ECN 594: Final Review

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Plan

1. **Review: Part 1 (Demand and Pricing)**

2. Practice problems: Part 1

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3. Review: Part 2 (Competition)

4. Practice problems: Part 2

5. Exam logistics

Final exam format

- **Duration:** 80 minutes

- **Allowed:** Calculator + two-sided cheat sheet (letter-size paper)

- **Coverage:** Cumulative (Lectures 1-12)

- **Structure:**
  - Short answer (T/F/NEI, definitions, quick calculations)
  - Longer problems (derivations, multi-step)

- Emphasis on Part 2 material, but Part 1 is included

Demand estimation: key concepts

- **Logit model:** $s_j = \frac{\exp(\delta_j)}{1 + \sum_k \exp(\delta_k)}$

- **Berry inversion:** $\ln(s_j) - \ln(s_0) = \delta_j$

- **Elasticities:**

$$\eta_{jj} = \alpha p_j (1 - s_j) \quad \text{(own)}$$
$$\eta_{jk} = -\alpha p_k s_k \quad \text{(cross)}$$

- **Consumer surplus:** $CS = \frac{1}{|\alpha|} \ln \left[\sum_j \exp(\delta_j) \right]$

- **IIA problem:** Substitution proportional to share, not similarity

T/F: Demand estimation

True, False, or Not Enough Information?

1. In logit, if product A is removed, its share goes disproportionately to similar products.

2. OLS estimation of α on price is biased toward zero.

3. The Berry inversion requires data on individual consumer choices.

4. Consumer surplus from logit depends on the log-sum of $\exp(\delta_j)$.

Take 2 minutes.

T/F: Demand estimation (solutions)

Solutions

1. **FALSE.** IIA: share goes proportionally to market shares, not similarity.

2. **TRUE.** High quality $\rightarrow$ high ξ $\rightarrow$ high price; omitted variable bias attenuates α.

3. **FALSE.** Berry inversion uses market-level shares only, not individual data.

4. **TRUE.** $CS = \frac{1}{|\alpha|} \ln \left[\sum_j \exp(\delta_j) \right]$ (the log-sum formula).

Identification and IVs

- **Problem:** $E[p_j \xi_j] \neq 0$ (price endogeneity)

- High unobserved quality $\rightarrow$ high price

- OLS: $\hat{\alpha}$ biased toward zero

- **Solution:** IVs correlated with p, uncorrelated with ξ
    - Hausman: prices in other markets
    - BLP: competitor characteristics
    - Cost shifters

Pricing and price discrimination

- **Lerner index:** $L = \frac{p-MC}{p} = \frac{1}{|\varepsilon|}$

- **Selection by indicators:**
    - Different prices for observable groups
    - Higher price in more inelastic market

- **Two-part tariff:** $F + p \times q$
    - Optimal: $p = MC$, $F = CS$

- **Self-selection:** IC binds for high type, IR for low type

T/F: Pricing

True, False, or Not Enough Information?

1. With perfect price discrimination, the firm captures all consumer surplus.

2. A firm using selection by indicators charges higher prices to groups with lower elasticity.

3. Under optimal two-part tariff, the firm makes zero variable profit.

4. In self-selection (versioning), the IC constraint binds for the low-type consumer.

Take 2 minutes.

T/F: Pricing (solutions)

Solutions

1. **TRUE.** Perfect price discrimination: firm extracts all surplus.

2. **TRUE.** More inelastic $\rightarrow$ higher markup (inverse elasticity rule).

3. **TRUE.** Optimal $p = MC$, so variable profit $= 0$; all profit comes from F.

4. **FALSE.** IC binds for HIGH type (they might want to pretend to be low type); IR binds for low type.

Plan

1. Review: Part 1 (Demand and Pricing)

2. **Practice problems: Part 1**

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3. Review: Part 2 (Competition)

4. Practice problems: Part 2

5. Exam logistics

Practice: Consumer surplus calculation

- **Question:** Logit model with $\alpha = -0.5$. Three products with:
    - $\delta_1 = 2.0$, $\delta_2 = 1.5$, $\delta_3 = 1.0$

- (a) Calculate total consumer surplus per consumer.

- (b) If product 3 is removed, what is the change in CS?

Take 4 minutes.

