR_2 = MC$$

- Equivalently, using elasticity rule:

$$\frac{p_1 - MC}{p_1} = \frac{1}{|\varepsilon_1|} \quad \text{and} \quad \frac{p_2 - MC}{p_2} = \frac{1}{|\varepsilon_2|}$$

- **Key implication:** Charge higher price in market with more inelastic demand

Worked example: Optimal pricing across markets

- **Question:**
- Two markets with $\varepsilon_1 = -2$ and $\varepsilon_2 = -4$
- $MC = 6$
- Find optimal prices in each market.

Take 3 minutes to solve this.

Worked example: Optimal pricing (solution)

- Using Lerner index: $\frac{p-MC}{p} = \frac{1}{|\varepsilon|}$

- **Market 1** ($\varepsilon_1 = -2$):

$$\frac{p_1 - 6}{p_1} = \frac{1}{2} \Rightarrow p_1 - 6 = 0.5p_1 \Rightarrow p_1 = 12$$

- **Market 2** ($\varepsilon_2 = -4$):

$$\frac{p_2 - 6}{p_2} = \frac{1}{4} \Rightarrow p_2 - 6 = 0.25p_2 \Rightarrow p_2 = 8$$

- Price is higher in more inelastic market (market 1)

Welfare effects of selection by indicators

- **Producer surplus:** Increases (that's why firms do it)
- **Consumer surplus:** Ambiguous
 - Some consumers pay more (inelastic market)
 - Some consumers pay less (elastic market)
 - Some consumers now served who weren't before
- **Total welfare:** Depends on whether new markets are served
 - If discrimination opens new markets → welfare may increase
 - If just redistributes → welfare may decrease

Key Points

1. **Log-sum formula:** $CS_i = \frac{1}{\alpha} \ln [\sum_j \exp(\delta_j)]$
2. **Red Bus / Blue Bus:** Logit overcounts value of similar products
3. **IIA:** Substitution proportional to share, not similarity
4. Demographics partially help; mixed logit fully relaxes IIA
5. **Price discrimination:** Different prices for same good
6. **Perfect PD:** Charge each consumer their WTP (benchmark)
7. **Selection by indicators:** Group pricing based on observables
8. Charge higher price in **more inelastic** market: $p = MC / (1 + 1/\epsilon)$

Next time

- **Lecture 5:** Two-Part Tariffs and Self-Selection
 - Two-part tariffs: $F + p \times q$
 - Self-selection: menu design, versioning
 - Incentive compatibility constraints