ECN 594: Midterm Review

Nicholas Vreugdenhil

January 4, 2026

Plan

1. **Demand estimation review**
2. Pricing review
3. Exam logistics
4. Preview of Part 2

Midterm format

- **Duration:** 80 minutes
- **Allowed:** Calculator + two-sided cheat sheet (letter-size paper)
- **Coverage:** Lectures 1-5 (all of Part 1)
- **Structure:**
  - Short answer questions (T/F/NEI, definitions, quick calculations)
  - Longer problems (derivations, pricing, elasticity calculations)

Plan

1. **Demand estimation review**
2. Pricing review
3. Exam logistics
4. Preview of Part 2

Why demand estimation?

- We need demand models to:
  1. Measure substitution patterns between products
  2. Compute price elasticities
  3. Evaluate policy (mergers, new products, price changes)
  4. Calculate consumer welfare
- The dimensionality problem: J products $\rightarrow$ J^2 elasticities
- Solution: characteristics-based models (Lancaster, BLP)

The logit model: key equations

- Utility: $u_{ij} = \delta_j + \varepsilon_{ij}$ where $\delta_j = x_j\beta + \alpha p_j + \xi_j$
- Choice probability (market share):

$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{J} \exp(\delta_k)}$$

- Outside option: $s_0 = \frac{1}{1+\sum_{k=1}^{J} \exp(\delta_k)}$
- **Berry inversion:**

$$\ln(s_j) - \ln(s_0) = \delta_j$$

Logit elasticities

- **Own-price elasticity:**

$$\eta_{jj} = \frac{\partial s_j}{\partial p_j} \cdot \frac{p_j}{s_j} = \alpha p_j (1 - s_j)$$

- **Cross-price elasticity:**

$$\eta_{jk} = \frac{\partial s_j}{\partial p_k} \cdot \frac{p_k}{s_j} = -\alpha p_k s_k$$

- Note: $\alpha < 0$ so own-price elasticity is negative

- **IIA problem:** Cross-elasticity depends only on k, not on similarity to j

Practice: Elasticity calculation

- **Question:** Suppose $\alpha = -0.5$, product A has price $p_A = 20$ and share $s_A = 0.1$.
- (a) Calculate the own-price elasticity of product A.
- (b) If product B has $p_B = 25$ and $s_B = 0.15$, what is the cross-price elasticity of A with respect to B's price?

Take 3 minutes.

Practice: Elasticity calculation (solution)

Solution

- **(a) Own-price elasticity:**

$$\eta_{AA} = \alpha p_A (1 - s_A) = (-0.5)(20)(1 - 0.1) = -9$$

- Demand is elastic (a 1% price increase reduces quantity by 9%)

- **(b) Cross-price elasticity:**

$$\eta_{AB} = -\alpha p_B s_B = -(-0.5)(25)(0.15) = 1.875$$

- A 1% increase in B's price increases A's share by 1.875%

The identification problem

- **Problem:** We observe equilibrium (p, q) pairs
- Can't tell if demand shifted or supply shifted
- **Price endogeneity:** High unobserved quality $\xi_j \rightarrow$ high price
  - OLS sees: high price, still high demand
  - Concludes: price doesn't matter much
  - Result: $\hat{\alpha}$ biased toward zero
- **Solution:** Instrumental variables
  - Need: correlated with price, uncorrelated with ξ
  - Examples: Hausman IVs, BLP IVs, cost shifters

Practice: Identification

- **True, False, or Not Enough Information:**
- (a) OLS estimation of demand typically underestimates the price coefficient (in absolute value).
- (b) Using prices of the same product in other geographic markets as an IV is valid because prices in other markets don't affect local demand.
- (c) The logit model solves the dimensionality problem by assuming all products are equally substitutable.

Take 3 minutes.

Practice: Identification (solutions)

Answers

- **(a) TRUE.** Price endogeneity biases α toward zero. Since $\alpha < 0$, this means $|\hat{\alpha}_{OLS}| < |\alpha_{true}|$.
- **(b) TRUE (with caveat).** Hausman IVs work because common cost shocks affect prices in all markets (relevance), but other markets' prices don't directly affect local demand (exclusion). Caveat: requires no common demand shocks.
- **(c) FALSE.** Logit solves dimensionality by using product characteristics. But the IIA problem means substitution is proportional to share, not similarity.

