ECN 594: Oligopoly Competition

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January 4, 2026

Welcome to Part 2

- Demand estimation and pricing

- **Models of competition and industry structure**
    - Oligopoly models (Cournot, Bertrand, Hotelling)

    - Entry and entry deterrence

    - Mergers

    - Vertical relationships

    - Collusion

- **HW2 released:** Merger simulation module

Plan

1. **Cournot competition (refresher in IO notation)**

2. Bertrand competition (refresher)

3. Cournot vs Bertrand: when does each apply?

4. Product differentiation: why it matters

5. Hotelling model

6. Connection to demand estimation

From ECN 532: Oligopoly models

- You covered Cournot and Bertrand in Hector's class

- Today: quick refresher in IO notation

- **New focus:** Market power measurement
    - Connecting oligopoly models to Lerner index

    - When does each model apply?

Cournot competition: setup

- *n* firms producing homogeneous goods

- Firms choose **quantities** simultaneously

- Inverse demand: $P = P(Q)$ where $Q = \sum_{i=1}^{n} q_i$

- Constant marginal cost: c

- Firm i profit: $\pi_i = P(Q) \cdot q_i - c \cdot q_i$

Cournot: first-order conditions

- Firm i maximizes profit taking q_{-i} as given:

$$\frac{\partial \pi_i}{\partial q_i} = P(Q) + P'(Q)q_i - c = 0$$

- Rearranging:

$$P(Q) - c = -P'(Q)q_i$$

- Divide by P:

$$\frac{P - c}{P} = \frac{-P'(Q)q_i}{P} = \frac{-P'(Q)Q}{P} \cdot \frac{q_i}{Q} = \frac{s_i}{|\varepsilon|}$$

- where $s_i = q_i/Q$ is firm i's market share

Cournot: Lerner index

- **Key result:** In Cournot equilibrium,

$$L_i = \frac{P - MC}{P} = \frac{s_i}{|\varepsilon|}$$

- **Interpretation:**
    - Markup depends on market share
    - Larger firms have more market power
    - More elastic demand $\rightarrow$ lower markup

- This connects to demand estimation from Part 1!

Worked example: Cournot with market power

- **Question:** Inverse demand is $P = 100 - Q$. Two symmetric firms with $MC = 10$.

- (a) Find equilibrium quantities and price.

- (b) Calculate the Lerner index for each firm.

- (c) Verify using the $L = s/|\varepsilon|$ formula.

Take 5 minutes.

Worked example: Cournot (solution)

Solution

- **(a)** FOC: $100 - 2q_i - q_j - 10 = 0$

- Symmetric: $q_1 = q_2 = q^*$, so $100 - 3q^* = 10 \Rightarrow q^* = 30$

- $Q = 60$, $P = 100 - 60 = 40$

- **(b)** $L = \frac{40-10}{40} = \frac{3}{4} = 0.75$

- **(c)** Market share: $s_i = 30/60 = 0.5$

- Elasticity: $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = (-1) \cdot \frac{40}{60} = -\frac{2}{3}$

- Check: $L = \frac{s_i}{|\varepsilon|} = \frac{0.5}{2/3} = 0.75$ ✓

Cournot with n symmetric firms

- General result for linear demand $P = a - bQ$:

- Symmetric equilibrium: $q^* = \frac{a-c}{b(n+1)}$

- Total quantity: $Q^* = \frac{n(a-c)}{b(n+1)}$

- Price: $P^* = \frac{a+nc}{n+1}$

- Lerner index: $L = \frac{1}{n|\varepsilon|}$ (symmetric case where $s_i = 1/n$)

- **Key insight:** As $n \to \infty$, $P \to MC$ (perfect competition)

Practice: Cournot comparative statics

- **True, False, or NEI:**

- (a) In Cournot equilibrium, a firm with a larger market share has a higher markup.

- (b) Adding a third firm to a Cournot duopoly always reduces industry profits.

- (c) In Cournot, all firms must have the same marginal cost.

Take 2 minutes.

Practice: Cournot comparative statics (solution)

Answers

- **(a) TRUE.** From $L_i = s_i/|\varepsilon|$, larger s_i means higher L_i.

