

# ECN 453: Collusion and Price Wars 1

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## Collusion and Price Wars: Motivation

- Adam Smith: “People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices”.



## Collusion and Price Wars: Motivation

- Competition creates an **externality**:
  - e.g. Consider Cournot: firms maximize their own profit not accounting that part of the increase in profits is coming at the expense of the other firms in the market.
- Firms could do better by establishing agreement between themselves to increase their **market power**.
- These agreements are generically referred to as **collusion**.
  - Organized cartels
  - Secret agreements
  - Tacit agreements
- In this part of the course we will discuss these practices in detail.

## Collusion and Price Wars: Motivation

- Types of agreement
  - Increase price
  - Decrease quantity
  - Territory agreements
  - Agreements about quality, advertising etc
- Key to many agreements: **dynamic considerations**
  - 'Dynamic' means agreements with a time dimension. E.g. 'Let's set the monopoly price together today otherwise I'll punish you tomorrow'.
- So, before talking about collusive agreements, we first need to understand how to solve games/strategic interactions with dynamics (**repeated games**).

## Plan

1. Present discounted value
2. Repeated games: method
3. Stability of collusive agreements

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## Present discounted value

- First need to introduce the idea of a **discount factor**, denoted  $\delta$
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  - $\delta = 0$ : don't value the future at all
  - $\delta = 1$ : value \$1 next period the same as \$1 today (e.g. don't care about waiting)
  - $0 < \delta < 1$ : e.g.  $\delta = 0.9$  value \$1 in the next period as 90c today.

## Present discounted value

- **Question:** What is the **present discounted value** of receiving \$5 repeatedly in every period (where there are infinite periods) if the discount factor is  $\delta < 1$ ?
- Value =  $5 + \delta 5 + \delta^2 5 + \dots$ 
  - Why is the  $\delta$  squared in the third term on the right-hand-side?

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- Value =  $5 + \delta 5 + \delta^2 5 + \dots$ 
  - Why is the  $\delta$  squared in the third term on the right-hand-side?
    - A: This is the value two periods in the future, so multiply it once to discount it back to one period in the future, and discount it again to get it back to the current period.
- **Important math equation:** (Geometric sum:)
  - Constant:  $c$
  - Discount value:  $\delta < 1$  then:
$$c + \delta c + \delta^2 c + \delta^3 c + \dots = \frac{c}{1 - \delta}$$
- So, overall, Value =  $\frac{c}{1 - \delta}$

# Plan

1. Present discounted value
2. **Repeated games: method**
3. Stability of collusive agreements

## Repeated games: prisoner's dilemma with $T = 1$

- Denote  $T$  as number of times the game is repeated. Here, consider  $T = 1$ .
- $(B, R)$  is the unique Nash equilibrium.
- Both firms would do much better if could somehow agree to play  $(T, L)$

		Player 2	
		L	R
		5	6
Player 1	T	5	0
	B	0	1
		6	1

## Repeated games: prisoner's dilemma with $T = 2$

- Strategies in repeated games: 'contingent strategies'
  - Contingent strategy: given all the actions in the previous periods (i.e, the history of actions), choose an action in the current period

		Player 2	
		L	R
		5	6
Player 1	T	5	0
	B	0	1
		6	1

## Repeated games: prisoner's dilemma with $T = 2$

- Is playing (B,R) in both periods a Nash equilibrium?

## Repeated games: prisoner's dilemma with $T = 2$

- Is playing (B,R) in both periods a Nash equilibrium?
- Yes! Are there any other equilibria?
- Let's consider one alternative called the **grim trigger strategy**.
- Idea: play (T,L) . If there is **any** deviation then play (B,R).

		Player 2	
		L	R
		5	6
Player 1	T	5	0
	B	0	1
		6	1

## Repeated games: prisoner's dilemma with $T = 2$

- **Grim trigger strategy:**

- Idea: play (T,L) in period 1.
- Threat: play (B,R) in period 2 contingent on either player deviating.
- Otherwise, play (T,L) again.
- Is this a NE?

		Player 2	
		L	R
		5	6
Player 1	T	5	0
	B	0	1
		6	1

## Repeated games: prisoner's dilemma with $T = 2$

- Check grim trigger strategy using backwards induction.
- At  $t=2$ : Always play (B,R)
- So, at  $t=1$ : cannot promise to play (T,L) in period 2 credibly.
- So, grim trigger is *not* a Nash equilibrium in the  $T = 2$  game.