Consumer surplus: log-sum formula

- Expected utility for consumer i:

$$E[\max_j u_{ij}] = \ln\left[\sum_{j=0}^{J} \exp(\delta_j)\right] + \text{constant}$$

- Consumer surplus (in dollars):

$$CS = \frac{1}{|\alpha|} \ln\left[\sum_{j=0}^{J} \exp(\delta_j)\right]$$

- Divide by $|\alpha|$ to convert utils to dollars

- **Application:** Welfare change from adding/removing products

The IIA problem

- **IIA:** Ratio of choice probabilities doesn't depend on other options

$$\frac{s_j}{s_k} = \frac{\exp(\delta_j)}{\exp(\delta_k)} = \exp(\delta_j - \delta_k)$$

- **Red Bus / Blue Bus:** Adding identical bus option incorrectly increases welfare
- **Problem:** Logit doesn't know similar products are close substitutes
- **Solutions:**
  - Demographic interactions (partial fix)
  - Mixed logit / random coefficients (full fix, beyond scope)

Quick summary: Logit elasticity formulas

Formula	**Expression**
Own-price elasticity	$\eta_{jj} = \alpha p_j (1 - s_j)$
Cross-price elasticity	$\eta_{jk} = -\alpha p_k s_k$
Berry inversion	$\delta_j = \ln(s_j) - \ln(s_0)$
Consumer surplus	$CS = \frac{1}{\lvert\alpha\rvert} \ln\left[\sum_j \exp(\delta_j)\right]$

- Remember: $\alpha < 0$, so own-price elasticity is negative
- Put these on your cheat sheet!

Practice: Berry inversion

- **Question:** A market has 3 products plus outside option.
- Observed shares: $s_0 = 0.4$, $s_1 = 0.3$, $s_2 = 0.2$, $s_3 = 0.1$
- (a) Compute the mean utilities δ_1, δ_2, δ_3.
- (b) Rank the products by consumer preference.

Take 2 minutes.

Practice: Berry inversion (solution)

Solution

- Using $\delta_j = \ln(s_j) - \ln(s_0)$:

$$\delta_1 = \ln(0.3) - \ln(0.4) = \ln(0.75) \approx -0.29$$
$$\delta_2 = \ln(0.2) - \ln(0.4) = \ln(0.5) \approx -0.69$$
$$\delta_3 = \ln(0.1) - \ln(0.4) = \ln(0.25) \approx -1.39$$

- **(b)** Ranking: Product 1 $>$ Product 2 $>$ Product 3
- All products have negative mean utility relative to outside option
- (Outside option is normalized to $\delta_0 = 0$)

Practice: Consumer surplus change

- **Question:** Market has $\delta_0 = 0$, $\delta_1 = 1$, $\delta_2 = 0.5$.
- Price coefficient $\alpha = -0.5$.
- If product 1 is removed from the market, what is the consumer surplus loss per consumer?

Take 3 minutes.

Practice: Consumer surplus change (solution)

Solution

- **Before removal:**

$$CS^{\text{before}} = \frac{1}{0.5} \ln(e^0 + e^1 + e^{0.5}) = 2 \ln(1 + 2.72 + 1.65) = 2 \ln(5.37) \approx 3.36$$

- **After removal:**

$$CS^{\text{after}} = \frac{1}{0.5} \ln(e^0 + e^{0.5}) = 2 \ln(1 + 1.65) = 2 \ln(2.65) \approx 1.95$$

- **CS Loss:** $3.36 - 1.95 = \$1.41$ per consumer

- This is a common HW1/exam question type!

