Solutions to Exercises From "Introduction to Industrial Organization"

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■ **1.1*** Empirical evidence from a sample of more than 600 U.K. firms indicates that, controlling for the quantity of inputs (that is, taking into account the quantity of inputs), firm output is increasing in the number of competitors and decreasing in market share and industry concentration.[2] How do these results relate to the ideas presented in the chapter?

Solution: In Section 1.2, we argued that one of the implications of market power is the decline of productive efficiency. Controlling for input levels, the level of output is a measure of productive efficiency. The number of competitors and the degree of concentration are measures of the degree of competition (concentration is an inverse indicator). The empirical evidence from U.K. firms is therefore consistent with the view presented in the text.

As to the third explanatory variable (market share), see the discussion in Chapter 9.

■ **2.1** "A price-taking firm selling in a market with a price greater than the firm's average cost should increase its output level." Comment.

Solution: In a competitive market, firms are price takers; optimal output is such that price equals marginal cost (or marginal revenue equals marginal cost). It is perfectly possible that price be equal to marginal cost and greater than average cost. In fact, if price is greater than the minimum of average cost, then the optimal output is such that price is greater than average cost. In summary, the sentence is wrong.

[1] I am grateful to Critian Dezso (New York University) for excellent assistance in preparing these solutions.

[2] Stephen J. Nickell, "Competition and Corporate Performance," *Journal of Political Economy* **104** (1996), 724–746.

■ **2.2*** Consider the following values of the price elasticity of demand:

	Cigarettes	0.5
zinha	U.S. luxury cars in U.S.	1.9
	Foreign luxury cars in U.S.	2.8

Based on these values, provide an estimate of the impact on revenues of a 10% increase in the price of each of the above three products.

Solution: Revenue is given by

$$R = PQ$$

Taking the derivative with respect to P and rearranging, we get

$$\begin{aligned}\frac{dR}{dP} &= Q + P\frac{dQ}{dP} \\ &= Q + Q\frac{P}{Q}\frac{dQ}{dP} \\ &= Q(1-\epsilon),\end{aligned}$$

where

$$\epsilon \equiv -\frac{dQ}{dP}\frac{P}{Q}$$

is the price elasticity of demand (see page 17). It follows that a 10% increase in price implies an increase in revenues from cigarette sales given by $10(1-.5) = 5\%$. In the case of U.S. luxury cars and foreign luxury cars, a 10% price increase would lead to a decrease in revenues of -9% and -18%, respectively.

■ **2.3** You own and operate a facility located in Taiwan that manufactures 64-megabit dynamic random-access memory chips (DRAMs) for personal computers (PCs). One year ago you acquired the land for this facility for $2 million, and used $3million of your own money to finance the plant and equipment needed for DRAM manufacturing. Your facility has a maximum capacity of 10 million chips per year. Your cost of funds is 10% per year for either borrowing and investing. You could sell the land, plant and equipment today for $8 million; you estimate that the land, plant, and equipment will gain 6% in value over the coming year. (Use a one-year planning horizon for this problem.)

In addition to the cost of land, plant, and equipment, you incur various operating expenses associated with DRAM production, such as energy, labor, raw materials, and packaging. Experience shows that these costs are $4 per chip, regardless of the number of chips produced during the year. In addition, producing DRAMs will cause you to incur fixed costs of $500,000 per year for items such as security, legal, and utilities.

(a) What is your cost function, $C(q)$, where q is the number of chips produced during the year?

Assume now that you can sell as many chips as you make at the going market price per chip of p.

(b) What is the minimum price, p, at which you would find it profitable to produce DRAMs during the coming year?

Solution:

(a) The \$5 million you originally spent for the land, plant, and equipment is a sunk expenditure and thus not an economic cost. However, there is a "user cost of capital" associated with the land, plant and equipment, based on its current market value of \$8 million and your cost of funds and the rate of depreciation or appreciation of the asset over the planning horizon. Your (opportunity) cost of investing \$8 million for one year is \$800,000, but these assets will appreciate by \$480,000 over the year, giving a (net) user cost of capital of \$320,000. (The depreciation rate is 6%.) This is a fixed cost of making DRAM's, to which we must add the other fixed costs of \$500,000 to get a combined fixed cost of \$820,000 for the year. The variable costs are a constant \$4 per chip, so the cost function is $C(Q) = 820,000 + 4Q$, in the range of $0 < Q < 10,000,000$. (One could also report that $C(0) = 0$, by definition, and that $C(Q)$ is infinite for $Q > 10,000,000$, since your maximum capacity is ten million chips per year. Of course, in practice there would likely be a way to push production beyond "rated capacity," at some cost penalty, but that is beyond the scope of this problem.)

(b) The average cost function is $AC(Q) = 820,000/Q + 4$, again up to ten million chips per year. This declines with Q, so the minimum AC is achieved at full capacity utilization. At ten million chips per year, the fixed costs come to \$0.082 per chip, so average costs are \$4.082 per chip. This is your minimum average cost, and thus the minimum price at which is makes sense to stay open for the year.

■ **2.4** Consider the following 1988 data on the costs of a Sprinter (Class 150/2) train:[3]

Capital cost	525,000
Annual costs (per unit)	
Depreciation (20 years)	26,300
Overhaul and maintenance	32,600
Stabling and cleaning	9,400
Total annual cost of	
2 drivers	20,200
2 guards	15,600
Mileage costs of rolling stock (per unit mile)	
Maintenance	0.15
Fuel	0.126

[3]Source: Data provided by British Rail to the Mergers and Monopolies Commission.

(Notes: (a) Annual costs assume a 90,000 mile benchmark annual use. (b) There are 145 seats on the train.)

Based on these numbers, answer the following questions:
(a) What is the average cost per train mile?
(b) What is the average cost per passenger mile? (Note: the average number of passengers during this time period was 45.)
(c) What is the marginal cost per train mile?
(d) What is the marginal cost per passenger mile?

Solution:

(a) Fixed costs are 26,300+32,600+9,400+20,200+15,600=104,100. (Note: the capital cost should not be included in the yearly cost, only its depreciation.) Average variable cost per train mile is constant at .15+.126=.276 per train mile. It follows that average cost per train mile is 104,100/m+.276, where m is the number of miles. Using the benchmark of 90,000 miles, this comes down to 1.157+.276=1.433. As these number suggests, this is a capital-intensive, strong-scale-economies technology.

(b) 1.433/45=.032 (approximately).

(c) Average variable cost per train mile is constant(see part a), thus equal to marginal cost: .126

(d) .276/45= .0061 (approximately).

■ **2.5** You are considering opening your own restaurant. To do so, you will have to quit your current job, which pays $46k per year, and cash in your life savings of $200k, which have been in a certificate of deposit paying 6% per year. You will need this $200k to purchase equipment for your restaurant operations. You estimate that you will have to spend $4k during the year to maintain the equipment so as to preserve its market value at $200k. Fortunately, you own a building suitable for the restaurant. You currently rent out this building on a month-by-month basis for $2500 per month. You anticipate that you will spend $50k for food, $40k for extra help, and $14k for utilities and supplies during the first year of operations. There are no other costs involved in this business.

