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Spatial dynamics of team sports exposed by Voronoi diagrams

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ABSTRACT

Team sports represent complex systems: players interact continuously during a game, and exhibit intricate patterns of interaction, which can be identified and investigated at both individual and collective levels. We used Voronoi diagrams to identify and investigate the spatial dynamics of players' behavior in Futsal. Using this tool, we examined 19 plays of a sub-phase of a Futsal game played in a reduced area (20 m^2) from which we extracted the trajectories of all players. Results obtained from a comparative analysis of player's Voronoi area (dominant region) and nearest teammate distance revealed different patterns of interaction between attackers and defenders, both at the level of individual players and teams. We found that, compared to defenders, larger dominant regions were associated with attackers. Furthermore, these regions were more variable in size among players from the same team but, at the player level, the attackers' dominant regions were more regular than those associated with each of the defenders. These findings support a formal description of the dynamic spatial interaction of the players, at least during the particular sub-phase of Futsal investigated. The adopted approach may be extended to other team

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behaviors where the actions taken at any instant in time by each of the involved agents are associated with the space they occupy at that particular time.

1. Introduction

Team sports can be viewed as complex systems in that the players, the agents of the system, interact continuously during a game (Davids, Araújo, & Shuttleworth, 2005; McGarry, Anderson, Wallace, Hughes, & Franks, 2002). Their interaction determines the occurrence of specific events during a game (Passos et al., 2008). Therefore, having a good understanding of this dynamic behavior would not only allow a better characterization of these systems but could also help coaches to anticipate some outcomes or events.

Players' interaction behavior can be assessed from a spatial perspective. For instance, players change their location continuously during a game as they adjust their relative positions according to the information they (can) perceive (Passos et al., 2008; Travassos, Araújo, Vilar, & McGarry, 2011), acting collectively as a result of phenomena such as cooperation and competition. Thus, players, collective behavior cannot be explained by the simple addition of behaviors from each player (Gréhaigne, Bouthier, & David, 1997); instead, players' behaviors should be considered in terms of the entire dynamic system that they compose (Glazier, 2010; McGarry, 2009; Passos et al., 2009), where both time (Araújo, Davids, & Hristovskic, 2006) and space (Davids, Handford, & Williams, 1994; Schöllhorn, 2003) need to be brought into the equation. Considering both space and time, it is possible to evaluate the spatial configuration players present during game play.

To illustrate, spatial configurations can be classified as random, regular or clustered. A configuration may be considered as random when players are at random distances from each other in the field, as regular when players are equally distant from each other in the field, and as clustered when we can identify different groups of players aggregated in different parts of the field (Fig. 1). These spatial distribution patterns can be easily identified by measuring interpersonal distances, in particular the minimum interpersonal distance, or nearest neighbor distance (Clark & Evans, 1954).

The spatial distribution of the players in a field, and hence the space within which players have to act, is dependent on a large number of constraints that change continuously throughout a game, with ball possession being an obvious one. In principle, the attacking team normally tries to free-up space while the defending team tries to tie-up space (Gréhaigne et al., 1997; McGarry et al., 2002). Therefore, in terms of nearness, it is expected that the interpersonal distance between players is kept greater by the attacker team and smaller by the defender team, resulting in more space for the attack. This relationship was already observed using surface area (Frencken, Lemmink, Delleman, & Visscher, 2011) and stretch index variables (Bourbousson, Sève, & McGarry, 2010).

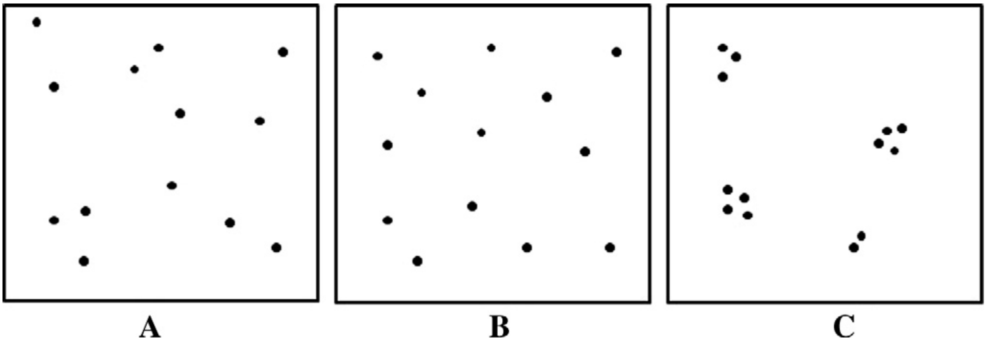

Fig. 1. Example of spatial distribution patterns (A) random, (B) regular and (C) clustered.

