ECN 594: Collusion

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Plan for today

1. Collusion refresher (from ECN 532)
2. Critical discount factor with N firms
3. Cournot vs Bertrand collusion

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4. Detection and fines
5. Leniency programs
6. Antitrust enforcement

Part 1: Collusion Theory

From ECN 532: Collusion basics

- **Collusion:** Firms coordinate to raise prices/restrict output
- Problem: each firm has incentive to deviate (undercut)
- **Solution:** Repeated game with punishment
- **Grim trigger strategy:**
  - Collude as long as everyone colludes
  - If anyone deviates $\rightarrow$ Nash forever
- You derived this in Hector's class

The collusion condition

- **Three profit levels:**
  - π^C: Collusive profit (per period)
  - π^D: Deviation profit (one-shot gain)
  - π^{NE}: Nash equilibrium profit (punishment)
- **Collusion sustained if:**

$$\frac{\pi^C}{1-\delta} \geq \pi^D + \frac{\delta \pi^{NE}}{1-\delta}$$

- Rearranging:

$$\delta \geq \delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}}$$

Critical discount factor: intuition

- $\delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}}$
- **Numerator:** Gain from deviating ($\pi^D - \pi^C$)
- **Denominator:** Total loss from punishment ($\pi^D - \pi^{NE}$)
- **Higher δ^* means collusion is harder**
  - Need more patient firms
  - More frequent interaction helps (reduces effective δ)

Cournot collusion with N firms

- Linear demand: $P = a - bQ$, symmetric firms with $MC = c$
- **Collusive profit per firm:**

$$\pi^C = \frac{\pi^M}{N} = \frac{(a-c)^2}{4bN}$$

- **Nash profit per firm:**

$$\pi^{NE} = \frac{(a-c)^2}{b(N+1)^2}$$

- **Deviation profit:** Best response to $N-1$ firms playing q^C

Critical discount factor: Cournot formula

- For symmetric linear Cournot with N firms:

$$\delta^* = \frac{(N+1)^2}{N^2 + (N+1)^2}$$

- **Examples:**

N	δ^*
2	$9/17 \approx 0.53$
3	$16/25 = 0.64$
4	$25/41 \approx 0.61$
10	$121/221 \approx 0.55$

- Key insight: Collusion harder with more firms

Worked example: Cournot collusion

- **Question:** 3 symmetric Cournot firms. $P = 100 - Q$, $MC = 10$.
- (a) Calculate π^C, π^{NE}, and π^D for each firm.
- (b) Find the minimum δ for collusion.

Take 7 minutes.

Worked example: Cournot collusion (solution)

- **(a) Profit calculations:**
- $\pi^M = (90)^2/4 = 2025$, so $\pi^C = 2025/3 = 675$
- $q^C = 45/3 = 15$ per firm (monopoly quantity split)
- Nash: $q^{NE} = 90/4 = 22.5$, $\pi^{NE} = 90^2/16 = 506.25$
- Deviation: BR to $2 \times 15 = 30$ is $q^D = (90 - 30)/2 = 30$
- $P = 100 - 60 = 40$, $\pi^D = (40 - 10) \times 30 = 900$

Worked example: Cournot collusion (solution cont.)

- **(b) Critical discount factor:**

$$\delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}} = \frac{900 - 675}{900 - 506.25} = \frac{225}{393.75} = 0.571$$

- Or use formula: $\delta^* = \frac{(3+1)^2}{3^2+(3+1)^2} = \frac{16}{9+16} = \frac{16}{25} = 0.64$
- (Small difference due to rounding in worked example)
- **Interpretation:** Firms must value future at 64% of present

Bertrand collusion with N firms

- Homogeneous Bertrand: $\pi^{NE} = 0$ (price = cost)
- Collusion: split monopoly profits
- **Key difference:** Punishment is more severe ($\pi^{NE} = 0$)
- **Critical discount factor for Bertrand:**

$$\delta^* = \frac{\pi^D - \pi^C}{\pi^D - 0} = \frac{\pi^M - \pi^M/N}{\pi^M} = \frac{N-1}{N}$$