Plan

1. Demand estimation review
2. **Pricing review**
3. Exam logistics
4. Preview of Part 2

Monopoly pricing: key equations

- Profit maximization: $\max_q \pi = p(q) \cdot q - c(q)$
- First-order condition: $MR = MC$
- **Lerner index:**

$$L = \frac{p - MC}{p} = \frac{1}{|\varepsilon|}$$

- **Interpretation:** Markup is inversely related to elasticity
- Equivalently: $p = \frac{MC}{1 - 1/|\varepsilon|}$

Practice: Lerner index

- **Question:** A monopolist faces demand $p = 100 - 2q$ and has constant marginal cost $MC = 20$.
- (a) Find the profit-maximizing price and quantity.
- (b) Calculate the Lerner index.
- (c) Verify using the elasticity formula.

Take 4 minutes.

Practice: Lerner index (solution)

Solution

- **(a)** $MR = 100 - 4q$. Set $MR = MC$: $100 - 4q = 20 \Rightarrow q = 20$
- Price: $p = 100 - 2(20) = 60$
- **(b)** Lerner index: $L = \frac{60-20}{60} = \frac{40}{60} = \frac{2}{3}$
- **(c)** Elasticity: $\varepsilon = \frac{dq}{dp} \cdot \frac{p}{q} = -\frac{1}{2} \cdot \frac{60}{20} = -1.5$
- Check: $L = \frac{1}{|\varepsilon|} = \frac{1}{1.5} = \frac{2}{3}$ ✓

Types of price discrimination

1. **Perfect price discrimination**
   - Charge each consumer their exact WTP
   - Benchmark; rarely feasible
2. **Selection by indicators**
   - Different prices for observable groups
   - Example: student discounts, geographic pricing
3. **Self-selection**
   - Design menu to induce type revelation
   - Example: versioning, bundling

Selection by indicators: inverse elasticity rule

- Two markets with different elasticities
- Apply Lerner in each market:

$$\frac{p_1 - MC}{p_1} = \frac{1}{|\varepsilon_1|} \quad \text{and} \quad \frac{p_2 - MC}{p_2} = \frac{1}{|\varepsilon_2|}$$

- **Key insight:** Charge higher price in more inelastic market
- Rearranging: $p = \frac{MC}{1-1/|\varepsilon|}$

Practice: Selection by indicators

- **Question:** A firm sells in two markets. Market A has $\varepsilon_A = -3$. Market B has $\varepsilon_B = -5$. Marginal cost is $MC = 10$.
- Find the optimal price in each market.

Take 3 minutes.

Practice: Selection by indicators (solution)

Solution

- Using $p = \frac{MC}{1-1/|\varepsilon|}$:
- **Market A:**

$$p_A = \frac{10}{1-1/3} = \frac{10}{2/3} = 15$$

- **Market B:**

$$p_B = \frac{10}{1-1/5} = \frac{10}{4/5} = 12.50$$

- Higher price in more inelastic market (A)

Two-part tariffs

- Structure: Payment $= F + p \cdot q$
- **Optimal (homogeneous consumers):**
  - Set $p = MC$ (maximize total surplus)
  - Set $F = CS(p = MC)$ (extract all surplus)
- **Result:** Firm captures entire surplus; efficient but inequitable
- **Heterogeneous consumers:** Tradeoff between extraction and participation

Self-selection: IC and IR constraints

- **IC (Incentive Compatibility):** Each type prefers their option

$$\mathrm{IC}_H : \quad v_H^F - p_F \geq v_H^S - p_S$$

- **IR (Individual Rationality):** Each type willing to participate

$$\mathrm{IR}_L : \quad v_L^S - p_S \geq 0$$

- **Optimal menu:** IC binds for H, IR binds for L

- H gets "information rent"; L gets zero surplus

Practice: Self-selection

- **Question:** Two products (Premium, Basic), two consumer types.

	Premium	Basic
High type	100	60
Low type	50	40

- $MC = 20$ for both. Equal numbers of each type.
- A firm considers: $p_P = 100$, $p_B = 40$.
- (a) What does each type buy?
- (b) Is this menu optimal? If not, what should change?

Take 4 minutes.

