ECN 453: Cournot Competition

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Static Models of Oligopoly: Cournot Competition

- Last time we studied Bertrand competition (competition where firms choose **prices**)
- Today we will study Cournot competition (competition when firms choose **quantities**)

Plan

1. Cournot competition: setup
2. Cournot competition: solution using a graph
3. Cournot competition: solution using math
4. Connection between Bertrand competition and Cournot competition

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Cournot competition: setup

- **Players**: Two firms (denote each by i where i = 1 or 2)
- **Strategies**: Each firm chooses output level q_1, q_2
  - Sell homogeneous (identical) products
- **Payoffs:** Each firm i's payoff is profit: $\pi_i = q_i(P(q_1 + q_2) - c)$
  - Prices are determined by a demand curve $P(Q)$ where $Q = q_1 + q_2$.
  - Marginal cost is c
  - Note: observe that the price that firm 1 gets $P(q_1 + q_2)$ is not just dependent on how much firm 1 produces q_1, but also how much firm 2 produces q_2
- Observe that the setup is the same as Bertrand <u>except</u> that firms now choose quantities.

Cournot competition: solution overview

- We will follow our 'usual steps' to find the equilibrium in Cournot competition:

1. Find the best response of firm 1 to firm 2's choice (denote this $q_1^*(q_2)$)
2. Find the best response of firm 2 to firm 1's choice (denote this $q_2^*(q_1)$)
3. Find where the two best reponse curves cross: this is the Nash equilibrium

- We will now see two equivalent ways of doing these three steps: a solution using a graph and a solution using math.

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Cournot competition: solution using a graph - preliminaries

- Suppose that firm 2 is producing q_2. How can we find firm 1's best response $q_1^*(q_2)$?
- Start by defining the **residual demand curve**.
  - This is denoted $d_1(q_2)$: defined as all possible combinations of Firm 1's quantity and price for *a given value of* q_2.
  - Idea: suppose that firm 2 produces q_2. The demand 'left over' for firm 1 is the residual demand curve.
  - Example: residual demand curve at $q_2 = 0$ is $P(q_1 + 0) = P(q_1)$.
- The residual demand curve has a **residual marginal revenue curve**: $r_1(q_2)$.
- The optimal quantity is then found by applying the monopoly solution *on the marginal revenue curve*: i.e. set $r_1(q_2) = MC$.

Cournot competition: solution using a graph - best responses

- Let's find the best responses of Firm 1 at two extremes. These two extremes will allow us to 'trace out' the best response curve for Firm 1. (Graphical version of this slide is on the next slide.)

- <u>Case 1:</u> Firm 2 produces $q_2 = 0$. We can show $q_1^*(0) = q^M$
  - If Firm 2 produces $q_2 = 0$ then the residual demand curve for Firm 1 is $P(q_1 + 0) = P(q_1)$.

  - So, Firm 1's demand curve is the entire market demand. Thus, this is equivalent to when Firm 1 is a monopolist and the optimal quantity is the monopoly quantity.

Cournot competition: solution using a graph - best responses

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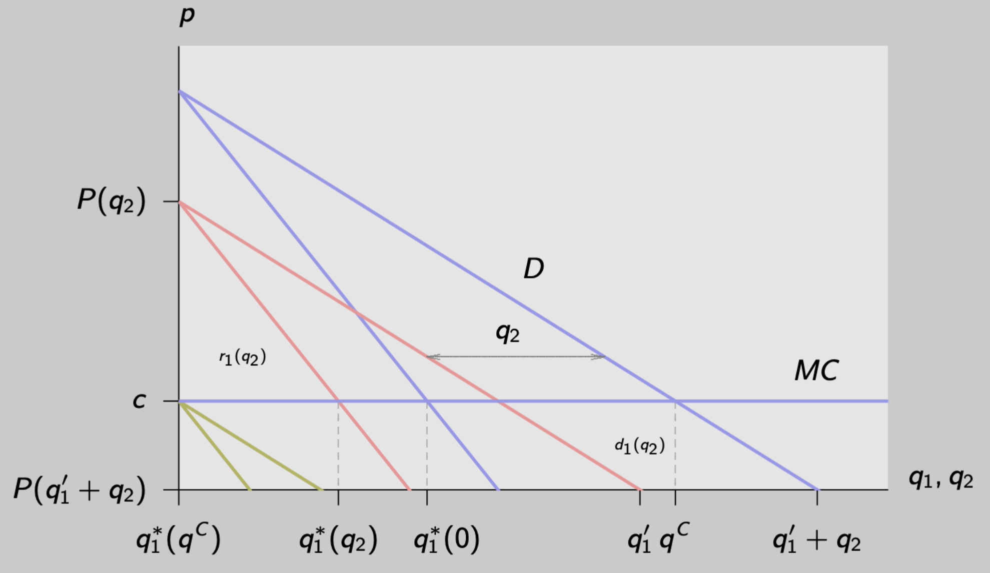
- Case 2: Firm 2 produces $q_2 = q^c$ (the perfect competition quantity). We can show $q_1^*(q^c) = 0$
  - On the following slide I draw the residual demand curve $d_1(q^c)$ (along with some other points).

