

ECN 453: Homework 2

Due: Start of class Monday 25th October.

You can work in groups of up to 3 people (hand in one solution per group).

1. Bertrand Competition (20 points)

Given:

$$\begin{aligned} Q &= 90 - 3p \\ p &= \min\{p_1, p_2\} \\ c_1 &= 15 \\ c_2 &= 10 \end{aligned}$$

a. Let's find monopoly price of firm 2 (p_2^M):

$$\begin{aligned} \pi_2 &= (30 - \frac{Q}{3}Q) - 10Q \\ \frac{d\pi_2}{dQ} &= 0 \implies \\ 30 - \frac{2Q}{3} - 10 &= 0 \implies Q_2^M = 30 \implies p_2^M = 20 \end{aligned}$$

So, we have a case of $p_1^M > p_2^M > c_1 > c_2$

Then NE will be such that: $p_1 = c_1$, $p_2 = c_1 - \epsilon \approx c_1$, where $c_1 = 15$

Profits of Firm 1 = 0 as $q_1 = 0$

Quantity of firm 2: $q_2 = 90 - 3 \times 15 = 45$

Profits of firm 2: $\pi_2 = (15 - 10) \times 45 = 225$

b. Let the firm with marginal cost of 20 be denoted as Firm 1, and the other two firms as Firm 2 and 3. Now we have the following case:

$$p_1^M > \underbrace{p_2^M = p_3^M = c_1}_{20} > \underbrace{c_2 = c_3}_{10}$$

In this case Firm 2 and 3 will drive down their price to their marginal cost, i.e. $p = 10$

Why? Suppose not, such that $p_2 \in (10, 20]$. Then firm 3 can set a price $p_3 - \epsilon$ such that Firm 3 captures the whole demand. Notice that at $p_2 = p_3 = 10$, both firms have no incentive to deviate.

- c. Price competition will not necessarily drive down prices to marginal costs.

2. Cournot Competition With Increasing Marginal Costs (25 points)

Given:

$$\underbrace{Q}_{q_1+q_2} = 10 - 0.5p \implies p = 20 - 2(q_1 + q_2)$$

$$c(q_i) = 10 + q_i^2$$

a.

$$\pi_1 = p(Q)q_1 - c(q_1) \implies \pi_1 = (20 - 2q_1 - 2q_2)q_1 - 10 - q_1^2$$

$$\frac{d\pi_1}{dq_1} = 0 \implies 20 - 4q_1 - 2q_2 - 2q_1 = 0 \implies q_1(q_2) = \frac{10 - q_2}{3}$$

By symmetry:

$$q_2(q_1) = \frac{10 - q_1}{3}$$

In equilibrium, $q_1 = q_2$, so:

$$q_1 = \frac{10 - q_1}{3} \implies q_1 = 2.5 = q_2 \implies p = 20 - 2(5) = 10 \implies \pi_i = 10 \times 2.5 - 10 - 2.5^2 = 8.75$$

b. Now:

$$\underbrace{Q}_{q_1+q_2+q_3} = 10 - 0.5p \implies p = 20 - 2(q_1 + q_2 + q_3)$$

$$c(q_1) = c(q_2) = 10 + q_i^2$$

$$c(q_3) = 10 + 2q_3^2$$

Solving for Firm 1 (Analogous for Firm 2):

$$\pi_1 = (20 - 2q_1 - 2q_2 - 2q_3)q_1 - 10 - q_1^2$$

$$\frac{d\pi_1}{dq_1} = 0 \implies$$

$$6q_1(q_2, q_3) = 20 - 2q_2 - 2q_3$$

By symmetry:

$$6q_2(q_1, q_3) = 20 - 2q_1 - 2q_3$$

Solving for Firm 3:

$$\pi_3 = (20 - 2q_1 - 2q_2 - 2q_3)q_3 - 10 - 2q_3^2$$

$$\frac{d\pi_3}{dq_3} = 0 \implies$$

$$8q_3(q_1, q_2) = 20 - 2q_1 - 2q_2$$

Since the problems of Firm 1 and 2 are identical, we can substitute $q_1 = q_2$ for the $q_1(q_2, q_3)$ equation:

$$6q_1 = 20 - 2q_1 - 2q_3 \implies$$

$$q_1 = 2.5 - 0.25q_3 \implies$$

$$q_2 = 2.5 - 0.25q_3$$

Plugging $q_1 = q_2$ into the $q_3(q_1, q_2)$ equation:

$$8q_3 = 20 - 4q_1 \implies$$

$$q_3 = 2.5 - 0.5q_1$$

Plugging this into q_1 :

$$q_1 = q_2 = 2.1428$$

$$q_3 = 1.429$$

3. Cournot Competition With Asymmetric Marginal Costs (30 points)

Given:

$$\underbrace{Q}_{q_1+q_2} = 450 - 2p \implies$$

$$p = 225 - \frac{(q_1 + q_2)}{2}$$

$$c_1 = 50$$

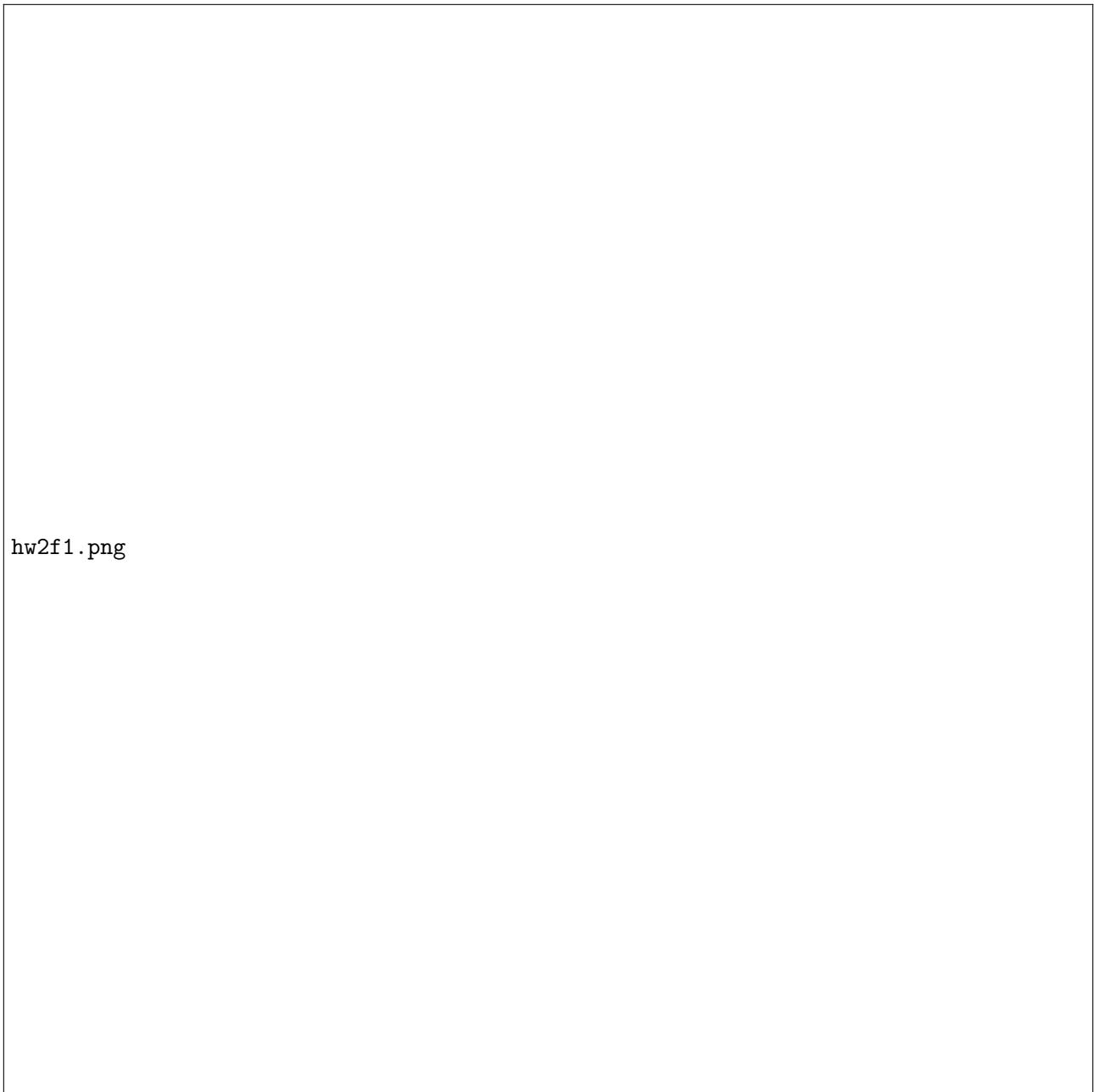
$$c_2 = 40$$

- a. In order to find the best responses, we can equate $MR = MC$ for both firms:

$$MR_i(q_i) = 225 - q_i - \frac{q_{-i}}{2} \implies$$

$$q_1(q_2) = 175 - \frac{q_2}{2}$$

$$q_2(q_1) = 185 - \frac{q_1}{2}$$



hw2f1.png

- b. Equate the two best response functions ($q_1(q_2)$ and $q_2(q_1)$) to find equilibrium quantities of 110 and 130 for Firm 1 and 2 respectively
- c. Refer to dashed lines in the graph above. Formed by adjusting the $MR = MC$ condition for Firm 1 with the new marginal cost.
- d. Firm 1's best response (after accounting for technological innovation):

