ECN 453: Game Theory 2

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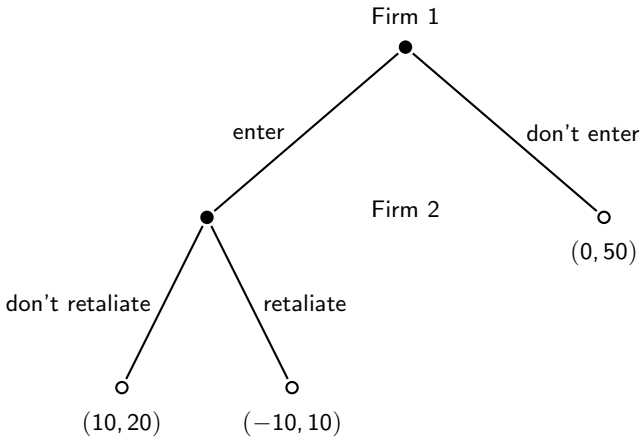
Sequential games

- Last time we studied games where players made decisions simultaneously.
- What if players make decisions one after the other (**sequentially**)?
  - Classic example: an industry that is currently a monopoly. There is a *potential entrant*. The timing is:
  - 1. Potential entrant decides whether to enter
  - 2. Incumbent decides whether to price aggressively *after* observing (potential) entry
- How should we model such behavior?
- Sequential games!

Plan

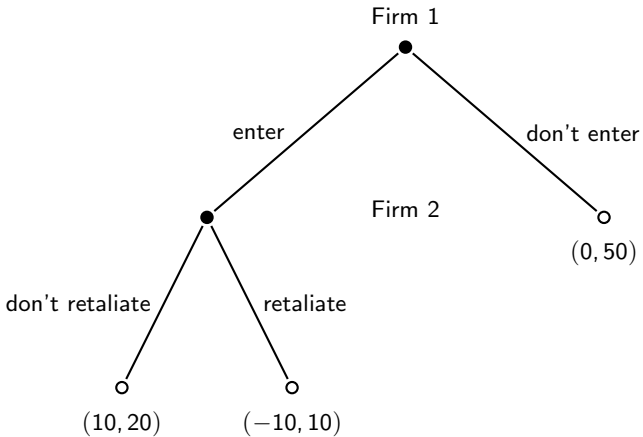
1. **Sequential games: setup and equilibrium**
2. Sequential games: the value of commitment
3. Sequential games: rationality assumption

Sequential games: setup



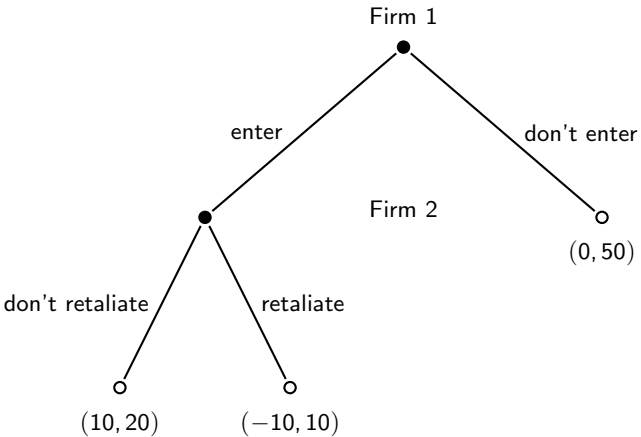
- **Components:**
- 'Extensive form' of a game
- Players

