ECN 453: Game Theory 1

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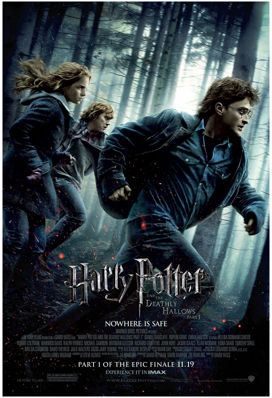
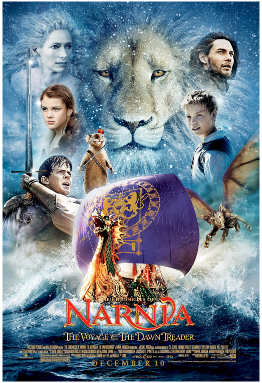
Strategic decision making

- So far, we have seen models of *independent* decision making.
- In other words, when firms made their optimal choices (for example, choosing optimal prices) they did not need to think about what other firms were doing.
  - For example, in monopoly, there were no other firms in the market!
- Today, we will study **game theory**.
- These are models where optimal choices are *strategic*: a firm's optimal choice depends on what other firms are doing.
- Much of what we will see in this lecture is review from previous courses. We will build on this review in future lectures.

Example of strategic decision making

- Two hollywood studios in 2010: Warner Bros and Fox, deciding when to release their blockbuster movies: Harry Potter, Chronicles of Narnia.





Example of strategic decision making (continued)

- Which month should the studios release their movie: December or November?

- Consider the decision of Warner Bros who are thinking about when to release the Harry Potter movie.

Example of strategic decision making (continued)

- Which month should the studios release their movie: December or November?
- Consider the decision of Warner Bros who are thinking about when to release the Harry Potter movie.
  - December is in the holidays so it's a better month to release Harry Potter if it is the only movie because it will get a bigger audience.
  - But: if Fox decides to release Narnia in December, the audience for the Harry Potter movie will be split.
  - So, if Fox decides to release Narnia in December, it will be better to release the Harry Potter movie in November.
- So: the decision of one movie studio about when to release the movie depends on the choices of the *other* movie studio. It is *strategic*.
  - Game theory can be used to model and understand strategic decisions like these.

Plan

1. **Simultaneous games: setup and Nash equilibrium**
2. Simultaneous games: dominant and dominated strategies

Simultaneous games: setup

- We will start by studying **simultaneous games**.
- These are games where choices are made at the same time.
  - By contrast, we will next study *sequential games* where firms makes choices one after the other.
- I will begin by defining some important language that economists use when talking about game theory.

Simultaneous games: setup

It's very important that you can identify the following components from the figure on the right.

Player 2

Player 1		L	R
	T	5, 5	3, 6
	B	6, 3	4, 4

- **Players**:

Simultaneous games: setup

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	B	3 6	4 4

- **Players**: Player 1 and Player 2
- **Strategies**

Simultaneous games: setup

It's very important that you can identify the following components from the figure on the right.

		Player 2	
		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

- **Players**: Player 1 and Player 2
- **Strategies**
  - Player 1 has strategies 'T' and 'B'
  - Player 2 has strategies 'L' and 'R'
- **Payoffs**:

Simultaneous games: setup

It's very important that you can identify the following components from the figure on the right.

		Player 2	
		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

- **Players**: Player 1 and Player 2
- **Strategies**
  - Player 1 has strategies 'T' and 'B'
  - Player 2 has strategies 'L' and 'R'
- **Payoffs**: the numbers in the matrix
  - Higher payoffs are better
  - Payoffs depend on the choices of *both players*
  - E.g. if Player 1 chooses 'T', Player 2 chooses 'R' then:
    - Player 1 receives payoff $= 3$
    - Player 2 receives payoff $= 6$.

Simultaneous games: setup

Player 2

Player 1		L	R
	T	5 5	6 3
	B	3 6	4 4

- Representing the elements (players, strategies, payoffs) in this form is called the 'normal form' of a game.