  - This residual demand curve intersects marginal cost at $q_1 = 0$; since the marginal revenue curve has the same vertical intercept it also intersects marginal cost at $q_1 = 0$. Hence $q_1 = 0$ is the optimal quantity.

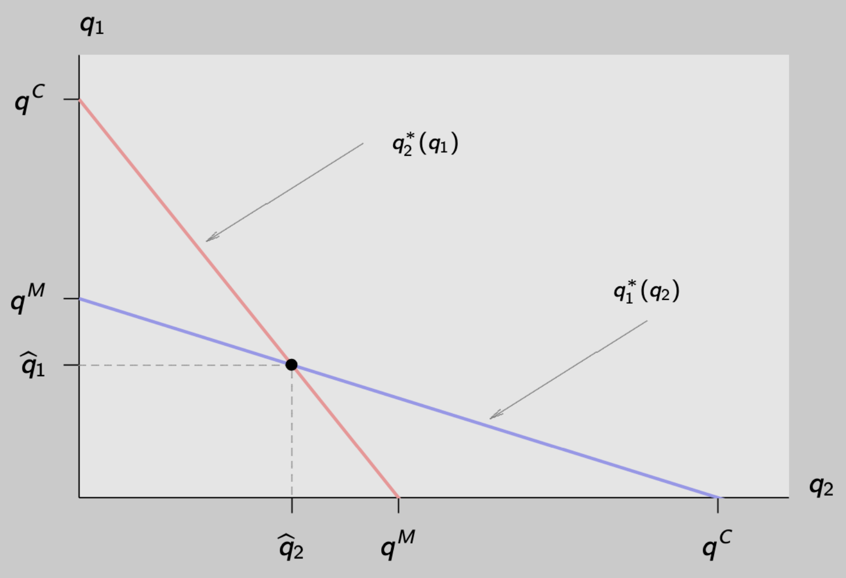
Cournot competition: solution using a graph - best responses



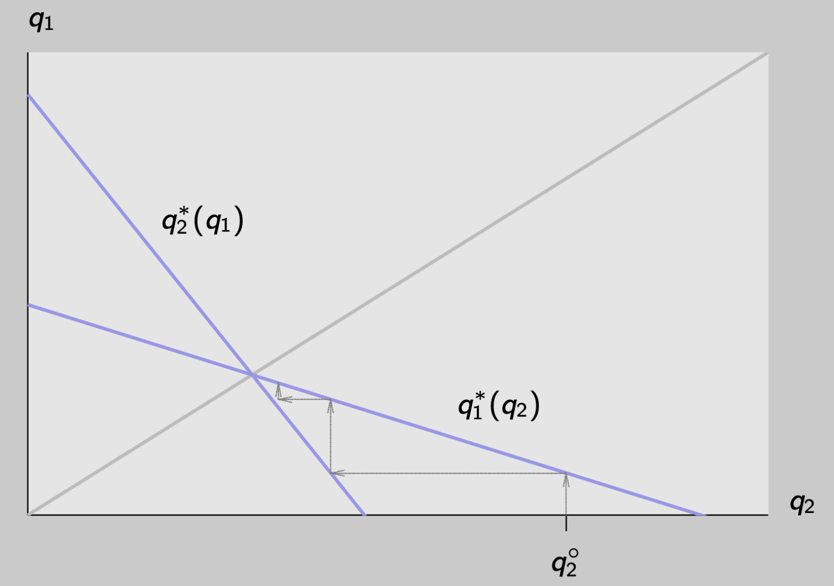
Cournot competition: solution using a graph - best responses and Nash equilibrium

- On the next slide I plot all of Firm 1's best responses $q_1^*(q_2)$ by drawing a line between these two extreme cases.
- By the same arguments for Firm 1, on the next slide I also trace out Firm 2's best responses $q_2^*(q_1)$.
- Finally, on the next slide, I plot the point where the curves cross: this is the Nash equilibrium (denoted $(\hat{q}_2, \hat{q}_1)$)

Cournot competition: solution using a graph - Nash equilibrium



Cournot competition: any initial solution converges to Nash equilibrium



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Warm up: maximizing profit with calculus for a monopolist

- 1. Write down profit (profit=TR-TC)
- 2. Take the derivative of profit with respect to q
- 3. Set the derivative equal to 0
- 4. Solve for q

Warm up: example from calculus.pdf

- To remember how to use these steps, let's do the example from the calculus practice material.
- **Question:** Suppose that demand is $q = 10 - 2p$ and marginal cost is 2.
- 1. What is the firms profit?
- 2. What level of output maximizes profit?