What are the economic costs of operating the restaurant during the first year? In other words, what level of revenues will you need to achieve in the first year to make the first year profitable in an economic sense?

Solution: There are three opportunity costs:

1. The salary you could earn if you do not quit: $46k.
2. The interest income your savings could earn if you do not cash in: $\$200k \times 0.06 = \$12$k.

3. The rent your building could earn if you do not use it for your restaurant: $\$2.5k \times 12 months = \$30$k.

There are four direct costs:

1. Maintaining the equipment: \$4k.
2. Food: \$50k.
3. Hiring extra help: \$40k.
4. Utilities and supplies: \$14k.

Note that the \$200k cost of the equipment is not an economic cost because it is essentially reversible. That is, you can always sell the equipment for its current market value as long as you maintain it. Only the interest you would have earned on the money tied up in the equipment and the cost to maintain it are economic costs.

Adding up opportunity and direct costs yields \$196k. This is the break-even revenue for first year of operations.

■ **2.6** Eurotunnel, the company that owns the tunnel linking England and France, earned an operating profit of £46 million during the first semester of 1998. However, subtracting interest payments (mainly from the construction of the tunnel), its bottom line was a loss of £130 during the same period.[4] Is it optimal to continue operating the tunnel, given all these losses?

Solution: The interest payments correspond to a cost (building the tunnel) that is sunk (literally!). It should therefore not be taken into consideration in the decision of whether or not to continue operations. However, if bankruptcy is a viable option for the owners of Eurotunnel, and if the situation is expected to remain the same (operating profit less than interest payments), then the optimal option is to declare bankruptcy.

■ **2.7*** 1998 was a turning point for Old McDonald's farm. Until then, the farm produced unprocessed tomato exclusively, selling its 100,000t for a profit margin of \$2.1/t. In January 1998, however, Old McDonald decided to start exporting processed tomato (tomato pulp) to Europe. At that time, the price of tomato pulp was \$6/t. In order to produce tomato pulp, Old McDonald bought a machine capable of processing 100,000t per year. The machine cost \$200,000 and was paid for with retained earnings that had been earning an 8% rate of return. This machine has a useful lifetime of 2 years. The market value of this machine drops to \$50,000 after one year of use (and zero after two years of use). In addition to the machine cost, there is a \$2.2/t harvesting and processing cost (mostly labor cost).

(a) Determine Old McDonald's average cost, marginal cost, and profit margin.

[4] *The Wall Street Journal Europe*, September 22, 1998.

A few months later, things turned bad for Old McDonald. In December 1998, the European Union increased its tariffs on imported tomato pulp, implying that the net price received by American exporters is now only \$5/t. It is not expected that this price will change in the future. One accountant consulting for Old McDonald stated that as margins have declined drastically the farmer had better sell the machine right away and go back to producing unprocessed tomato. Old McDonald is trying to decide whether to take this consultant's advice.

(b) What would you advise Old McDonald to do?

(c) Would your advice change if the price of unprocessed tomato were expected to be \$0.50/t higher than described above? Explain why or why not.

Solution:

(a) The user cost of capital corresponding to the machine is given by 8% times \$200,000 plus $(200,000 - 50,000)$, or simply \$166,000. Divided by 100,000t this gives \$1.66/t. Adding labor costs of \$2.2/t, this gives a total of \$3.86/t, the average cost. Marginal cost is \$2.2/t up to 100,000t/year, infinity thereafter. The profit margin is therefore \$6-\$2.2=\$3.8/t (up to 100,000t).

(b) We are considering the option of continuing to produce tomato pulp versus the option of producing unprocessed tomato. There are two opportunity costs that need to be accounted for. First, by selling tomato pulp the farmer is foregoing the chance of selling unprocessed tomato. This opportunity cost amounts to the the margin on unprocessed tomato, or \$2.1/t. The second opportunity cost is that of the machine — the user cost of capital. Since the machine is now worth only \$50,000 and will last for one more year, the user cost of capital is given by 50,000 plus 8% times 50,000 plus, or \$54,000, which corresponds to \$.54/t. The average economic profit, that is, including all imputed costs is, \$5 (price) - 2.2 (labor) - .54 (cost of capital) - 2.1 (margin on unprocessed tomato) = \$.16. Since this is positive, the firm should continue operating the machine and sell tomato pulp.

(c) By a calculation analogous to the one above, we conclude that the farmer is better off by switching to unprocessed tomato.

■ **2.8*** Las-O-Vision is the sole producer of holographic TVs, 3DTVs. The daily demand for 3DTVs is $D(p) = 10200 - 100p$. The cost of producing q 3DTVs per day is $q^2/2$ (note this implies that MC $= q$).

(a) What is Las-O-Vision's total revenue schedule?

(b) What is Las-O-Vision's marginal revenue schedule?

(c) What is the profit-maximizing number of 3DTVs for Las-O-Vision to produce each day? What price does Las-O-Vision charge per 3DTV? What is its daily profit?

Solution:

(a) Total Revenue is given by $p(x) \cdot x$, that is, the revenue that Las-O-Vision receives when it sells x units. To get $p(x)$, we invert the demand function $x = 10,200 - 100p$ by solving for p in terms of x, or $p(x) = 102 - x/100$. Substituting this into our total revenue equation, we obtain $TR(x) = (102 - x/100) \cdot x = 102x - x^2/100$.

(b) Marginal revenue is the derivative of Total Revenue with respect to x, so $MR(x) = 102 - x/50$; or, since our demand equation is linear in x, we can obtain it by recalling that the marginal revenue curve is twice as steep as the inverse demand curve and starts at the same point on the vertical axis.

(c) The profit maximizing quantity, x^* is that quantity at which marginal cost and marginal revenue are equal. Setting $MR(x) = MC$, we have $102 - x^*/50 = x^*$, or $x^* = 100$. The profit maximizing prices is that which generates $x^* = 100$ in sales or, substituting into the inverse demand function calculated in (a), $p(100) = 102 - (100/100) = 101$. When selling 100 units, Las-O-Vision generates Total revenues equal to $TR(100) = 102 \cdot 100 - 1002/100 = \$10,100$. Its total cost is 1002/2=5000. Therefore its total profit when it sells 100 units is $10,100 - 5000 = \$5,100$.

■ **2.9*** You own a private parking lot near U.C. Berkeley with a capacity of 600 cars. The demand for parking at this lot is estimated to be $Q = 1000 - 2p$, where Q is the number of customers with monthly parking passes and p is the monthly parking fee per car.

(a) Derive your marginal revenue schedule.

(b) What price generates the greatest revenues?