Practice: Consumer surplus (solution)

> **Solution**
>
> - **(a)** $CS = \frac{1}{|\alpha|} \ln\left[1 + \sum_j \exp(\delta_j)\right]$
>   - $= \frac{1}{0.5} \ln[1 + e^{2.0} + e^{1.5} + e^{1.0}]$
>   - $= 2 \times \ln[1 + 7.39 + 4.48 + 2.72] = 2 \times \ln(15.59)$
>   - $= 2 \times 2.75 = 5.5$
>
> - **(b)** Without product 3:
>   - $CS' = 2 \times \ln[1 + 7.39 + 4.48] = 2 \times \ln(12.87) = 5.11$
>   - $\Delta CS = 5.11 - 5.5 = -0.39$
>
> - Removing product 3 reduces CS by 0.39 per consumer

Practice: Demand and pricing

- **Question 1:** Logit model with $\alpha = -0.4$. Product A has $p_A = 30$, $s_A = 0.15$. Calculate the own-price elasticity.

- **Question 2:** Two markets with $\varepsilon_1 = -2$, $\varepsilon_2 = -3$, $MC = 10$. Find optimal prices.

- **Question 3:** A firm uses two-part tariff. Individual demand is $q = 20 - 2p$, $MC = 2$. Find optimal F and p.

Take 5 minutes.

Practice: Demand and pricing (solutions)

Solutions

- **Q1:** $\eta_{AA} = \alpha p_A (1 - s_A) = (-0.4)(30)(0.85) = -10.2$

- **Q2:** Using $p = \frac{MC}{1 - 1/|\varepsilon|}$:
    - $p_1 = 10/(1 - 0.5) = 20$
    - $p_2 = 10/(1 - 1/3) = 15$

- **Q3:** Optimal $p = MC = 2$. At $p = 2$: $q = 16$.
    - $CS = \frac{1}{2} \times 16 \times 8 = 64$ (triangle)
    - Optimal $F = 64$

Plan

1. Review: Part 1 (Demand and Pricing)

2. Practice problems: Part 1

   _____

3. **Review: Part 2 (Competition)**

4. Practice problems: Part 2

5. Exam logistics

Oligopoly models

- **Cournot:** Firms choose quantities
    - Lerner: $L = s_i/|\varepsilon|$
    - Positive profits

- **Bertrand (homogeneous):** $P = MC$, zero profits

- **Hotelling:** $p^* = c + t$ (markup = transport cost)

- Cournot applies with capacity constraints

- Product differentiation creates pricing power

T/F: Oligopoly

True, False, or Not Enough Information?

1. In Cournot, adding more firms always increases total output.

2. The Bertrand paradox says that two firms are enough for competitive pricing.

3. In Hotelling with linear transport costs, the equilibrium markup equals the transport cost.

4. Cournot firms have higher markups when demand is more elastic.

Take 2 minutes.

T/F: Oligopoly (solutions)

Solutions

1. **TRUE.** More firms $\rightarrow$ more competition $\rightarrow$ higher Q, lower P.

2. **TRUE.** With homogeneous products, $N = 2$ gives $P = MC$.

3. **TRUE.** In symmetric Hotelling: $p^* = c + t$.

4. **FALSE.** More elastic demand $\rightarrow$ *lower* markup (from $L = s/|\varepsilon|$).

Entry and mergers

- **Free entry:** $\pi(N^*) \geq F > \pi(N^* + 1)$

- **Entry deterrence:** Requires credible commitment
    - Capacity building

    - Solve by backward induction (SPE)

- **Mergers:** Internalize substitution $\rightarrow$ higher prices

- **Merger simulation:** Change ownership matrix, resolve FOCs

- **HHI:** $= \sum s_i^2 \times 10000$

Vertical relationships and collusion

- **Double marginalization:**
    - Two markups $\rightarrow$ price too high
    - Solutions: integration, two-part tariff

- **Collusion:**
    - $\delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}}$
    - Cournot: $\delta^* = \frac{(N+1)^2}{N^2 + (N+1)^2}$
    - Bertrand: $\delta^* = \frac{N-1}{N}$

- **Leniency programs:** Create "race to report"

Plan

1. Review: Part 1 (Demand and Pricing)

2. Practice problems: Part 1

   _____

3. Review: Part 2 (Competition)

4. **Practice problems: Part 2**

5. Exam logistics

Practice: Entry deterrence

- **Question:** Monopolist incumbent faces potential entrant.

