ECN 594: Oligopoly Competition

Nicholas Vreugdenhil

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Welcome to Part 2

- Demand estimation and pricing
- **Models of competition and industry structure**
  - Oligopoly models (Cournot, Bertrand, Hotelling)
  - Entry and entry deterrence
  - Mergers
  - Vertical relationships
  - Collusion
- **HW2 released:** Merger simulation module

Plan

1. **Cournot and Bertrand competition**
2. Product differentiation and Hotelling model

From ECN 532: Oligopoly models

- You covered Cournot and Bertrand in Hector's class
- Today: quick refresher in IO notation
- **New focus:** Market power measurement
  - Connecting oligopoly models to Lerner index
  - When does each model apply?

Cournot competition: setup

- n firms producing homogeneous goods
- Firms choose **quantities** simultaneously
- Inverse demand: $P = P(Q)$ where $Q = \sum_{i=1}^{n} q_i$
- Constant marginal cost: c
- Firm i profit: $\pi_i = P(Q) \cdot q_i - c \cdot q_i$

Cournot: first-order conditions

- Firm i maximizes profit taking q_{-i} as given:

$$\frac{\partial \pi_i}{\partial q_i} = P(Q) + P'(Q)q_i - c = 0$$

- Rearranging:

$$P(Q) - c = -P'(Q)q_i$$

- Divide by P:

$$\frac{P - c}{P} = \frac{-P'(Q)q_i}{P} = \frac{-P'(Q)Q}{P} \cdot \frac{q_i}{Q} = \frac{s_i}{|\varepsilon|}$$

- where $s_i = q_i / Q$ is firm i's market share

Cournot: Lerner index

- **Key result:** In Cournot equilibrium,

$$L_i = \frac{P - MC}{P} = \frac{s_i}{|\varepsilon|}$$

- **Interpretation:**
  - Markup depends on market share
  - Larger firms have more market power
  - More elastic demand $\rightarrow$ lower markup
- This connects to demand estimation from Part 1!

Worked example: Cournot with market power

- **Question:** Inverse demand is $P = 100 - Q$. Two symmetric firms with $MC = 10$.
- (a) Find equilibrium quantities and price.
- (b) Calculate the Lerner index for each firm.
- (c) Verify using the $L = s/|\varepsilon|$ formula.

Take 5 minutes.

Worked example: Cournot (solution)

Solution

- **(a)** FOC: $100 - 2q_i - q_j - 10 = 0$
- Symmetric: $q_1 = q_2 = q^*$, so $100 - 3q^* = 10 \Rightarrow q^* = 30$
- $Q = 60,\ P = 100 - 60 = 40$
- **(b)** $L = \frac{40-10}{40} = \frac{3}{4} = 0.75$
- **(c)** Market share: $s_i = 30/60 = 0.5$
- Elasticity: $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = (-1) \cdot \frac{40}{60} = -\frac{2}{3}$
- Check: $L = \frac{s_i}{|\varepsilon|} = \frac{0.5}{2/3} = 0.75$ ✓

Cournot with n symmetric firms

- General result for linear demand $P = a - bQ$:
- Symmetric equilibrium: $q^* = \frac{a-c}{b(n+1)}$
- Total quantity: $Q^* = \frac{n(a-c)}{b(n+1)}$
- Price: $P^* = \frac{a+nc}{n+1}$
- Lerner index: $L = \frac{1}{n|\varepsilon|}$ (symmetric case where $s_i = 1/n$)
- **Key insight:** As $n \to \infty$, $P \to MC$ (perfect competition)

Practice: Cournot comparative statics

- **True, False, or NEI:**
- (a) In Cournot equilibrium, a firm with a larger market share has a higher markup.
- (b) Adding a third firm to a Cournot duopoly always reduces industry profits.
- (c) In Cournot, all firms must have the same marginal cost.

Take 2 minutes.