Your fixed costs of operating the parking lot, such as the monthly lease paid to the landlord and the cost of hiring an attendant, are $25,000 per month. In addition, your insurance company charges you $20 per car per month for liability coverage, and the City of Berkeley charges you $30 per car per month as part of its policy to discourage the use of private automobiles.

(c) What is your profit-maximizing price?

Solution:

(a) Solving for p gives $p = 500 - Q/2$. Using the "twice-the-slope" formula for marginal revenue associated with a linear demand curve, we then have $MR = 500 - Q$.

Alternatively, one could directly write down the revenue function, $R(Q) = p(Q) * Q$, and plus in for $p(Q) = 500 - Q/2$ to get $R(Q) = (500 - Q/2)Q = 500Q - Q2/2$, then differentiate with respect to Q to get $MR(Q) = 500 - Q$.

(b) Revenues are maximized when marginal revenues equal zero. Setting $MR = 0$ gives $500 - Q = 0$, or $Q = 500$. Then solving for price using the demand curve gives $p = 250$.

(c) The (monthly) cost function here is $C(Q) = 25,000 + 50Q$. Marginal cost per car is simply \$50. Setting $MR = MC$ gives $500 - Q = 50$, or $Q^* = 450$. Using the demand curve to solve for the price that goes along with this quantity gives $p^* = \$275$.

To confirm that this is indeed the profit-maximizing price, you also should check that it is not optimal to shut down, i.e., that your economic profits are positive in comparison with shutting down. This can be done by directly calculating profits, which are given by $\pi^* = p^*Q^* - C(Q^*) = \$275(450) - \$50(450) - \$25,000 = \$76,250$. Another way to check profitability is to calculate the "contribution to fixed costs" generated by your customers. This contribution is \$225 per customer times 450 customers, or \$101,250, which easily exceeds the fixed costs of \$25,000 per month.

■ **2.10***** You are one of two companies bidding to try to win a large construction project. Call your bid B. You estimate that your costs of actually performing the work required will be \$800k. You are risk neutral.[5] You will win if and only if your bid is lower than that of the other bidder. You are not sure what bid your rival will submit, but you estimate that the rival's bid is uniformly distributed between \$1m and \$2m.[6] What bid should you submit?

Solution: A risk-neutral bidder will use a bidding strategy that maximizes the expected value of its bid B. This entails picking a bid value B that balances two offsetting effects—changes in the value of winning due to changes in the bid (the larger your bid is the more valuable the contract is) and changes in the chances of winning due to changes in B (the larger your bid is the less likely you are to win). Formally, the expected value of a bid B can be expressed $E[B] = (B - 800,000) \cdot \text{Prob}(B < B_r)$, in which B_r is the rival bidder's bid and $\text{Prob}(B < Br)$ is the probability that its bid, B, is less than its rival's bid. The first term in this equation is simply the payoff when a bidder wins. The second term is its chances of winning (which requires that $B < Br$).

In this problem, the focal bidder believes that its rival's bid can be anywhere between \$1m and \$2m so $\text{Prob}(B < B_r) = 1 - (B - 1,000,000)/1,000,000$ for all bids between bid \$1m and \$2m ($\text{Prob}(B < B_r) = 1$ for all bids less than \$1m since it believes that the rival never bids below \$1m and $\text{Prob}(B < B_r) = 0$ for all bids greater than \$2m since it believes that the rival never bids above \$2m). Substituting this expression into the expected value of the bid B we obtain: $E[B] = (B - 800,000)[1 - (B - 1,000,000)/1,000,000]$.

From this expression it is clear that the bidder's payoff goes up with B but that its chance of winning declines with B. Picking the optimal B entails finding the maximum of $E[B]$, which we can easily obtain by taking the derivative of $E[B]$, setting it equal to zero and solving for B^*. This bid will be the point at which the two effects of changing B just offset each other. Dropping the zeros we have

$$\frac{\partial}{\partial B} E[B] \quad = \quad (1 - (B^* - 1)) - (B^* - .8) = 0$$

[5] We say that an agent is risk neutral if he or she is indifferent between receiving 100 for sure and receiving 0 or 200 with probability 50% each. More generally, a risk-neutral agent only cares about the expected value of each outcome.

[6] By "uniformly distributed between a and b" we mean that all values between a and b are equally likely.

$$\begin{aligned} B^* - .8 &= 1 - B^* + 1 \\ B^* &= 1.4\text{m} \end{aligned}$$

An alternative approach to this problem is to construct a demand function from the information you have about the market. You can then solve the problem in the same way as you would with more straightforward problems in which you are given an explicit demand function (i.e., set $MR = MC$ and solve for Q^*, then solve for B^*). To see this approach, note that the bid the firm submits is just like a price. The higher its bid, the lower its expected demand will be. In this case, demand falls as the price goes up because the firm's chance of winning is falling. Formally, expected demand, Q, at any level B is equal to $Q = 1 - \text{Prob}(B < B_r) = 1 - (B - 1,000,000)/1,000,000 = 2 - B/1,000,000$. As this equation indicates, when the firm's bid is equal to 1,000,000 demand will be 1 unit. That is, the firm is sure to win the contract. As its bid (the price) increases, demand falls to some fraction of a unit until at 2,000,000 demand is zero. Since the contract is a winner take all item, the idea of fractional units is not really correct, but if there were say N consumers instead of a single consumer, and the firm was bidding against other firms for the business of each consumer, the aggregate demand function would then be $Q_N = N(2 - B/1,000,000) = 2N - B(N/1,000,000)$. This is like a simple linear demand function.

To continue with this approach, we need to invert the demand function and solve for B to get $B = 2,000,000 - 1,000,000 \cdot Q$. The bidder's total revenue is then $BQ = (2,000,000 - 1,000,000 \cdot Q)Q$. Taking the derivative of this total revenue function, we find that the marginal revenue of the firm is $2,000,000 - 2 \cdot 1,000,000 \cdot Q$. As we would expect, the marginal revenue curve has twice the slope of the inverse de-mand curve. We can then set this marginal revenue equal to the marginal cost of 800,000 to get Q^*:

$$\begin{aligned} 800,000 &= 2,000,000 - 2 \cdot 1,000,000 \cdot Q \\ 2 \cdot 1,000,000 \cdot Q &= 1,200,000 \\ Q^* &= 1.2/2 = .6 \end{aligned}$$

Substituting this value into our inverse demand function, we obtain the optimal bid of $B(.6) = 2,000,000 - (.6)1,000,000 = 1,400,000$.

■ **3.1** Explain why the assumption of profit maximization is or is not reasonable?

Solution: The answer to this question is given by Section 3.1 in the book. The main reson why we might think that the assumption of profit maximization is not reasonable is that the firm managers are frequently not the firm owners; and the goals of managers frequently differs from those of the owners. However, it can be argued that the discipline imposed by the shareholders, the labor market, the product market and the capital market are sufficient to enforce profit maximization. In particular, the threat of a takeover has been found to have a significant effect on value maximization.