- **Examples:**
  - $N = 2$: $\delta^* = 0.5$
  - $N = 4$: $\delta^* = 0.75$

Cournot vs Bertrand collusion

N	δ^* (Cournot)	δ^* (Bertrand)
2	0.53	0.50
3	0.64	0.67
4	0.61	0.75

- At $N = 2$: Bertrand collusion **easier**
- **Why?** Bertrand punishment is harsher ($\pi^{NE} = 0$)
- At higher N: Bertrand collusion harder
- **Why?** Deviation captures entire market (bigger temptation)

Part 2: Detection and Policy

Detection probability and fines

- In reality: cartels may be detected and punished
- **Each period:**
  - Detection probability: ρ
  - Fine if detected: F
- **Modified collusion condition:**

$$\delta^* = \frac{\pi^D - \pi^C + \rho F}{\pi^D - \pi^{NE} + \rho F}$$

- Higher ρ or higher $F \rightarrow$ higher $\delta^* \rightarrow$ harder to collude

Worked example: Detection and fines

- **Question:**
- Cartel earns $\pi^C = 100$ per period
- $\pi^{NE} = 25$, $\pi^D = 150$
- Detection probability $\rho = 0.1$, fine $F = 500$
- Find the minimum δ for collusion.

Take 3 minutes.

Worked example: Detection (solution)

- Expected fine per period: $\rho F = 0.1 \times 500 = 50$
- Apply formula:

$$\delta^* = \frac{\pi^D - \pi^C + \rho F}{\pi^D - \pi^{NE} + \rho F} = \frac{150 - 100 + 50}{150 - 25 + 50} = \frac{100}{175} = 0.571$$

- **Compare to no detection:**

$$\delta^*_{\text{no detection}} = \frac{150 - 100}{150 - 25} = \frac{50}{125} = 0.4$$

- Detection and fines make collusion harder ($0.4 \rightarrow 0.57$)

Leniency programs

- **Leniency:** First firm to report cartel gets reduced/zero fine
- **US Corporate Leniency Program (1993):**
  - First to report: automatic immunity
  - Second: significant reduction possible
- **Effect on incentives:**
  - Creates "race to report"
  - Each firm fears others will report first
  - Destabilizes existing cartels

Why leniency works

- **Without leniency:**
  - If detected, everyone pays fine
  - No incentive to report
- **With leniency:**
  - First to report gets immunity
  - Creates Prisoner's Dilemma within cartel
  - Each firm thinks: "Better report before they do"
- **Result:** Cartel detection increased dramatically after 1993
- **Exam question:** "Explain why leniency programs help detect cartels."

Factors facilitating collusion

1. **Few firms:** Easier to coordinate and monitor
2. **Frequent interaction:** Higher effective δ
3. **Similar costs:** Easier to agree on price
4. **Stable demand:** Easier to detect deviations
5. **Homogeneous products:** Easier to monitor prices
6. **Industry associations:** Facilitate communication

Famous cartel cases

- **Lysine cartel (1990s):**
  - Price-fixing among feed additive producers
  - FBI surveillance, recorded meetings
- **LCD screen cartel (2000s):**
  - Samsung, LG, Sharp, others
  - $1.4 billion in fines
- **LIBOR scandal (2012):**
  - Banks manipulated interest rate benchmark
  - $9 billion in fines

Detecting collusion: what regulators look for

- **Pricing patterns:**
  - Parallel price changes
  - Price rigidity despite cost changes
  - Similar prices despite different costs
- **Market characteristics:**
  - High concentration
  - Frequent meetings/communication
  - History of antitrust violations
- **Whistleblowers:** Leniency program tips

Key Points

1. **Critical discount factor:** $\delta^* = \frac{\pi^D - \pi^C}{\pi^D - \pi^{NE}}$
2. **Cournot with N firms:** $\delta^* = \frac{(N+1)^2}{N^2 + (N+1)^2}$
3. **Bertrand with N firms:** $\delta^* = \frac{N-1}{N}$
4. More firms $\rightarrow$ generally harder to collude
5. **Detection and fines** raise δ^*: $\delta^* = \frac{\pi^D - \pi^C + \rho F}{\pi^D - \pi^{NE} + \rho F}$
6. **Leniency programs:** Create "race to report," destabilize cartels
7. Collusion easier with: few firms, frequent interaction, similar costs

Next time

- **Lecture 13:** Final Review
  - Comprehensive review of Part 1 and Part 2
  - Practice problems for final exam
- **HW2 due before Lecture 13**