    - If entrant stays out: Monopolist earns 100

    - If entrant enters and incumbent accommodates: Both earn 30

    - If entrant enters and incumbent fights: Both earn -10

    - Entry cost: 20

- (a) Draw the game tree.

- (b) Find the SPE. Will entry occur?

Take 4 minutes.

Practice: Entry deterrence (solution)

Solution

- **(a) Game tree:** Entrant moves first (Enter/Stay Out), then Incumbent moves (Accommodate/Fight)

- **(b) SPE by backward induction:**
  - If entry occurs: Incumbent chooses Accommodate ($30 > -10$)
  - Entrant anticipates: Enter $\rightarrow$ payoff $30 - 20 = 10$; Stay Out $\rightarrow 0$
  - Entrant enters since $10 > 0$

- **SPE:** (Enter, Accommodate)

- **Key insight:** "Fight" threat is not credible—incumbent won't follow through

Practice: Collusion

- **Question:** Four identical Bertrand firms consider colluding.

- (a) What is the minimum discount factor for collusion?

- (b) If detection probability is $\rho = 0.2$ and fine is $F = 2\pi^M$, does collusion become easier or harder?

Take 4 minutes.

Practice: Collusion (solution)

Solution

- **(a)** Bertrand formula: $\delta^* = \frac{N-1}{N} = \frac{3}{4} = 0.75$

- **(b)** Detection and fines:
    - $\pi^C = \pi^M/4$, $\pi^D = \pi^M$, $\pi^{NE} = 0$

    - $\rho F = 0.2 \times 2\pi^M = 0.4\pi^M$

    - $\delta^* = \frac{\pi^D - \pi^C + \rho F}{\pi^D - \pi^{NE} + \rho F}$

    - $= \frac{\pi^M - 0.25\pi^M + 0.4\pi^M}{\pi^M - 0 + 0.4\pi^M} = \frac{1.15\pi^M}{1.4\pi^M} = 0.82$

- Higher δ^* ($0.75 \rightarrow 0.82$) $\rightarrow$ collusion **harder**

Practice: Competition

- **Question 1:** Cournot duopoly. $P = 120 - Q$, $MC = 20$. Find equilibrium P, Q, and Lerner index.

- **Question 2:** Market has shares 50%, 30%, 20%. Top two firms merge. Calculate ΔHHI.

- **Question 3:** Bertrand duopoly. What is the minimum δ for collusion?

Take 5 minutes.

Practice: Competition (solutions)

Solutions

- **Q1:** FOC: $120 - 2q_i - q_j = 20$. Symmetric: $q^* = 100/3 \approx 33.3$.
    - $Q = 66.7$, $P = 53.3$
    - $L = (53.3 - 20)/53.3 = 0.625$ (or $s/|\varepsilon| = 0.5/0.8 = 0.625$)

- **Q2:** Shortcut: $\Delta HHI = 2 \times 50 \times 30 = 3000$
    - Pre: $50^2 + 30^2 + 20^2 = 3800$
    - Post: $80^2 + 20^2 = 6800$

- **Q3:** Bertrand with $N = 2$: $\delta^* = \frac{N-1}{N} = \frac{1}{2} = 0.5$

T/F: Vertical and collusion

True, False, or Not Enough Information?

1. Double marginalization makes the final price too high for both the manufacturer and consumers.

2. Resale price maintenance is always anticompetitive.

3. The critical discount factor for Bertrand collusion increases with the number of firms.

4. Leniency programs help detect cartels by creating a "race to confess."

Take 2 minutes.

T/F: Vertical and collusion (solutions)

Solutions

1. **TRUE.** DM hurts manufacturer profits AND raises consumer prices—both sides lose.

2. **FALSE.** RPM can be procompetitive (prevents free-riding) or anticompetitive (facilitates collusion).

3. **TRUE.** Bertrand: $\delta^* = (N-1)/N$, which increases in N.

4. **TRUE.** First to report gets immunity $\rightarrow$ Prisoner's Dilemma within cartel.