■ **3.2** Should firms have their own catering services or should they outsource it? What are the main trade-offs? Are their other alternatives in addition to "make or buy"?

Solution: The answer to this question is given by Section 3.2 in the book.

■ **3.3** Two parts in an automobile taillight are the plastic exterior cover and the light bulb. Which of these parts is a car company more likely to manufacture in-house? Why?

Solution: Light bulbs are a generally used homogeneous good. External suppliers enjoy economies of scale and specialization and supply the entire industry. In contrast, the plastic exterior cover must be custom-designed and manufactured for each make and model. Because it requires more Relationship Specific Investment (RSI), it is more likely to be made in-house.

■ **3.4** There are three main suppliers of commercial jet engines, Pratt & Whitney, General Electric, and Rolls-Royce. All three maintain extensive support staff at major (and many minor) airports throughout the world. Why doesn't one firm service each airport? Why do all three feel they need to provide service and support operations worldwide themselves? Why don't they subcontract this work? Why don't they leave it entirely to the airlines?

Solution: Jet engines are marvelously idiosyncratic. The knowledge, tools and parts needed to service one family (brand) of engines do not transfer fully across brands. One firm does not typically service each airport because the economies of scale (across brands) are small and the economies of specialization (within brand) are large. The only thing worse for an airline than an AOG (an aircraft sitting on the ground with a broken engine) is an aircraft flying with a broken engine or two. To ensure their reputation and revenues and to avoid ex post hold up, airlines demand before purchasing an aircraft that engine makers pre-commit capital to ensure that parts and service are available at major stations worldwide. Because the skills to do this are RSIs, and because the engine owner's reputation is at stake, to sell engines and credibly commit to keeping them running, each manufacturer must provide service and support at major stations.

Subcontracting would be difficult because of the RSI required (the subcontractor would fear hold-up) and because a poor subcontractor would impose a negative externality on the manufacturer. When the jet goes down, the manufacturer's reputation will suffer on a scale beyond any contractual penalty a subcontractor could likely be held to, so the work is not usually subcontracted. In addition, the manufacturers benefit directly from direct feedback within the firm on the performance of the engines they produce. This information may flow more readily within the firm than across firms.

Some airlines with sufficient scale do perform their own routine engine maintenance at their own maintenance bases. However, the airlines cannot efficiently do emergency engine repairs away from an airline's main bases. While there are enough GE engines going through Karachi International Airport to justify an on-site GE technical support staff, most airlines do not have enough flights through Karachi to justify the investment. The economies of scale in non-routine work are site and engine specific, not generally airline specific.

■ **3.5** The Smart car was created as a joint venture between Daimler-Benz AG and Swatch Group AG. Although Micro Compact Car AG (the name of the joint venture) was originally jointly owned, in November of 1998 Daimler-Benz AG took complete control by buying Swatch's share.[7] The deal put an end to a very stressed relationship between Daimler and Swatch. What does Section 3.2 suggest as to what the sources of strain might have been?

Solution: Section 3.2 suggests that, when two parties invest in specific assets and contracts are incomplete, the equilibrium solution is inefficient in every situation short of vertical integration. (See also the mathematical supplement corresponding to this section.) It is likely that some of this happened in the "stressed relationship" between Daimler and Swatch. Since none of the parties was in complete control (and ownership) of the future developments in the joint venture, the incentives for each party to invest were less than efficient.

■ **3.6** Why do television networks have a few "owned and operated" stations but work through independent affiliates in most geographic locations?

Solution: See Exercise 3.7.

[7] *The Wall Street Journal Europe*, November 5, 1998.

■ **3.7** Empirical evidence from franchise retailing suggests that, even when stores have similar characteristics, the mother company resorts to a mix between company-owned stores and franchised ones.[8] How can this be justified?

Solution: Franchisers face a problem in judging the performance of their franchisees. Keeping some retail locations in-house provides the parent company with a baseline of more readily accessible and less biased information against which the performance of the franchises can be measured. This information then helps to set standards in negotiating and administering future franchise contracts. Franchising the majority of retail locations limits the parent's direct financial outlay and exposure. Franchisers might also have an interest in direct control of locations that could have a particularly strong impact on its brand or reputation.

■ **3.8** The U.K. Body Shop franchise network consists of three types of stores: franchised, company owned and partnership stores. All stores that are distant from headquarters by more than 300 miles are franchised. More than half of the company-owned stores are within 100 miles of headquarters.[9] How can you explain these fact?

Solution: Owning a store has the advantages of vertical integration discussed in Section 3.2. However, it also has the problem that it requires increased monitoring by the store owner. We would expect the costs from monitoring to be lower the closer the store is to headquarters. Consequently, we would expect vertical integration to be more likely when the store is located closer to headquarters. The empirical evidence seems consistent with this hypothesis.

■ **3.9** Explain why Intel has maintained, if not increased, its competitive advantage with respect to rivals. Indicate the explanatory power of the different causes considered in the text (impediments to imitation, causal ambiguity, strategy, history).

Solution: This is a complex question. In fact, as argued in this chapter, this is *the* question in strategy. A good source for the particular case of Intel is the HBS case "Intel Corporation: 1968–1997," No. 9–797–137 (Rev. October 21, 1998).

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[8] See, for example,

Affuso, Luisa (1998), "An Empirical Study on Contractual Heterogeneity Within the Firm: The "Vertical Integration-Franchise Contracts" Mix," University of Cambridge..

[9] Source:

Watts, Christopher F (1995), "The Determinants of Organisational Choice: Franchising and Vertical Integration," M.Sc. dissertation, University of Southampton..

■ **3.10***** Suppose that a firm's profits are given by $\pi = \alpha + \phi(e) + \epsilon$, where α denotes the intensity of product market competition, e effort by the manager, and ϵ a random shock. The function $\phi(e)$ is increasing and concave, that is, $\phi' > 0$ and $\phi'' < 0$.

In order for the firm to survive, it must be that profits are greater than $\underline{\pi}$. The manager's payoff is $\beta > 0$ if the firm survives and zero if it is liquidated, that is, if profits fall short of the minimum target. The idea is that if the firm is liquidated, then the manager loses his job and the rents associated with it.

Suppose that ϵ is normally distributed with mean μ and variance σ^2, and that $\mu > \underline{\pi}$. Show that increased product market competition (lower α) induces greater effort by the manager, that is, $\frac{\partial e}{\partial \alpha} < 0$.