$$q_1(q_2) = 181 - \frac{q_2}{2}$$

Firm 2's best response (same as old one):

$$q_2(q_1) = 185 - \frac{q_1}{2}$$

By equating the two best responses, we get equilibrium quantities of:

$$\begin{aligned} q_1 &= 118 \\ q_2 &= 126 \\ p &= 225 - \frac{118 + 126}{2} = 103 \end{aligned}$$

Therefore, Firm 1's profits (under new tech):

$$\pi_1^{new} = (p - c_1) \times q_1 = (103 - 44)118 = 6962$$

Firm 1's profits (under old tech):

$$\pi_1^{old} = (p - c_1) \times q_1 = (105 - 50)110 = 6050$$

$$\text{where } p = 225 - \frac{110 + 130}{2} = 105$$

Therefore, Firm 1 would be willing to pay up to $\pi_1^{new} - \pi_1^{old} = 912$

4. Stackelberg Competition (25 points)

Given:

$$p = 100 - 4 \underbrace{Q}_{q_1 + q_2}$$

$$c_1 = c_2 = 20$$

- a. Solve problem backwards. Firm 2 solves problem taking Firm 1's output as given:

$$\pi_2 = (100 - 4q_1 - 4q_2)q_2 - 20q_2$$

$$\frac{d\pi_2}{dq_2} = 0 \implies$$

$$100 - 4q_1 - 8q_2 - 20 = 0 \implies$$

$$4q_2(q_1) = 40 - 2q_1$$

Now let's look at Firm 1's problem. Firm 1 now solves the problem taking Firm 2's best response into account:

$$\pi_1 = (100 - 4q_1 - 4q_2(q_1))q_1 - 20q_1$$

$$\pi_1 = (60 - 2q_1)q_1 - 20q_1$$

$$\frac{d\pi_1}{dq_1} = 0 \implies$$

$$60 - 4q_1 - 20 = 0 \implies$$

$$q_1 = 10 \implies q_2(q_1) = 5$$