Lesson 8: Interpretations and Visualizations of Odds Ratios

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Last time to this time

- Used the Wald test and Wald 95% confidence interval to interpret coefficients in a fitted model
- This time: Interpret using odds ratio

Learning Objectives

1. Interpret odds ratios from fitted simple logistic regression model for a continuous explanatory variable.

2. Interpret odds ratios from fitted simple logistic regression model for a binary explanatory variable.

3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.

4. Report the odds ratio using a table and/or a forest plot.

So far we've looked at the association using the log-odds scale

For a *population* simple logistic regression model with a continuous predictor

$$\text{logit}(\pi(X)) = \beta_0 + \beta_1 \cdot X$$

- β_0: log-odds when X is 0
- β_1: increase in log-odds for every 1 unit increase in X

For our *fitted* simple logistic regression model with a continuous predictor *w/ data*

$$\text{logit}(\widehat{\pi}(X)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot X$$

- $\widehat{\beta}_0$: estimated log-odds of $Y = 1$ when X is 0.
- $\widehat{\beta}_1$: estimated increase in log-odds of $Y = 1$ for every 1 unit increase in X
- Can use expected instead of estimated

Example: From last class

```
1  bc_reg = glm(Late_stage_diag ~ Age_c, data = bc, family = binomial)
2  tidy(bc_reg, conf.int=T) %>% gt() %>% tab_options(table.font.size = 35) %>%
3    fmt_number(decimals = 3)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	−0.989	0.023	−42.637	0.000	−1.035	−0.944
Age_c	0.057	0.003	17.780	0.000	0.051	0.063

- For our *fitted* simple logistic regression model with age as a predictor

$$\text{logit}(\widehat{\pi}(Age^c)) = -0.989 + 0.057 \cdot Age^c$$

mean age when $Age^c = 0$

- $\widehat{\beta}_0$: The estimated log-odds is -0.989 when age is 61.71 years (95% CI: -1.035, -0.944)

- $\widehat{\beta}_1$: The estimated increase in log-odds is 0.057 for every 1 year increase in age (95% CI: 0.051, 0.063).

Poll Everywhere Question 1

We typically interpret our results using odds ratios
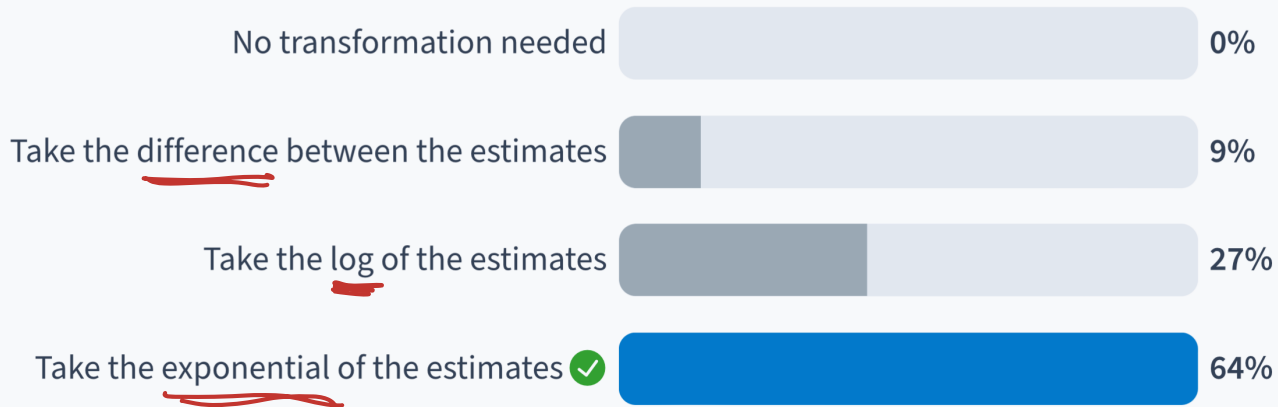
For our *fitted* simple logistic regression model with a continuous predictor

$$\text{logit}(\widehat{\pi}(X)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot X$$

- How do we go from interpretations of $\widehat{\beta}_0$ and $\widehat{\beta}_1$ using log odds to odds ratios?
- We will need to take the exponential of our model:
  - $\exp(\widehat{\beta}_0)$: expected odds that $Y = 1$ when X is 0.
  - $\exp(\widehat{\beta}_1)$: expected odds ratio that $Y = 1$ for every 1 unit increase in X
- **Important distinction:**
  - We take the **inverse logit** to find our **predicted probability**
  - We take the **exponential** to interpret the **odds/odds ratios**

$$\widehat{\pi} = \frac{e^{\widehat{\beta}_0 + \widehat{\beta}_1 X}}{1 + e^{\widehat{\beta}_0 + \widehat{\beta}_1 X}} \qquad inv \; logit$$

$$\widehat{OR}_X = e^{\widehat{\beta}_1} \qquad 2$$

Intro/Recap of Interpreting Fitted Model

- Interpret coefficients from fitted logistic regression model
  - **Goodness-of-fit of model should be assessed before summarizing findings** (have not covered yet)
  - In this lecture: assume model fits data well
- The interpretation of the coefficients involves two issues:
  - The functional relationship between the dependent variable and the independent variable (link function)
  - Unit of change for the independent variable
- We will learn the interpretation for
  - Binary independent variable
  - Categorical independent variable with multiple groups
    - We looked at this for our race and ethnicity variable
  - Continuous independent variable

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1. Interpret odds ratios from fitted simple logistic regression model for a continuous explanatory variable.

2. Interpret odds ratios from fitted simple logistic regression model for a binary explanatory variable.

3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.

4. Report the odds ratio using a table and/or a forest plot.

Coefficient interpretation: Continuous Independent Variable

- For simplicity, we assume the linear relationship between logit and continuous variable x

- Again using simple logistic regression model to illustrate the interpretation of $\widehat{\beta}$ for a continuous variable x

$$\text{logit}(\widehat{\pi}(X)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot X$$

- The estimated slope coefficient, $\widehat{\beta}_1$, is the **expected change in the log odds for 1 unit increase in** x
  - Additional attention should be paid to picking a meaningful units of change in x

How do we get the odds ratio for X's coefficient?

For $\exp(\widehat{\beta}_1)$

- We compare $X = x$ and $X = x + 1$,
- So we have

$$\text{logit}(\widehat{\pi}(X = x)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot x$$

and

$$\text{logit}(\widehat{\pi}(X = x + 1)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot (x + 1)$$

- And...

$$\text{logit}(\widehat{\pi}(X = x + 1)) - \text{logit}(\widehat{\pi}(X = x))$$
$$= \widehat{\beta}_0 + \widehat{\beta}_1 \cdot (x + 1) - [\widehat{\beta}_0 + \widehat{\beta}_1 \cdot x]$$
$$= \widehat{\beta}_0 + \widehat{\beta}_1 \cdot x + \widehat{\beta}_1 - \widehat{\beta}_0 - \widehat{\beta}_1 \cdot x$$
$$= \widehat{\beta}_1$$

$\Delta X = 1$ 1 unit inc

- Thus,

$\log \text{odds} = \text{logit}(\widehat{\pi})$

$\log \text{odds} \left(\frac{\pi}{1-\pi} \right)$

$$\widehat{\beta}_1 = \text{logit}(\widehat{\pi}(X = x + 1)) - \text{logit}(\widehat{\pi}(X = x))$$

$$\widehat{\beta}_1 = \log \left(\frac{\widehat{\pi}(X = x + 1)}{1 - \widehat{\pi}(X = x + 1)} \right) - \log \left(\frac{\widehat{\pi}(X = x)}{1 - \widehat{\pi}(X = x)} \right)$$

$$\widehat{\beta}_1 = \log \left(\frac{\dfrac{\widehat{\pi}(X = x + 1)}{1 - \widehat{\pi}(X = x + 1)}}{\dfrac{\widehat{\pi}(X = x)}{1 - \widehat{\pi}(X = x)}} \right)$$

$\log(a) - \log(b)$
$= \log \left(\dfrac{a}{b} \right)$

$$\exp\left[\widehat{\beta}_1\right] = \exp\left[\log \left(\frac{\text{odds}_{X=x+1}}{\text{odds}_{X=x}} \right) \right]$$

$$\exp\left[\widehat{\beta}_1\right] = \frac{\text{odds}_{X=x+1}}{\text{odds}_{X=x}}$$

$\widehat{OR}$ for 1 unit inc of x

Example: Interpretation of Age Coefficient/OR

- $\widehat{\beta}_1$ is 0.057, suggesting that one year increase in age is associated with 0.057 increase in log odds of receiving a late stage breast cancer diagnosis

- $\exp\left(\widehat{\beta}_1\right)$ is 1.06 suggesting that one year increase in age is associated with 1.06 times the odds of receiving a late stage breast cancer diagnosis

- For continuous covariates in logistic regression model, it is helpful to subtract 1 from the odds ratio and multiply by 100 to obtain the percentage change in odds for 1-unit increase.

  - The estimated OR for age is 1.06, suggesting that a 1-year increase in age is associated with a 6% increase in the predicted odds of late stage diagnosis in the patient population

$$0.06 \longrightarrow 6\%$$

Example: Age and Late Stage Diagnosis (1/5)

> **Odds ratio from logistic regression**
>
> Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

Needed steps:

1. Fit the regression model -

2. Transform the coefficients into odds ratios ←

3. Interpret the odds ratio ·

Example: Age and Late Stage Diagnosis (2/5)

Odds ratio from logistic regression

Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

---

1. Fit the regression model