Solution: The manager's payoff is given by

$$P = \beta\, \mathcal{P}\left(\alpha + \phi(e) + \epsilon > \underline{\pi}\right) - e,$$

where $\mathcal{P}(x > y)$ is the probability that $x > y$. Since ϵ is normally distributed, we have

$$P = \beta\ \left(1 - F(\underline{\pi} - \alpha - \phi(e))\right) - e,$$

where $F(x)$ is the probability that ϵ is less than x (cumulative distribution function). Taking the derivative with respect to e, the manager's choice of effort level, we get

$$\frac{d\,P}{d\,e} = \beta\ f\left(\underline{\pi} - \alpha - \phi(e)\right)) - 1,$$

where $f(x)$ is the density function of ϵ. Since $\mu > \underline{\pi}$, $\mu > \underline{\pi} - \alpha - \phi(e)$. Therefore $f\left(\underline{\pi} - \alpha - \phi(e)\right))$ is in the increasing portion of f. It follows that an increase in α leads to a decrease in $f\left(\underline{\pi} - \alpha - \phi(e)\right))$; and this, in turn, implies a lower $\frac{d\,P}{d\,e}$. Finally, a lower $\frac{d\,P}{d\,e}$ implies a lower value of e. In words, a decrease in the degree of competition (higher α) decreases the marginal benefit from managerial effort ($\frac{d\,P}{d\,e}$), and ultimately leads to a lower effort of managerial effort (e).

■ **4.1** What are the assumptions regarding player rationality implicit in solving a game by elimination of dominated strategies? Contrast this with the case of dominant strategies.

Solution: When applying the iterated elimination of dominated strategies one implicitly assumes that each player is rational and believes that the other player is rational. With dominant strategies the only assumption needed is that players are rational, utility-maximizing agents, *regardless* of their beliefs about other players.

■ **4.2** The UK Office of Fair Trading has recently unveiled a plan that will offer immunity from prosecution to firms who blow the whistle on their co-cartel conspirators. In the U.S., this tactic has proven extremely successful: since its introduction in 1993, the total amount of fines for anti-competitive behavior has increased twentyfold.

Show how the tactic initiated by the U.S. Department of Justice and soon to be followed by the Office of Fair Trading changes the rules of the game played between firms in a secret cartel.

Solution: Prior to the introduction of the plan, each cartel firm would have two options: (a) to stick by the agreement or (b) to deviate and set lower prices. With the introduction of the plan, the firm has a third option: (c) to blow the whistle. Let α be the probability that the DOJ discovers the price conspiracy. High values of α imply a low expected value from (a). The same is true of (b), though probably to a lesser extent. Finally, (c) is invariant to the value of α. We would thus expect that, for high values of α, (c) is the best strategy.

With the introduction of the plan, the firms now play a second prisoner's dilemma type of game. Before, it was whether to price high or price low. Now, it's whether to blow the whistle or not. Firm would be better off if neither of them blew the whistle. However, if α is high, the bblowing the whistle is a dominat strategy.

■ **4.3** Figure 1 represents a series of two-player games which illustrate the rivalry between Time magazine and Newsweek. Each magazine's strategy consists of choosing a cover story: "Impeachment" or "Financial crisis" are the two choices.[10]

The first version of the game corresponds to the case when the game is symmetric (Time and Newsweek are equally well positioned). As the payoff matrix suggests, "Impeachment" is a better story but payoffs are lower when both magazines choose the same story. The second version of the game corresponds to the assumption that Time is a more popular magazine (Time's payoff is greater then Newsweek's when both magazines cover the same story). Finally, the third version of the game illustrates the case when the magazines are sufficiently different that some readers will buy both magazines even if they cover the same story.

For each of the three versions of the game,

(a) Determine whether the game can be solved by dominant strategies.

(b) Determine all Nash equilibria.

(c) Indicate clearly which assumptions regarding rationality are required in order to reach the solutions in (a) and (b).

Solution:

(i) Impeachment is a dominat strategy for both players. It follows that (Impeachment, Impeachment) is the unique Nash equilibrium. All we need to assume to reach this conclusion is that players are rational and know their own payoffs.

[10] In each cell, the first number is the payoff for the row player (Time).

Newsweek

Time		Impeachment	Financial Crisis
	Impeachment	35, 35	70, 30
	Financial Crisis	30, 70	15, 15

(i) Time and Newsweek are evenly matched

Newsweek

Time		Impeachment	Financial Crisis
	Impeachment	42, 28	70, 30
	Financial Crisis	30, 70	18, 12

(ii) Time is more popular than Newsweek

Newsweek

Time		Impeachment	Financial Crisis
	Impeachment	42, 28	70, 50
	Financial Crisis	50, 70	30, 20

(iii) Some customers will buy both magazines

Figure 1: The cover-story game.

(ii) Impeachment is a dominant strategy for Time, but not for Newsweek. Given that Time chosses Impeachment, Financial Crisis is the optimal choice for Newsweek. It follows that (Impeachment, Financial Crisis) is the unique Nash equilibrium. This solution assumes that Time is rational and knows its payoffs; and Newsweek is rational, knows the payoffs for both players, and believes Time is a rational player.

(iii) There are no dominant strategies in this game. There are two Nash equilibria (in pure strategies): (Impeachment, Financial Crisis) and (Fiancial Crisis, Impeachment). In this context, the concept of Nash equilibrium pressuposes that players know the payoffs of both players; moreover, it is common knowledge (I expect that you expect that I expect...) that the particular equilibrium will be played.

■ **4.4*** In the movie "E.T.," a trail of Reese's Pieces, one of Hershey's chocolate brands, is used to lure the little alien out of the woods. As a result of the publicity created by this scene, sales of Reese's Pieces trebled, allowing Hershey to catch up with rival Mars.

Universal Studio's original plan was to use a trail of Mars' M&Ms. However, Mars turned down the offer, presumably because it thought $1m was a very high price. The makers of "E.T." then turned to Hershey, who accepted the deal.

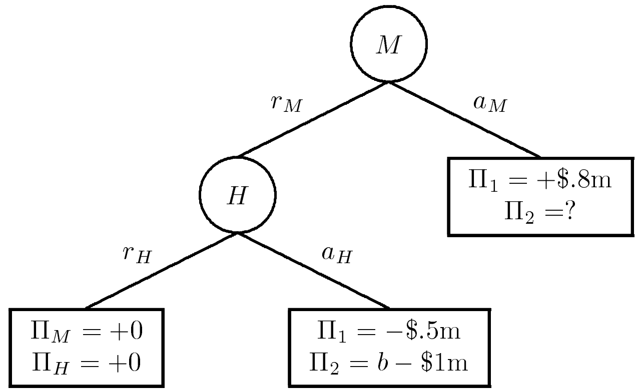

Figure 2: Mars vs Hershey. a_i and r_i signify acceptance and rejection by firm i, respectively.

Suppose that the publicity generated by having M&Ms included in the movie would increase Mars' profits by $800,000. Suppose moreover that Hershey's increase in market share cost Mars a loss of $500,000. Finally, let b be the benefit for Hershey's from having its brand be the chosen one.

Describe the above events as a game in extensive form. Determine the equilibrium as a function of b. If the equilibrium differs from the actual events, how do you think they can be reconciled?