Practice: Cournot comparative statics (solution)

Answers

- **(a) TRUE.** From $L_i = s_i/|\varepsilon|$, larger s_i means higher L_i.
- **(b) TRUE.** More firms $\Rightarrow$ lower price $\Rightarrow$ lower industry profit. Each firm's profit falls, and total profit falls because P is closer to MC.
- **(c) FALSE.** Asymmetric costs work fine. Low-cost firms have higher q_i and s_i, hence higher margins.

Bertrand competition: setup

- n firms producing **homogeneous** goods
- Firms choose **prices** simultaneously
- Consumers buy from lowest-price firm
- If tie: split demand equally
- Constant marginal cost: c

Bertrand: the paradox

- **Nash equilibrium:** $p_1 = p_2 = c$ (marginal cost pricing!)
- **Why?**
  - If $p_i > p_j > c$: firm i can undercut and capture entire market
  - Undercutting continues until $p = c$
- **The "paradox":**
  - Only 2 firms, but competitive outcome!
  - Zero profits with just 2 competitors
  - Seems unrealistic for most markets

Cournot vs Bertrand: summary

	Cournot	**Bertrand**
Strategic variable	Quantities	Prices
Equilibrium price	$P > MC$	$P = MC$
Profits	Positive	Zero
Lerner index	$L = s/\|\varepsilon\|$	$L = 0$

- Which model is "right"?
- Answer: depends on the industry!

Escaping the Bertrand paradox

1. **Capacity constraints** (Kreps-Scheinkman)
   - Can't serve entire market at low price
   - Leads to Cournot outcome
2. **Product differentiation** (today's main topic!)
   - Consumers have preferences
   - Not all switch to lowest price
3. **Repeated interaction** (collusion, covered later)
   - Firms can sustain $P > MC$ through punishment strategies

When does each model apply?

- **Cournot applies when:**
  - Capacity constraints matter
  - Firms commit to production before selling
  - Quantities are hard to adjust quickly
- Examples: manufacturing, airlines (seat capacity)
- **Bertrand applies when:**
  - Prices adjust quickly
  - No capacity constraints
  - Homogeneous products
- Examples: online retail, commodities

Industry examples: which model?

Industry	**Model**	**Why?**
Airlines	Cournot	Capacity (planes, gates) committed
Cement	Cournot	Production committed; transport costs
Gasoline stations	Bertrand	Prices adjust daily; commodity
Online retail	Bertrand	Instant price changes; no capacity
Smartphones	Differentiated	Product differentiation dominates

- In practice: most markets have differentiated products!

Practice: Cournot vs Bertrand

- **Question:** Two firms have $MC = 20$. Market demand is $P = 100 - Q$.
- (a) Find equilibrium price under Cournot.
- (b) Find equilibrium price under Bertrand.
- (c) Which model yields higher consumer surplus? Why?

Take 3 minutes.

Practice: Cournot vs Bertrand (solution)

Solution

- **(a) Cournot:** Using $P^* = \frac{a+nc}{n+1} = \frac{100+2(20)}{3} = 46.67$
- Or: $q^* = \frac{100-20}{3} = 26.67$, $Q = 53.33$, $P = 46.67$
- **(b) Bertrand:** $P = MC = 20$
- **(c)** Bertrand has higher CS:
  - Bertrand: $CS = \frac{1}{2}(100-20)(80) = 3200$
  - Cournot: $CS = \frac{1}{2}(100-46.67)(53.33) \approx 1422$
- Lower price $\Rightarrow$ higher quantity $\Rightarrow$ higher CS

Kreps-Scheinkman (1983): resolving the puzzle

- Two-stage game:
  1. Stage 1: Firms choose capacities (quantities)
  2. Stage 2: Firms compete in prices
- **Result:** Equilibrium outcome = Cournot!
- **Intuition:**
  - Capacity choice commits firms
  - Price competition is constrained by capacity
  - Undercutting is limited by what you can produce
- Key insight: commitment matters

Plan

1. Cournot and Bertrand competition

2. **Product differentiation and Hotelling model**

Why differentiation matters

- Bertrand paradox: $P = MC$ with homogeneous products
- **Solution:** Product differentiation!
- If products are different, consumers don't all buy from lowest-price firm
- Firms have some pricing power
- This is exactly what we modeled in Part 1 (logit demand)
- Now: a classic spatial model of differentiation

