

Chapter 3: Independent Events

Meike Niederhausen and Nicky Wakim

2023-10-02

Table of contents

- Learning objectives
- Revisiting our coin toss
- Independent Events
- Example of two dice
- Independence of 3 Events
- Probability at least one smoker

Learning objectives

1. Define independence of 2-3 events given probability notation
2. Calculate whether two or more events are independent

Revisiting our coin toss

Question: Which of the following sequences of coin tosses of heads (H) and tails (T) is more likely to happen, assuming the coin is fair?

HTTHHHTHTHHTTTH

or

HTTTTTTTTHTTTTTT

Independent Events

Definition: Independence

Events A and B are **independent** if

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B).$$

Notation: For shorthand, we sometimes write

$$A \perp\!\!\!\perp B,$$

to denote that A and B are independent events.

Example of two dice

Example 1

Two dice (red and blue) are rolled. Let A = event a total of 7 appears, and B = event red die is a six. Are events A and B independent?

Independence of 3 Events

Definition: Independence of 3 Events

Events A , B , and C are *mutually independent* if

1. • $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$
 - $\mathbb{P}(A \cap C) = \mathbb{P}(A) \cdot \mathbb{P}(C)$
 - $\mathbb{P}(B \cap C) = \mathbb{P}(B) \cdot \mathbb{P}(C)$
2. $\mathbb{P}(A \cap B \cap C) = \mathbb{P}(A) \cdot \mathbb{P}(B) \cdot \mathbb{P}(C)$

Remark:

On your homework you will show that $(1) \not\Rightarrow (2)$ and $(2) \not\Rightarrow (1)$.

Probability at least one smoker

Example 2

Suppose you take a random sample of n people, of which people are smokers and non-smokers independently of each other. Let

- A_i = event person i is a smoker, for $i = 1, \dots, n$, and
- p_i = probability person i is a smoker, for $i = 1, \dots, n$.

Find the probability that at least one person in the random sample is a smoker.

