ECN 594: Utility Models and Demand

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Plan for today

1. Why do we need demand models in IO?
2. Ordinal vs cardinal utility
3. Random utility framework
4. From individual choice to market demand
5. The dimensionality problem

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1. **Why do we need demand models in IO?**
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Why is estimating demand useful?

- Quantifying market power (think: Lerner index)
- Effects of a merger on prices
- Value of new goods
- Any question about consumer welfare
- Numerous other applications: school choice, health insurance, etc.

Why is estimating demand useful?

- Recall from Part 1: Lerner index $L = (p - MC)/p = 1/|\varepsilon|$
- To compute market power, we need the **demand elasticity** ε
- How do we get the elasticity?

Why is estimating demand useful?

- Recall from Part 1: Lerner index $L = (p - MC)/p = 1/|\varepsilon|$
- To compute market power, we need the **demand elasticity** ε
- How do we get the elasticity?
- We need to **estimate demand**
- This is one of the key empirical methods in IO

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Ordinal vs cardinal utility

- **Ordinal utility:** only rankings matter, not magnitudes
  - $U(A) > U(B)$ means I prefer A to B
  - But "$U(A) = 2 \times U(B)$" has no meaning
- This is the standard microeconomics assumption
- **Problem:** We want to measure consumer surplus in \$!
  - "How much better off are consumers from this policy?"
  - Need utility to have a meaningful *scale*

Quasi-linear utility makes utility cardinal

- **Solution:** Assume quasi-linear utility

$$U = u(\text{goods}) + y$$

  where y is income (or "money left over")

- Take the derivative with respect to income:

$$\frac{\partial U}{\partial y} = 1$$

- The **marginal utility of income is constant** (and equals 1)

- This means we can measure utility in dollars!

Why this matters

- With quasi-linear utility:
    - 1 util = \$1
    - Consumer surplus has a meaningful interpretation
    - We can add up utility across consumers
- This is the standard assumption in IO demand estimation
- (In contrast to general equilibrium models where income effects matter)

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Random utility framework (refresher from ECN 565)

- Consumer i chooses among J products
- Utility of consumer i for product j:

$$u_{ij} = V_{ij} + \varepsilon_{ij}$$

- V_{ij}: deterministic component (observed by econometrician)
- ε_{ij}: random component (taste shock)

Random utility framework

- Consumer i chooses product j if:

$$u_{ij} > u_{ik} \quad \text{for all } k \neq j$$

- Choice probability:

$$P(i \text{ chooses } j) = P(u_{ij} > u_{ik} \text{ for all } k)$$

- Different assumptions on ε_{ij} give different models:
  - Type I Extreme Value $\rightarrow$ **Logit**
  - Normal $\rightarrow$ **Probit**
- You covered this in ECN 565; we'll apply it to IO

Discrete choice in IO

- In IO, we typically write:

$$u_{ij} = x_j \beta - \alpha p_j + \xi_j + \varepsilon_{ij}$$

- x_j: observed product characteristics (size, horsepower, etc.)
- p_j: price
- ξ_j: unobserved product quality
- ε_{ij}: idiosyncratic taste shock
- $\alpha > 0$: price coefficient (enters negatively!)

What is ξ_j?

- ξ_j = unobserved product quality
- Examples:
  - Brand equity ("I just like Toyota")
  - Advertising effects
  - Design/style
  - Reputation
- Key insight: firms *observe* ξ_j when setting prices!
- This creates an endogeneity problem (more on this next lecture)

Why isn't income in the utility function?

- You might expect: $u_{ij} = x_j\beta + \alpha(y_i - p_j) + \xi_j + \varepsilon_{ij}$

- But we write: $u_{ij} = x_j\beta - \alpha p_j + \xi_j + \varepsilon_{ij}$

- **Why?**

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- But we write: $u_{ij} = x_j\beta - \alpha p_j + \xi_j + \varepsilon_{ij}$
- **Why?**
- With quasi-linear utility, income enters linearly
- When comparing alternatives, income *cancels out*:

$$\begin{aligned} u_{ij} - u_{ik} &= [x_j\beta + \alpha(y_i - p_j) + \xi_j + \varepsilon_{ij}] \\ &\quad - [x_k\beta + \alpha(y_i - p_k) + \xi_k + \varepsilon_{ik}] \\ &= (x_j - x_k)\beta - \alpha(p_j - p_k) + (\xi_j - \xi_k) + (\varepsilon_{ij} - \varepsilon_{ik}) \end{aligned}$$

- Only *differences* matter for choice, and y_i drops out

What does α mean?