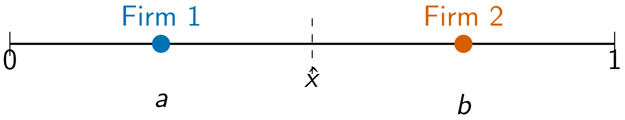
Hotelling model: setup

- Consumers uniformly distributed on $[0, 1]$ ("Main Street")
- Two firms located at positions a and b on $[0, 1]$
- Consumer at location x has utility:

$$u_j = v - p_j - t|x - \ell_j|$$

- v: base value of product
- p_j: price of firm j
- t: transport cost per unit distance
- $|x - \ell_j|$: distance to firm j

Hotelling: graphical intuition



- Consumers to the left of $\hat{x}$ buy from Firm 1
- Consumers to the right of $\hat{x}$ buy from Firm 2
- $\hat{x}$ is the "indifferent consumer"

Finding the indifferent consumer

- Consumer at $\hat{x}$ is indifferent between firms:

$$v - p_1 - t|\hat{x} - a| = v - p_2 - t|b - \hat{x}|$$

- With $a = 0$ and $b = 1$ (firms at endpoints):

$$\begin{aligned} v - p_1 - t\hat{x} &= v - p_2 - t(1 - \hat{x}) \\ p_2 - p_1 &= t(1 - 2\hat{x}) \\ \hat{x} &= \frac{1}{2} + \frac{p_2 - p_1}{2t} \end{aligned}$$

- Demand for firm 1: $D_1 = \hat{x}$

- Demand for firm 2: $D_2 = 1 - \hat{x}$

Hotelling: equilibrium prices

- Firm 1 maximizes: $\pi_1 = (p_1 - c) \cdot \hat{x}(p_1, p_2)$
- FOC: $\frac{\partial \pi_1}{\partial p_1} = \hat{x} + (p_1 - c)\frac{\partial \hat{x}}{\partial p_1} = 0$
- With $\frac{\partial \hat{x}}{\partial p_1} = -\frac{1}{2t}$:

$$\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{p_1 - c}{2t} = 0$$

- Symmetric equilibrium ($p_1 = p_2 = p^*$):

$$p^* = c + t$$

- **Markup = transport cost!**

Hotelling: interpretation

- $p^* = c + t$: Firms charge above marginal cost
- **Transport cost t measures differentiation**
  - High t: products very different $\rightarrow$ high markup
  - Low t: products similar $\rightarrow$ low markup
  - $t \rightarrow 0$: products identical $\rightarrow$ Bertrand ($p \rightarrow c$)
- **No Bertrand paradox:** Differentiation creates pricing power
- Each firm gets half the market: $D_1 = D_2 = 1/2$
- Profit: $\pi = (p^* - c) \cdot \frac{1}{2} = \frac{t}{2}$

Worked example: Hotelling

- **Question:** Two ice cream vendors on a beach of length 1 mile. Transport cost $t = 2$ dollars per mile. Marginal cost $c = 1$.
- (a) Find the equilibrium price.
- (b) If firm 1 raises price to $p_1 = 4$, what is its market share?
- (c) Calculate firm 1's demand elasticity at the equilibrium.

Take 4 minutes.

Worked example: Hotelling (solution)

Solution

- **(a)** $p^* = c + t = 1 + 2 = 3$
- **(b)** At $p_1 = 4$, $p_2 = 3$:

$$\hat{x} = \frac{1}{2} + \frac{3-4}{2(2)} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

- Firm 1's market share falls to 25%
- **(c)** At equilibrium: $D_1 = 1/2$, $\frac{\partial D_1}{\partial p_1} = -\frac{1}{2t} = -\frac{1}{4}$

$$\varepsilon_1 = \frac{\partial D_1}{\partial p_1} \cdot \frac{p_1}{D_1} = -\frac{1}{4} \cdot \frac{3}{1/2} = -1.5$$

