

ECN 594: Midterm Review

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Plan for today

1. Review: Demand estimation
2. Practice problems: Demand

3. Review: Pricing and price discrimination
4. Practice problems: Pricing
5. Exam logistics

Midterm format

- **Duration:** 70 minutes
- **Allowed:** Calculator + two-sided cheat sheet (letter-size paper)
- **Coverage:** Lectures 1-5 (all of Part 1)
- **Structure:**
 - Short answer questions (T/F/NEI, definitions, quick calculations)
 - Longer problems (derivations, pricing, elasticity calculations)

Part 1: Demand Estimation Review

Why demand estimation?

- We need demand models to:
 1. Measure substitution patterns between products
 2. Compute price elasticities
 3. Evaluate policy (mergers, new products, price changes)
 4. Calculate consumer welfare
- The dimensionality problem: J products $\rightarrow J^2$ elasticities
- Solution: characteristics-based models (Lancaster, BLP)

The logit model: key equations

- Utility: $u_{ij} = \delta_j + \varepsilon_{ij}$ where $\delta_j = x_j \beta + \alpha p_j + \xi_j$
- Choice probability (market share):

$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^J \exp(\delta_k)}$$

- Outside option: $s_0 = \frac{1}{1 + \sum_{k=1}^J \exp(\delta_k)}$

- **Berry inversion:**

$$\ln(s_j) - \ln(s_0) = \delta_j$$

Logit elasticities

- Own-price elasticity:

$$\eta_{jj} = \frac{\partial s_j}{\partial p_j} \cdot \frac{p_j}{s_j} = \alpha p_j (1 - s_j)$$

- Cross-price elasticity:

$$\eta_{jk} = \frac{\partial s_j}{\partial p_k} \cdot \frac{p_k}{s_j} = -\alpha p_k s_k$$

- Note: $\alpha < 0$ so own-price elasticity is negative
- **IIA problem:** Cross-elasticity depends only on k , not on similarity to j

Practice: Elasticity calculation

- **Question:** Suppose $\alpha = -0.5$, product A has price $p_A = 20$ and share $s_A = 0.1$.
- (a) Calculate the own-price elasticity of product A.
- (b) If product B has $p_B = 25$ and $s_B = 0.15$, what is the cross-price elasticity of A with respect to B's price?

Take 3 minutes.

Practice: Elasticity calculation (solution)

- **(a) Own-price elasticity:**

$$\eta_{AA} = \alpha p_A (1 - s_A) = (-0.5)(20)(1 - 0.1) = -9$$

- Demand is elastic (a 1% price increase reduces quantity by 9%)

- **(b) Cross-price elasticity:**

$$\eta_{AB} = -\alpha p_B s_B = -(-0.5)(25)(0.15) = 1.875$$

- A 1% increase in B's price increases A's share by 1.875%

The identification problem

- **Problem:** We observe equilibrium (p, q) pairs
- Can't tell if demand shifted or supply shifted
- **Price endogeneity:** High unobserved quality $\xi_j \rightarrow$ high price
 - OLS sees: high price, still high demand
 - Concludes: price doesn't matter much
 - Result: $\hat{\alpha}$ biased toward zero
- **Solution:** Instrumental variables
 - Need: correlated with price, uncorrelated with ξ
 - Examples: Hausman IVs, BLP IVs, cost shifters

Practice: Identification

- **True, False, or Not Enough Information:**
- (a) OLS estimation of demand typically underestimates the price coefficient (in absolute value).
- (b) Using prices of the same product in other geographic markets as an IV is valid because prices in other markets don't affect local demand.
- (c) The logit model solves the dimensionality problem by assuming all products are equally substitutable.

Take 3 minutes.

Practice: Identification (solutions)

- **(a) TRUE.** Price endogeneity biases α toward zero. Since $\alpha < 0$, this means $|\hat{\alpha}_{OLS}| < |\alpha_{true}|$.
- **(b) TRUE (with caveat).** Hausman IVs work because common cost shocks affect prices in all markets (relevance), but other markets' prices don't directly affect local demand (exclusion). Caveat: requires no common demand shocks.
- **(c) FALSE.** Logit solves dimensionality by using product characteristics. But the IIA problem means substitution is proportional to share, not similarity.

Consumer surplus: log-sum formula

- Expected utility for consumer i :

$$E[\max_j u_{ij}] = \ln \left[\sum_{j=0}^J \exp(\delta_j) \right] + \text{constant}$$

- Consumer surplus (in dollars):

$$CS = \frac{1}{|\alpha|} \ln \left[\sum_{j=0}^J \exp(\delta_j) \right]$$

- Divide by $|\alpha|$ to convert utils to dollars
- **Application:** Welfare change from adding/removing products

The IIA problem

- **IIA:** Ratio of choice probabilities doesn't depend on other options

$$\frac{s_j}{s_k} = \frac{\exp(\delta_j)}{\exp(\delta_k)} = \exp(\delta_j - \delta_k)$$

- **Red Bus / Blue Bus:** Adding identical bus option incorrectly increases welfare
- **Problem:** Logit doesn't know similar products are close substitutes
- **Solutions:**
 - Demographic interactions (partial fix)
 - Mixed logit / random coefficients (full fix, beyond scope)

Part 2: Pricing Review

Monopoly pricing: key equations

- Profit maximization: $\max_q \pi = p(q) \cdot q - c(q)$
- First-order condition: $MR = MC$
- **Lerner index:**

$$L = \frac{p - MC}{p} = \frac{1}{|\varepsilon|}$$

- **Interpretation:** Markup is inversely related to elasticity
- Equivalently: $p = \frac{MC}{1 - 1/|\varepsilon|}$

Practice: Lerner index

- **Question:** A monopolist faces demand $p = 100 - 2q$ and has constant marginal cost $MC = 20$.
- (a) Find the profit-maximizing price and quantity.
- (b) Calculate the Lerner index.
- (c) Verify using the elasticity formula.

