ECN 594: The Logit Demand Model

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Announcements

- **Homework 1 released today**
- Due: Feb 4 (before Lecture 6)
- Demand estimation using Python and `pyblp`
- Start early!

Plan for today

1. Logit model derivation
2. Adding the outside option
3. Berry (1994) inversion
4. Elasticity formulas
5. IIA problem (preview)

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Recap: Random utility

- Consumer i chooses among J products
- Utility:

$$u_{ij} = x_j \beta - \alpha p_j + \xi_j + \varepsilon_{ij}$$

- Consumer chooses the product with highest utility
- Last time: we left ε_{ij} unspecified
- Today: we assume ε_{ij} has a specific distribution

The logit assumption

- Assume ε_{ij} is i.i.d. **Type I Extreme Value**
- Also called Gumbel distribution
- CDF: $F(\varepsilon) = \exp(-\exp(-\varepsilon))$
- Why this assumption?
  - Gives us **closed-form** choice probabilities!
  - Computationally tractable

Logit choice probabilities

- Define **mean utility**:

$$\delta_j = x_j\beta - \alpha p_j + \xi_j$$

- So utility is: $u_{ij} = \delta_j + \varepsilon_{ij}$

- With Type I Extreme Value errors, the probability that consumer chooses j:

$$P(\text{choose } j) = \frac{\exp(\delta_j)}{\sum_{k=1}^{J} \exp(\delta_k)}$$

- This is the **logit** formula

Logit choice probabilities: intuition

- Choice probability:

$$P(\text{choose } j) = \frac{\exp(\delta_j)}{\sum_{k=1}^{J} \exp(\delta_k)}$$

- Higher $\delta_j \Rightarrow$ higher probability of choosing j
- If $\delta_j \gg \delta_k$ for all $k \neq j$, then $P(\text{choose } j) \approx 1$
- If all δ_j are equal, then $P(\text{choose } j) = 1/J$

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The outside option

- Problem: Our formula doesn't allow consumers to "not buy"
- We need an **outside option** (product $j = 0$)
- Utility of outside option:

$$u_{i0} = \varepsilon_{i0}$$

- We normalize: $\delta_0 = 0$
- All other utilities are *relative* to this outside option

Logit with outside option

- With the outside option, the share of product j is:

$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{J} \exp(\delta_k)}$$

- And the share of the outside option is:

$$s_0 = \frac{1}{1 + \sum_{k=1}^{J} \exp(\delta_k)}$$

- Note: $s_0 + \sum_{j=1}^{J} s_j = 1$ (shares sum to 1)

Why does the outside option matter?

- Without outside option: if all prices rise, consumers still buy
- With outside option: if all prices rise, consumers can exit the market
- This affects elasticities!
  - Firms face competition from "not buying"
  - Limits market power
- Practical question: what is s_0?
  - Often computed as: (potential market size - total sales) / potential market size

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Berry (1994) inversion: the key insight

- We observe: market shares s_j
- We want: mean utilities δ_j (to estimate β, α)
- **Problem:** How do we get δ_j from s_j?

Berry (1994) inversion: the key insight

- We observe: market shares s_j
- We want: mean utilities δ_j (to estimate β, α)
- **Problem:** How do we get δ_j from s_j?
- **Berry's insight:** Take the log of shares!

Berry (1994) inversion

- Start with:

$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{J} \exp(\delta_k)}, \quad s_0 = \frac{1}{1 + \sum_{k=1}^{J} \exp(\delta_k)}$$

- Take logs:

$$\ln(s_j) = \delta_j - \ln\left(1 + \sum_{k=1}^{J} \exp(\delta_k)\right)$$
$$\ln(s_0) = -\ln\left(1 + \sum_{k=1}^{J} \exp(\delta_k)\right)$$

- Subtract:

$$\ln(s_j) - \ln(s_0) = \delta_j$$

Berry (1994) inversion: the estimating equation

- We have: $\ln(s_j) - \ln(s_0) = \delta_j$
- Substitute $\delta_j = x_j\beta - \alpha p_j + \xi_j$:

$$\ln(s_j) - \ln(s_0) = x_j\beta - \alpha p_j + \xi_j$$

- This is a **linear regression**!
- LHS: can compute from observed shares
- RHS: product characteristics, price, and an error term
- This is how we'll estimate demand

Why is this so useful?