Hotelling: why transport cost matters

- **High t:** Strong differentiation
  - Consumers have strong location preferences
  - Firm can raise price without losing many customers
  - Example: specialty restaurants vs fast food
- **Low t:** Weak differentiation
  - Consumers nearly indifferent across locations
  - Small price cut steals many customers
  - Approaches Bertrand as $t \to 0$
- In demand estimation: $1/\alpha$ plays similar role to t

Practice: T/F on Hotelling

- **True, False, or NEI:**
- (a) In Hotelling equilibrium, firms always split the market equally.
- (b) A monopolist in Hotelling should locate at the center of the line.
- (c) Higher transport costs benefit consumers because products are more differentiated.

Take 2 minutes.

Practice: T/F on Hotelling (solution)

Answers

- **(a) TRUE** (with caveat). If firms are at endpoints and have same costs, yes. With asymmetric locations or costs, shares differ.
- **(b) TRUE.** Monopolist minimizes total transport costs by locating at center. This maximizes consumer value and thus WTP.
- **(c) FALSE.** Higher t means higher prices ($p^* = c + t$) and higher transport costs. Both hurt consumers.

Welfare in Hotelling

- **Total welfare** = Consumer surplus + Profits
- Transport costs are deadweight loss
- **Socially optimal locations:** $a = 1/4$, $b = 3/4$
  - Minimizes total transport costs
- **Equilibrium locations:** Both firms at $1/2$ (minimum differentiation)
  - Firms want to capture more customers
  - But this increases total transport costs
- "Principle of minimum differentiation" (but fragile)

Connection to demand estimation

- Hotelling is a specific **differentiated Bertrand** model
- Location $\leftrightarrow$ product characteristics
- Transport cost $\leftrightarrow$ preference heterogeneity
- **Logit demand** generalizes this idea:
  - Products differ in characteristics space
  - Consumers have heterogeneous preferences
  - Price competition with differentiated products
- Next lectures: how to use demand estimates for merger simulation

From Hotelling to logit: the connection

Hotelling	**Logit**
Location on line	Product characteristics
Transport cost t	Sensitivity $1/\lvert\alpha\rvert$
Consumer position	Consumer preferences
Linear utility	Logit choice probabilities
2 products	J products

- Logit demand lets us do empirical Hotelling-style analysis
- Estimate α from data $\rightarrow$ calibrate "differentiation"

Oligopoly models: summary

- **Homogeneous products:**
  - Cournot (quantity): $L = s_i / |\varepsilon|$
  - Bertrand (price): $P = MC$
- **Differentiated products:**
  - Hotelling/Differentiated Bertrand: $P > MC$
  - Markup depends on differentiation (transport cost)
- **Empirical approach:**
  - Estimate demand (logit)
  - Assume pricing behavior (usually differentiated Bertrand)
  - Back out marginal costs

Preview: merger simulation

- **Key question:** If two firms merge, what happens to prices?
- **Approach:**
  1. Estimate demand $\rightarrow$ get elasticities
  2. Assume Bertrand pricing $\rightarrow$ back out MC
  3. Change ownership structure
  4. Solve new Bertrand equilibrium
  5. Compare prices and welfare
- This is exactly what HW2 will do!
- Will cover in detail in Lecture 10

Key Points

1. **Cournot:** Firms choose quantities; $L = s_i / |\varepsilon|$
2. **Bertrand (homogeneous):** $P = MC$, zero profits
3. **Bertrand paradox:** Only 2 firms but competitive outcome
4. Cournot applies with capacity constraints; Bertrand with flexible prices
5. **Product differentiation** creates pricing power
6. **Hotelling:** $p^* = c + t$ (markup = transport cost)
7. Higher t (more differentiation) $\rightarrow$ higher markup
8. Hotelling connects to logit demand from Part 1

Next time

- **Lecture 9:** Entry and Market Structure
  - Free entry condition
  - Entry deterrence: limit pricing, excess capacity
  - Strategic entry barriers