Take 4 minutes.

Practice: Lerner index (solution)

- **(a)** $MR = 100 - 4q$. Set $MR = MC$: $100 - 4q = 20 \Rightarrow q = 20$
- Price: $p = 100 - 2(20) = 60$
- **(b)** Lerner index: $L = \frac{60-20}{60} = \frac{40}{60} = \frac{2}{3}$
- **(c)** Elasticity: $\varepsilon = \frac{dq}{dp} \cdot \frac{p}{q} = -\frac{1}{2} \cdot \frac{60}{20} = -1.5$
- Check: $L = \frac{1}{|\varepsilon|} = \frac{1}{1.5} = \frac{2}{3} \checkmark$

Types of price discrimination

1. Perfect price discrimination

- Charge each consumer their exact WTP
- Benchmark; rarely feasible

2. Selection by indicators

- Different prices for observable groups
- Example: student discounts, geographic pricing

3. Self-selection

- Design menu to induce type revelation
- Example: versioning, bundling

Selection by indicators: inverse elasticity rule

- Two markets with different elasticities
- Apply Lerner in each market:

$$\frac{p_1 - MC}{p_1} = \frac{1}{|\varepsilon_1|} \quad \text{and} \quad \frac{p_2 - MC}{p_2} = \frac{1}{|\varepsilon_2|}$$

- **Key insight:** Charge higher price in more inelastic market
- Rearranging: $p = \frac{MC}{1-1/|\varepsilon|}$

Practice: Selection by indicators

- **Question:** A firm sells in two markets. Market A has $\varepsilon_A = -3$. Market B has $\varepsilon_B = -5$. Marginal cost is $MC = 10$.
- Find the optimal price in each market.

Take 3 minutes.

Practice: Selection by indicators (solution)

- Using $p = \frac{MC}{1-1/|\varepsilon|}$:

- **Market A:**

$$p_A = \frac{10}{1 - 1/3} = \frac{10}{2/3} = 15$$

- **Market B:**

$$p_B = \frac{10}{1 - 1/5} = \frac{10}{4/5} = 12.50$$

- Higher price in more inelastic market (A)

Two-part tariffs

- Structure: $\text{Payment} = F + p \cdot q$
- **Optimal (homogeneous consumers):**
 - Set $p = MC$ (maximize total surplus)
 - Set $F = CS(p = MC)$ (extract all surplus)
- **Result:** Firm captures entire surplus; efficient but inequitable
- **Heterogeneous consumers:** Tradeoff between extraction and participation

Self-selection: IC and IR constraints

- **IC (Incentive Compatibility):** Each type prefers their option

$$\text{IC}_H : v_H^F - p_F \geq v_H^S - p_S$$

- **IR (Individual Rationality):** Each type willing to participate

$$\text{IR}_L : v_L^S - p_S \geq 0$$

- **Optimal menu:** IC binds for H, IR binds for L

- H gets “information rent”; L gets zero surplus

Practice: Self-selection

- **Question:** Two products (Premium, Basic), two consumer types.

	Premium	Basic
High type	100	60
Low type	50	40

- $MC = 20$ for both. Equal numbers of each type.
- A firm considers: $p_P = 100$, $p_B = 40$.
- (a) What does each type buy?
- (b) Is this menu optimal? If not, what should change?

Take 4 minutes.

Practice: Self-selection (solution)

- **(a) Consumer choices:**
- High type: $CS_P = 100 - 100 = 0$, $CS_B = 60 - 40 = 20$
- High type buys Basic! (IC violated)
- Low type: $CS_P = 50 - 100 = -50$, $CS_B = 40 - 40 = 0$
- Low type buys Basic
- **(b) Not optimal.** Need to lower p_P so IC binds.
- IC binds when: $100 - p_P = 60 - 40 = 20 \Rightarrow p_P = 80$
- Optimal menu: $p_P = 80$, $p_B = 40$

Exam Logistics

Exam format reminder

- **Date:** Monday, Feb 9 (Lecture 7)
- **Duration:** 70 minutes
- **Allowed:** Calculator + two-sided cheat sheet
- **Coverage:**
 - Demand: logit, Berry inversion, elasticities, IVs, IIA, CS
 - Pricing: monopoly, Lerner index, price discrimination
 - Two-part tariffs, self-selection (IC/IR)
- **Format:** Mix of T/F/NEI, short answer, and problems

What to put on your cheat sheet

- **Key formulas:**
 - Logit share, Berry inversion
 - Own and cross elasticities
 - Log-sum formula for CS
 - Lerner index
 - Two-part tariff optimal conditions
- **Worked examples:** Similar to practice problems
- **Key intuitions:**
 - Why price endogeneity biases α toward zero
 - Why IC binds for high type, IR for low type

Key Points

1. **Logit:** $s_j = \exp(\delta_j) / [1 + \sum \exp(\delta_k)]$
2. **Berry inversion:** $\ln(s_j) - \ln(s_0) = \delta_j$
3. **Elasticities:** Own: $\alpha p_j(1 - s_j)$; Cross: $-\alpha p_k s_k$
4. **IVs needed** because $E[p_j \xi_j] \neq 0$; bias toward zero
5. **Lerner:** $L = (p - MC) / p = 1 / |\varepsilon|$
6. **Selection by indicators:** Higher price in inelastic market
7. **Two-part tariff:** $p = MC, F = CS$
8. **Self-selection:** IC binds for high type, IR for low type

Good luck on the midterm!

- **Office hours:** [TBD]
- **Practice exam:** Posted on course website
- Questions?