# Homework 6

**BSTA 550**

2023-11-16

## Directions

Please turn in this homework on Sakai. Please submit your homework in pdf format. You can type your work on your computer or submit a photo of your written work or any other method that can be turned into a pdf. Please let me know if you greatly prefer to submit a physical copy. We can work out another way for you to turn in homework.

**Try to complete all of the problems listed below at some point this quarter! You may want to save some of them for studying later!** Only turn in the ones listed in the "Turn In" column. Please submit problems in the order they are listed.

The more work you include that shows your thought process, the more I can give you feedback.

| Chapter | Turn In | Extra Problems |
|---|---|---|
| Calculus Review | | NTB # 1 |
| 24 | TB # 19, 20* | # 2, 3, 7, 17, 18, 22, 23 |
| 25 | TB # 18, NTB # 2 | # 1, 4, 8, 17, 23, 24 |
| 26** | TB # 12, NTB # 3, 4 | # 7, 9, 19, 20 |
| 27 | TB # 12*** | # 6, 8, 13, 17 |

* (Ch 24) Also find the cdf $F_X(x)$

** Although within Chapter 26, these exercises are primarily practicing the material from Chapter 25.

** For Ch 27 # 12, in order to find the conditional densities in parts (a) and (b), you will need to calculate $f_Y(y)$ for the specific regions of $y$ specified. After finding the conditional densities in parts (a) and (b), also calculate the conditional probabilities below. Please submit these together with your other work in parts (a) and (b):

- Find $\mathbb{P}[0.5 < X < 3 | Y = 4]$.
- Find $\mathbb{P}[0.5 < X < 3 | Y = 7]$.

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Non-textbook problems (NTB):

1. Calculus Review

   a. $$\int_0^y c(x+y)dx$$

   b. $$\frac{d}{dx}\left(\frac{4}{9}x^2y^2 + \frac{5}{9}xy^4\right)$$

   c. $$\frac{d}{dy}\left(\frac{4}{9}x^2y^2 + \frac{5}{9}xy^4\right)$$

   d. $$\int_0^y 2e^{-x}e^{-y}dx$$

   e. $$\int_0^\infty xye^{-(x+y)}dy$$

   f. $$\int_x^{2x} 2e^{-(x+3y)}dy$$

   g. Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$ by integrating with respect to $x$.
   h. Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$ by integrating with respect to $y$.
   i. Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $y = x - 1$ by integrating with respect to $x$.
   j. Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $y = x - 1$ by integrating with respect to $y$.

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with common pdf $f_X(x)$ and cdf $F_X(x)$. Find the pdf for the random variable $Z$, where $Z = max(X_1, X_2, \ldots, X_n)$.

3. Let $X$ and $Y$ be independent random variables with respective pdf's $f_X(x) = \frac{1}{5}$, for $0 \leq x \leq 5$, and $f_Y(y) = 2e^{-2y}$, for $y > 0$.

a. Find the joint distribution $f_{X,Y}(x, y)$.
b. Find the probability that $X$ is less than $Y$.
c. Let $Z$ be the random variable that is the smaller of $X$ and $Y$. Find the cumulative distribution function for $Z$.
d. Find the pdf for Z.

4. Suppose that the random variables $X$ and $Y$ have joint density $f_{X,Y}(x, y)$, for $0 < x < 1$, and $\frac{1}{2} < y < 1$. Set up the equation for the cdf of $Z$, where $Z = X/Y$.

   *Hint: First determine what the possible values for $Z$* are. Then make a sketch of the domain of the joint pdf and shade in the region representing the cdf of Z for different values of $z$. Make sure to pay close attention to how the region we need to integrate over changes as $z$ changes. The cdf has two different cases depending on the value of $z$. Plug in specific values of $z$ and shade in the region representing the cdf to see why two different cases are needed.

## Some select answers

Selected answers (or hints) not provided at the end the book:

- Calculus Review
  - (a) $c(\frac{y^2}{2} + y^2)$
  - (b) $\frac{8}{9}xy^2 + \frac{5}{9}y^4$
  - (c) $\frac{8}{9}x^2y + \frac{20}{9}xy^3$
  - (d) $-2e^{-2y} + 2e^{-y}$
  - (e) $xe^{-x}$
  - (f) $-\frac{2}{3}(e^{-7x} - e^{-4x})$
  - (g) $\frac{9}{2}$
  - (h) $\frac{9}{2}$
  - (i) $\frac{9}{2}$
  - (j) $\frac{9}{2}$
- Chapter 24
  - # 2: (a) Discrete (b) Discrete (c) Continuous

- # 22:

$$f_X(x) = \begin{cases} 0 & x < 0 \\ \frac{7x}{4} & 0 \le x \le 1 \\ 0 & 1 < x < 7 \\ \frac{1}{8} & 7 \le x \le 8 \\ 0 & x > 8 \end{cases}$$

- Chapter 25
  - # 4: 7/16
  - # 8: (a) $\frac{25}{228}$ (b) $f_X(x) = \frac{1}{12}(x+1)$, for $0 \le x \le 4$ (c) $f_Y(y) = \frac{3}{76}(y^2+1)$, for $0 \le y \le 4$
  - # 18: 5/6
  - # 24: (a) $f_X(x) = -2e^{-2x} + 2e^{-x}$, for $x \ge 0$ (b) $f_Y(y) = 2e^{-2y}$, for $y \ge 0$
- Chapter 26
  - # 12: (b) $\frac{233}{256}$ (c) $\frac{65}{256}$ (d) $\frac{1}{512}$
  - # 20: (a) Yes. (b) $\frac{15}{16}$
  - NTB # 3: (b) 0.09999546 (d) $f_Z(z) = \left(\frac{11}{5} - \frac{2z}{5}\right)e^{-2z}$, for what values of $z$?
- Chapter 27
  - # 6: $f_{X|Y}(x|y) = \frac{e^{-x/4-y/5}}{4(e^{-y/5}-e^{-9y/20})}$, for $0 < x < y$
  - # 8: $f_{X|Y}(x|y) = \frac{1-x^2}{1-y-\frac{(1-y)^3}{3}}$, for $0 \le x, 0 \le y, x+y \le 1$
  - # 12: (a) $f_{X|Y}(x|y) = \frac{1}{2}$ (c) $\frac{4}{7}$