```
1  bc_reg = glm(Late_stage_diag ~ Age_c, data = bc, family = binomial)
2  summary(bc_reg)
```

```
Call:
glm(formula = Late_stage_diag ~ Age_c, family = binomial, data = bc)

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.989422   0.023205  -42.64   <2e-16 ***
Age_c        0.056965   0.003204   17.78   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 11861  on 9999   degrees of freedom
```

```
Residual deviance: 11510  on 9998  degrees of freedom
AIC: 11514

Number of Fisher Scoring iterations: 4
```

Example: Age and Late Stage Diagnosis (3/5)

> **Odds ratio from logistic regression**
>
> Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

2. Transform the coefficients into odds ratios • Option 1: `tidy()`

glm output

$$e^{\hat{\beta_1}} \quad e^{\hat{\beta_0}}$$

```
1  tidy_bc_reg = tidy(bc_reg, conf.int=T, exponentiate = T)
2  tidy_bc_reg %>% gt() %>% tab_options(table.font.size = 35) %>%
3    fmt_number(decimals = 3)
```

$\widehat{OR}$ $\widehat{OR}_{CL}$ $\widehat{OR}_{CH}$

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	0.372	0.023	−42.637	0.000	0.355	0.389
Age_c	1.059	0.003	17.780	0.000	1.052	1.065

```
1  tidy_bc_reg$conf.low # I prefer tidy() bc now I can grab each component
```
```
[1] 0.3551931 1.0520321
```

Example: Age and Late Stage Diagnosis (4/5)

> ## Odds ratio from logistic regression
>
> Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

2. Transform the coefficients into odds ratios • Option 2: `logistic.display()`