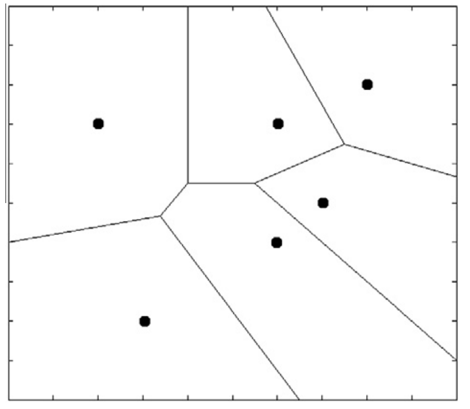

Fig. 2. Example of a Voronoi diagram generated for the set of points represented in the figure.

An alternative method to study the spatial relation established between players at each instant of a game is the Voronoi diagram (Dirichlet, 1850; Voronoi, 1907), which is a spatial construction that allows a spatial partitioning of the field area into cells, each associated with each of the players, according to their positions (Fig. 2). These cells result from applying a very simple nearest-neighbor rule: each player, represented by the coordinates of his/her location in the field, is associated to all parts of the field that are nearer to that player than to any other player (see Okabe, Boots, Sugihara, & Chiu, 2000).

Other researchers have already recognized that Voronoi diagrams may provide a potent tool to study the players' spatial distribution in team sports and to define players' and teams' dominant regions. Notably, Voronoi diagrams have been applied in a variety of game settings, including real soccer games (Taki, Hasegawa, & Fukumura, 1996), electronic soccer games (Kim, 2004), robotic soccer (Law, 2005), and real hockey games (Fujimura & Sugihara, 2005). When real games were considered, dominant regions were calculated considering more than just players' location. In particular, Taki et al. (1996) focused on players' direction and speed, whereas Fujimura and Sugihara (2005) focused on the players' distance from the ball and distance to the goal. In all these studies it was shown that the position of the ball affects the location of the players and hence the size of their respective dominant regions.

In spite of the aforementioned advances in the analysis of spatial patterns of behavior in team sports, an important dimension has not been considered. In fact, when analyzing systems of interacting agents, it is necessary to measure their degree of complexity (Harbourne & Stergiou, 2009; Stergiou, Buzzi, Kurz, & Heidel, 2003), because this is key to understanding the emergence of successful performances in dynamical movement systems (Bartlett, Wheat, & Robins, 2007; Davids, Glazier, Araújo, & Bartlett, 2003). To assess the complexity of a system, one may use the nonlinear measure suggested by Pincus in 1991, the Approximate Entropy (ApEn), which quantifies the regularity (predictability) of a variable measured in the system under study. When this variable expresses the state of the system (Harbourne & Stergiou, 2009), its regularity is directly proportional to the system's complexity, i.e., lower values of ApEn indicate more regularity and hence lower complexity.

Thus, the main goal of the present paper was to characterize the spatial interaction dynamics of players in team sports in order to understand how players from two opposite teams coordinate their location in the field during a game and how they define and adjust their dominant regions throughout the game. We expected that players from the attacking team would present greater interpersonal distances, greater dominant regions, and greater regularity overtime in terms of space area when in ball possession.

2. Methods

In this study we considered 19 experimental plays of Futsal, in which 15 male senior players participated (23.25 ± 1.96 years old). Participants were treated in agreement with the ethical standards of American Psychological Association (APA). Plays represented the sub-phase of Futsal of 5 vs 4 + GK performed in half field (20 m width × 20 m depth) where all players occupied fixed initial positions. This is a common scenario in Futsal when the team losing the game has ball possession and aims to score where, due to numerical disadvantage, the defender team retracts their positions to their respective half field. Accordingly, in each play, the aim of the attacker team is to score while the defender team tries to avoid it, and each play ends whenever the attack loses ball possession.