Practice: Double marginalization

- **Question:** Manufacturer ($MC = 5$) sells to retailer who sells to consumers. Demand: $q = 50 - p$.

- (a) Find integrated monopolist's price and profit.

- (b) Find prices with vertical separation.

Take 4 minutes.

Practice: Double marginalization (solution)

Solution

- **(a) Integrated:**
    - $MR = 50 - 2q = 5 \Rightarrow q = 22.5$, $p = 27.5$
    - $\pi = (27.5 - 5) \times 22.5 = 506.25$

- **(b) Separated:**
    - Retailer: $q = (50 - w)/2$, $p = (50 + w)/2$
    - Manufacturer: $\max_w (w - 5)(50 - w)/2$
    - FOC: $w = 27.5$
    - $p = (50 + 27.5)/2 = 38.75$, $q = 11.25$

- Price 41% higher, quantity 50% lower with separation

Plan

1. Review: Part 1 (Demand and Pricing)

2. Practice problems: Part 1

   _____

3. Review: Part 2 (Competition)

4. Practice problems: Part 2

5. **Exam logistics**

What's on the final

- **Part 1 topics:**
    - Logit, elasticities, IVs, consumer surplus
    - Lerner index, price discrimination, two-part tariffs
    - Self-selection (IC/IR)

- **Part 2 topics:**
    - Cournot, Bertrand, Hotelling
    - Entry, entry deterrence
    - Mergers, HHI, merger simulation
    - Double marginalization, vertical restraints
    - Collusion (δ^* formulas, leniency)

Comprehensive summary table

Topic	Key Formula	Key Insight
Logit	$s_j = \exp(\delta_j)/[1 + \Sigma \exp(\delta_k)]$	IIA: proportional substitution
Elasticities	$\eta_{jj} = \alpha p_j(1 - s_j)$	More elastic $\rightarrow$ lower markup
Two-part tariff	$p = MC$, $F = CS$	Extract surplus via fixed fee
Cournot	$L = s/\lvert \varepsilon \rvert$	Positive profits, $\uparrow N \rightarrow\downarrow$ profit
Bertrand	$P = MC$	Zero profits (paradox)
Hotelling	$p^* = c + t$	Differentiation = pricing power
Merger	$\Delta HHI = 2s_1 s_2 \times 10000$	Internalize substitution
Collusion	$\delta^* = (\pi^D - \pi^C)/(\pi^D - \pi^{NE})$	Patient firms can collude

Common exam mistakes to avoid

1. **Confusing IC and IR:** IC binds for HIGH type, IR for LOW type

2. **Sign errors on α:** $\alpha < 0$, so $|\alpha| = -\alpha$

3. **Forgetting $\times 10000$ for HHI:** Shares as percentages

4. **SPE vs Nash:** SPE requires backward induction

5. **Collusion formulas:** Cournot $\neq$ Bertrand; know both

6. **Double marginalization:** Two markups, not one doubled

Key formulas for your cheat sheet

- Logit: $s_j = \exp(\delta_j)/[1 + \Sigma \exp(\delta_k)]$

- Berry: $\ln(s_j) - \ln(s_0) = \delta_j$

- Elasticities: $\eta_{jj} = \alpha p_j(1 - s_j)$, $\eta_{jk} = -\alpha p_k s_k$

- Lerner: $L = 1/|\varepsilon|$ (monopoly), $L = s_i/|\varepsilon|$ (Cournot)

- HHI: $\Delta = 2s_1 s_2 \times 10000$

- Hotelling: $p^* = c + t$

- Collusion: $\delta^* = (\pi^D - \pi^C)/(\pi^D - \pi^{NE})$

Final tips

1. Work through practice exam carefully

2. Review HW1 and HW2 problems

3. Know the intuition behind formulas, not just formulas

4. Show your work for partial credit

5. Manage your time (80 minutes goes fast!)

Time management strategy

- **80 minutes total** for entire exam

- **Suggested allocation:**
    - Short answer (T/F, definitions): 15-20 minutes
    - Calculation problems: 15-20 minutes each
    - Leave 5-10 minutes for review