```
1  logistic.display(bc_reg) # Cannot grab each component in this
```

```
Logistic regression predicting Late_stage_diag : 1 vs 0

                    OR(95%CI)            P(Wald's test) P(LR-test)
Age_c (cont. var.)  1.06  (1.05,1.07)    < 0.001        < 0.001

Log-likelihood = -5754.8442
No. of observations = 10000
AIC value = 11513.6884
```
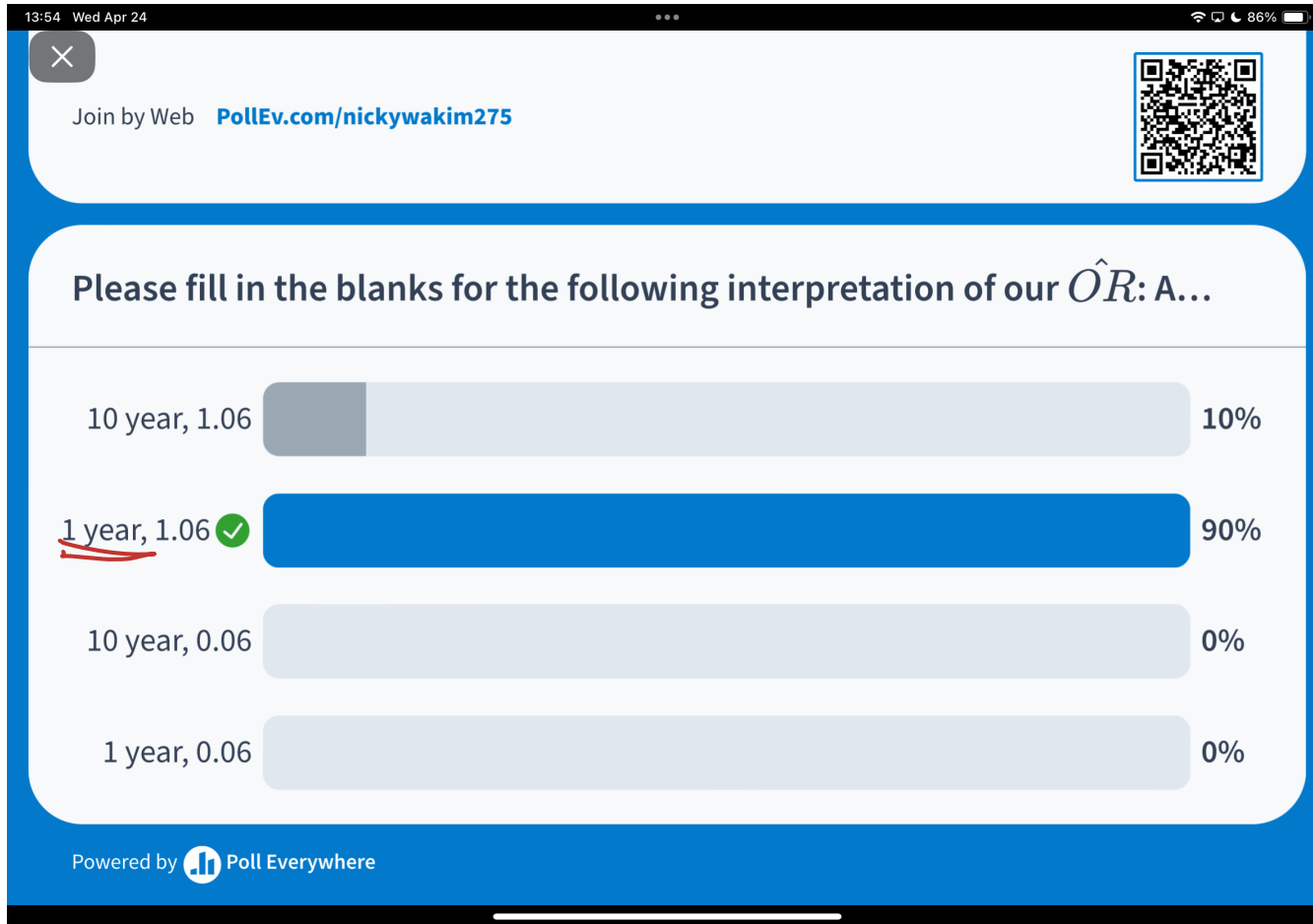
Poll Everywhere Question 1

Example: Age and Late Stage Diagnosis (5/5)

> **Odds ratio from logistic regression**
>
> Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

3. Interpret the odds ratio

For every one year increase in age, there is an estimated 5.86% increase in the ~~estimated~~ odds of late stage breast cancer diagnosis (95% CI: 5.2%, 6.53%).

6%

look @ my qmd

Transformations of continuous variable to make more interpretable

- Sometimes a change in "1" unit may not be considered clinically interesting
  - For example, a 1 year increase in age or a 1 mm Hg increase in systolic blood pressure may be too small for a meaningful change in log odds
  - Instead, we may be interested to find out the log odds change for a increase of 10 years in age or 10 mm Hg in systolic blood pressure
  - On the other hand, if the range of x is small (say 0-1), than a change in 1 unit of x is too large to be meaningful
- We should be able to compute and interpret coefficients for a continuous independent covariate x for an arbitrary change of "c" units in x

0.1 inc

Transformations of continuous variable to make more interpretable

- The estimated log odds ratio for a change of c units in x can be obtained from

$$\hat{g}(x + c) - \hat{g}(x) = c\hat{\beta}_1$$

- $\widehat{OR}(c) = \exp\left(c\hat{\beta}_1\right)$

- The 95% CI for $\widehat{OR}(c)$ is:

$$\exp\left(c\hat{\beta}_1 \pm 1.96 \cdot c \cdot SE_{\hat{\beta}_1}\right)$$

- The c is chosen to be a clinically meaningful unit change in x

- The value of c should be clearly specified in all tables and calculations

  - Because the estimated OR and the corresponding CI depends on the choice of c value

Example: 10 year increase in age and Late Stage Diagnosis

- What if we are interested in learning the OR corresponding to 10-year increase in age?