Simultaneous games: best responses

Player 2

Player 1		L	R
	T	5, 5	3, 6
	B	6, 3	4, 4

- **Best response**:

Simultaneous games: best responses

		Player 2	
		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

- **Best response**: the optimal strategy for a player *given* the choice of the other player.

- Notation: "best response of player 1 given that player 2 chooses R" written as:

$$BR_1(R)$$

- We can find the best response by finding the strategy with the highest payoff given a choice by the other player.

- **Example:** $BR_1(R) = B$.

- Why? Given player 2 plays R, player 1 gets:
  - Payoff=3 if play T $<$ payoff=4 if play B.

Simultaneous games: best responses

		Player 2	
		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

- Let's find some more best responses.
- $BR_1(L) = B$
  - Payoff=5 if play T $<$ payoff=6 if play B.
- $BR_2(T) = R$
  - Payoff=5 if play L $<$ payoff=6 if play R.
- $BR_2(B) = R$
  - Payoff=3 if play L $<$ payoff=4 if play R.

Simultaneous games: Nash equilibrium

- Where do we expect the game to end up? A **Nash equilibrium**:

Simultaneous games: Nash equilibrium

- Where do we expect the game to end up? A **Nash equilibrium**:

  A pair of strategies constitutes a Nash equilibrium if no player can unilaterally change its strategy in a way that improves its payoff.

- So, a Nash equilibrium occurs when each player is choosing a best response to the other player's choice.

Simultaneous games: Nash equilibrium

- In practice we can find a Nash equilibrium using the following steps:

1. Find the best responses for player 1 and circle them in the payoff matrix.
2. Find the best responses for player 2 and circle them in the payoff matrix.
3. If a box in the payoff matrix has two circles, it is a Nash equilibrium.

- We will now go through these steps for our example game in the previous slide.

Simultaneous games: Nash equilibrium

- Step 1: find the best responses for player 1 and circle them in the payoff matrix.

		Player 2	
		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

Simultaneous games: Nash equilibrium

- Step 1: find the best responses for player 1 and circle them in the payoff matrix.

- Step 2: find the best responses for player 2 and circle them in the payoff matrix.

Player 2

Player 1	L	R
T	5, 5	3, 6
B	6, 3	4, 4

Simultaneous games: Nash equilibrium

- Step 1: find the best responses for player 1 and circle them in the payoff matrix.

- Step 2: find the best responses for player 2 and circle them in the payoff matrix.

- Step 3: if a box in the payoff matrix has two circles, it is a Nash equilibrium.

- So, (B, R) is a Nash equilibrium.
  - The notation (B, R) means player 1 chooses B, and player 2 chooses R.

		Player 2	
		L	R
Player 1	T	5 5	6 3
	B	3 6	4 4

Simultaneous games: Nash equilibrium

- It's possible to have *multiple* Nash equilbria in a game.

- When this happens, our simple model makes no predictions about which equilibrium will be observed - all we can say is that the game will end up in *one* of the equilbria.

- An example of a game with multiple equilibria is on the right.
  - I have circled the best responses and the two Nash equilibria are at (T, L) and (B, R)

Player 2

Player 1		L	R
	T	2 1	0 0
	B	0 0	1 2

Simultaneous games: prisoner's dilemma

- Let's look at the first game that we looked at today (rewritten on the right). This game is an example of the *prisoner's dilemma*.
- Let's look at it again with the players and strategies relabelled (but with the same payoffs).
  - The strategies are now to choose prices: a high price 'H' or to choose a low price 'L'.
  - The interpretation of the payoffs is now profits.
  - (Note: we could come up with demand, costs, specific prices, that generate these profits, but I will ignore this for now.)

		Firm 2	
		H	L
Firm 1	H	5 5	6 3
	L	3 6	4 4

Simultaneous games: prisoner's dilemma

- What can we learn from the prisoner's dilemma?

- From the *joint* perspective of both firms, the optimal strategies are to choose high prices (H,H). Joint profits will be $= 10$.

- But, from the *individual* perspective of each firm, there is an incentive to *deviate* from (H, H).
    - A firm gets payoff $6 > 5$ from choosing L when the other firm chooses H.

- Therefore, the game results in a nash equilibrium of (L,L): here, joint profits are $= 8$. This is lower than what is *jointly* best for the firms!

