ECN 594: Homework 1 Solutions

Demand Estimation

Solution Key

Question 1: Basic Logit Model

(a) Berry Inversion

Starting from the logit choice probability:

$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{J} \exp(\delta_k)}$$
$$s_0 = \frac{1}{1 + \sum_{k=1}^{J} \exp(\delta_k)}$$

Taking the ratio:

$$\frac{s_j}{s_0} = \exp(\delta_j)$$

Taking logs:

$$\ln(s_j) - \ln(s_0) = \delta_j$$

Substituting $\delta_j = \beta_0 + \beta_1 \cdot \text{sugar}_j + \alpha \cdot p_j + \xi_j$:

$$\boxed{\ln(s_j) - \ln(s_0) = \beta_0 + \beta_1 \cdot \text{sugar}_j + \alpha \cdot p_j + \xi_j}$$

This is a linear regression with dependent variable $\ln(s_j) - \ln(s_0)$ and error term ξ_j.

(b) OLS Estimation

Parameter	OLS Estimate
$\hat{\beta}_0$ (constant)	-2.9665
$\hat{\beta}_1$ (sugar)	0.0463
$\hat{\alpha}$ (price)	-10.2036

(c) OLS Bias

OLS is likely biased because prices are **endogenous**:

- Firms set higher prices for products with higher unobserved quality (ξ_j)
- This creates positive correlation: $\text{Cov}(p_j, \xi_j) > 0$
- Since ξ_j is in the error term, we have $\text{Cov}(p_j, \text{error}) > 0$
- This causes **upward bias** in α (makes it less negative than the true value)

Intuition: Products consumers like for unobserved reasons (brand, taste, advertising) charge higher prices. OLS interprets high prices with high shares as "price doesn't hurt demand much" when really it's unobserved quality driving both.

(d) 2SLS Estimation

Parameter	OLS	2SLS
$\hat{\beta}_0$ (constant)	-2.9665	-2.8915
$\hat{\beta}_1$ (sugar)	0.0463	0.0483
$\hat{\alpha}$ (price)	-10.2036	-10.9449

As expected, OLS is **less negative** than 2SLS (-10.20 vs -10.94), confirming upward bias. The 2SLS estimate suggests consumers are more price sensitive than OLS indicates.

(e) Own-Price Elasticities

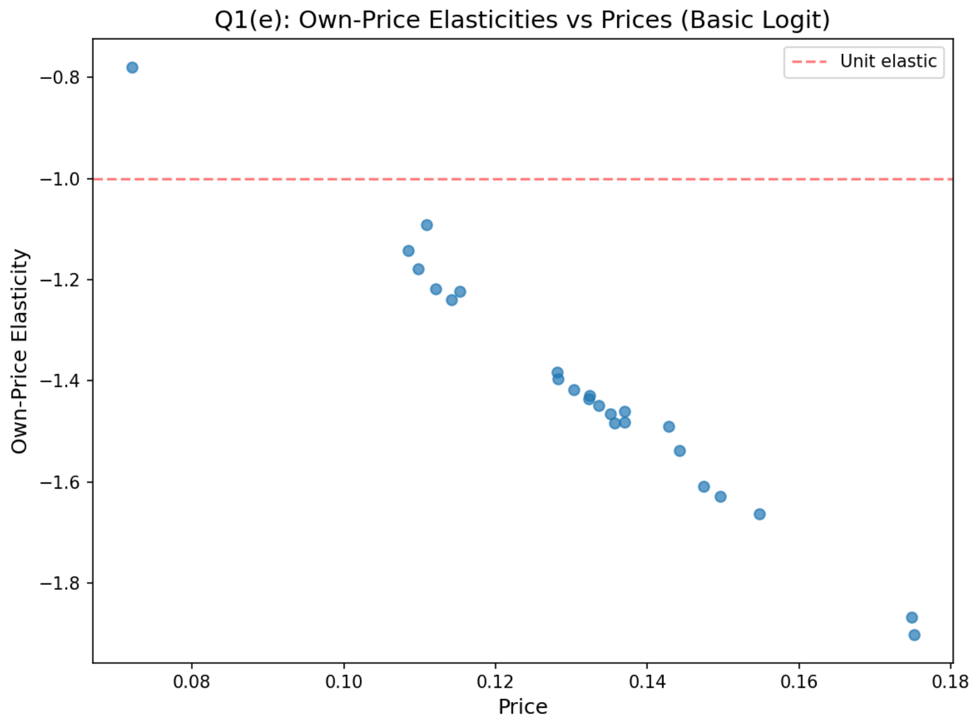

Figure 1: Own-price elasticities vs. prices for basic logit model (market C01Q1)

There is a **negative linear relationship** between prices and elasticities.