```
1  bc_reg = glm(Late_stage_diag ~ Age_c, data = bc, family = binomial)
2  tidy(bc_reg, conf.int=T) %>% gt() %>% tab_options(table.font.size = 35) %>%
3    fmt_number(decimals = 3)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	−0.989	0.023	−42.637	0.000	−1.035	−0.944
Age_c	0.057	0.003	17.780	0.000	0.051	0.063

$$10 \cdot 0.057$$

Example: 10 year increase in age and Late Stage Diagnosis

- What if we are interested in learning the OR corresponding to 10-year increase in age?

$$\widehat{OR}\,(10) = \exp\left(10 \cdot \hat{\beta}_1\right) = \exp\left(0.56965\right) = 1.767$$

- The 95% CI for $\widehat{OR}\,(10)$ is:

$$\begin{aligned}
\widehat{OR}\,(10) &= \exp\left(10 \cdot \hat{\beta}_1 \pm 1.96 \cdot 10 \cdot SE_{\hat{\beta}_1}\right) \\
&= \exp\left(10 \cdot 0.056965 \pm 1.96 \cdot 10 \cdot 0.003204\right) \\
&= (1.66,\ 1.88)
\end{aligned}$$

Example: 10 year increase in age and Late Stage Diagnosis

- What if we are interested in learning the OR corresponding to 10-year increase in age?

```
1  bc2 = bc %>% mutate(Age_c_10 = Age_c/10)
2  bc_reg_10 = glm(Late_stage_diag ~ Age_c_10, data = bc2, family = binomial)
3  tidy(bc_reg_10, conf.int=T, exponentiate = T) %>% gt() %>% tab_options(table.font.s
4    fmt_number(decimals = 3)
```

1 unit inc = 10 yr inc

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	0.372	0.023	−42.637	0.000	0.355	0.389
Age_c_10	1.768	0.032	17.780	0.000	1.661	1.883

$e^{0.57}$

Last Note About Continuous Independent Variable

- Notice that the logistic regression model suggests that logit is linear in the covariate

- The model implies the additional risk of late stage breast cancer diagnosis for a 40 year-old compared to a 30 year-old is the same as the additional risk of late stage breast cancer diagnosis for a 60 year-old compared to a 50-year-old

- This assumption may not be realistic

- To address this, we may consider using higher order terms (e.g., x^2, x^3,...) or other nonlinear transformation(e.g., $log(x)$)

- Categorize the continuous variable may be another option
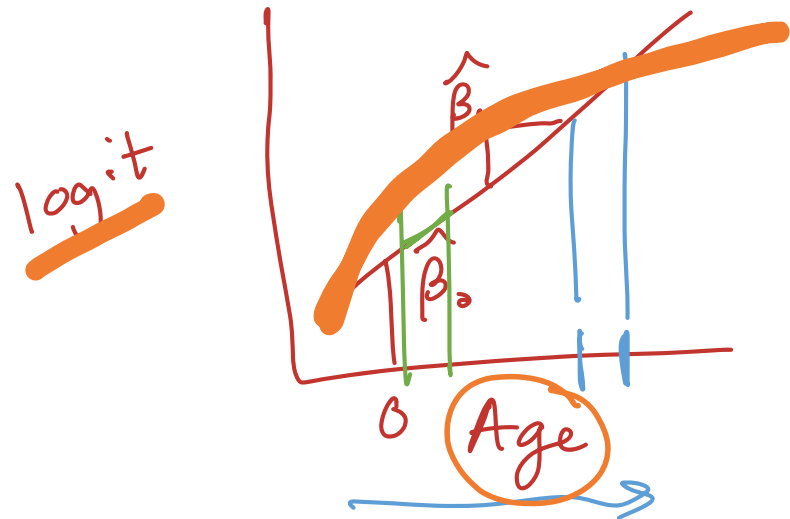
How do we get the odds for the intercept?

For $\exp\left(\widehat{\beta}_0\right)$

- When $X = 0$, we have

$$\text{logit}(\widehat{\pi}(X = 0)) = \widehat{\beta}_0 + \widehat{\beta}_1(0)$$

- Thus,

$$\widehat{\beta}_0 = \text{logit}(\widehat{\pi}(X))$$
$$\exp\left[\widehat{\beta}_0\right] = \exp\left[\text{logit}(\widehat{\pi}(X))\right]$$
$$\exp\left[\widehat{\beta}_0\right] = \exp\left[\log\left(\frac{\widehat{\pi}(X)}{1 - \widehat{\pi}(X)}\right)\right]$$
$$\exp\left[\widehat{\beta}_0\right] = \frac{\widehat{\pi}(X)}{1 - \widehat{\pi}(X)}$$

odds when $X = 0$

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3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.