Two fixed digital video cameras running at 25 Hz were used to capture players' movements during each play. Individual player trajectories were extracted from the recorded videos using TACTO software (see more in Duarte et al., 2010; Fernandes & Malta, 2007) and transformed into real coordinates (x, y) using a direct linear transformation method (2D-DLT) (Abdel-Aziz & Karara, 1971). The 19 plays had, on average, 848 (± 374) frames (corresponding to approximately 34.2 (± 14.94) s), with a minimum of 315 and a maximum of 1558 frames (approximately 12.6 and 62.4 s, respectively).

In the present work two variables were considered to describe this system, namely the individual players' dominant region, as defined by the respective Voronoi cell, and the minimum interpersonal distance between teammates. The minimum interpersonal distance between all teammates (N), here designated nearest teammate distance ($\mathrm{Dist}_{\mathrm{NT}}$), was calculated at each frame (f), considering the Euclidean distances between all pairs of players of a team (A), as described below.

$$\mathrm{Dist}_{\mathrm{NT}}(\mathrm{A})^f = \min_{\substack{i,j \\ i \neq j}} \left\{ \sqrt{(x_i^f - x_j^f)^2 + (y_i^f - y_j^f)^2} \right\}, \; i, j = 1, \ldots, N$$

As for players' individual dominant region, we considered the respective Voronoi cells and calculated their area ($\mathrm{Area}_{\mathrm{DR}}$) as described next.

The field was mapped with a grid of 20 × 20 positions. At each frame (f), the area of the DR of player k (k[1,M]) is the sum of all grid positions (i,j) (where i = 1,...,20 and j = 1,...,20) that are closer to that player than to any other player. This can be mathematically defined as presented below,

$$\mathrm{Area}_{\mathrm{DR}}(\mathrm{k})^f = \sum_{i=1}^{20} \sum_{j=1}^{20} I_{(i,j)} \qquad k = 1, \ldots, M$$

where I(i,j) is a Boolean function that takes value 1 if player k is the closest player to the grid position (i,j) and 0 otherwise:

$$I_{(i,j)} = \begin{cases} 1 & \text{if} \quad \sqrt{(i - x_k^f)^2 + (j - y_k^f)^2} < \sqrt{(i - x_m^f)^2 + (j - y_m^f)^2}, \forall\, m \neq k,\; m = 1, \ldots, M \\ 0 & \text{otherwise} \end{cases}$$

Grid points that are equidistant to two or more players constitute the boundaries of their respective regions and are therefore not added to the corresponding areas.

For each player and team we investigated how the size of their dominant regions changes over time and how the sizes in question relate to each other. MATLAB routines were written to generate, at each frame, the Voronoi diagram associated to the spatial distribution of the players, and to calculate the size of the dominant region ($\mathrm{Area}_{\mathrm{DR}}$), according to the descriptions given above.

The regularity of time series data from $\mathrm{Area}_{\mathrm{DR}}$ and $\mathrm{Dist}_{\mathrm{NT}}$ was measured using the $\mathrm{ApEn}_{\mathrm{RatioRandom}}$ (Fonseca, Passos, Davids, Araújo, & Milho, 2012), which is a normalized measure of Pincus (1991) approximate entropy (ApEn), obtained by dividing the ApEn of the original series, Y, by the average ApEn of 100 random series of the same size of Y. This measure allows the comparison of entropy values calculated in series of varying lengths. A value of $\mathrm{ApEn}_{\mathrm{RatioRandom}}$ of approximately 0.2 indicates regularity (high predictability), whereas 1 indicates low regularity (high unpredictability) (Fonseca et al., 2012).

We used descriptive statistics (Mean (M) ± Standard Deviation (SD)) and inferential statistics (ANOVA, t-test and paired t-test) to compare the spatial behavioral complexity between players, teams, and teams by play, respectively.

2.1. Reliability

From all the plays, one of them was randomly selected and the data trajectories of the players were re-digitized by the same researcher. Data were then assessed for accuracy and reliability using technical error of measurement (TEM) and coefficient of reliability (R), respectively (Goto & Mascie-Taylor, 2007). The TEM yielded values of 0.137 m (0.23%) and the coefficient of reliability was equal to .984.