Cournot competition: solution using math

- **Question:** Suppose that we have the Cournot competition setup. Assume that demand is $Q = 10 - 2p$ where $Q = q_1 + q_2$ and marginal cost is 2.
- 1. What is the profit of Firm 1 and Firm 2?
- 2. What are the best responses of Firm 1 and Firm 2?
- 3. What is the Nash equilibrium?

Cournot competition: solution using math

- **Solution:** 1. What is the profit of Firm 1 and Firm 2?
- Firm 1:
- $\pi_1 = q_1(P(q_1 + q_2) - c)) = q_1(5 - 0.5(q_1 + q_2) - 2) = q_1(3 - 0.5(q_1 + q_2))$
- Firm 1:
- $\pi_2 = q_2(P(q_1 + q_2) - c)) = q_2(5 - 0.5(q_1 + q_2) - 2) = q_2(3 - 0.5(q_1 + q_2))$

Cournot competition: solution using math

- **Solution:** 2. What are the best responses of Firm 1 and Firm 2?
- Start with Firm 1 and follow the 'maximizing profit' steps from before.
- Note that when maximizing profit for Firm 1, treat Firm 2's quantity choice q_2 as a constant.
- First, take the derivative:

$$\frac{d\pi_1}{dq_1} = 3 - q_1 - 0.5q_2$$

- Set the derivative $= 0$ to maximize profit:

$$0 = 3 - q_1 - 0.5q_2$$

- Rearrange for q_1: this is Firm 1's best response to q_2.

$$q_1^*(q_2) = 3 - 0.5q_2$$

Cournot competition: solution using math

- **Solution:** 2. What are the best responses of Firm 1 and Firm 2?

- Use exactly the same arguments to get Firm 2's best response:

$$q_2^*(q_1) = 3 - 0.5q_1$$

Cournot competition: solution using math

- **Solution:** 3. What is the Nash equilibrium?

- We now have two equations (the best responses) and two unknowns (the Nash equilibrium quantities q_1, q_2)

- Substitute Firm 2's best response equation into Firm 1's best response equation:

$$q_1 = 3 - 0.5(3 - 0.5q_1)$$

- Rearranging for q_1 (and substituting this q_1 into Firm 2's best response to get q_2)

$$q_1 = 2$$
$$q_2 = 2$$

- So, $(q_1, q_2) = (2, 2)$ is the Nash equilibrium.

Cournot competition: solution reprinted with more math sub-steps

1. Find the best response of firm 1 to firm 2's choice (denote this $q_1^*(q_2)$)
    1.1 Write down Firm 1's profit.

    1.2 Take the derivative of Firm 1's profit with respect to q_1 and set this derivative $= 0$.

    1.3 Rearrange for q_1

2. Find the best response of firm 2 to firm 1's choice (denote this $q_2^*(q_1)$)
    2.1 Write down Firm 2's profit.

    2.2 Take the derivative of Firm 2's profit with respect to q_2 and set this derivative $= 0$.

    2.3 Rearrange for q_2

3. Find where the two best reponse curves cross: this is the Nash equilibrium
    3.1 We have two equations (the two best response curves) and two unknowns (the Nash equilibrium q_1 and q_2).

    3.2 Solve these two equations for q_1 and q_2: this is the Nash equilibrium.

Cournot competition: general solution with two identical firms; p196

- **Setup:**
- Two identical firms with (inverse) demand $P(Q) = a - bQ$ where $Q = q_1 + q_2$ and a, b are constants. Marginal cost $= c$.
- **Solution:**
- Best responses:

$$q_1^*(q_2) = \frac{a-c}{2b} - \frac{q_2}{2}$$
$$q_2^*(q_1) = \frac{a-c}{2b} - \frac{q_1}{2}$$

- Nash equilibrium:

$$q_1 = q_2 = \frac{a-c}{3b}$$

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Summary of key points*

- Know the assumptions behind Cournot competition (quantity competition)
- Know how to solve a Cournot model using a graph
- Very important! Know how to solve a Cournot model using math (including: take derivative of profit, get the best responses, get the Nash equilibrium etc)

*To clarify, all the material in the slides, problem sets, etc is assessable unless stated otherwise, but I hope this summary might be a useful place to start when studying the material.