Chapter 11: Expected Values of Sums of Discrete RVs

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Learning Objectives

1. Calculate the mean (expected value) of *sums of* discrete random variables

Revisiting our two card draw

Example 1

Suppose you draw 2 cards from a standard deck of cards *with* replacement. Let X be the number of hearts you draw. Find $\mathbb{E}[X]$.

Recall Binomial RV with $n = 2$:

$$p_X(x) = \binom{2}{x} p^x (1-p)^{2-x} \text{ for } x = 0, 1, 2$$

What if we draw A LOT of cards?

Example 2

What is the expected number of hearts in Example 1 if you draw 200 cards?

Recall Binomial RV with $n = 200$:

$$p_X(x) = \binom{200}{x} p^x (1-p)^{200-x}$$

$$\text{for } x = 0, 1, 2, \ldots, 200$$

Sum of discrete RVs

Theorem 11.1: Sum of discrete RVs

For discrete r.v.'s X_i and constants $a_i, i = 1, 2, \ldots, n$,

$$\mathbb{E}\left[\sum_{i=1}^{n} a_i X_i\right] = \sum_{i=1}^{n} a_i \mathbb{E}[X_i].$$

Remark: The theorem holds for infinitely r.v.'s X_i as well.

- For two RVs, X and Y:
  - We can say $E[X + Y] = E[X] + E[Y]$
  - … and constant numbers a and b, we can also say $E[aX + bY] = aE[X] + bE[Y]$
  - We can also also say $E[X - Y] = E[X] - E[Y]$, since $b = -1$

Corollaries from Thm 11.1

Corollary 11.1.1

For a discrete r.v. X, and constants a and b,

$$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b.$$

Corollary 11.1.2

If $X_i, i = 1, 2, \ldots, n$, are identically distributed r.v.'s, then

$$\mathbb{E}\left[\sum_{i=1}^{n} X_i\right] = n\mathbb{E}[X_1].$$

Revisiting our ghost!

Example 3

The ghost is trick-or-treating at a different house now. In this case it is known that the bag of candy has 10 chocolates, 20 lollipops, and 30 laffy taffies. The ghost grabs a handful of five pieces of candy. How many pieces of chocolate do we expect the ghost to take?

Cost of hotel rooms

Example 4

A tour group is planning a visit to the city of Minneapolis and needs to book 30 hotel rooms. The average price of a room is $200. In addition, there is a 10% tourism tax for each room. What is the expected cost for the 30 hotel rooms?