Sequential games: setup



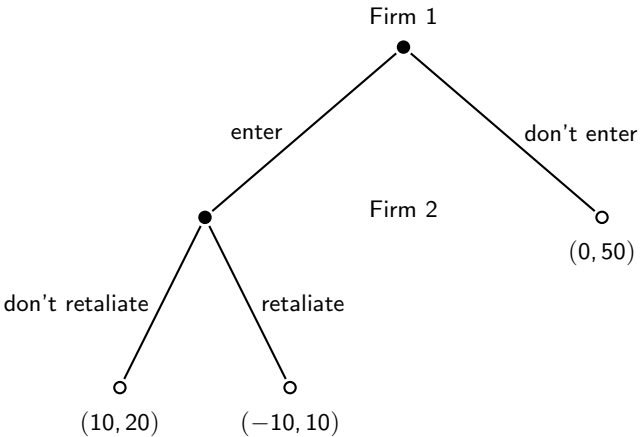
- **Components:**
- 'Extensive form' of a game
- Players (Firm 1 and Firm 2)
- Strategies

Sequential games: setup



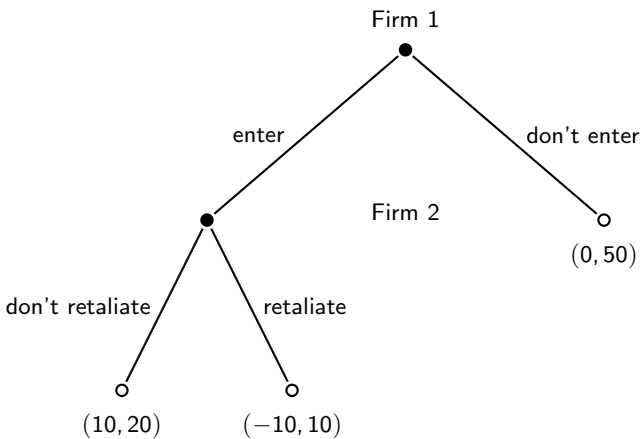
- **Components:**
- 'Extensive form' of a game
- Players (Firm 1 and Firm 2)
- Strategies (enter, don't enter, don't retaliate, retaliate)
- Payoffs
- Decision nodes

Sequential games: setup



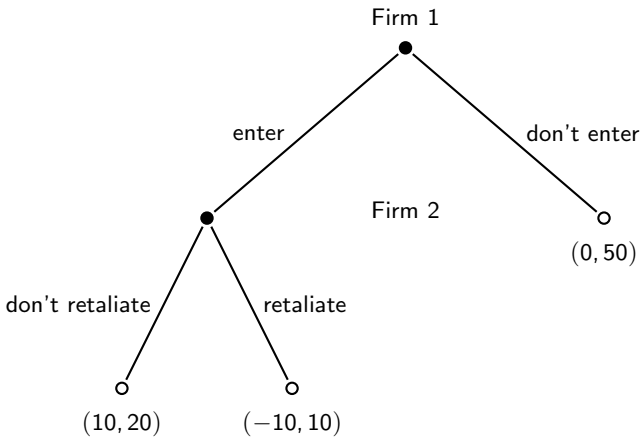
- **Components:**
- 'Extensive form' of a game
- Players (Firm 1 and Firm 2)
- Strategies (enter, don't enter, don't retaliate, retaliate)
- Payoffs
- Decision nodes (black circles where firm 1 and firm 2 choose)

Sequential games: equilibrium



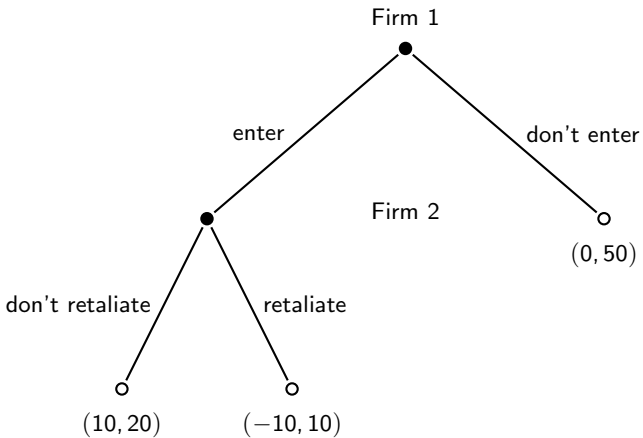
- Solve by **backwards induction**:

Sequential games: equilibrium



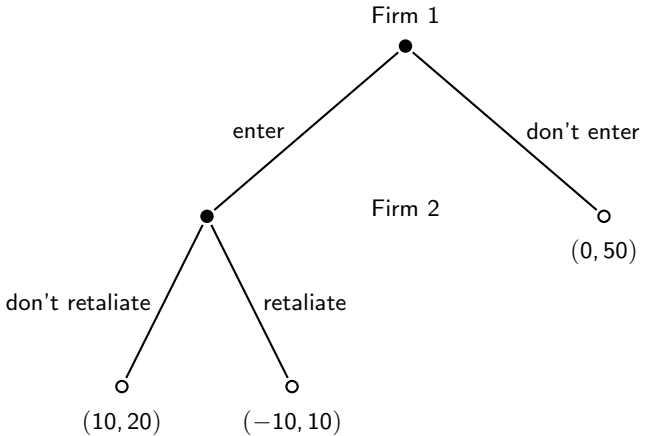
- Solve by **backwards induction**:
- Start at the *final* decision node, find the optimal decision, move back to the previous decision node *given* the action in the final node etc
- Deriving the equilibrium strategies (i.e. a strategy at each decision node) in this way produces what is called a **subgame-perfect equilibrium**

Sequential games: equilibrium



- **Questions:**
- 1. Find the Nash equilibria
- 2. Find the subgame-perfect equilibria

Sequential games: equilibrium



- **Questions:**
- 1. Find the Nash equilibria: (enter, don't retaliate); (don't enter, retaliate)
- 2. Find the subgame-perfect equilibria (enter, don't retaliate)
- What can we learn from this particular game: the Nash equilibrium (don't enter, retaliate) is **not credible**.
  - Backwards induction allowed us to rule out this 'unreasonable equilibrium'

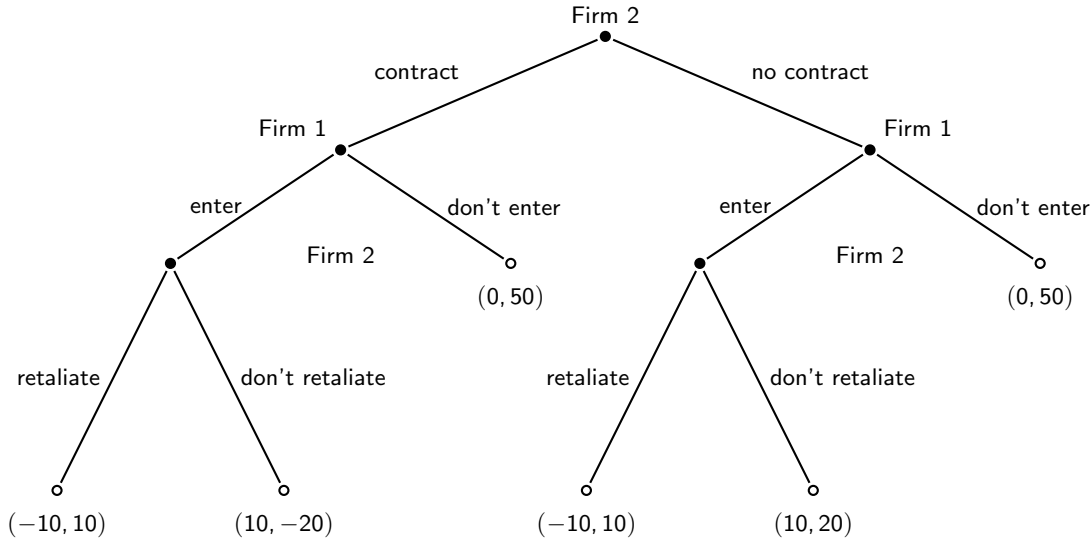
Plan

1. Sequential games: setup and equilibrium
2. **Sequential games: the value of commitment**
3. Sequential games: rationality assumption

Sequential games: the value of commitment

- We saw before that the equilibrium (don't enter, retaliate) could be ruled out.
- This equilibrium relied on firm 2 making the not credible threat to play retaliate if firm 1 enters.
- What if firm 1 could 'tie its hands' and **commit** to playing 'retaliate' if firm 1 enters?
  - E.g. firm 2 writes a contract that if firm 1 chooses enter then firm 2 must choose retaliate.
  - Contract: if firm 2 were to choose 'not retaliate' then they would incur a penalty of 40, lowering their payoff to -20.