4. Report the odds ratio using a table and/or a forest plot.

Coefficient Interpretation: Binary Independent Variable

- Independent variable x is a binary variable (x can take values: 0 or 1)

- We are fitting the simple logistic regression model:

$$\text{logit}\,(\pi(X)) = \beta_0 + \beta_1 \cdot I(X = 1)$$

ref = 0

→ Yes / No
A / B

$I(X = Yes)$ or $I(X = B)$
ref = N   ref = A

- The logit difference is β_1 for binary independent variable

  - β_1 represents the change/difference in the logit for $x = 1$ vs. $x = 0$

- It will be much easier to understand if we can interpret the coefficient using odds ratio (OR)

Binary: How do we interpret the coefficient? (I)

- For individuals with $X = 0$: $\quad X = a$

$$\text{logit}\left(\pi(X=0)\right) = \beta_0 + \beta_1 \times (0) = \beta_0$$

$$I(X=1) = 0$$
$$I(X=b) = 0$$
$$X = a$$

- For individuals with $X = 1$: $\quad X = b$

$$\text{logit}\left(\pi(X=1)\right) = \beta_0 + \beta_1 \times (1) = \beta_0 + \beta_1$$

$$I(X=1) = 1$$
$$I(X=b) = 1$$

- To solve for β_1, we take the difference of the logits:

$$\text{logit}\left(\pi(X=1)\right) - \text{logit}\left(\pi(X=0)\right) = (\beta_0 + \beta_1) - (\beta_0) = \beta_1$$

Binary: How do we interpret the coefficient? (II)

$$\text{logit}\left(\pi(X=1)\right) - \text{logit}\left(\pi(X=0)\right) = (\beta_0 + \beta_1) - (\beta_0) = \beta_1$$

$$\beta_1 = logit\left(\pi(X=1)\right) - logit\left(\pi(X=0)\right)$$

$$\beta_1 = log\left(\frac{\pi(X=1)}{1-\pi(X=1)}\right) - log\left(\frac{\pi(X=0)}{1-\pi(X=0)}\right)$$

$$\beta_1 = log\left(\frac{\frac{\pi(X=1)}{1-\pi(X=1)}}{\frac{\pi(X=0)}{1-\pi(X=0)}}\right)$$

$$\exp(\beta_1) = \frac{\frac{\pi(X=1)}{1-\pi(X=1)}}{\frac{\pi(X=0)}{1-\pi(X=0)}}$$

$odds_2$ $odds_B \leftarrow$

$= OR$

odd_0 $odds_A \leftarrow$

Review of Odds Ratio

- Odds for a subject with $X = 1$:

$$\text{odds}_1 = \frac{\pi(X = 1)}{1 - \pi(X = 1)}$$

- Odds for a subject with $X = 0$:

$$\text{odds}_0 = \frac{\pi(X = 0)}{1 - \pi(X = 0)}$$

- Odds Ratio for $X = 1$ vs. $X = 0$:

$$OR = \frac{\dfrac{\pi(X = 1)}{1 - \pi(X = 1)}}{\dfrac{\pi(X = 0)}{1 - \pi(X = 0)}}$$

How does this relate to a 2x2 table?

- 2x2 table with the respective logistic functions in each cell

Outcome Variable (Y)	Independent Variable (X)	
	$X = 1$	$X = 0$
$Y = 1$	$\pi(1) = \dfrac{\exp(\beta_0 + \beta_1)}{1 + \exp(\beta_0 + \beta_1)}$	$\pi(0) = \dfrac{\exp(\beta_0)}{1 + \exp(\beta_0)}$
$Y = 0$	$1 - \pi(1) = \dfrac{1}{1 + \exp(\beta_0 + \beta_1)}$	$1 - \pi(0) = \dfrac{1}{1 + \exp(\beta_0)}$
Total	1.0	1.0

Recall:

$\pi(1) = \pi(X = 1)$
$= P(Y = 1 | X = 1)$

$\pi(0) = \pi(X = 0)$
$= P(Y = 1 | X = 0)$

Poll Everywhere Question 2

True or False: The estimated odds ratio from a contingency table between two binary variables is the same as the estimated odds ratio from simple logistic regression of those variables.

↳ cont variables OR more than 1 var.

True ✓ — 60%

False — 40%

How does this relate to a 2x2 table?

$$OR = \frac{a/c}{b/d} = \frac{\left(\dfrac{\exp(\beta_0 + \beta_1)}{1 + \exp(\beta_0 + \beta_1)}\right)}{\left(\dfrac{1}{1 + \exp(\beta_0 + \beta_1)}\right)} \Big/ \frac{\left(\dfrac{\exp(\beta_0)}{1 + \exp(\beta_0)}\right)}{\left(\dfrac{1}{1 + \exp(\beta_0)}\right)} = \frac{\exp(\beta_0 + \beta_1)}{\exp(\beta_0)} = e^{\beta_1}$$

$$e^{\beta_0 + \beta_1 - (\beta_0)}$$

$$\frac{e^x}{e^y} = e^{x-y}$$

	1	0
1	a	b
0	c	d

- Simple relationship between coefficient and odds ratio is a primary reason why we report OR for categorical data analysis.

- For binary independent variable x, **OR computed in logistic regression** model is the **same as OR computed using contingency table**

Example: Binary age and Late Stage Diagnosis (I)

> **Odds ratio from logistic regression**
>
> What is the odds ratio of late stage breast cancer diagnosis for older individuals (>65 years old) compared to younger individuals (≤65 years old)?

- Two options to calculate this value:
  - **Option 1:** Calculate $\widehat{OR}$ from 2x2 contingency table *practice*
    - Refer to Lesson 3 for this process
  - **Option 2:** Calculate $\widehat{OR}$ from logistic regression

Needed steps for Option 2:

1. Fit the regression model
2. Transform the coefficients into odds ratios
3. Interpret the odds ratio ⟵

Example: Binary age and Late Stage Diagnosis (I)

> **Odds ratio from logistic regression**
>
> What is the odds ratio of late stage breast cancer diagnosis for older individuals (>65 years old) compared to younger individuals (≤65 years old)?

1. Fit the regression model

```
1  bc3 = bc %>% mutate(Age_binary = ifelse(Age > 65, 1, 0))
2  age_bin_glm = glm(Late_stage_diag ~ Age_binary, data = bc3, family = binomial)
```

Example: Binary age and Late Stage Diagnosis (I)

> **Odds ratio from logistic regression**
>
> What is the odds ratio of late stage breast cancer diagnosis for older individuals (>65 years old) compared to younger individuals (≤65 years old)?