- In our utility function: $u_{ij} = x_j\beta - \alpha p_j + \xi_j + \varepsilon_{ij}$
- α captures the **marginal utility of income**
- Since we normalize the scale of utility, α tells us how much consumers dislike paying
- Higher $\alpha \rightarrow$ more price-sensitive consumers
- This is why we can measure consumer surplus in \$ later

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From individual choice to market demand

- Individual choice probability:

$$P(i \text{ chooses } j)$$

- Market share = average of individual choice probabilities:

$$s_j = \int P(i \text{ chooses } j)\, dF(i)$$

- If all consumers have the same preferences (up to ε_{ij}):

$$s_j = P(\text{consumer chooses } j)$$

- With heterogeneous preferences, we integrate over consumer types

The outside option

- Important: Consumers can choose **not to buy** any product
- This is the **outside option** (product 0)
- Utility of outside option:

$$u_{i0} = \varepsilon_{i0}$$

- We normalize non-idiosyncratic components to 0
- All other utilities are *relative* to the outside option
- Why this matters:
  - If prices rise, consumers can "exit" the market
  - This affects elasticities and market power

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The dimensionality problem

- Suppose we have J products in the market
- What do we need to describe demand fully?

The dimensionality problem

- Suppose we have J products in the market
- What do we need to describe demand fully?
- **Own-price elasticities:** J elasticities
- **Cross-price elasticities:** $J \times (J-1)$ elasticities
- Total: J^2 elasticities!
- For $J = 100$ products: 10,000 elasticities to estimate

The dimensionality problem

- This is a fundamental problem in demand estimation
- Solutions:
  1. **Aggregate demand:** Estimate a single elasticity
     - But this ignores substitution between products
  2. **AIDS/Rotterdam:** Estimate a demand system
     - Works for few products, but doesn't scale
  3. **Discrete choice:** <u>Model based on product characteristics</u>
     - The solution we'll use in this course

Characteristics-based models

- **Key idea:** Products are bundles of characteristics

- Consumers have preferences over characteristics, not products

$$u_{ij} = x_j \beta - \alpha p_j + \xi_j + \varepsilon_{ij}$$

- Instead of J^2 elasticities, we estimate:
    - K coefficients on characteristics (β)
    - 1 price coefficient (α)

- Cross-price elasticities come out of the model structure

- Products with similar characteristics are close substitutes

Example: Cars

- Product characteristics x_j:
  - Size (length, weight)
  - Horsepower
  - Fuel efficiency (MPG)
  - Air conditioning, etc.
- A Honda Civic and Toyota Corolla have similar characteristics
- $\Rightarrow$ They are close substitutes
- A Honda Civic and BMW 7-Series have different characteristics
- $\Rightarrow$ They are not close substitutes
- This structure comes from the model, not from estimating J^2 elasticities

Looking ahead

- **Next lecture:** The logit demand model
  - Assume ε_{ij} is Type I Extreme Value
  - Get closed-form choice probabilities
  - Derive elasticity formulas
- **Later:** Identification and instrumental variables
  - Why is price endogenous?
  - How do we solve this problem?
- **Later:** Estimation with `pyblp`

Summary

- Demand estimation is central to IO
    - Elasticities $\rightarrow$ market power $\rightarrow$ policy analysis
- **Quasi-linear utility** makes utility cardinal (measurable in \$)
- **Random utility framework:** $u_{ij} = V_{ij} + \varepsilon_{ij}$
- Income cancels out due to quasi-linearity
- **Dimensionality problem:** J products $\rightarrow$ J^2 elasticities
- **Solution:** Characteristics-based models
    - Products are bundles of characteristics
    - Substitution patterns come from model structure