3. Results

When looking at changes in the minimum interpersonal distance between teammate players ($Dist_{NT}$) and area of the dominant region ($Area_{DR}$) across each play, we found that, on average, players from the attacker team tend to be further from each other in comparison with players from the defender team, as expected (see the exemplar single play depicted in Fig. 3). Consequently, the space

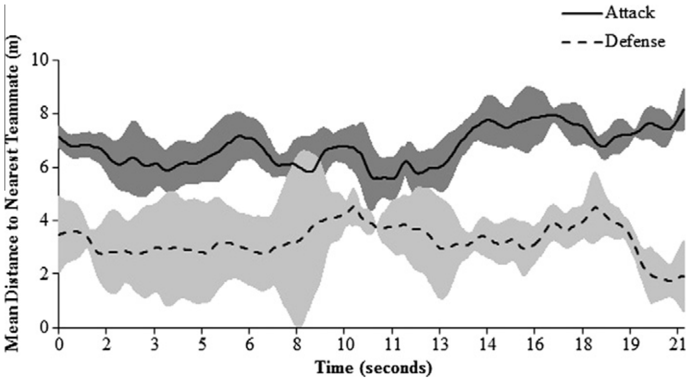

Fig. 3. Mean distance to nearest teammate distance, across time, for the attacker and defender teams in a randomly selected play (error bars represent the standard deviation).

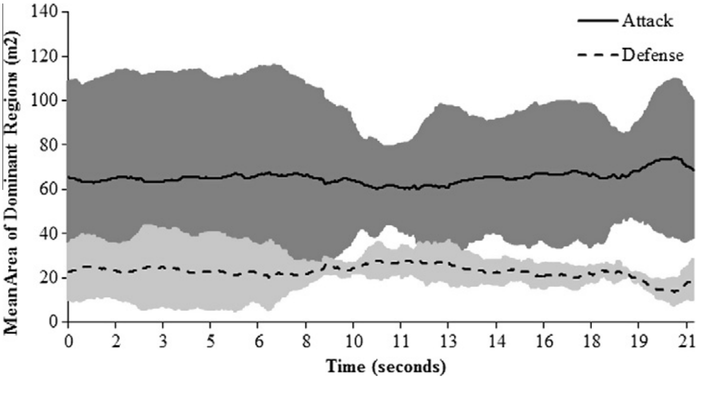

Fig. 4. Mean area of the dominant region, across time, for the attacker and defender teams in a randomly selected play (error bars represent the standard deviation).

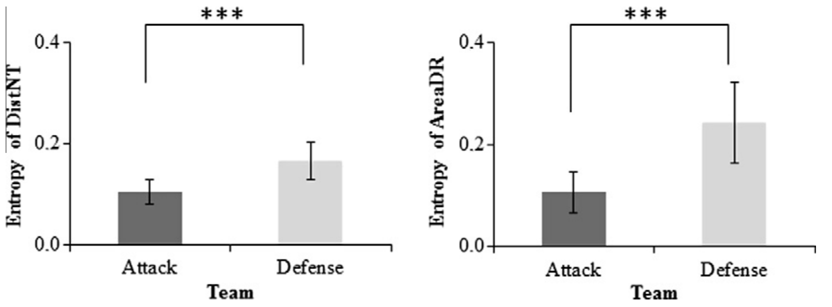

Fig. 5. Comparison of the mean entropy of the distance to nearest teammate ($Dist_{NT}$) and area of the dominant region ($Area_{DR}$) between teams in the same play. Error bars represent the standard deviation ($^{***}p < .001$).

occupied by each player is, on average, greater for the team with the ball (attacker team) in comparison with the defender team (see the exemplar single play depicted in Fig. 4).

When comparing the amount of variability within each team for both variables, it is clear that the attacker team shows less variability than the defender team in the $Dist_{NT}$ and more variability than the defender team in the $Area_{DR}$, as shown by the error bars in Fig. 3 and Fig. 4, respectively, This tendency was observed in all plays, suggesting that, in comparison to what was found in the defender team, the area occupied by the attacker team is much more variable within each frame, whereas the minimum interpersonal distance is less variable. In Figs. 3 and 4, the moment captured at time 10 s corresponds to the exact moment (observed by visual inspection) when the ball is received by an attacker inside the defensive structure, which is, according to Futsal's literature, a critical occurrence for the defender team (Lucena, 2007). As a consequence, all defenders were trying to close the space around the ball carrier and avoid the attacker team to score, and both $Dist_{NT}$ and $Area_{DR}$, presented particularly low variability.