Practice: Self-selection (solution)

Solution

- **(a) Consumer choices:**
- High type: $CS_P = 100 - 100 = 0$, $CS_B = 60 - 40 = 20$
- High type buys Basic! (IC violated)
- Low type: $CS_P = 50 - 100 = -50$, $CS_B = 40 - 40 = 0$
- Low type buys Basic
- **(b) Not optimal.** Need to lower p_P so IC binds.
- IC binds when: $100 - p_P = 60 - 40 = 20 \Rightarrow p_P = 80$
- Optimal menu: $p_P = 80$, $p_B = 40$

Practice: Two-part tariff

- **Question:** A gym has demand $q = 20 - p$ from each consumer.
- Marginal cost is $MC = 4$.
- (a) What is the optimal two-part tariff (F, p)?
- (b) What is the firm's profit per customer?
- (c) What would happen if firm charged $p = 10$ and $F = 50$?

Take 3 minutes.

Practice: Two-part tariff (solution)

Solution

- **(a)** Optimal: $p = MC = 4$
- At $p = 4$: $q = 20 - 4 = 16$
- Consumer surplus: $CS = \frac{1}{2}(20 - 4)(16) = 128$
- Optimal fee: $F = 128$
- **(b)** Profit per customer: $F + (p - MC)q = 128 + 0 = 128$
- **(c)** At $p = 10$: $q = 10$, $CS = \frac{1}{2}(20 - 10)(10) = 50$
- $F = 50$ is feasible, but profit $= 50 + (10 - 4)(10) = 110 < 128$
- Lesson: $p > MC$ leaves money on the table!

Practice: Backing out marginal cost

- **Question:** Firm observes price $p = 50$ and elasticity $\varepsilon = -2$.
- (a) What is the firm's marginal cost?
- (b) If the elasticity were actually -4, what would MC be?
- (c) What does this say about the importance of accurate elasticity estimates?

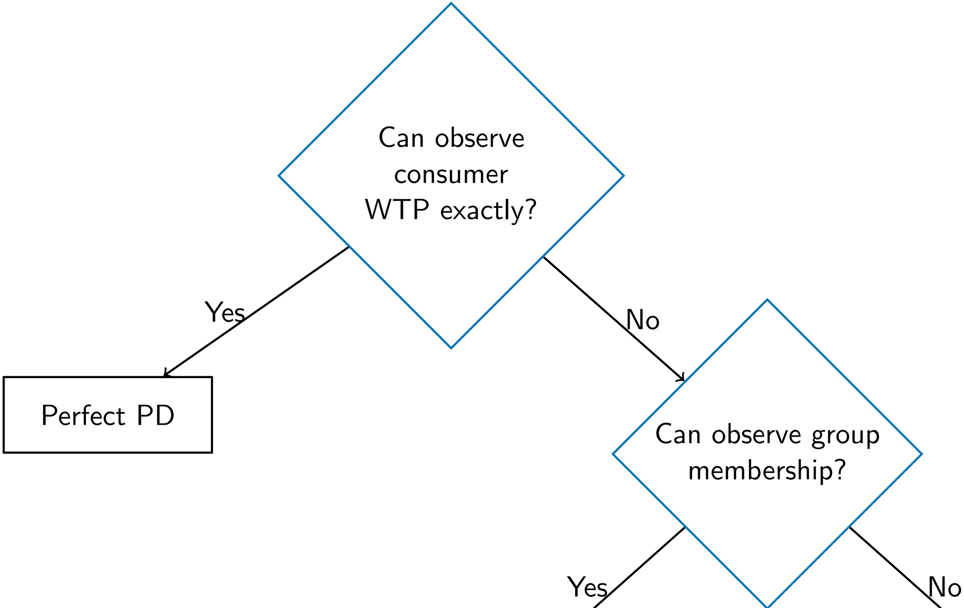
Take 3 minutes.

Practice: Backing out marginal cost (solution)

Solution

- Using Lerner: $MC = p\left(1 - \frac{1}{|\varepsilon|}\right)$
- **(a)** $MC = 50(1 - 1/2) = 50(0.5) = 25$
- **(b)** $MC = 50(1 - 1/4) = 50(0.75) = 37.5$
- **(c)** Key insight: inferred MC is very sensitive to elasticity!
  - Doubling $|\varepsilon|$ increased inferred MC by 50%
  - This matters for merger analysis: need accurate elasticities

Price discrimination: Which type?