Solution:

As can be seen from Figure 2, if $b > \$1,000,000$ then Hershey's equilibrium strategy is to accept the offer; likewise, Mars' equilibrium strategy is to accept the offer. If $b < \$1,000,000$, however, then the equilibrium strategies is for both firms to turn down the offer.

This differs from what actually happened (Mars rejected the offer, whereas Hershey accepted it). One possible explanation is that Mars underestimated either its own benefits from having M&Ms featured in the movie, or Hershey's benefits, or both.

■ **4.5** Hernan Cortéz, the Spanish navigator and explorer, is said to have burnt his ships upon arrival to Mexico. By so doing, he effectively eliminated the option of him and his soldiers returning to their homeland. Discuss the strategic value of this action knowing the Spanish colonists were faced with potential resistance from the Mexican natives.

Japan

		Low	High
		3	4
U.S.	Low	4	2
		2	1
	High	3	1

Figure 3: The HDTV game: each country chooses a high or a low level of R&D on HDTV.

Solution: By eliminating the option of turning back, Hernan Cortez established a credible commitment regarding his future actions, that is, to fight the Mexican natives should they attack. Had Cortez not made this move, natives could have found it better to attack, knowing that instead of bearing losses the Spaniards would prefer to withdraw.

■ **4.6** Consider the following game depicting the process of standard setting in high-definition television (HDTV).[11] The U.S. and Japan must simultaneously decide whether to invest a high or a low value into HDTV research. Each country's payoffs are summarized in Figure 3.

(a) Are there any dominant strategies in this game? What is the Nash equilibrium of the game? What are the rationality assumptions implicit in this equilibrium?

(b) Suppose now the U.S. has the option of committing to a strategy ahead of Japan's decision. How would you model this new situation? What are the Nash equilibria of this new game?

(c) Comparing the answers to (a) and (b), what can you say about the value of commitment for the U.S.?

(d) "When pre-commitment has a strategic value, the player that makes that commitment ends up 'regretting' its actions, in the sense that, given the rivals' choices, it could achieve a higher payoff by choosing a different action." In light of your answer to (b), how would you comment this statement?

Solution: (a) For the United States investing, a low value in HDTV research is a dominant strategy. The Nash equilibrium of the game is given by the U.S. choosing Low and Japan choosing High. The rationality assumptions implicit in this solution are that both players are rational and, moreover, Japan belives the U.S. acts rationally.

(b) See Figure 3. (See also Section 4.2.) By solving backwards, with get the following Nash equilibrium: U.S. chooses High, Japan chooses Low.

(c) Comparing the answers from a. and b. we can see that the value of commitment to the U.S. is 1 that is, 3 minus 2.

[11]This exercise is adapted from

Dixit, Avinash K., and Barry J. Nalebuff (1991), *Thinking Strategically*, New York: W W Norton..

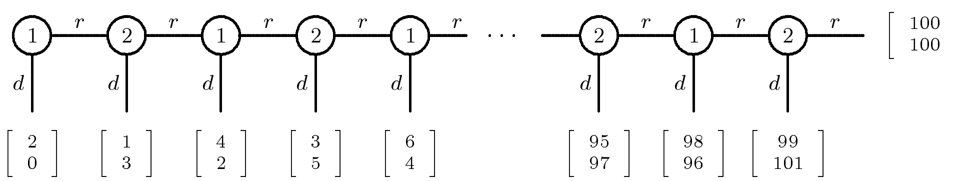

Figure 4: The centipede game. In the payoff vectors, the top number is Player 1's payoff, the bottom one Player 2's.

(d) *Given* that Japan chooses Low, the U.S. would be better off by choosing Low as well. However, it must be the case that the cost of switching from High to Low is so high that the U.S. won't do it (ex post). Otherwise, the commitment to stick to High would not be credible.

■ **4.7** Consider a one-shot game with two equilibria and suppose this game is repeated twice. Explain in words why there may be equilibria in the two-period game which are different from the equilibria of the one-shot game.

Solution: When the game is repeated twice the strategy space for each player becomes more complex. Each player's strategy specifies the action to be taken in period 1 as well as the action to be taken in period 2 *as a function of the outcome in period 1.* The possibility of linking period 2's actions to past actions allows for equilibrium outcomes that would not be attainable in the corresponding one-shot game (for example, the use of a 'punishment' action in period 2 if one of the players deviates from the designated period 1 payoff-maximizing action).

■ **4.8**** Consider the game in Figure 4.[12] Show, by backward induction, that rational players choose d at every node of the game, yielding a payoff of 2 for Player 1 and zero for Player 2. Is this equilibrium reasonable? What are the rationality assumptions implicit in it?

Solution: [IMPORTANT NOTE: there is a typo in the game tree: the payoffs in the second and third to last nodes should be increased by 2.]

[12]This game was first proposed by

Rosenthal, Robert (1981), "Games of Perfect Information, Predatory Pricing and the Chain-Store Paradox," *Journal of Economic Theory* **25**, 92–100.

Starting from the right-most node, we observe that Player 2's strategy, if that node is reached, is to play d, in which case its gets 101, whereas Player 1 gets 99. This implies that, in the second to last node, Player 1 is better off choosing d. In fact, by choosing r, Player 1 expects to get 99 (see sentence above) instead of 100 from d. And so forth. We conclude that the unique sub-game perfect Nash equilibrium is for each player to play d whenever it is called upon to make a move. The outcome of this equilibrium is Player 1 getting 2 and Player 2 getting 0.

Obviously, one might question whether this result is reasonable or not. Here, the implicit assumption is that each player is rational, believes that the other player is rational, believes that the other player believes that the first player is rational, and so forth.

To see how important this assumption is, suppose that Player 1 chooses r in the first period. Since this is not according to the equilibrium, Player 2 may not conjecture that Player 1 is not rational. But then chosing d may no longer be in Player 2's best interest. But then chosing r may be, after all, a rational strategy by Player 1 in the first place.

■ **5.1** "The degree of monopoly power is limited by the elasticity of demand." Comment.

Solution: Optimal monopoly pricing leads to the following relation between the price-cost margin and demand elasticity: $(p - MC)/p = 1/\epsilon$, where p is price, MC marginal cost, and e demand elasticity. It follows that the greater the value of ϵ the lower the value of $(p - MC)$ and the lower monopoly profits. A monopolist facing a very elastic demand curve makes profits at the level of a competitive firm.

■ **5.2** A firm sells one million units at a price of \$100 each. The firm's marginal cost is constant at \$40, and its average cost (at the output level of one million units) is \$90. The firm estimates that its elasticity of demand is constant at 2.0. Should the firm raise price, lower price, or leave price unchanged? Explain.

Solution: Optimal monopoly pricing leads to the following relation between the price, marginal cost, and demand elasticity: $(p - MC)/p = 1/\epsilon$, where p is price, MC is marginal cost, and ϵ is the elasticity of demand. In this problem, we have $(p - MC)/p = (100 - 40)/100$ or 0.6, which is greater than $1/\epsilon = 1/2 = 0.5$. This tells us that the price/cost margin is too high, so a lower price (\$80) would be optimal. It would be a mistake to use AC rather than MC for the purposes of calculating the price/cost margin.

