ECN 594: Oligopoly Competition

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Welcome to Part 2

- Part 1: Demand estimation and pricing
- **Part 2: Models of competition and industry structure**
    - Oligopoly models (Cournot, Bertrand, Hotelling)
    - Entry and entry deterrence
    - Mergers
    - Vertical relationships
    - Collusion
- **HW2 released:** Merger simulation module

Plan for today

1. Cournot competition (refresher in IO notation)
2. Bertrand competition (refresher)
3. Cournot vs Bertrand: when does each apply?

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4. Product differentiation: why it matters
5. Hotelling model
6. Connection to demand estimation

Part 1: Cournot and Bertrand Competition

From ECN 532: Oligopoly models

- You covered Cournot and Bertrand in Hector's class
- Today: quick refresher in IO notation
- **New focus:** Market power measurement
    - Connecting oligopoly models to Lerner index
    - When does each model apply?

Cournot competition: setup

- n firms producing homogeneous goods
- Firms choose **quantities** simultaneously
- Inverse demand: $P = P(Q)$ where $Q = \sum_{i=1}^{n} q_i$
- Constant marginal cost: c
- Firm i profit: $\pi_i = P(Q) \cdot q_i - c \cdot q_i$

Cournot: first-order conditions

- Firm i maximizes profit taking q_{-i} as given:

$$\frac{\partial \pi_i}{\partial q_i} = P(Q) + P'(Q)q_i - c = 0$$

- Rearranging:

$$P(Q) - c = -P'(Q)q_i$$

- Divide by P:

$$\frac{P - c}{P} = \frac{-P'(Q)q_i}{P} = \frac{-P'(Q)Q}{P} \cdot \frac{q_i}{Q} = \frac{s_i}{|\varepsilon|}$$

- where $s_i = q_i / Q$ is firm i's market share

Cournot: Lerner index

- **Key result:** In Cournot equilibrium,

$$L_i = \frac{P - MC}{P} = \frac{s_i}{|\varepsilon|}$$

- **Interpretation:**
  - Markup depends on market share
  - Larger firms have more market power
  - More elastic demand $\rightarrow$ lower markup
- This connects to demand estimation from Part 1!

Worked example: Cournot with market power

- **Question:** Inverse demand is $P = 100 - Q$. Two symmetric firms with $MC = 10$.
- (a) Find equilibrium quantities and price.
- (b) Calculate the Lerner index for each firm.
- (c) Verify using the $L = s/|\varepsilon|$ formula.

Take 5 minutes.

Worked example: Cournot (solution)

- **(a)** FOC: $100 - 2q_i - q_j - 10 = 0$
- Symmetric: $q_1 = q_2 = q^*$, so $100 - 3q^* = 10 \Rightarrow q^* = 30$
- $Q = 60$, $P = 100 - 60 = 40$
- **(b)** $L = \frac{40-10}{40} = \frac{3}{4} = 0.75$
- **(c)** Market share: $s_i = 30/60 = 0.5$
- Elasticity: $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = (-1) \cdot \frac{40}{60} = -\frac{2}{3}$
- Check: $L = \frac{s_i}{|\varepsilon|} = \frac{0.5}{2/3} = 0.75$ ✓

Bertrand competition: setup

- n firms producing **homogeneous** goods
- Firms choose **prices** simultaneously
- Consumers buy from lowest-price firm
- If tie: split demand equally
- Constant marginal cost: c

Bertrand: the paradox

- **Nash equilibrium:** $p_1 = p_2 = c$ (marginal cost pricing!)
- **Why?**
  - If $p_i > p_j > c$: firm i can undercut and capture entire market
  - Undercutting continues until $p = c$
- **The "paradox":**
  - Only 2 firms, but competitive outcome!
  - Zero profits with just 2 competitors
  - Seems unrealistic for most markets

Cournot vs Bertrand: summary

	Cournot	**Bertrand**
Strategic variable	Quantities	Prices
Equilibrium price	$P > MC$	$P = MC$
Profits	Positive	Zero
Lerner index	$L = s/\lvert\varepsilon\rvert$	$L = 0$

- Which model is "right"?
- Answer: depends on the industry!

When does each model apply?

- **Cournot applies when:**
  - Capacity constraints matter
  - Firms commit to production before selling
  - Quantities are hard to adjust quickly
- Examples: manufacturing, airlines (seat capacity)
- **Bertrand applies when:**
  - Prices adjust quickly
  - No capacity constraints
  - Homogeneous products
- Examples: online retail, commodities

Kreps-Scheinkman (1983): resolving the puzzle

- Two-stage game:
  1. Stage 1: Firms choose capacities (quantities)
  2. Stage 2: Firms compete in prices
- **Result:** Equilibrium outcome = Cournot!
- **Intuition:**
  - Capacity choice commits firms
  - Price competition is constrained by capacity
  - Undercutting is limited by what you can produce
- Key insight: commitment matters

Part 2: Product Differentiation

Why differentiation matters

- Bertrand paradox: $P = MC$ with homogeneous products
- **Solution:** Product differentiation!
- If products are different, consumers don't all buy from lowest-price firm
- Firms have some pricing power
- This is exactly what we modeled in Part 1 (logit demand)
- Now: a classic spatial model of differentiation