2. Transform the coefficients into odds ratios

```
1  age_bin_tidy = tidy(age_bin_glm, conf.int=T, exponentiate = T)
2  age_bin_tidy %>% gt() %>%
3    tab_options(table.font.size = 35) %>%
4    fmt_number(decimals = 3)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	0.297	0.031	−39.608	0.000	0.280	0.315
Age_binary	1.875	0.045	13.928	0.000	1.716	2.048

≥65 yo
<65 yo

Poll Everywhere Question 3

Join by Web   PollEv.com/nickywakim275

Please fill in the blanks for the following interpretation of our $\hat{OR}$: The estimated odds of late stage breast cancer among individuals over 65 years old is 1.87 (95% CI: (_____, 2.05)) times _____than individuals 65 years or younger.

higher

♥ 10

- 0.29, higher

- 0.29, lower

- 1.72, higher

- 1.72, lower

Powered by Poll Everywhere

Example: Binary age and Late Stage Diagnosis (I)

> **Odds ratio from logistic regression**
>
> What is the odds ratio of late stage breast cancer diagnosis for older individuals (>65 years old) compared to younger individuals (≤65 years old)?

3. Interpret the odds ratio

The estimated odds of late stage breast cancer among individuals over 65 years old is 1.87 (95% CI: (1.72, 2.05)) times that of individuals 65 years or younger.

Learning Objectives

1. Interpret odds ratios from fitted simple logistic regression model for a continuous explanatory variable.

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3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.

4. Report the odds ratio using a table and/or a forest plot.

Coefficient Interpretation: Multi-group Categorical Variable

- Independent variable x is a multi-level categorical variable

- Let's say X takes values: a, b, c, or d

- We are fitting the simple logistic regression model:

$$\text{logit}\,(\pi(X)) = \beta_0 + \beta_1 \cdot I(X = b) + \beta_2 \cdot I(X = c) + \beta_3 \cdot I(X = d)$$

a ref

  - Where a is our reference group

- The logit difference is β_1 for binary independent variable

  - β_1 represents the change/difference in the logit for $x = b$ vs. $x = a$

- It will be much easier to understand if we can interpret the coefficient using odds ratio (OR)

β_3 : diff X = d vs. X = a

Coefficient Interpretation: Multi-group Categorical Variable

We are fitting the simple logistic regression model with reference group a:

$$\text{logit}\left(\pi(X)\right) = \beta_0 + \beta_1 \cdot I(X = b) + \beta_2 \cdot I(X = c) + \beta_3 \cdot I(X = d)$$

- β_0: the log-odds of event $Y = 1$ for group a
- β_1: the difference in log-odds of event $Y = 1$ comparing group b to group a
- β_2: the difference in log-odds of event $Y = 1$ comparing group c to group a
- β_3: the difference in log-odds of event $Y = 1$ comparing group d to group a

Multi-level categorical: How do we interpret the coefficient? (II)

$$\text{logit}\left(\pi(X = c)\right) - \text{logit}\left(\pi(X = a)\right) = (\beta_0 + \beta_1 \cdot 0 + \beta_2 \cdot 1 + \beta_3 \cdot 0) - (\beta_0 + \beta_1 \cdot 0 + \beta_2 \cdot 0 + \beta_3 \cdot 0) = \beta_2$$

$$\beta_2 = \text{logit}\left(\pi(X = c)\right) - \text{logit}\left(\pi(X = a)\right)$$

$$\beta_2 = \log\left(\frac{\pi(X = c)}{1 - \pi(X = c)}\right) - \log\left(\frac{\pi(X = a)}{1 - \pi(X = a)}\right)$$

$$\beta_2 = \log\left(\frac{\dfrac{\pi(X = c)}{1 - \pi(X = c)}}{\dfrac{\pi(X = a)}{1 - \pi(X = a)}}\right)$$

$$\exp\left(\beta_2\right) = \frac{\dfrac{\pi(X = c)}{1 - \pi(X = c)}}{\dfrac{\pi(X = a)}{1 - \pi(X = a)}} \qquad \frac{odds_c}{odds_a} \;\; \begin{array}{l}\text{of } Y = 1 \\ \text{for } c\end{array}$$

Coefficient Interpretation: Multi-group Categorical Variable

We are fitting the simple logistic regression model with reference group a:

$$\text{logit}\left(\pi(X)\right) = \beta_0 + \beta_1 \cdot I(X = b) + \beta_2 \cdot I(X = c) + \beta_3 \cdot I(X = d)$$

- $\exp\left(\beta_0\right)$: the odds of event $Y = 1$ for group a
- $\exp\left(\beta_1\right)$: the odds of event $Y = 1$ for group b is $\exp\left(\beta_1\right)$ times the odds of event $Y = 1$ for group a
- $\exp\left(\beta_2\right)$: the odds of event $Y = 1$ for group c is $\exp\left(\beta_2\right)$ times the odds of event $Y = 1$ for group a
- $\exp\left(\beta_3\right)$: the odds of event $Y = 1$ for group d is $\exp\left(\beta_3\right)$ times the odds of event $Y = 1$ for group a

How do we pick the reference group?

- The choice can be more apparent for multi-group categorical independent variables within studies
- For example, if we want to evaluate the association between clinical response and four treatments.
  - The treatment variable has 4 categories: "active treatment A", "active treatment B", "active treatment C" and "Placebo treatment"
  - The investigator is interested in comparing each of the three active treatment with the placebo treatment
  - Then the placebo treatment should be picked as the reference group

Example: Late stage diagnosis and race and ethnicity

- Chose Non-Hispanic White individuals as reference group

- Underlying health disparities linked to racism in healthcare and in clinical studies

- There is evidence that white individuals receive a certain standard of care that is not paralleled for POC Mateo and Williams (2021)

Race/Ethnicity	Breast Cancer Diagnosis		Total
	Early Stage	Late Stage	
Non-Hispanic White	5,321	1,980	7,301
Non-Hispanic Black	683	357	1,040
Non-Hispanic Asian/Pacific Islander	556	234	790
Hispanic-Latinx	575	271	846
Non-Hispanic American Indian/Alaska Native	17	6	23
Total	7,152	2,848	10,000

SE

Example: Late stage diagnosis and race and ethnicity

Odds ratio from logistic regression

What is the odds ratio of late stage breast cancer diagnosis for Non-Hispanic Asian/Pacific Islander individuals compared to Non-Hispanic White individuals?

Needed steps:

1. Fit the regression model

2. Transform the coefficients into odds ratios

3. Interpret the odds ratio

Example: Late stage diagnosis and race and ethnicity

<div style="background-color: #f0ad4e; color: white; padding: 10px;">

Odds ratio from logistic regression

</div>

What is the odds ratio of late stage breast cancer diagnosis for Non-Hispanic Asian/Pacific Islander individuals compared to Non-Hispanic White individuals?

1. Fit the regression model