■ **5.3** A recent study estimates the long-run demand elasticity of AT&T in the period 1988–1991 to be around 10.[13] Assuming the estimate is correct, what does this imply in terms of AT&T's market power?

Solution: A demand elasticity of 10 implies that AT&T's demand is very elastic. In fact, the author of the study that produced this estimate computes the welfare loss due to AT&T's market power to be less than 1% of sales volume.

■ **5.4** Sprint currently offers long-distance telephone service to residential customers at a price of 8c per minute. At this price, Sprint sells 200 million minutes of calling per day. Sprint believe that its marginal cost per minute of calling is 5c. So, Sprint's residential long-distance telephone service business is contributing $6 million per day towards overhead/fixed costs.

Based on a statistical study of calling patterns, Sprint estimates that it faces a constant elasticity of demand for long-distance calling by residential customers of 2.0.

(a) Based on this information, should Sprint raise, lower, or leave unchanged its price?

(b) How much additional contribution to overhead, if any, can Sprint obtain by optimally adjusting its price?

Solution:

(a) Given the elasticity of demand for long-distance, the optimal price is given by $p = MC\epsilon/(\epsilon - 1)$. The optimal price is thus $0.05 \cdot 2/1 = .10$. Sprint should raise its price from 0.08 to 0.10.

(b) The demand curve in this case has constant elasticity. The general formula for demand with constant elasticity is $q = Qp^{-\epsilon}$, where A is a positive constant. We can find A in this problem by substituting p and Q ($p = 8$; $Q = 200$) into the formula. The result is that $A = 12,800$. Substituting the optimal value $p = .10$ into the above, $q = (12,800)(.10)^{-2}$, gives 128 million minutes a day.

The contribution to fixed cost is $128(.10 - .05) = \$6.4$m. Repricing yields higher profits of $400,000 per day.

■ **5.5** After spending 10 years and $1.5 billion, you have finally gotten Food and Drug Administration (FDA) approval to sell your new patented wonder drug, which reduces the aches and pains associated with aging joints. You will market this drug under the brand name of Ageless. Market research indicates that the elasticity of

13

Ward, Michael R. (1995), "Measurements of Market Power in Long Distance Telecommunications," Federal Trade Commission, Bureau of Economics Staff Report..

demand for Ageless is 1.25 (at all points on the demand curve). You estimate the marginal cost of manufacturing and selling one more dose of Ageless is $1.

(a) What is the profit-maximizing price per dose of Ageless?

(b) Would you expect the elasticity of demand you face for Ageless to rise or fall when your patent expires?

Solution:

(a) Our general markup rule states that $(p - MC)/p = 1/\epsilon$, where ϵ is the elasticity of demand facing the firm at the point on the demand curve at which the firm operates. With a constant elasticity of demand and constant marginal cost, as in this problem, we can use this formula to solve directly for the profit-maximizing price, p^*. Here we get $(p^* - 1)/p^* = 1/1.25$. Solving for the optimal price gives $p^* = \$5$. Equivalently, one can directly use the other version of the markup formula, $p = MC\epsilon/(\epsilon - 1)$, to get $p = 1 \times 1.25/(1.25 - 1)$, which again gives $p^* = \$5$. Of course, the R&D expenditures are now sunk and thus do not enter into the pricing decision.

(b) The level of demand for Ageless must fall now that there are many very close substitutes in the form of generic versions. Hopefully, your brand will still allow you to command a premium price, but surely at any given price you will sell less as a result of the presence of the generic competition.

The elasticity of demand for Ageless will very likely rise now that closer substitutes are available. Customers will presumably be more price sensitive, and thus will induce you to set a lower price.

■ **5.6** Is the Windows operating system an essential facility? What about the Intel Pentium microprocessor? To what extent does the discussion in Section **??** on essential facilities (vertical integration, access pricing) apply to the above examples?

Solution: [Note: this is a very controversial question and not all economists agree on a single answer.] Both Microsoft (the producer of the Windows operating system) and Intel (the producer of the Intel Pentium microprocessor) provide computer makers with essential components, without which the machines could not function. Nevertheless, strictly speaking, we cannot say that their output represents an essential facility. The discussion in section 5.3 applies to monopolists. The crucial difference from the examples presented in the section is the fact that Microsoft and Intel are not monopolists: computer makers always have the option of switching to another provider of components.

However, the widespread use of the Windows operating system, and the fact that Windows is only supplied by Microsoft, implies that the latter's position is much closer to the one of a monopolist than is Intel's. Even though Intel's chip design is very close to being an industry standard, Intel is not the only company supplying microprocessors with that desing. Hence, the Windows operating system is closer to what is called an essential facility than Intel's Pentium processor.

■ **6.1** The technology of book publishing is characterized by a high fixed cost (typesetting the book) and a very low marginal cost (printing). Prices are set at much higher levels than marginal cost. However, book publishing yields a normal rate of return. Are these facts consistent with profit maximizing behavior by publishers? Which model do you think describes this industry best?

Solution: The model of monopolistic competition is probably the best approximation to describing this industry. The model of monopolistic competition shows that price-making, profit-maximizing behavior is consistent with a zero-profit long-run equilibrium. The strong scale economies in book publishing imply that the gap between price and marginal cost is particularly high.

■ **6.2** The market for laundry detergent is monopolistically competitive. Each firm owns one brand, and each brand has effectively differentiated itself so that is has some market power (i.e., faces a downward sloping demand curve). Still, no brand earns economic profits, because entry causes the demand for each brand to shift in until the seller can just break even. All firms have identical cost functions, which are U-shaped.

Suppose that the government does a study on detergents and finds out they are all alike. The public is notified of these findings and suddenly drops allegiance to any brand. What happens to price when this product that was brand-differentiated becomes a commodity? What happens to total sales? What happens to the number of firms in the market?

Solution: Based on the information provided, it seems that the initial situation in this market is like the long-run equilibrium of the monopolistic competition model; see Figure 6.3. The government's announcement has turned a differentiated product into a homogeneous one. In terms of the graph in Figure 6.3, this implies a flattening of the demand curve faced by each firm and a new long-run equilibrium where d (now horizontal) is tangent to the AC curve. At this new long-run equilibrium, price is given by p'_{LR} and each firm's output is given by q'_{LR}.

Clearly, the new equilibrium implies a lower price and a higher output per firm: $p'_{LR} < p_{LR}$ and $q'_{LR} > q_{LR}$.

Suppose that price were to drop from p_{LR} to p'_{LR} *without changing the degree of product differentiation or the number of firms*. This would imply an output per firm equal to q'_{SR}, where q'_{SR} is greater than q_{LR} but lower than q'_{LR}. If we take into account the disappearance of product differentiation (and continue with the same number of firms), then the output per firm would be less than q'_{SR}. Whatever the exact value is, each firm would be losing money ($p'_{LR} < AC$). Therefore, in the post-announcement long-run equilibrium, some firms will need to exit the market.