This is a **bug** (limitation) of the logit model. The formula $\eta_{jj} = \alpha \cdot p_j \cdot (1 - s_j)$ mechanically ties elasticity to price. Since $\alpha < 0$ and $(1 - s_j) \approx 1$ for small shares, elasticity is roughly proportional to price.

This relationship is imposed by functional form, not learned from data. In reality, some high-priced products may have inelastic demand (luxury goods), while some low-priced products may be elastic (commodities).

Question 2: Logit with Demographic Interactions

(a) Interpretation of α_{inc}

α_{inc} captures how income affects price sensitivity.

Total price coefficient: $\alpha_i = \alpha_0 + \alpha_{inc} \cdot \text{income}_i$

Expected sign: Positive

Reasoning:

- $\alpha_0 < 0$ (everyone dislikes higher prices)
- Higher-income consumers are less price sensitive
- If $\alpha_{inc} > 0$, high-income consumers have α_i closer to zero (less negative)

Example: If $\alpha_0 = -5$ and $\alpha_{inc} = 2$:

- Low income (= 0): $\alpha = -5$ (very price sensitive)
- High income (= 1): $\alpha = -3$ (less price sensitive)

(b) Estimation Results

Parameter	Estimate
$\hat{\beta}_0$ (constant)	-3.7933
$\hat{\alpha}_0$ (price)	-3.1154
$\hat{\beta}_{0,inc}$ (constant × income)	4.9601
$\hat{\alpha}_{inc}$ (price × income)	-37.9653
$\hat{\beta}_1$ (sugar × income)	0.1336

Note: The large negative α_{inc} is an artifact of how income is scaled in the data.

(c) Elasticities with Demographics

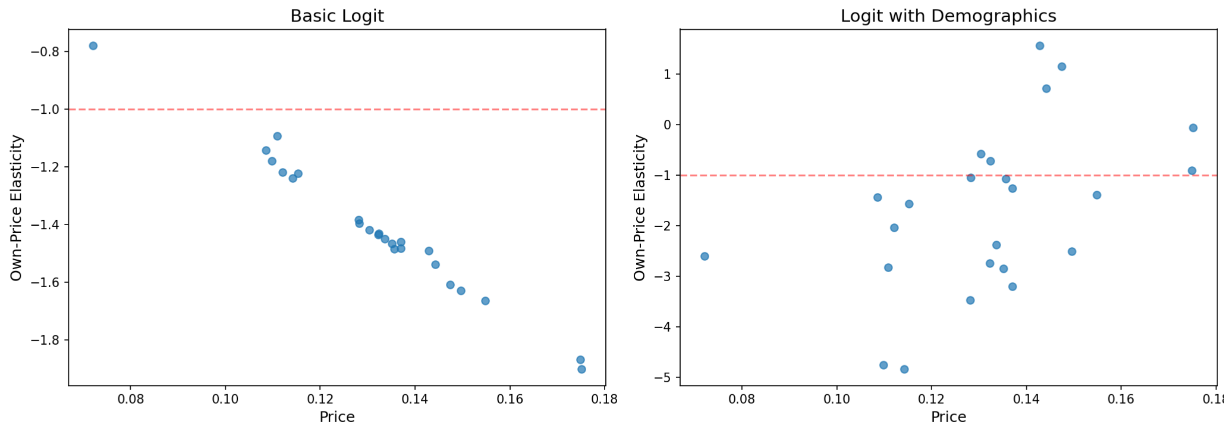

Figure 2: Own-price elasticities: basic logit (left) vs. logit with demographics (right)

The tight linear relationship is now **broken**—the pattern is scattered.

Why?