Midterm preparation checklist

- ☐ **Formulas:** Logit shares, Berry inversion, elasticities, log-sum
- ☐ **Derivations:** Can derive $MR = MC$, Lerner from FOC
- ☐ **Identification:** Why OLS biased, what makes good IVs
- ☐ **IIA:** Red bus/blue bus, when it matters
- ☐ **Pricing:** Lerner, inverse elasticity rule for segments
- ☐ **Two-part tariffs:** $p = MC$, $F = CS$; heterogeneity tradeoff
- ☐ **Self-selection:** IC/IR constraints, which binds for whom
- ☐ **Practice:** Worked through HW1 and practice exam

Plan

1. Demand estimation review
2. Pricing review
3. **Exam logistics**
4. Preview of Part 2

Exam format reminder

- **Date:** Monday, Feb 9 (Lecture 7)
- **Duration:** 80 minutes
- **Allowed:** Calculator + two-sided cheat sheet
- **Coverage:**
  - Demand: logit, Berry inversion, elasticities, IVs, IIA, CS
  - Pricing: monopoly, Lerner index, price discrimination
  - Two-part tariffs, self-selection (IC/IR)
- **Format:** Mix of T/F/NEI, short answer, and problems

What to put on your cheat sheet

- **Key formulas:**
  - Logit share, Berry inversion
  - Own and cross elasticities
  - Log-sum formula for CS
  - Lerner index
  - Two-part tariff optimal conditions
- **Worked examples:** Similar to practice problems
- **Key intuitions:**
  - Why price endogeneity biases α toward zero
  - Why IC binds for high type, IR for low type

Key Points

1. **Logit:** $s_j = \exp(\delta_j) / [1 + \Sigma \exp(\delta_k)]$
2. **Berry inversion:** $\ln(s_j) - \ln(s_0) = \delta_j$
3. **Elasticities:** Own: $\alpha p_j (1 - s_j)$; Cross: $-\alpha p_k s_k$
4. **IVs needed** because $E[p_j \xi_j] \neq 0$; bias toward zero
5. **Lerner:** $L = (p - MC) / p = 1 / |\varepsilon|$
6. **Selection by indicators:** Higher price in inelastic market
7. **Two-part tariff:** $p = MC$, $F = CS$
8. **Self-selection:** IC binds for high type, IR for low type

Good luck on the midterm!

- **Office hours:** [TBD]
- **Practice exam:** Posted on course website
- Questions?

Plan

1. Demand estimation review
2. Pricing review
3. Exam logistics
4. **Preview of Part 2**

What's coming after the midterm

- **Part 2: Models of Competition**
  - Oligopoly models (Cournot, Bertrand, Hotelling)
  - Entry and entry deterrence
  - Mergers and merger simulation
  - Vertical relationships
  - Collusion
- **Connection to Part 1:** We'll use demand estimation to analyze competition
- **HW2:** Merger simulation using your demand estimates

Refresher: Cournot competition

- You covered this in ECN 532 (Hector's class)
- **Setup:** n firms choose quantities simultaneously
- **Key result:** Lerner index in Cournot equilibrium

$$L_i = \frac{P - MC}{P} = \frac{s_i}{|\varepsilon|}$$

- **Interpretation:**
  - Markup depends on market share s_i
  - More elastic demand $\rightarrow$ lower markup
- This connects oligopoly theory to our demand estimation!

Refresher: Bertrand competition

- **Setup:** Firms choose prices simultaneously (homogeneous goods)
- **Key result:** $P = MC$ (even with just 2 firms!)
- **The Bertrand Paradox:**
  - Two firms enough for perfectly competitive outcome
  - Seems unrealistic—most oligopolies have positive profits
- **Escaping the paradox:**
  - Product differentiation (Hotelling)
  - Capacity constraints
  - Repeated interaction (collusion)

Why this matters for Part 2

- **Mergers:** Understanding how firms compete tells us merger effects
  - Cournot merger: internalize quantity competition
  - Bertrand merger: internalize price competition
- **Entry:** Market structure determines profitability
- **Policy:** Antitrust authorities use these models
- **HW2 preview:** You'll simulate a merger using demand estimates

We'll cover this in detail after the midterm!