Finally, it is not clear what will happen to total output. On the one hand, each firm's output goes up. On the other hand, the number of firms goes down. Which effect dominates depends on how consumers value product differentiation and how the demand curve shifts as a result of the government announcement.

■ **6.3**** Show that, in a long-run equilibrium with free entry and equal access to the best available technologies, the comparison of price to the minimum of average cost or the comparison of price to marginal cost are equivalent tests of allocative efficiency. In other words, price is greater than the minimum of average costs if and only if price is greater than marginal cost.

Show, by example, that the same is not true in general.

Solution: We first show the following **fact:** *marginal cost is greater than average cost if and only if average cost is increasing.* To see this, notice that Average Cost is given by the ratio Cost / Output. Taking the derivative with respect to Output q, we get

$$\frac{d\,AC}{d\,q} = \frac{d}{d\,q}\frac{C}{q} = \frac{\frac{d\,C}{q}q - C}{q^2} = (MC - AC)/q,$$

which shows the fact.

In the long-run equilibrium of an industry with equal access, each firm will be producing at a point in the left-hand portion of its Average Cost curve. Given the above fact, it follows that marginal cost is lower than or equalt to average cost. Since there is free entry, price is equal to average cost. Specifically, either price is equal to the minimum of average cost and equal to marginal cost; or price is greater than the minimum of average cost and greater than marginal cost.

The same is not true, for example, in a short-run equilibrium. Consider the case of perfect competition. and suppose that price is greater than the minimum of average cost. Since firms are price takers, price is equal to marginal cost. So, the comparison price minus marginal cost is zero whereas price minus the minimum of average cost is positive.

■ **7.1** According to Bertrand's theory, price competition drives firms' profits down to zero even if there are only two competitors in the market. Why don't we observe this in practice very often?

Solution: Section 7.2 suggests three possible explanations: (a) product differentiation, (b) dynamic competition, (c) capacity constraints.

■ **7.2** Three criticisms are frequently raised against the use of the Cournot oligopoly model: (i) firms normally choose prices, not quantities; (ii) firms don't normally take their decisions simultaneously; (iii) firms are frequently ignorant of their rivals' costs; in fact, they do not use the notion of Nash equilibrium when making their strategic decisions.

How would you respond to these criticisms? (Hint: in addition to this chapter, you may want to refer to Chapter **??**.)

Solution:

(i) If firms are capacity constraint, then price competition "looks like" like quatitiy competition. See Section 7.2.

(ii) If there are significant information lags, then sequential decisions "look like" simultaneous decisions. See Chapter 4 (first section).

(iii) The last section of Chapter 7 presents an argument for the relevance of Nash equilibrium which only requires each firm to know its own profit function.

■ **7.3** Which model (Cournot, Bertrand) would you think provides a better approximation to each of the following industries: oil refining, internet access, insurance. Why?

Solution: Capacity constraints seem relatively more important in oil refining and relatively less important in insurance. Given the discussion in Section 7.4, one would be inclined to select the Cournot model for oil refining and the Bertrand model for insurance. Internet access is an intermediate case between the previous two.

■ **7.4*** Two firms, CS nd LC, make identical goods, GPX units, and sell them in the same market. The demand in the market is $Q = 1200 - P$. Once a firm has built capacity, it can produce up to its capacity each period with a marginal cost of $MC = 0$. Building a unit of capacity costs 2400 (for either CS or LC) and a unit of capacity lasts four years. The interest rate is zero. Once production occurs each period, the price in the market adjusts to the level at which all production is sold. (In other words, these firms engage in quantity competition, not price competition.)

(a) If CS knew that LC were going to build 100 units of capacity, how much would CS want to build? If CS knew that LC were going to build x units of capacity, how much would CS want to build (that is, what is CS's best response function in capacity)?

(b) If CS and LC each had to decide how much capacity to build without knowing the other's capacity decision, what would the one-shot Nash equilibrium be in the amount of capacity built?

Solution:

(a) If LC builds 100 units of capacity, then CS faces a residual demand of $Q_{\text{CS}} = Q - 100 = 1100 - p$. Its marginal revenue (contribution) is then $MR_{\text{CS}} = 1100 - 2Q_{\text{CS}}$. Equating this marginal revenue with CS's capacity costs of 600 yields the optimal capacity for CS as $Q^*_{\text{CS}} = 250$ units.

The generalization of this is to solve for CS's residual demand as a function of LC's capacity Q_{LC}. That is, $Q_{\text{CS}} = Q - Q_{\text{LC}} = 1200 - Q_{\text{LC}} - p$. CS's total revenue is then equal to $TR_{\text{CS}} = pQ_{\text{CS}} = (1200 - Q_{\text{LC}} + Q_{\text{CS}})Q_{\text{CS}}$ and its marginal revenue can be obtained by taking the derivative of TR_{CS} with respect to Q_{CS} (treating Q_{LC} as a constant). This yields $MR_{\text{CS}} = 1200 - Q_{\text{LC}} - 2Q_{\text{CS}}$. Equating this marginal revenue to marginal cost and solving for Q_{CS} yields $Q_{\text{CS}} = 300 - Q_{\text{LC}}/2$ as CS's optimal capacity in response to any capacity decision by LC.

(b) Since the two firms are symmetric, LC's best response to CS is analogous to CS's best response to LC, or $Q_{\text{LC}} = 300 - Q_{\text{CS}}/2$. A Nash equilibrium requires that $Q^*_{\text{LC}} = 300 - Q^*_{\text{CS}}/2$ and $Q^*_{\text{CS}} = 300 - Q^*_{\text{LC}}/2$. Substituting Q^*_{LC} into Q^*_{CS} and solving for Q^*_{CS} yields $Q^*_{\text{CS}} = 200$. Substituting this amount into the LC's best response function yields $Q^*_{\text{LC}} = 200$. At these capacities the market price is $p = 1200 - 200 - 200 = 800$. Each firm's profits are then $(800 - 600)(200) = \$40,000$.

■ **7.5*** Consider a market for a homogeneous product with demand given by $Q = 37.5 - P/4$. There are two firms, each with constant marginal cost equal to 40.

a) Determine output and price under a Cournot equilibrium.

b) Compute the efficiency loss as a percentage of the efficiency loss under monopoly.

Solution: (a) Duopolist i's profit is given by

$$\pi_i = q_i p(Q) - C(q_i) = q_i[150 - 4(q_i + q_j)] - 40q_i,$$

where the term in the square brackets comes from the demand function. The first order condition for profit maximization is given by:

$$150 - 4(q_i + q_j) - 4q_i - 40 = 0. \tag{1}$$

By symmetry, we have $q_i = q_j = 9.166$. Also, $p = 150 - 8q_i = 76.666$.