# ECN 453: Hotelling Competition

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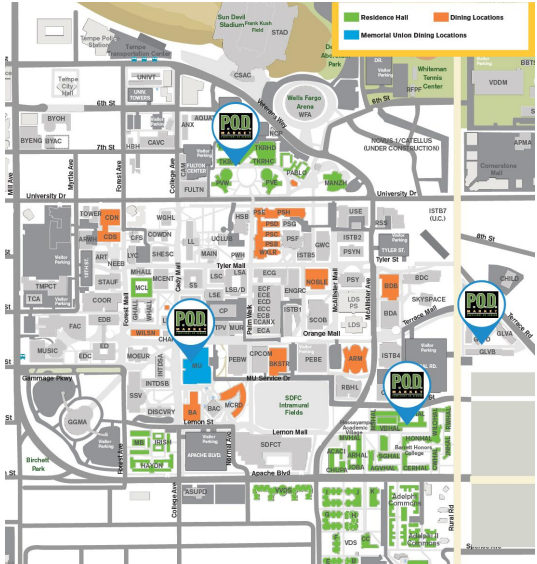
# Static Models of Oligopoly: Hotelling Competition

- So far we have studied three models of competition with **homogeneous** products.
- In many markets firms sell **differentiated** (heterogeneous) products.
- How should we model competition in these markets?
- Today we will see one model: **Hotelling competition**.

# Static Models of Oligopoly: Hotelling Competition

- Hotelling competition relates to selling products with **horizontal differentiation**.
- 'Horizontal differentiation' relates to characteristics where there is no clear quality ordering
- Examples:
  - Geographical differentiation (one supermarket might be closer to where you live than another)
  - Product characteristics that appeal to different 'tastes' (Pepsi vs Coke)
  - Ice-cream vendors on a 1 mile beach (this was the original example given when this model was written)
- Contrast: 'Vertical differentiation' - consumers agree on which is the better product (i.e. quality differentiation)

# Static Models of Oligopoly: Horizontal Differentiation On Campus



# Plan

1. Hotelling Competition: Setup
2. Hotelling Competition: Solution
3. Hotelling Competition: Application

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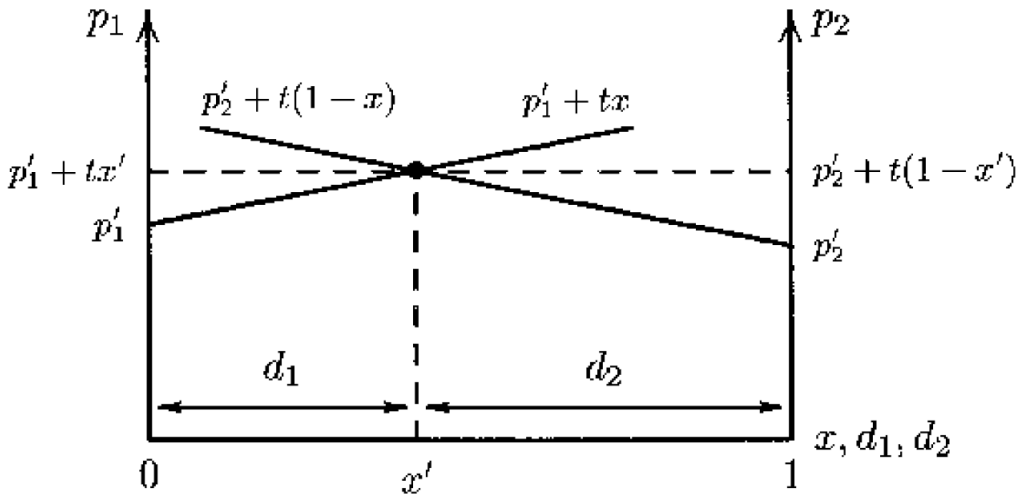
# Hotelling Competition: Setup - Preliminaries

- Let's review one math preliminary before getting into the model.
- Remember the uniform distribution?
- Uniform distribution: $x \sim U[0,1]$
  - If we pick a point $s \in [0,1]$, $Pr(x \leq s) = s$

# Hotelling Competition: Setup

- **Players**
- Consumers are distributed $x \in U[0,1]$
  - E.g. x refers to a consumer's 'address'
- Firm 1 is located at address $x = 0$ and Firm 2 is located at address $x = 1$.
- Consumer at location x pays a *transport cost* equal to $t$ multiplied by the distance between its address and the Firm's address when they buy from a particular Firm (as well as price)
  - Consumer at address $x$ pays transport cost $tx$ to get to Firm 1 and $t(1-x)$ to get to Firm 2.
- **Strategies**
  - Firm 1 and Firm 2 choose prices $p_1$ and $p_2$
- **Payoffs**
  - Firm 1 has marginal cost $c_1$ and Firm 2 has marginal cost $c_2$
  - Firms maximize their profits

# Hotelling Competition: Setup - graph of consumer's payoffs



# Hotelling Competition: Setup

- Before solving this model, let's first think about how the math relates to some real-world applications:

- **Supermarket competition**: here there are two supermarkets and the 'distance' cost might be e.g. cost of driving

- **Coke vs Pepsi (or other products)**: consumers have different tastes - if prices are equal will just choose the brand that is closest to their tastes.

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# Hotelling Competition: Solution

- Broadly, we will follow our 'usual steps' to solve these models of competition:
- 1. Get the payoffs (profits)
- 2. Find the best responses
- 3. Solve for the Nash equilibrium

# Hotelling Competition: Solution - 1. Get the payoffs

- **Note**: for simplicity, we will assume that consumers always choose to buy *something* (i.e. ignore the possibility that the firms might set prices so high some consumers may choose to buy from either firm)

- A consumer with address $x'$ is indifferent between the two firms if:

$$tx' + p_1 = t(1 - x') + p_2$$

- So, consumers to the left of $x'$ buy from Firm 1 and consumers to the right of $x'$ buy from Firm 2.
  - So, since consumers are uniform distributed Firm 1's demand is $x'$ and Firm 2's demand is $1 - x'$.