- **(b) TRUE.** More firms $\Rightarrow$ lower price $\Rightarrow$ lower industry profit. Each firm's profit falls, and total profit falls because P is closer to MC.

- **(c) FALSE.** Asymmetric costs work fine. Low-cost firms have higher q_i and s_i, hence higher margins.

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1. Cournot competition (refresher in IO notation)

2. **Bertrand competition (refresher)**

3. Cournot vs Bertrand: when does each apply?

4. Product differentiation: why it matters

5. Hotelling model

6. Connection to demand estimation

Bertrand competition: setup

- *n* firms producing **homogeneous** goods

- Firms choose **prices** simultaneously

- Consumers buy from lowest-price firm

- If tie: split demand equally

- Constant marginal cost: c

Bertrand: the paradox

- **Nash equilibrium:** $p_1 = p_2 = c$ (marginal cost pricing!)

- **Why?**

    - If $p_i > p_j > c$: firm i can undercut and capture entire market

    - Undercutting continues until $p = c$

- **The "paradox":**

    - Only 2 firms, but competitive outcome!

    - Zero profits with just 2 competitors

    - Seems unrealistic for most markets

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Cournot vs Bertrand: summary

	Cournot	**Bertrand**
Strategic variable	Quantities	Prices
Equilibrium price	$P > MC$	$P = MC$
Profits	Positive	Zero
Lerner index	$L = s/|\varepsilon|$	$L = 0$

- Which model is "right"?

- Answer: depends on the industry!

Escaping the Bertrand paradox

1. **Capacity constraints** (Kreps-Scheinkman)
   - Can't serve entire market at low price
   - Leads to Cournot outcome

2. **Product differentiation** (today's main topic!)
   - Consumers have preferences
   - Not all switch to lowest price

3. **Repeated interaction** (collusion, covered later)
   - Firms can sustain $P > MC$ through punishment strategies

When does each model apply?

- **Cournot applies when:**

  - Capacity constraints matter

  - Firms commit to production before selling

  - Quantities are hard to adjust quickly

- Examples: manufacturing, airlines (seat capacity)

- **Bertrand applies when:**

  - Prices adjust quickly

  - No capacity constraints

  - Homogeneous products

- Examples: online retail, commodities

Industry examples: which model?

Industry	Model	Why?
Airlines	Cournot	Capacity (planes, gates) committed
Cement	Cournot	Production committed; transport costs
Gasoline stations	Bertrand	Prices adjust daily; commodity
Online retail	Bertrand	Instant price changes; no capacity
Smartphones	Differentiated	Product differentiation dominates

- In practice: most markets have differentiated products!

Practice: Cournot vs Bertrand

- **Question:** Two firms have $MC = 20$. Market demand is $P = 100 - Q$.

- (a) Find equilibrium price under Cournot.

- (b) Find equilibrium price under Bertrand.

- (c) Which model yields higher consumer surplus? Why?

Take 3 minutes.

Practice: Cournot vs Bertrand (solution)

Solution

- **(a) Cournot:** Using $P^* = \frac{a+nc}{n+1} = \frac{100+2(20)}{3} = 46.67$

- Or: $q^* = \frac{100-20}{3} = 26.67$, $Q = 53.33$, $P = 46.67$

- **(b) Bertrand:** $P = MC = 20$

- **(c)** Bertrand has higher CS:
    - Bertrand: $CS = \frac{1}{2}(100-20)(80) = 3200$
    - Cournot: $CS = \frac{1}{2}(100-46.67)(53.33) \approx 1422$

- Lower price $\Rightarrow$ higher quantity $\Rightarrow$ higher CS

Kreps-Scheinkman (1983): resolving the puzzle

- Two-stage game:

    1. Stage 1: Firms choose capacities (quantities)

    2. Stage 2: Firms compete in prices

- **Result:** Equilibrium outcome = Cournot!