```
1  RE_glm = glm(Late_stage_diag ~ Race_Ethnicity, data = bc,
2                   family = binomial)
```

Example: Late stage diagnosis and race and ethnicity

Odds ratio from logistic regression

What is the odds ratio of late stage breast cancer diagnosis for Non-Hispanic Asian/Pacific Islander individuals compared to Non-Hispanic White individuals?

2. Transform the coefficients into odds ratios

```
1  RE_tidy = tidy(RE_glm, conf.int=T, exponentiate = T)
2  RE_tidy %>% gt() %>%
3    tab_options(table.font.size = 35) %>%
4    fmt_number(decimals = 3)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept) *Odds for NH White*	0.372	0.026	−37.553	0.000	0.353	0.392
Race_EthnicityHispanic-Latino	0.968	0.082	−0.398	0.691	0.822	1.135
Race_EthnicityNH American Indian/Alaskan Native	0.948	0.476	−0.111	0.911	0.342	2.287
Race_EthnicityNH Asian/Pacific Islander	1.131	0.082	1.497	0.134	0.961	1.327
Race_EthnicityNH Black	1.405	0.070	4.826	0.000	1.223	1.611

Example: Late stage diagnosis and race and ethnicity

> **Odds ratio from logistic regression**
>
> What is the odds ratio of late stage breast cancer diagnosis for Non-Hispanic Asian/Pacific Islander individuals compared to Non-Hispanic White individuals?

3. Interpret the odds ratio

The estimated odds of late stage breast cancer among Non-Hispanic Asian/Pacific Islander individuals is 1.13 (95% CI: (0.96, 1.33)) times that of Non-Hispanic White individuals.

What if you want to compare other groups?

- What if we want to estimate OR comparing Non-Hispanic Asian Pacific Islander to Non-Hispanic Black individuals?

- **Option 1:** Change reference group and refit the model (maybe the easiest option)

- **Option 2:** Estimate OR using fitted coefficients ($\widehat{\beta}$'s) in the current model:

$$\log\left(OR(\text{NH API}, \text{NH B})\right) = \text{logit}\left(\pi\left(X = \text{NH API}\right)\right) - \text{logit}\left(\pi\left(X = \text{NH B}\right)\right)$$
$$= [\beta_0 + \beta_3 \cdot 1] - [\beta_0 + \beta_4 \cdot 1]$$
$$\log\left(\widehat{OR}(\text{NH API}, \text{NH B})\right) = \widehat{\beta}_3 - \widehat{\beta}_4$$
$$\widehat{OR}(\text{NH API}, \text{NH B}) = \exp\left(\widehat{\beta}_3 - \widehat{\beta}_4\right)$$

Poll Everywhere Question 5

What if you want to compare other groups? Option 1

```
1  bc3 = bc %>%
2    mutate(Race_Ethnicity = relevel(Race_Ethnicity, ref = "NH Black"))
3  RE_glm2 = glm(Late_stage_diag ~ Race_Ethnicity, data = bc3,
4                 family = binomial)
5  tidy(RE_glm2, conf.int=T, exponentiate = T) %>% gt() %>%
6    tab_options(table.font.size = 38) %>%
7    fmt_number(decimals = 3)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	0.523	0.065	−9.934	0.000	0.459	0.594
Race_EthnicityNH White	0.712	0.070	−4.826	0.000	0.621	0.818
Race_EthnicityHispanic-Latino	0.689	0.102	−3.664	0.000	0.564	0.840
Race_EthnicityNH American Indian/Alaskan Native	0.675	0.479	−0.819	0.413	0.242	1.641
Race_EthnicityNH Asian/Pacific Islander	0.805	0.102	−2.131	0.033	0.659	0.982

Learning Objectives

1. Interpret odds ratios from fitted simple logistic regression model for a continuous explanatory variable.

2. Interpret odds ratios from fitted simple logistic regression model for a binary explanatory variable.

3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.

4. Report the odds ratio using a table and/or a forest plot.

How to present odds ratios: Table

- `tbl_regression()` in the `gtsummary` package is helpful for presenting the odds ratios in a clean way