- Solving the above equation for $x'$ implies:

$$q_1 = 0.5 + \frac{p_1 - p_2}{2t}$$

$$q_2 = 0.5 + \frac{p_2 - p_1}{2t}$$

# Hotelling Competition: Solution - 1. Get the payoffs

- On the previous slide we found demand for each firm. Use this demand to get the payoffs:

$$\pi_1 = q_1(p_1 - c_1) = (0.5 + \frac{p_1 - p_2}{2t})(p_1 - c_1)$$

$$\pi_2 = q_2(p_2 - c_2) = (0.5 + \frac{p_2 - p_1}{2t})(p_2 - c_2)$$

# Hotelling Competition: Solution - 2. Get the best responses

- Take the derivative of the payoffs with respect to price and set to 0 to maximize profit:
- Firm 1:

$$0.5 - \frac{p_2 - p_1}{2t} - \frac{1}{2t}(p_1 - c_1) = 0$$

- So:

$$p_1 = 0.5(c_1 + t + p_2)$$

- Firm 2:

$$0.5 - \frac{p_1 - p_2}{2t} - \frac{1}{2t}(p_2 - c_2) = 0$$

- So:

$$p_2 = 0.5(c_2 + t + p_1)$$

# Hotelling Competition: Solution - 3. Solve for the Nash equilibrium

- Setting $c_1 = c_2 = c$ and noting that since Firm 1 and Firm 2 are now identical, in equilibrium $p_1 = p_2$.

- So:

$$p_1 = p_2 = c + t$$

# Hotelling Competition: Solution Discussion

- Consider the solution:

$$p_1 = p_2 = c + t$$

- The transportation costs $t$ (which index how differentiated products are to consumers) govern how intense competition is.

- If $t = 0$ consumers flock to the product with the lowest cost and we get the Bertrand solution. So, in the model, product differentiation ($t > 0$) solves the 'Bertrand Trap'.

- Using the model solution, we can relate this idea to 'market power' (the ability of firms to price their products above marginal cost):

- **The greater the degree of product differentiation, the greater the degree of market power.**

# Hotelling Competition: Solution Discussion

- What if firms could choose their location (product positioning) and then set prices?
  - Full solution is a little too complicated to go into in detail, but I'll discuss the main intuition

- Depends on the interplay between two effects (consider the case of moving from opposite ends of the uniform distribution the center at 0.5):
  - Direct effect: Holding prices fixed, moving closer to the center increases demand and profits

  - Strategic effect: Moving closer to the rival leads the rival to decrease its prices, which in turn decreases a firm's profits.

# Hotelling Competition: Solution Discussion

- Examples of these effects:

- *Example:* retail banking in Europe. Prices (interest rates) determined at the country level, so strategic effect is low. So, expect banks to locate branches close to the center.

- *Example:* ice-cream vendors on a beach. Consumers choose solely based on price - expect them to locate far from each other.

# Hotelling Competition: Solution Discussion

- Overall:

- **If price competition is very intense, then firms tend to locate far apart (high degree of differentation).**

- **If price competition is not very intense, then firms tend to locate close to each other (low degree of differentiation).**

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# Hotelling Competition: Application - Strategic Trade Policy

- **Background**: Airplane producers: US (Boeing); EU (Airbus)
- In 2005 US sued EU before the World Trade Organization, accusing the EU of subsidizing Airbus.
- We'd like to know: what was the impact of these subsidies on Boeing's profitability?

# Hotelling Competition: Application - Strategic Trade Policy

- **Setup**: Suppose that we have the Hotelling setup from before, where we found the best responses were:

$$\begin{aligned} \text{Firm 1:} \quad & p_1 = 0.5(c_1 + t + p_2) \\ \text{Firm 2:} \quad & p_2 = 0.5(c_2 + t + p_1) \end{aligned}$$

- **Questions:**

- 1. What is the Nash equilibrium?

- 2. Starting from the case where costs are the same ($c_1 = c_2$) what is the change in Boeing's profits (Firm 1) for a \$1 increase in subsidies for Airbus (Firm 2)?

# Hotelling Competition: Application - Strategic Trade Policy: Solution

- 1. Nash equilibrium (where $i$ is a particular firm and $j$ is its rival) is:

$$p_i = \frac{2}{3}c_i + \frac{1}{3}c_j + t$$

- 2. Note that profit is: $\pi_i = \frac{1}{18t}(3t + c_j - c_i)^2$
- Compute $\frac{d\pi_1}{dc_2} =$.
- Substitute in $c_1 = c_2$ and we get that the derivative here is $-\frac{1}{3}$.
- So, \$1 increase in subsidies results in a 66c decrease in Boeing's profit.

# Summary of key points*

- Know what 'horizontal differentiation' is
- Know how to setup, interpret, and solve the Hotelling model with two firms
- Understand the 'strategic' vs 'direct' effects as firms change location

*To clarify, all the material in the slides, problem sets, etc is assessable unless stated otherwise, but I hope this summary might be a useful place to start when studying the material.

# Question

- **Setup:**
- Suppose that consumers are distributed uniformly between 0 and 1.
- Firm 1 is located at 0 and Firm 2 is located at 0.75.
- Marginal costs are equal for both firms and equal to $c$.
- Constant transport costs equal to $td$ where $d$ is distance and $t$ is a number.
- Firms choose prices and compete under Hotelling competition.
- **Question:**
- 1. Find demand for each firm
- 2. Find the best responses
- 3. Find the Nash equilibrium prices