- **Intuition:**

    - Capacity choice commits firms

    - Price competition is constrained by capacity

    - Undercutting is limited by what you can produce

- Key insight: commitment matters

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4. **Product differentiation: why it matters**

5. Hotelling model

6. Connection to demand estimation

Why differentiation matters

- Bertrand paradox: $P = MC$ with homogeneous products

- **Solution:** Product differentiation!

- If products are different, consumers don't all buy from lowest-price firm

- Firms have some pricing power

- This is exactly what we modeled in Part 1 (logit demand)

- Now: a classic spatial model of differentiation

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5. **Hotelling model**
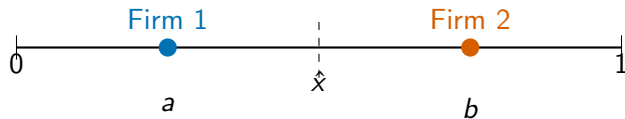
6. Connection to demand estimation

Hotelling model: setup

- Consumers uniformly distributed on $[0, 1]$ ("Main Street")

- Two firms located at positions a and b on $[0, 1]$

- Consumer at location x has utility:

$$u_j = v - p_j - t|x - \ell_j|$$

- v: base value of product

- p_j: price of firm j

- t: transport cost per unit distance

- $|x - \ell_j|$: distance to firm j

Hotelling: graphical intuition

- Consumers to the left of $\hat{x}$ buy from Firm 1

- Consumers to the right of $\hat{x}$ buy from Firm 2

- $\hat{x}$ is the "indifferent consumer"

Finding the indifferent consumer

- Consumer at $\hat{x}$ is indifferent between firms:

$$v - p_1 - t|\hat{x} - a| = v - p_2 - t|b - \hat{x}|$$

- With $a = 0$ and $b = 1$ (firms at endpoints):

$$v - p_1 - t\hat{x} = v - p_2 - t(1 - \hat{x})$$
$$p_2 - p_1 = t(1 - 2\hat{x})$$
$$\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

- Demand for firm 1: $D_1 = \hat{x}$

- Demand for firm 2: $D_2 = 1 - \hat{x}$

Hotelling: equilibrium prices

- Firm 1 maximizes: $\pi_1 = (p_1 - c) \cdot \hat{x}(p_1, p_2)$

- FOC: $\frac{\partial \pi_1}{\partial p_1} = \hat{x} + (p_1 - c)\frac{\partial \hat{x}}{\partial p_1} = 0$

- With $\frac{\partial \hat{x}}{\partial p_1} = -\frac{1}{2t}$:

$$\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{p_1 - c}{2t} = 0$$

- Symmetric equilibrium ($p_1 = p_2 = p^*$):

$$p^* = c + t$$

- **Markup = transport cost!**

Hotelling: interpretation

- $p^* = c + t$: Firms charge above marginal cost

- **Transport cost t measures differentiation**

    - High t: products very different $\rightarrow$ high markup

    - Low t: products similar $\rightarrow$ low markup

    - $t \rightarrow 0$: products identical $\rightarrow$ Bertrand ($p \rightarrow c$)

- **No Bertrand paradox:** Differentiation creates pricing power

- Each firm gets half the market: $D_1 = D_2 = 1/2$

- Profit: $\pi = (p^* - c) \cdot \frac{1}{2} = \frac{t}{2}$

Worked example: Hotelling

- **Question:** Two ice cream vendors on a beach of length 1 mile. Transport cost $t = 2$ dollars per mile. Marginal cost $c = 1$.

- (a) Find the equilibrium price.

- (b) If firm 1 raises price to $p_1 = 4$, what is its market share?

- (c) Calculate firm 1's demand elasticity at the equilibrium.

Take 4 minutes.