- Before Berry (1994): demand estimation was hard
  - Needed individual-level data
  - Or complicated estimation methods
- After Berry (1994): can use **market-level data**
  - Just need: shares, prices, characteristics
  - Run a regression!
- This made demand estimation much more practical

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Logit elasticities

- Given shares $s_j = \frac{\exp(\delta_j)}{1+\sum_k \exp(\delta_k)}$

- We can derive price elasticities:

$$\eta_{jj} = \frac{\partial s_j}{\partial p_j}\frac{p_j}{s_j} = -\alpha p_j(1 - s_j) \quad \text{(own-price)}$$

$$\eta_{jk} = \frac{\partial s_j}{\partial p_k}\frac{p_k}{s_j} = \alpha p_k s_k \quad \text{(cross-price)}$$

- Note: $\alpha > 0$, so own-price elasticity is **negative** (as expected)

Worked example: Logit elasticities

- **Question:**
- Suppose $\alpha = 0.5$, product j has price $p_j = 20$ and market share $s_j = 0.1$.
- Compute the own-price elasticity for product j.

Take 2 minutes to solve this.

Worked example: Logit elasticities (solution)

- Own-price elasticity formula:

$$\eta_{jj} = -\alpha p_j (1 - s_j)$$

- Plug in: $\alpha = 0.5$, $p_j = 20$, $s_j = 0.1$

$$\begin{aligned} \eta_{jj} &= -0.5 \times 20 \times (1 - 0.1) \\ &= -0.5 \times 20 \times 0.9 \\ &= -9 \end{aligned}$$

- Interpretation: A 1% price increase $\Rightarrow$ 9% decrease in quantity

Worked example: Cross-price elasticity

- Now compute the cross-price elasticity with product k
- Given: $\alpha = 0.5$, $p_k = 25$, $s_k = 0.05$

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- Now compute the cross-price elasticity with product k

- Given: $\alpha = 0.5$, $p_k = 25$, $s_k = 0.05$

- Cross-price elasticity formula:

$$\eta_{jk} = \alpha p_k s_k$$

- Plug in:

$$\begin{aligned}\eta_{jk} &= 0.5 \times 25 \times 0.05 \\ &= 0.625\end{aligned}$$

- Interpretation: A 1% increase in $p_k \Rightarrow 0.625\%$ increase in s_j

Properties of logit elasticities

- Own-price: $\eta_{jj} = -\alpha p_j(1 - s_j)$
  - Higher price $\Rightarrow$ more elastic demand
  - Higher share $\Rightarrow$ less elastic demand
- Cross-price: $\eta_{jk} = \alpha p_k s_k$
  - Only depends on p_k and s_k (not on product j!)
  - This will be important...

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The IIA problem

- Look at the cross-price elasticity again:

$$\eta_{jk} = \alpha p_k s_k$$

- This doesn't depend on product j at all!
- Implication: All products have the **same** cross-elasticity with product k
- Is this realistic?

Example: BMW price increase

- Suppose BMW raises its price
- Who substitutes away from BMW?
- According to logit: consumers substitute to other products **in proportion to their market shares**
- So: same fraction go to Mercedes as to Honda Civic!
- This doesn't seem right...

IIA: Independence of Irrelevant Alternatives

- This property is called **IIA**
- Formally: ratio of choice probabilities is independent of other options

$$\frac{s_j}{s_k} = \frac{\exp(\delta_j)}{\exp(\delta_k)} = \exp(\delta_j - \delta_k)$$

- Adding or removing other products doesn't change this ratio
- This is a limitation of the basic logit model

Looking ahead

- We'll return to the IIA problem in Lecture 4
  - The famous "red bus / blue bus" example
- **Next lecture:** Identification and IVs
  - Why is price endogenous?
  - How do we solve this problem?
- **Later:** Allowing for heterogeneous preferences
  - This helps with IIA

Summary

- **Logit model**: ε_{ij} is Type I Extreme Value
- **Share equation**: $s_j = \frac{\exp(\delta_j)}{1+\sum_k \exp(\delta_k)}$
- **Berry inversion**: $\ln(s_j) - \ln(s_0) = \delta_j$
  - Turns demand estimation into a linear regression
- **Elasticities**:
  - Own: $\eta_{jj} = -\alpha p_j(1 - s_j)$
  - Cross: $\eta_{jk} = \alpha p_k s_k$
- **IIA**: Cross-elasticities don't depend on similarity
  - A limitation we'll address later