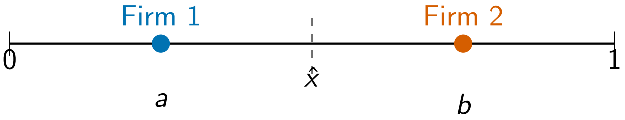
Hotelling model: setup

- Consumers uniformly distributed on $[0, 1]$ ("Main Street")
- Two firms located at positions a and b on $[0, 1]$
- Consumer at location x has utility:

$$u_j = v - p_j - t|x - \ell_j|$$

- v: base value of product
- p_j: price of firm j
- t: transport cost per unit distance
- $|x - \ell_j|$: distance to firm j

Hotelling: graphical intuition



- Consumers to the left of $\hat{x}$ buy from Firm 1
- Consumers to the right of $\hat{x}$ buy from Firm 2
- $\hat{x}$ is the "indifferent consumer"

Finding the indifferent consumer

- Consumer at $\hat{x}$ is indifferent between firms:

$$v - p_1 - t|\hat{x} - a| = v - p_2 - t|b - \hat{x}|$$

- With $a = 0$ and $b = 1$ (firms at endpoints):

$$\begin{aligned} v - p_1 - t\hat{x} &= v - p_2 - t(1 - \hat{x}) \\ p_2 - p_1 &= t(1 - 2\hat{x}) \\ \hat{x} &= \frac{1}{2} + \frac{p_2 - p_1}{2t} \end{aligned}$$

- Demand for firm 1: $D_1 = \hat{x}$

- Demand for firm 2: $D_2 = 1 - \hat{x}$

Hotelling: equilibrium prices

- Firm 1 maximizes: $\pi_1 = (p_1 - c) \cdot \hat{x}(p_1, p_2)$
- FOC: $\frac{\partial \pi_1}{\partial p_1} = \hat{x} + (p_1 - c)\frac{\partial \hat{x}}{\partial p_1} = 0$
- With $\frac{\partial \hat{x}}{\partial p_1} = -\frac{1}{2t}$:

$$\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{p_1 - c}{2t} = 0$$

- Symmetric equilibrium $(p_1 = p_2 = p^*)$:

$$p^* = c + t$$

- **Markup = transport cost!**

Hotelling: interpretation

- $p^* = c + t$: Firms charge above marginal cost
- **Transport cost t measures differentiation**
  - High t: products very different $\rightarrow$ high markup
  - Low t: products similar $\rightarrow$ low markup
  - $t \rightarrow 0$: products identical $\rightarrow$ Bertrand ($p \rightarrow c$)
- **No Bertrand paradox:** Differentiation creates pricing power
- Each firm gets half the market: $D_1 = D_2 = 1/2$
- Profit: $\pi = (p^* - c) \cdot \frac{1}{2} = \frac{t}{2}$

Worked example: Hotelling

- **Question:** Two ice cream vendors on a beach of length 1 mile. Transport cost $t = 2$ dollars per mile. Marginal cost $c = 1$.
- (a) Find the equilibrium price.
- (b) If firm 1 raises price to $p_1 = 4$, what is its market share?
- (c) Calculate firm 1's demand elasticity at the equilibrium.

Take 4 minutes.

Worked example: Hotelling (solution)

- **(a)** $p^* = c + t = 1 + 2 = 3$
- **(b)** At $p_1 = 4$, $p_2 = 3$:

$$\hat{x} = \frac{1}{2} + \frac{3-4}{2(2)} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

- Firm 1's market share falls to 25%
- **(c)** At equilibrium: $D_1 = 1/2$, $\frac{\partial D_1}{\partial p_1} = -\frac{1}{2t} = -\frac{1}{4}$

$$\varepsilon_1 = \frac{\partial D_1}{\partial p_1} \cdot \frac{p_1}{D_1} = -\frac{1}{4} \cdot \frac{3}{1/2} = -1.5$$

Welfare in Hotelling

- **Total welfare** = Consumer surplus + Profits
- Transport costs are deadweight loss
- **Socially optimal locations:** $a = 1/4$, $b = 3/4$
  - Minimizes total transport costs
- **Equilibrium locations:** Both firms at $1/2$ (minimum differentiation)
  - Firms want to capture more customers
  - But this increases total transport costs
- "Principle of minimum differentiation" (but fragile)

Connection to demand estimation

- Hotelling is a specific **differentiated Bertrand** model
- Location $\leftrightarrow$ product characteristics
- Transport cost $\leftrightarrow$ preference heterogeneity
- **Logit demand** generalizes this idea:
  - Products differ in characteristics space
  - Consumers have heterogeneous preferences
  - Price competition with differentiated products
- Next lectures: how to use demand estimates for merger simulation

Key Points

1. **Cournot:** Firms choose quantities; $L = s_i / |\varepsilon|$
2. **Bertrand (homogeneous):** $P = MC$, zero profits
3. **Bertrand paradox:** Only 2 firms but competitive outcome
4. Cournot applies with capacity constraints; Bertrand with flexible prices
5. **Product differentiation** creates pricing power
6. **Hotelling:** $p^* = c + t$ (markup = transport cost)
7. Higher t (more differentiation) $\rightarrow$ higher markup
8. Hotelling connects to logit demand from Part 1

Next time

- **Lecture 9:** Entry and Market Structure
  - Free entry condition
  - Entry deterrence: limit pricing, excess capacity
  - Strategic entry barriers