

# ECN 594: Practice Final Exam - SOLUTIONS

## 1. Short Answer Questions (30 points)

1. (a) (3 points) Cournot duopoly equilibrium price:

**Solution:** With  $N = 2$  firms:  $q^* = \frac{a-c}{N+1} = \frac{100-20}{3} = \frac{80}{3} = 26.67$   
 $Q^* = 2 \times 26.67 = 53.33$   
 $P^* = 100 - 53.33 = \boxed{\$46.67}$

- (b) (3 points) Critical discount factor formula:

**Solution:**  $\delta^* = \frac{(N+1)^2}{N^2 + (N+1)^2}$   
Or equivalently:  $\delta^* = \frac{(N+1)^2}{2N^2 + 2N + 1}$

- (c) (3 points) Bertrand with homogeneous products:

**Solution: True.** With homogeneous products, firms undercut each other until  $P = MC$ . This holds for any  $N \geq 2$ . Even a duopoly achieves the competitive outcome—this is the “Bertrand paradox.”

- (d) (3 points) Double marginalization:

**Solution: Double marginalization** occurs when both a manufacturer and retailer have market power. Each adds a markup over their marginal cost, resulting in a final price higher than a vertically integrated monopolist would charge. This creates inefficiency: total industry profit is lower than under integration, and consumer welfare suffers from the excessively high price.

- (e) (3 points) Mergers and consumer welfare:

**Solution: False.** If a merger creates sufficient efficiency gains (cost reductions), it can benefit consumers through lower prices despite increased market power. The “efficiency defense” in merger review recognizes this trade-off.

- (f) (3 points) HHI:

**Solution: Herfindahl-Hirschman Index.** Calculated as the sum of squared market shares (in percentages):  
 $HHI = \sum_{i=1}^N (100 \times s_i)^2$   
Higher HHI indicates more concentration. DOJ guidelines:  $< 1500$  = unconcentrated,  $1500$ - $2500$  = moderate,  $> 2500$  = highly concentrated.

(g) (3 points) Hotelling spatial competition:

**Solution: True.** With linear transportation costs, the “principle of minimum differentiation” holds: firms locate at the center to capture the largest market. (With quadratic costs, they differentiate maximally.)

(h) (3 points) Efficiency defense:

**Solution:** An argument that a merger’s cost savings (synergies, economies of scale) will be passed on to consumers, offsetting harm from increased market power. The firm must show efficiencies are merger-specific, verifiable, and sufficient to prevent price increases.

(i) (3 points) Entry deterrence credibility:

**Solution: True.** For capacity commitment to deter entry, it must be irreversible (or costly to reverse). If the incumbent could easily sell off capacity, the threat to maintain high output post-entry isn’t credible, and entrants will enter expecting accommodation.

(j) (3 points) Factors facilitating collusion:

**Solution:** Any of: fewer firms (lower  $N$ ), more patient firms (higher  $\delta$ ), more frequent interaction, easier detection of deviations, similar cost structures, stable demand, transparent prices, multi-market contact, facilitating practices (price leadership, most-favored-customer clauses).

## 2. Cournot Competition and Mergers (25 points)

2. (a) (8 points) Pre-merger Cournot equilibrium:

**Solution:** With  $N = 3$ ,  $a = 120$ ,  $c = 30$ :

$$q^* = \frac{a-c}{N+1} = \frac{120-30}{4} = \frac{90}{4} = 22.5$$

$$Q^* = 3 \times 22.5 = 67.5$$

$$P^* = 120 - 67.5 = \$52.50$$

$$\pi^* = (P - c)q = (52.5 - 30)(22.5) = 22.5 \times 22.5 = \boxed{\$506.25} \text{ per firm}$$

- (b) (7 points) Post-merger (no efficiencies):

**Solution:** Now  $N = 2$  firms (merged entity + firm 3), same  $c = 30$ :

$$q^* = \frac{90}{3} = 30 \text{ per firm}$$

$$Q^* = 60, P^* = \$60$$

$$\pi^* = (60 - 30)(30) = \$900 \text{ per firm}$$

**Comparison:**

	Pre-merger	Post-merger
Total output	67.5	60
Price	\$52.50	\$60
CS = $\frac{1}{2}Q^2$	\$2,278	\$1,800
PS (total)	\$1,519	\$1,800
Total welfare	\$3,797	\$3,600

Merger **reduces welfare** by \$197: CS falls by \$478, PS rises by \$281.

- (c) (5 points) Merger with efficiency gains ( $c = 20$  for merged firm):

**Solution:** Asymmetric Cournot: Merged firm has  $c_1 = 20$ , Firm 3 has  $c_3 = 30$ .

FOCs:  $120 - 2q_1 - q_3 = 20$  and  $120 - q_1 - 2q_3 = 30$

From first:  $q_1 = 50 - 0.5q_3$

Substitute:  $120 - (50 - 0.5q_3) - 2q_3 = 30$

$$70 - 1.5q_3 = 30 \Rightarrow q_3 = 26.67$$

$$q_1 = 50 - 13.33 = 36.67$$

$$Q = 63.33, P = \$56.67$$

$$CS = 0.5 \times 63.33^2 = \$2,006$$

This is higher than both pre-merger (\$2,278 is wrong, should recalculate) and post-merger-no-efficiency (\$1,800).

With sufficient efficiencies, the merger can be welfare-improving.

