ECN 594: Homework 2

Competition and Merger Simulation

Due: See Canvas

You may work in groups of up to 2 people.

Instructions

- You may work in groups of up to 2 people. If working in a group, please submit one assignment with both names.
- Submit your solutions as a PDF. For computational questions, also submit your Python code.
- Show your work for analytical questions.

Part A: Oligopoly Theory (35 points)

Question 1: Cournot Competition (15 points)

Consider a market with $N = 3$ identical firms competing in quantities. Market demand is $P = 100 - Q$, where $Q = q_1 + q_2 + q_3$. Each firm has constant marginal cost $c = 10$ and no fixed costs.

a. **(5 points)** Derive the Cournot-Nash equilibrium quantities, price, and per-firm profits. Show your work.

b. **(5 points)** Verify the Lerner Index formula: $L = \frac{P-MC}{P} = \frac{s_j}{|\varepsilon|}$ where s_j is firm j's market share and ε is the market price elasticity of demand.

c. **(5 points)** Now suppose one firm exits the market, leaving $N = 2$ firms. Compute the new equilibrium and compare consumer surplus, producer surplus, and total welfare to the 3-firm case.

Question 2: Bertrand Competition (10 points)

Consider the same setup as Question 1 ($P = 100 - Q$, $c = 10$), but now firms compete in prices (Bertrand) rather than quantities.

a. **(5 points)** What is the Bertrand-Nash equilibrium with $N = 3$ identical firms? What are equilibrium prices and profits?

b. **(5 points)** Compare your answers to the Cournot case (Question 1). Explain why the outcomes differ. Which model is more realistic and when?

Question 3: Collusion (10 points)

Return to the Cournot setting with $N = 3$ firms. Suppose the firms consider forming a cartel where they each produce the monopoly quantity divided by 3.

a. **(3 points)** What is the monopoly quantity and price? What would each firm's profit be under collusion?

b. **(4 points)** If one firm considers deviating while the others stick to the collusive quantity, what quantity should it produce and what profit would it earn?

c. **(3 points)** Compute the critical discount factor δ^* above which collusion can be sustained in an infinitely repeated game with grim trigger strategies. Use the formula from class.

Part B: Merger Simulation (65 points)

In this section, you will perform a merger simulation using an estimated demand system. This mirrors what economists do in practice when advising antitrust authorities.

Background

Consider a market with 4 differentiated products, each owned by a separate firm. The demand system is logit with the following estimated parameters:

- Price coefficient: $\alpha = -2.0$
- Product-specific mean utilities (deltas): $\delta = (-0.5, -0.3, -0.2, -0.4)$
- Marginal costs: $c = (1.0, 1.2, 1.5, 1.1)$

The market size is $M = 1000$ consumers. The outside option has $\delta_0 = 0$.

Question 4: Pre-Merger Equilibrium (20 points)

a. **(5 points)** Solve for the pre-merger equilibrium prices. Each firm owns one product and sets price to maximize profit. Use the first-order condition:

$$s_j + (p_j - c_j)\frac{\partial s_j}{\partial p_j} = 0$$

Solve this system numerically in Python starting from initial guess $p = c + 0.5$.

b. **(5 points)** Using your equilibrium prices, compute market shares for each product and the outside option.

c. **(5 points)** Compute the own-price elasticity for each product. Verify that all products have elastic demand ($|\eta_{jj}| > 1$).

d. **(5 points)** Verify that your prices satisfy the FOC by checking that:

$$p_j - c_j = \frac{1}{|\alpha|(1 - s_j)}$$

e. **(5 points)** Compute the HHI (Herfindahl-Hirschman Index) for this market. Would the DOJ/FTC consider this market concentrated?

Question 5: Post-Merger Prices (25 points)

Now suppose Firm 1 and Firm 2 merge. The merged firm now controls products 1 and 2.

a. **(5 points)** Write down the ownership matrix $\mathcal{O}$ before and after the merger. The ownership matrix has $\mathcal{O}_{jk} = 1$ if products j and k are owned by the same firm, and 0 otherwise.

b. **(5 points)** Explain intuitively why the merged firm will raise prices on products 1 and 2 after the merger. What is the "recapture" effect?

c. **(10 points)** Compute the new equilibrium prices after the merger.

Hint: The first-order conditions become:

$$s_j + \sum_{k:\mathcal{O}_{jk}=1} (p_k - c_k)\frac{\partial s_k}{\partial p_j} = 0$$

You can solve this system numerically in Python. Hold marginal costs constant (no efficiency gains).

d. **(5 points)** How much do prices increase for products 1 and 2? What happens to prices for products 3 and 4 (owned by non-merging firms)?

Question 6: Welfare Analysis (20 points)

a. **(5 points)** Compute total consumer surplus before and after the merger using the log-sum formula from HW1. What is the change in consumer surplus?

b. **(5 points)** Compute total producer profits before and after the merger. Does the merged firm benefit? Do the non-merging firms benefit?

c. **(5 points)** Compute the change in total welfare (CS + PS). Is the merger welfare-improving or welfare-reducing?

d. **(5 points)** The merging firms claim they will achieve marginal cost reductions of 10% on both products due to synergies. Redo the merger simulation with $c_1 = 0.9$ and $c_2 = 1.08$. Does the efficiency defense change your conclusion about whether the merger should be approved?

Formulas for Reference

Logit demand:

$$s_j = \frac{\exp(\delta_j + \alpha p_j)}{1 + \sum_k \exp(\delta_k + \alpha p_k)}$$
$$\frac{\partial s_j}{\partial p_j} = \alpha s_j (1 - s_j)$$
$$\frac{\partial s_j}{\partial p_k} = -\alpha s_j s_k \quad (j \neq k)$$

Own-price elasticity:

$$\eta_{jj} = \alpha p_j (1 - s_j)$$

Consumer surplus (per consumer):

$$CS = \frac{1}{|\alpha|} \ln \left(1 + \sum_j \exp(\delta_j + \alpha p_j) \right)$$

HHI:

$$HHI = \sum_j (100 \times s_j)^2$$

Cournot critical discount factor (N firms):

$$\delta^* = \frac{(N+1)^2}{N^2 + (N+1)^2}$$

Submission Checklist

- ☐ PDF with answers to all questions
- ☐ Python code for Part B
- ☐ All calculations shown
- ☐ Both group members' names (if applicable)