Sequential games: the value of commitment



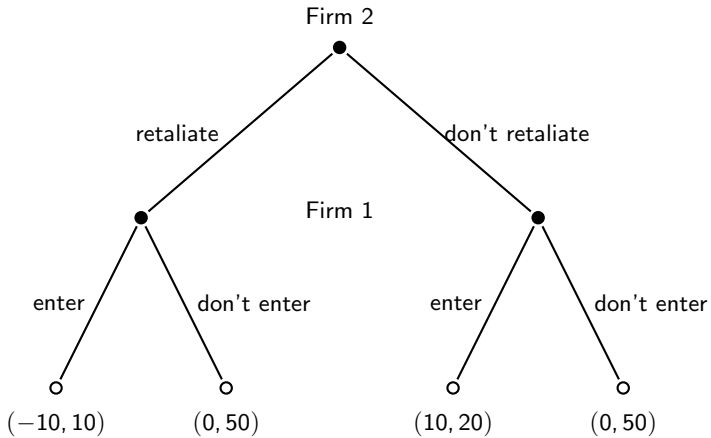
Sequential games: the value of commitment

- The subgame-perfect Nash equilibrium of the previous game is (don't enter, retaliate) and firm 2 now gets a payoff of 50.
- This relied on the (credible) commitment to play retaliate if firm 1 entered.
- This is an example of a broader category of firm strategies called **entry deterrence**

Sequential games: the value of commitment

- Two points:
- 1. A **credible commitment** may have significant strategic value
- 2. If we believe that firm 2 is credibly committed to playing retaliate then we could equivalently reformulate the game *with player 2 moving first* (see next slide...)

Sequential games: the value of commitment



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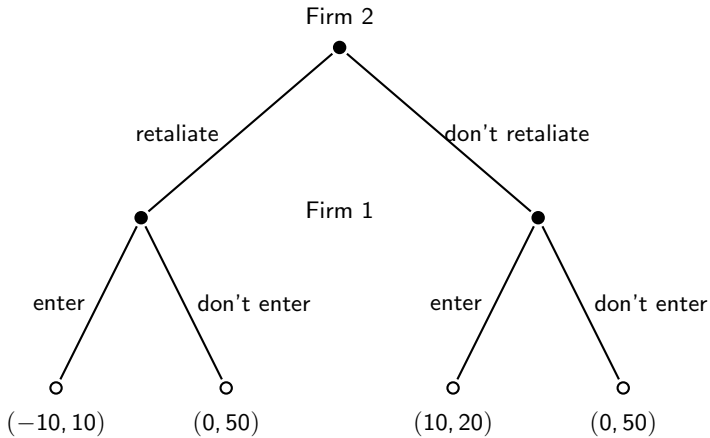
- 1. Find the Nash equilibria
- 2. Find the subgame-perfect equilibria

Sequential games: the value of commitment



- **Questions:**
- 1. Find the Nash equilibria (enter, don't retaliate), (don't enter, retaliate)
- 2. Find the subgame-perfect equilibria (don't enter, retaliate)

Sequential games: the value of commitment

- We used **entry deterrence** as an example of the value of commitment.
- Some other examples (beyond just economics)
- Dr Strangelove/Mutually Assured Destruction
- Rebel Without A Cause; how to win games of chicken

Sequential games: long-run vs short-run

- Sequential behavior arises naturally as well when thinking about short-run vs long-run decisions. We will see this later in the course.