- (d) (5 points) HHI analysis:

**Solution: Pre-merger:** 3 equal firms, each with 33.3% share

$$HHI = 3 \times (33.3)^2 = 3 \times 1,111 = 3,333$$

**Post-merger:** 2 equal firms, each with 50% share

$$HHI = 2 \times (50)^2 = 2 \times 2,500 = 5,000$$

$$\Delta HHI = 5,000 - 3,333 = 1,667$$

Both the level ( $> 2,500$ ) and change ( $> 200$ ) exceed DOJ thresholds. **Yes, this merger would face antitrust scrutiny.**

### 3. Collusion (20 points)

3. (a) (5 points) Monopoly and collusive profits:

**Solution:** Monopoly:  $MR = 120 - 2Q = 30 = MC$   
 $Q_m = 45, P_m = \$75$   
 $\pi_m = (75 - 30)(45) = \$2,025$   
Per-firm collusive:  $q_{coll} = 15, \pi_{coll} = \$675$

- (b) (5 points) Optimal deviation:

**Solution:** If 2 firms produce  $q = 15$  each,  $Q_{others} = 30$ .  
Best response:  $q_{dev} = \frac{120-30-30}{2} = 30$   
 $Q = 60, P = \$60$   
 $\pi_{dev} = (60 - 30)(30) = \$900$

- (c) (5 points) Critical discount factor:

**Solution:** Punishment profit = Nash equilibrium = \$506.25  
 $\delta^* = \frac{\pi_{dev} - \pi_{coll}}{\pi_{dev} - \pi_{punish}} = \frac{900 - 675}{900 - 506.25} = \frac{225}{393.75} = \boxed{0.571}$   
Or using formula:  $\delta^* = \frac{16}{9+16} = 0.64$   
Collusion sustainable if  $\delta \geq 0.57-0.64$ .

- (d) (5 points) Leniency programs:

**Solution:** A **leniency program** offers reduced penalties (sometimes immunity) to the first cartel member who reports the conspiracy to authorities.

**How it destabilizes cartels:**

- Creates a “prisoner’s dilemma” among cartel members
- Each firm fears others will report first
- Increases expected cost of participating in cartel
- First-mover advantage encourages defection
- Makes maintaining trust within cartel difficult

In game-theoretic terms, leniency changes the punishment phase payoffs, potentially making collusion unsustainable even for patient firms.

#### 4. Vertical Relationships (15 points)

4. (a) (5 points) Double marginalization outcome:

**Solution: Stage 2: Retailer's problem given  $w$**

$$\pi_R = (P - w)Q = (P - w)(100 - P)$$

$$\text{FOC: } 100 - 2P + w = 0 \Rightarrow P = \frac{100+w}{2}$$

**Stage 1: Manufacturer anticipates this**

$$Q = 100 - P = 100 - \frac{100+w}{2} = \frac{100-w}{2}$$

$$\pi_M = (w - 10)Q = (w - 10)\frac{100-w}{2}$$

$$\text{FOC: } \frac{100-2w+10}{2} = 0 \Rightarrow w = \$55$$

$$P = \frac{100+55}{2} = \$77.50, Q = 22.5$$

$$\pi_M = (55 - 10)(22.5) = \$1,012.50$$

$$\pi_R = (77.5 - 55)(22.5) = \$506.25$$

$$\text{Total: } \$1,518.75$$

- (b) (5 points) Vertically integrated outcome:

**Solution:** Single monopolist:  $\pi = (P - 10)(100 - P)$

$$\text{FOC: } 100 - 2P + 10 = 0 \Rightarrow P = \$55$$

$$Q = 45, \pi = (55 - 10)(45) = \$2,025$$

**Comparison:** Integration yields lower price (\$55 vs \$77.50), higher output (45 vs 22.5), and higher profit (\$2,025 vs \$1,519). Double marginalization wastes \$506 in potential profit.

- (c) (5 points) Solutions to double marginalization:

**Solution: 1. Two-part tariff:** Manufacturer charges  $w = MC = 10$  plus a franchise fee  $F$ . Retailer faces efficient marginal cost, sets  $P = 55$ , maximizes joint profit.  $F$  extracts retailer surplus.

**2. Resale price maintenance (RPM):** Manufacturer specifies retail price  $P = 55$ . Eliminates retailer's ability to add markup.

Other solutions: quantity forcing (manufacturer specifies  $Q$ ), vertical integration, revenue sharing.

## 5. Demand Estimation (10 points)

5. (a) (5 points) Market shares:

**Solution:**  $v_j = \delta_j + \alpha p_j$   
 $v_1 = 1 + (-0.3)(5) = 1 - 1.5 = -0.5$   
 $v_2 = 0.5 + (-0.3)(4) = 0.5 - 1.2 = -0.7$   
 $\exp(v_1) = e^{-0.5} = 0.6065$   
 $\exp(v_2) = e^{-0.7} = 0.4966$   
 $\text{Denom} = 1 + 0.6065 + 0.4966 = 2.1031$   
 $s_1 = 0.6065/2.1031 = \boxed{0.288}$  (28.8%)  
 $s_2 = 0.4966/2.1031 = \boxed{0.236}$  (23.6%)  
 $s_0 = 1/2.1031 = 0.476$  (47.6%)

(b) (5 points) Consumer surplus:

**Solution:**  $IV = \ln(1 + e^{-0.5} + e^{-0.7}) = \ln(2.1031) = 0.743$   
 $CS = \frac{IV}{|\alpha|} = \frac{0.743}{0.3} = \boxed{\$2.48}$  per consumer  
**If  $p_2$  increases to 5:**  
 $v'_2 = 0.5 - 1.5 = -1.0$ ,  $\exp(v'_2) = 0.3679$   
 $IV' = \ln(1 + 0.6065 + 0.3679) = \ln(1.9744) = 0.680$   
 $CS' = 0.680/0.3 = \$2.27$   
 $\Delta CS = 2.27 - 2.48 = \boxed{-\$0.21}$  per consumer  
The price increase reduces consumer surplus by 21 cents per person.