To better understand and characterize the system under study, we measured the regularity of $Dist_{NT}$ and $Area_{DR}$, at both player and team levels and within each play, using a normalized measure of the ApEn due to presence of signals with varying lengths (for more detail see Fonseca et al., 2012). At a player level, the regularity of the $Dist_{NT}$ and $Area_{DR}$ was calculated separately for each player in all plays. We found that the regularity of both variables was significantly different between at least two players ($Dist_{NT}$: $F(9,180) = 9.5$, $p < .001$; $Area_{DR}$: $F(9,180) = 12.5$, $p < .001$), being this difference only found between opponent players. This means that players within a team had similar behavioral patterns regarding proximity to their teammates and management of their dominant regions. At a team level, the regularity of the same two variables was compared between the teams (Defender vs Attacker) and significant differences were found in both variables ($Dist_{NT}$: 0.165 ± 0.048 vs 0.106 ± 0.043, $p < .001$; $Area_{DR}$: 0.264 ± 0.135 vs 0.114 ± 0.061, $p < .001$). In addition, and having shown a team effect, we tested the effect of the play in the spatial interacting behavior between teams. Hence, for the same two variables, we considered, for each play and for each team, the median entropy. Our results were consistent with what was shown above, suggesting that, within a play, $Dist_{NT}$ and $Area_{DR}$ were significantly more regular for the attacker team in comparison with the defender team ($t(18) = 8.26$, $p < .001$; $t(18) = 8.86$, $p < .001$, respectively) (Fig. 5).

4. Discussion

The aim of this study was to characterize the spatial dynamics of players and teams in Futsal using Voronoi diagrams. We considered the minimum interpersonal distance between teammates ($Dist_{NT}$) and the area of the dominant region of each player ($Area_{DR}$) as variables that can be considered to characterize the individual and collective behavior of the players. Both variables mentioned above appear to capture some interesting characteristics of this system of interactions, namely, players from the team with the ball, are further apart from each other whereas defenders are closer from each other. This spatial organization has direct influence on the dominant region defined by each player. These individual dominant regions were defined using Voronoi diagrams and they appear to be greater for the attacker team and smaller for the defender team. These results are in agreement with what was theoretically expected (McGarry et al., 2002). The spatial behavior assessed by these two variables did not present significant differences between players of the same team as their actions are, to some extent, regulated by their goal as a team, which is scoring and avoiding a score for the attacker and defender teams respectively.

Moreover, we found that the $Area_{DR}$ and $Dist_{NT}$ present, across time, lower regularity in the defender team implying that their behaviour was more unpredictable than the interaction behavior observed in the attacker team. This greater unpredictability associated to the defender team may be justified by the fact that the players on this team are constantly adjusting their spatial organization to protect the goal in function of what the attacker team does (Frencken et al., 2011). On the other hand, the attacker team explores the free space in a more regular way, possibly acting according to the trained coordination patterns that are assumed to increase chances of scoring.

Voronoi diagrams can then be considered to measure individual and team dominant regions. The observed signals of this variable appear to capture particular phases of the game, such as when the ball is received by an attacker inside the defensive structure, presenting behavioral patterns that may be used to describe and explain the performance outcome (Glazier, 2010; McGarry, 2009). Unlike other authors, in this paper, we did not consider any factor to weight players' Voronoi regions, so their areas were simply based on the position of the players, which, according to our results, are naturally influenced by ball possession. However, there are other factors, such as players' individual characteristics (Cordovil et al., 2009), distance from ball (Fujimura & Sugihara, 2005), motion direction, speed and acceleration (Fujimura & Sugihara, 2005; Taki et al., 1996), that are likely to determine players' actions and hence their spatial distribution in the field. In future work, some of the mentioned constraints could be considered to weight the distances used in the calculation of the dominant regions.

In addition, future research in this topic could consider other sub-phases of the game (e.g., 5 vs 5, counter-attack, corners) and study players' spatial configurations (e.g., attacker team vs defender team) in order to formally describe their spatial behavior and compare these with the principles that regulate them. With the same reasoning, the definition of players' spatial profiles for different game scenarios could be of much interest to the training processes (Travassos, Araújo, Correia, & Esteves, 2010).

5. Conclusion

In conclusion, we showed that Voronoi diagrams can be used to characterize players' spatial interaction behavior in Futsal. The interpersonal relationship between players and teams is well described by the variables considered and the quantification of their predictability was able to capture the interaction behavior between and within teams during performance.

This analysis can be further applied to other team sports to describe individual and collective behavior, identify patterns of coordination in different sub-phases of a game, and compare spatial patterns of coordination between teams of different levels of expertise.

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