		Player 2	
		L	R
		5	6
Player 1	T	5	0
	B	0	1
		6	1

## Repeated games: prisoner's dilemma with $T = \infty$

- As before, playing  $(B, R)$  every period is a Nash equilibrium.
- But now let's consider the grim trigger strategy again:
  - Choose  $(T, L)$  if the history has been  $(T, L)$  for every period in the past.
  - Choose  $(B, R)$  otherwise.
- **Question:** Is the grim trigger strategy a Nash equilibrium?

		Player 2	
		L	R
		5	6
Player 1	T	5	0
	B	0	1
		6	1

## Repeated games: prisoner's dilemma with $T = \infty$

- **Question:** Is the grim trigger strategy a Nash equilibrium?

- Equilibrium payoff:

- $\Pi = 5 + \delta 5 + \delta^2 5 + \dots = \frac{5}{1-\delta}$

- Deviation payoff:

- $\Pi' = 6 + \underbrace{\delta 1 + \delta^2 1 + \dots}_{\text{Grim trigger punishment}} = 6 + \frac{\delta}{1-\delta}$

		Player 2	
		L	R
		5	6
Player 1	T	5	0
	B	0	1
		6	1

- Is a NE if no incentive to deviate:  $\Pi \geq \Pi'$ 
  - Substitute in payoffs from above:  $\delta \geq 1/5$
  - Idea: if  $\delta$  high enough, deviation doesn't pay.

## Repeated games

- Why does the equilibrium in the repeated game differ from the one period game?
- **Because players can react to other players' past actions, repeated games allow for equilibrium outcomes that would not be an equilibrium in the corresponding one-shot game.**
- In this class, the main application of repeated games will be on collusive agreements. Many other economic examples (typically, when agents cannot sign an enforceable contract):
  - Economic interactions when reputation is important
  - International agreements (e.g. WTO, Kyoto)
  - Informal economic relationships built on trust

## Repeated games: steps to solve

- Steps to show that a particular strategy (e.g. grim trigger) is a Nash equilibrium
- 1. Find the payoff from the proposed equilibrium using the present discounted value formula
  2. Find the payoff from *unilaterally deviating* using the present discounted value formula
  3. Determine whether the payoff from the proposed equilibrium is greater than the payoff from deviating. If the payoff is higher, then it is an equilibrium.

## Repeated games: prisoner's dilemma with $T = \infty$

- **Question:** For what discount values  $\delta$  is the grim trigger strategy a Nash equilibrium in the following game?

		Player 2	
		L	R
		6	7
Player 1	T	6	0
	B	0	2
		7	2

# Plan

1. Present discounted value
2. Repeated games: method
3. **Stability of collusive agreements**

# Stability of collusive agreements

- **Setup (same as Bertrand):**
  - Homogeneous product.
  - Two firms with the same constant MC.
  - Firms set prices simultaneously. If firms set the same price then they split demand equally.
  - Note: If game is played once  $\rightarrow$  Bertrand equilibrium
- **Question:** if game is played repeatedly for infinite periods with discount factor  $\delta$ , for what values of  $\delta$  is the follow *grim trigger strategy* an equilibrium?
  - Set  $p = p^M$  if  $p = p^M$  in the past.
  - Set  $p = MC$  otherwise
- (Note: there might be other Nash equilibria, but we'll just check this one.)

## Stability of collusive agreements: solution

- Equilibrium net present value:
$$\Pi = 0.5\pi^M + \delta 0.5\pi^M + \delta^2 0.5\pi^M + \dots = 0.5\pi^M \frac{1}{1-\delta}$$
- Net present value from deviating (and undercutting the rival):
$$\Pi' = \pi^M + \underbrace{\delta 0 + \delta^2 0 + \dots}_{\text{Punishment: revert back to marginal cost}} = \pi^M + \frac{0}{1-\delta}$$
- Nash equilibrium condition:
$$\Pi \geq \Pi'$$
, substituting in payoffs,  $\delta \geq 0.5$ .
- **Note:** if discount factor is sufficiently high, then there exists a NE of the repeated game where firms set the monopoly price every period under the 'threat' that if any firm deviates, both firms revert to pricing at the marginal cost level forever.