- **If stuck on a problem:**
    - Write what you know (partial credit!)
    - Move on and come back later
    - Don't let one question sink you
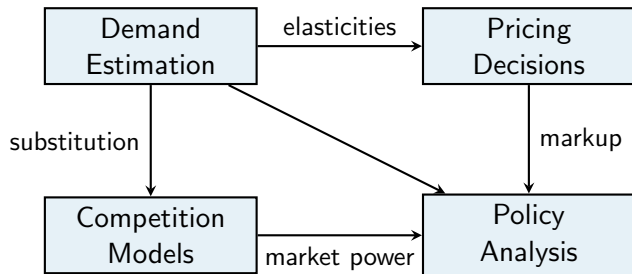
Final exam checklist

Before the exam, make sure you can:

- ☐ Calculate logit elasticities and consumer surplus

- ☐ Solve for optimal two-part tariff

- ☐ Identify when IC vs IR binds in self-selection

- ☐ Solve Cournot and Bertrand equilibria

- ☐ Find free entry equilibrium (N^*)

- ☐ Solve entry deterrence games by backward induction

- ☐ Calculate ΔHHI for mergers

- ☐ Derive double marginalization prices

- ☐ Calculate critical discount factors for collusion

The big picture: what is IO about?

- **Central question:** When do markets work well, and when do they fail?

- **Our approach:**
    - Estimate demand to understand consumer behavior
    - Model firm behavior (pricing, entry, competition)
    - Evaluate policy interventions (mergers, regulation)

- **Key insight:** Market power is pervasive—perfect competition is the exception, not the rule

- Understanding imperfect competition is essential for business strategy AND public policy

How the pieces fit together


```
┌─────────────┐  elasticities  ┌─────────────┐
│   Demand    │ ─────────────▶ │   Pricing   │
│ Estimation  │                │  Decisions  │
└─────────────┘                └─────────────┘
      │                              │
substitution                       markup
      │                              │
      ▼                              ▼
┌─────────────┐                ┌─────────────┐
│ Competition │                │   Policy    │
│   Models    │ market power ▶ │  Analysis   │
└─────────────┘                └─────────────┘
```

- Demand estimation → elasticities → pricing power

- Competition models → predict firm behavior

- Both feed into policy: merger review, antitrust, regulation

The IO economist's toolkit

Tool	What it tells us
Logit demand	Consumer preferences, substitution patterns
Elasticities	Price sensitivity, competitive pressure
Lerner index	Markup, market power
Cournot/Bertrand	Oligopoly pricing, profit levels
Entry models	Market structure, barriers to entry
Merger simulation	Price effects, welfare consequences
Collusion theory	Cartel stability, deterrence policy

Where IO is used in practice

- **Antitrust agencies** (DOJ, FTC, EC)
    - Merger review, cartel prosecution
    - Economic testimony in court cases

- **Tech companies** (Amazon, Google, Uber)
    - Pricing algorithms, marketplace design
    - Demand forecasting, A/B testing

- **Consulting firms** (Compass Lexecon, Analysis Group)
    - Litigation support, damages estimation
    - Regulatory analysis

- **Central banks and regulators**
    - Banking competition, consumer protection

Five things to remember from this course

1. **Demand matters:** Can't understand markets without understanding consumers

2. **Market power is everywhere:** Firms set prices above cost—the question is how much

3. **Competition takes many forms:** Cournot, Bertrand, Hotelling capture different realities

4. **Structure affects conduct:** Entry, mergers, and vertical relationships shape competition

5. **Policy has real effects:** Antitrust, regulation, and leniency programs change outcomes

Key Points: Part 1

1. Logit demand $+$ Berry inversion for estimation

2. IVs needed: price endogeneity biases α toward zero

3. IIA problem: demographics partially help

4. Lerner index: $L = 1/|\varepsilon|$

5. Two-part tariff: $p = MC$, $F = CS$

6. Self-selection: IC binds for H, IR binds for L

Key Points: Part 2

1. Cournot: $L = s/|\varepsilon|$; Bertrand: $P = MC$

2. Hotelling: $p^* = c + t$

3. Free entry until $\pi(N^*) \geq F > \pi(N^* + 1)$

4. Entry deterrence requires credible commitment (SPE)

5. Merger simulation: change ownership matrix

6. Double marginalization: two markups hurt everyone

7. Collusion: $\delta^* = (\pi^D - \pi^C)/(\pi^D - \pi^{NE})$

Good luck on the final!

- Questions before the exam?