- E.g. first choose (long-run) investment in capacity, second, choose (short-run) prices

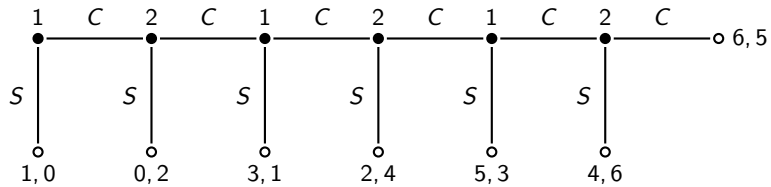
Plan

1. Sequential games: setup and equilibrium
2. Sequential games: the value of commitment
3. **Sequential games: rationality assumption**

Sequential games: rationality assumption

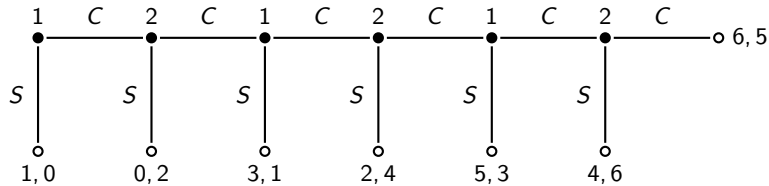
- To what extent should we believe the predictions from sequential games?
- Similar to simultaneous games, we need to make predictions about what strategies other players will choose, and to do this we implicitly assume other players are *rational* (they will correctly be able to solve the game etc..)
- When might we be worried about this assumption?

Sequential games: rationality assumption - centipede game



- 1. Solve this game by backwards induction
- 2. Is this equilibrium reasonable?
- 3. What are the implicit rationality assumptions?

Sequential games: rationality assumption - centipede game



- 1. Solve this game by backwards induction: Play S at each node
- 2. Is this equilibrium reasonable?: Maybe not... (in experiments, people tend to play C at each node)
- 3. What are the implicit rationality assumptions? That both players solve the game with backwards induction.

Summary of key points*

- Know how to identify the components of a sequential game
- Know how to compute the equilibrium of a sequential game via backwards induction
- Convert a sequential game to normal form and vice-versa
- Understand the pre-commitment may be valuable because it makes threats credible (e.g. in entry deterrence)
- Understand the implicit assumption of rationality that we make when solving sequential games

*To clarify, all the material in the slides, problem sets, etc is assessable unless stated otherwise, but I hope this summary might be a useful place to start when studying the material.

Ex. 7.7 HDTV Standards

- **Question:** US and Japan simultaneously decide whether to invest a high or low value into HDTV research. If both countries choose 'low' payoffs are (4,3) for the US and Japan respectively. If US chooses low level and Japan a high level, payoffs are (2,4). If US chooses high level and Japan low, payoffs are (3,2). If both countries chooses a high level, payoffs are (1,1).

- 1. Are there any dominant strategies in this game? What is the Nash equilibrium? What are the implicit rationality assumptions?

- 2. Suppose the US now has the option of committing to a strategy ahead of Japan. How would you model this situation? What are the subgame-perfect Nash equilbria of this game?

- 3. Comparing your answers to 1. and 2., what can you say about the value of commitment for the US?

Price match guarantee in the market for luxury cars

- Two firms are competing on prices. Each firm can choose to set a high price p_H or a low price p_L, where $p_H > p_L$. Profits are given by:

Firm 1 \ Firm 2	p_H	p_L
p_H	100, 100	120, 0
p_L	0, 120	70, 70

- 1. Draw the extensive form if firm 1 moves first. Solve for the subgame perfect equilibrium.

- 2. Suppose firm 1 offers consumers to match its price with the lowest price in the market. Solve for the SPE of the modified game (Hint: modify the game to three stages, allowing firm 1 to make a move in the third stage only in the case where it chose p_H in the firms stage and firm 2 chose p_L in the second stage.)