```
1  library(gtsummary)
2  tbl_regression(RE_glm, exponentiate = TRUE) %>%
3  as_gt() %>% # allows us to use tab_options()
4  tab_options(table.font.size = 38)
```

Characteristic	OR[1]	95% CI[1]	p-value
Race/Ethnicity			
NH White	—	—	
Hispanic-Latino	0.97	0.82, 1.14	0.7
NH American Indian/Alaskan Native	0.95	0.34, 2.29	>0.9
NH Asian/Pacific Islander	1.13	0.96, 1.33	0.13
NH Black	1.40	1.22, 1.61	<0.001

[1] OR = Odds Ratio, CI = Confidence Interval

How to present odds ratios: Forest Plot Setup

```r
1  library(broom.helpers)
2  RE_tidy = tidy_and_attach(RE_glm, conf.int=T, exponentiate = T) %>%
3    tidy_remove_intercept() %>%
4    tidy_add_reference_rows() %>% tidy_add_estimate_to_reference_rows() %>%
5    tidy_add_term_labels()
6  glimpse(RE_tidy)
```

```
Rows: 5
Columns: 16
$ term   <chr> "Race_EthnicityNH White", "Race_EthnicityHispanic-Latin…
$ variable   <chr> "Race_Ethnicity", "Race_Ethnicity", "Race_Ethnicity", "…
$ var_label   <chr> "Race_Ethnicity", "Race_Ethnicity", "Race_Ethnicity", "…
$ var_class   <chr> "factor", "factor", "factor", "factor", "factor"
$ var_type   <chr> "categorical", "categorical", "categorical", "categoric…
$ var_nlevels   <int> 5, 5, 5, 5, 5
$ contrasts   <chr> "contr.treatment", "contr.treatment", "contr.treatment"…
$ contrasts_type <chr> "treatment", "treatment", "treatment", "treatment", "tr…
$ reference_row <lgl> TRUE, FALSE, FALSE, FALSE, FALSE
$ label   <chr> "NH White", "Hispanic-Latino", "NH American Indian/Alas…
$ estimate   <dbl> 1.0000000, 0.9678002, 0.9484848, 1.1310170, 1.4046741
$ std.error   <dbl> NA, 0.08224948, 0.47558680, 0.08224988, 0.07041472
$ statistic   <dbl> NA, -0.3979312, -0.1112089, 1.4968682, 4.8257715
$ p.value   <dbl> NA, 6.906809e-01, 9.114507e-01, 1.344276e-01, 1.394623e…
$ conf.low   <dbl> NA, 0.8223138, 0.3417844, 0.9612074, 1.2226824
$ conf.high   <dbl> NA, 1.135332, 2.286596, 1.327092, 1.611466
```

# How to present odds ratios: Forest Plot

```
1  ggplot(data=RE_tidy, aes(y=label, x=estimate, xmin=conf.low, xmax=conf.high)) +
2    geom_point(size = 3) +  geom_errorbarh(height=.2) +
3    geom_vline(xintercept=1, color='#C2352F', linetype='dashed', alpha=1) +
4    theme_classic() +
5    labs(x = "OR (95% CI)", y = "Race and ethnicity",
6         title = "Odds ratios of Late Stage \n Breast Cancer Diagnosis") +
7    theme(axis.title = element_text(size = 25), axis.text = element_text(size = 25),
```
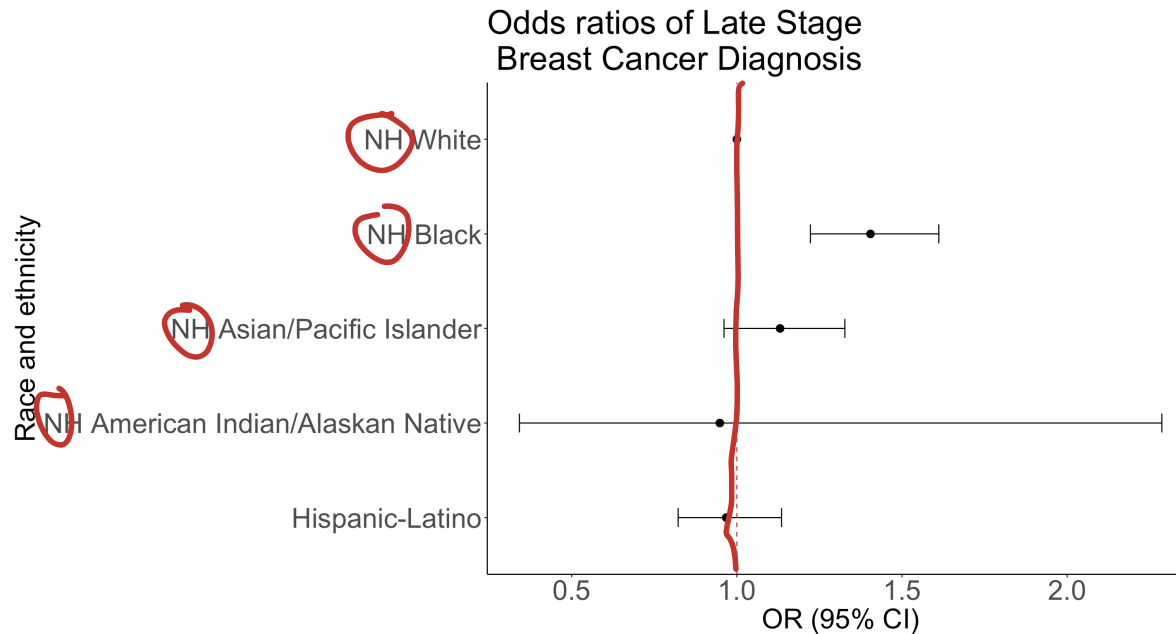
References

Mateo, Camila M., and David R. Williams. 2021. "Racism: A Fundamental Driver of Racial Disparities in Health-Care Quality." *Nature Reviews Disease Primers* 7 (1): 1–2. https://doi.org/10.1038/s41572-021-00258-1.

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