

$$\neg(A \leftrightarrow B) \quad \vdash \quad \neg A \leftrightarrow B.$$

Apologies for not being able to show this during our tutorial! It's a little trickier than exam standard and (you'll see) harder than the other direction.

1. $\neg(A \leftrightarrow B)$ --- given

2. $\neg A$	ass.
3. $\neg B$	ass.
4. A	ass.
5. \perp	$\perp I(2,4)$
6. B	$\perp E(5)$
7. $A \rightarrow B$	$\rightarrow I(4,6)$
8. B	ass.
9. \perp	$\perp I(3,8)$
10. A	$\perp E(9)$
11. $B \rightarrow A$	$\rightarrow I(8,10)$
12. $A \leftrightarrow B$	$\leftrightarrow I(7,11)$
13. \perp	$\perp I(1,12)$
14. B	PC(3,13)

15. $\neg A \rightarrow B$ $\rightarrow I(2,14)$

16. B	ass.
17. A	ass.
18. B	ass.
19. A	$\checkmark(17)$
20. $B \rightarrow A$	$\rightarrow I(18,19)$
21. A	ass.
22. B	$\checkmark(16)$
23. $A \rightarrow B$	$\rightarrow I(21,22)$
24. $A \leftrightarrow B$	$\leftrightarrow I(20,23)$
25. \perp	$\perp I(1,24)$
26. $\neg A$	$\neg I(17,25)$

27. $B \rightarrow \neg A$ $\rightarrow I(16,26)$

28. $\neg A \leftrightarrow B$ $\leftrightarrow I(15,27)$

I want to show

$$\neg A \rightarrow B$$

$$B \rightarrow \neg A$$

using $\rightarrow I$.

use PC. I don't have a good way to get B directly from $\neg(A \leftrightarrow B), \neg A$. (*)

← almost the same strategy.

(*) This is because of all the \neg symbols on the outside.

The only way to remove \neg is through contradiction. This is why I didn't use PC for the overall strategy (Think about this for 30 seconds).

$$A \leftrightarrow \neg B \quad \vdash \neg(A \leftrightarrow B).$$

This is the easier one. \uparrow

This is of the form $\neg F$, so we'll try $\neg I$.

1. $A \leftrightarrow \neg B$ given

2. $A \leftrightarrow B$	ass.
3. B	ass.
4. A	$\leftrightarrow E(3,2)$
5. $\neg B$	$\leftrightarrow E(4,1)$
6. \perp	$\perp I(3,5)$
7. $\neg B$	$\neg I(3,6)$
8. A	$\leftrightarrow E(7,1)$
9. B	$\leftrightarrow E(8,2)$
10. \perp	$\neg E(9,7)$
11. $\neg(A \leftrightarrow B)$	$\neg I(2,10)$

\leftarrow deliberately being inconsistent here