Worked example: Hotelling (solution)

Solution

- **(a)** $p^* = c + t = 1 + 2 = 3$

- **(b)** At $p_1 = 4$, $p_2 = 3$:

$$\hat{x} = \frac{1}{2} + \frac{3-4}{2(2)} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

- Firm 1's market share falls to 25%

- **(c)** At equilibrium: $D_1 = 1/2$, $\frac{\partial D_1}{\partial p_1} = -\frac{1}{2t} = -\frac{1}{4}$

$$\varepsilon_1 = \frac{\partial D_1}{\partial p_1} \cdot \frac{p_1}{D_1} = -\frac{1}{4} \cdot \frac{3}{1/2} = -1.5$$

Hotelling: why transport cost matters

- **High t:** Strong differentiation
    - Consumers have strong location preferences
    - Firm can raise price without losing many customers
    - Example: specialty restaurants vs fast food

- **Low t:** Weak differentiation
    - Consumers nearly indifferent across locations
    - Small price cut steals many customers
    - Approaches Bertrand as $t \to 0$

- In demand estimation: $1/\alpha$ plays similar role to t

Practice: T/F on Hotelling

- **True, False, or NEI:**

- (a) In Hotelling equilibrium, firms always split the market equally.

- (b) A monopolist in Hotelling should locate at the center of the line.

- (c) Higher transport costs benefit consumers because products are more differentiated.

Take 2 minutes.

Practice: T/F on Hotelling (solution)

Answers

- **(a) TRUE** (with caveat). If firms are at endpoints and have same costs, yes. With asymmetric locations or costs, shares differ.

- **(b) TRUE.** Monopolist minimizes total transport costs by locating at center. This maximizes consumer value and thus WTP.

- **(c) FALSE.** Higher t means higher prices ($p^* = c + t$) and higher transport costs. Both hurt consumers.

Welfare in Hotelling

- **Total welfare** $=$ Consumer surplus $+$ Profits

- Transport costs are deadweight loss

- **Socially optimal locations:** $a = 1/4$, $b = 3/4$
    - Minimizes total transport costs

- **Equilibrium locations:** Both firms at $1/2$ (minimum differentiation)
    - Firms want to capture more customers
    - But this increases total transport costs

- "Principle of minimum differentiation" (but fragile)

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Connection to demand estimation

- Hotelling is a specific **differentiated Bertrand** model

- Location $\leftrightarrow$ product characteristics

- Transport cost $\leftrightarrow$ preference heterogeneity

- **Logit demand** generalizes this idea:
    - Products differ in characteristics space

    - Consumers have heterogeneous preferences

    - Price competition with differentiated products

- Next lectures: how to use demand estimates for merger simulation

From Hotelling to logit: the connection

Hotelling	Logit
Location on line	Product characteristics
Transport cost t	Sensitivity $1/|\alpha|$
Consumer position	Consumer preferences
Linear utility	Logit choice probabilities
2 products	J products

- Logit demand lets us do empirical Hotelling-style analysis

- Estimate α from data $\rightarrow$ calibrate "differentiation"

Oligopoly models: summary

- **Homogeneous products:**
  - Cournot (quantity): $L = s_i / |\varepsilon|$
  - Bertrand (price): $P = MC$

- **Differentiated products:**
  - Hotelling/Differentiated Bertrand: $P > MC$
  - Markup depends on differentiation (transport cost)

- **Empirical approach:**
  - Estimate demand (logit)
  - Assume pricing behavior (usually differentiated Bertrand)
  - Back out marginal costs

Preview: merger simulation

- **Key question:** If two firms merge, what happens to prices?

- **Approach:**

  1. Estimate demand $\rightarrow$ get elasticities

  2. Assume Bertrand pricing $\rightarrow$ back out MC

  3. Change ownership structure

  4. Solve new Bertrand equilibrium

  5. Compare prices and welfare

- This is exactly what HW2 will do!

- Will cover in detail in Lecture 10

Key Points

1. **Cournot:** Firms choose quantities; $L = s_i / |\varepsilon|$

2. **Bertrand (homogeneous):** $P = MC$, zero profits

3. **Bertrand paradox:** Only 2 firms but competitive outcome

4. Cournot applies with capacity constraints; Bertrand with flexible prices

5. **Product differentiation** creates pricing power

6. **Hotelling:** $p^* = c + t$ (markup = transport cost)

7. Higher t (more differentiation) $\rightarrow$ higher markup

8. Hotelling connects to logit demand from Part 1

Next time

- **Lecture 9:** Entry and Market Structure

    - Free entry condition

    - Entry deterrence: limit pricing, excess capacity

    - Strategic entry barriers