- Basic logit: all consumers have same α, so elasticity mechanically depends on price
- With demographics: different consumers have different α_i
- Aggregate elasticity depends on distribution of consumer types and sorting
- Products attracting high-income (less price-sensitive) consumers may have lower elasticities even at high prices

This is more realistic—the model now lets data reveal the price-elasticity relationship.

(d) Does Adding Demographics Help with IIA?

Partially yes:

- Different consumer types have different substitution patterns
- At the aggregate level, this creates richer substitution

Still limited:

- For any individual consumer type, IIA still holds
- A consumer with specific income still has logit substitution patterns

What would fully address IIA:

- Random coefficients (mixed logit / full BLP)
- Allows idiosyncratic preferences even conditional on demographics
- Substitution then depends on characteristic similarity, not just shares

Question 3: Consumer Surplus

(a) Consumer Surplus Formula

For Type 1 EV errors, expected maximum utility has closed form. The "inclusive value" is:

$$IV_t = \ln\left(1 + \sum_{j=1}^{J_t} \exp(\delta_{jt})\right)$$

Consumer surplus in monetary units:

$$\boxed{E[CS_t] = \frac{1}{|\alpha|} \ln\left(1 + \sum_{j=1}^{J_t} \exp(\delta_{jt})\right)}$$

Notes:

- CS is relative to having only the outside option
- The "1" inside the log is from outside option ($\delta_0 = 0$)
- Each product j contributes $\exp(\delta_{jt})$ to the sum

(b) CS Calculation for C01Q1

Using 2SLS estimates with $\hat{\alpha} = -10.9449$:

Scenario	Expected CS per Consumer
All products available	\$0.0538
Without F1B04	\$0.0526
Change from removing F1B04	−\$0.0011

Consumers lose about \$0.0011 per person when F1B04 is removed.

(c) Products with Largest/Smallest Impact

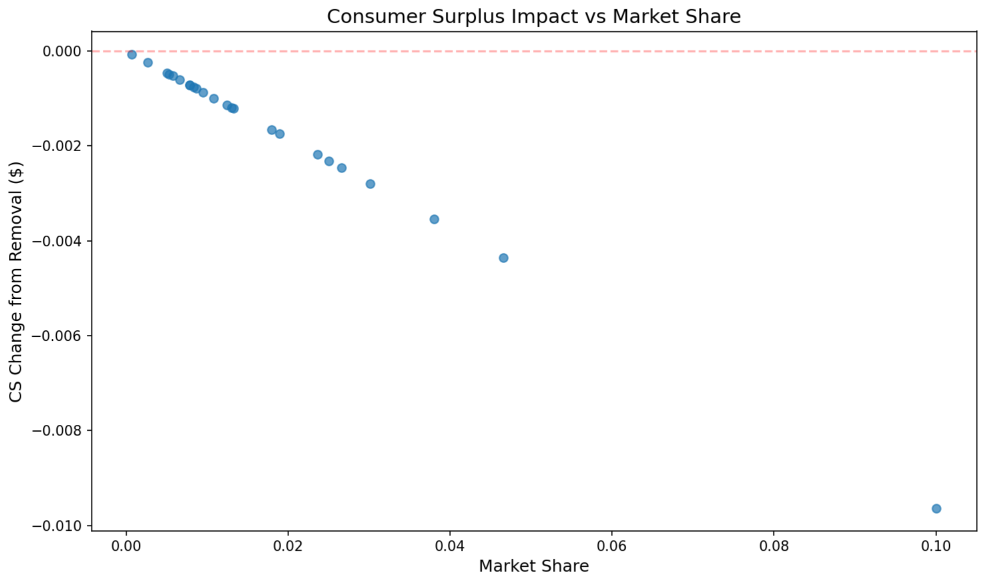

Figure 3: Consumer surplus impact vs. market share

Product whose removal most harms consumers: F2B19 (market share 10.0%, CS change −$0.0096).

Product whose removal least harms consumers: F4B10 (market share 0.07%, CS change −$0.0001).

Intuition: From the log-sum formula, products with high $\exp(\delta)$ have larger impact on inclusive value. High δ means:

- Lower price (since $\alpha < 0$)
- Desirable characteristics
- High unobserved quality

This makes sense: removing a popular product that many consumers choose hurts welfare more than removing a niche product few consumers buy.

Key relationship: CS change is roughly proportional to market share, since share $\propto \exp(\delta)$.