		Firm 2	
		H	L
Firm 1	H	5 5	6 3
	L	3 6	4 4

Simultaneous games: prisoner's dilemma

- The prisoners dilemma shows the *'conflict between individual incentives and joint incentives'*.

- It is an example of why competition might be good for consumers, but bad for firms.
  - If both firm 1 and firm 2 were the same firm (e.g. a monopoly) they would choose high prices - this would maximize profits but consumers would have to pay high prices.

  - When the firms are choosing individually ('competing') they undercut each other and choose lower prices - this lowers joint profits for firms but consumers do better because they pay lower prices.

  - We will study competition a lot more in the second part of the course.

Simultaneous games: finding Nash Equilibria in complicated games, p164-p165

Player 2

Player 1		L	M	R
	T	1 2	2 0	3 0
	M	1 1	1 1	0 1
	B	1 0	0 2	2 2

Simultaneous games: finding Nash Equilibria in complicated games, p164-p165

- Nash equilibrium at (B,R).

Player 2

Player 1	L	M	R
T	2, 1	0, 2	0, 3
M	1, 1	1, 1	1, 0
B	0, 1	2, 0	2, 2

Plan

1. Simultaneous games: setup and Nash equilibrium
2. **Simultaneous games: dominant and dominated strategies**

Dominant and dominated strategies

- We give names to certain types of strategies.
- One is a **dominant strategy**:
  - *A dominant strategy yields a player the highest payoff regardless of the other players choices.*
- One is a **dominated strategy**:
  - *A dominanted strategy yields a player a payoff which is lower than that of a different strategy, regardless of what the other players do.*
- Note: for many games, there are no dominant or dominated strategies.

Dominant and dominated strategies: prisoner's dilemma

		Firm 2	
		H	L
Firm 1	H	5 5	6 3
	L	3 6	4 4

- As an example of dominant and dominated strategies, let's revisit the prisoner's dilemma.
- Are there dominant and dominated strategies in this game?

Dominant and dominated strategies: prisoner's dilemma

		Firm 2	
		H	L
Firm 1	H	5 5	6 3
	L	3 6	4 4

- As an example of dominant and dominated strategies, let's revisit the prisoner's dilemma.
- Are there dominant and dominated strategies in this game?
- Yes!
- For Firm 1: L is the dominant strategy, H is the dominated strategy
- For Firm 2: L is the dominant strategy, H is the dominated strategy

Dominated strategies: example

Player 2

Player 1	L	C	R
T	1, 2	1, 0	1, 1
M	0, 0	0, 3	0, 0
B	2, 0	-2, 1	2, 2

- Let's solve the game from before using *iterated deletion of dominated strategies*.
  - Essentially, keep deleting dominated strategies until we reach a Nash equilibrium.

Dominated strategies: example

		Player 2		
		L	C	R
Player 1	T	2 1	0 1	1 1
	M	0 0	3 0	0 0
	B	0 2	1 -2	2 2

- Let's solve the game from before using *iterated deletion of dominated strategies*.
  - Essentially, keep deleting dominated strategies until we reach a Nash equilibrium.
  - For player 1, M is a dominated action. Eliminate it (cross it out) of the game.
  - Given the above, C is dominated for player 2. Eliminate it from the game.
  - Given the above, T is dominated for player 1. Eliminate it.
  - Finally, given the above, L is a dominated strategy for player 2. Eliminate it.
  - What is left: Nash equilbrium at (B,R).

Summary of key points*

- Know how to identify the components of a simultaneous game: players, strategies, payoffs
- Know how to compute best responses
- Compute Nash equilibrium from best responses, and understand the the predictions of a Nash equilibrium rely on assumptions about whether the players are choosing *rationally*
- Understand the prisoner's dilemma illustrates the 'conflict between individual incentives and joint incentives'.
- Know what a 'dominant' and 'dominated' strategy are.
- Solve games using iterated deletion of dominated strategies.

*To clarify, all the material in the slides, problem sets, etc is assessable unless stated otherwise, but I hope this summary might be a useful place to start when studying the material.