Chapter 36: Sums of Independent Normal RVs

Meike Niederhausen and Nicky Wakim

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Learning Objectives

1. Calculate probability of a sample mean using a Normally distributed population

Sum of Normal RVs

Theorem 1

Let $X \sim N(\mu, \sigma^2)$, and let $Y = aX + b$, where a and b are constants. Then

$$Y \sim N(a\mu + b, a^2\sigma^2)$$

$E(Y) = E(aX+b)$
$= aE(X)+b$
$Var(Y) = Var(aX+b)$
$= a^2 Var(X)$

Theorem 2

Let $X_i \sim N(\mu_i, \sigma_i^2)$ be independent normal rv's, for $i = 1, 2, \ldots, n$. Then

$$\sum_{i=1}^{n} X_i \sim N\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$$

b/c ind, we can sum variances

linearity
always take sum of exp val

Special Cases

"independent & identically distributed"

$\underbrace{\text{identically distributed}}_{\text{same mean \& variance}}$

1. Let $X_i \sim N(\mu, \sigma^2)$ be iid normal rv's, for $i = 1, 2, \ldots, n$. Then

$$\sum_{i=1}^{n} X_i \sim N(\underbrace{n\mu}_{\sum \mu_i = n\mu}, n\sigma^2)$$

2. Let $X_i \sim N(\mu, \sigma^2)$ be iid normal rv's, for $i = 1, 2, \ldots, n$. Then

$$\rightarrow \quad \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \sim N(\mu, \sigma^2/n)$$

even for small n (not normal approx)

3. Let $X \sim N(\mu_X, \sigma_X^2)$, and $Y \sim N(\mu_Y, \sigma_Y^2)$. Then

$$X - Y \sim N(\mu_X - \mu_Y, \sigma_X^2 + \sigma_Y^2)$$

$(-1)^2$

$\mathrm{var}(X-Y) = \mathrm{var}(X) + \mathrm{var}(-1 \cdot Y)$

$\mathrm{var}(X) + \mathrm{var}(Y)$

Detecting and solving sums of Normal RVs from a word problem

$$X_i \sim N(\mu = 16, \sigma = 3)$$

Example 1

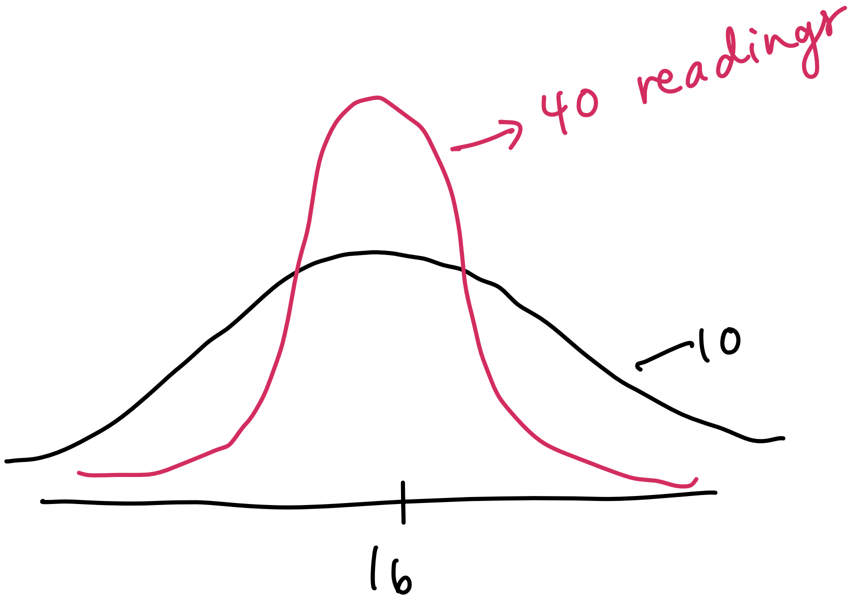
Glaucoma is an eye disease that is manifested by high intraocular pressure (IOP). The distribution of IOP in the general population is approximately normal with mean 16 mmHg and standard deviation 3 mmHg.

1. Suppose a patient has 40 IOP readings. What is the probability that their average reading is greater than 20.32 mmHg, assuming their eyes are healthy?

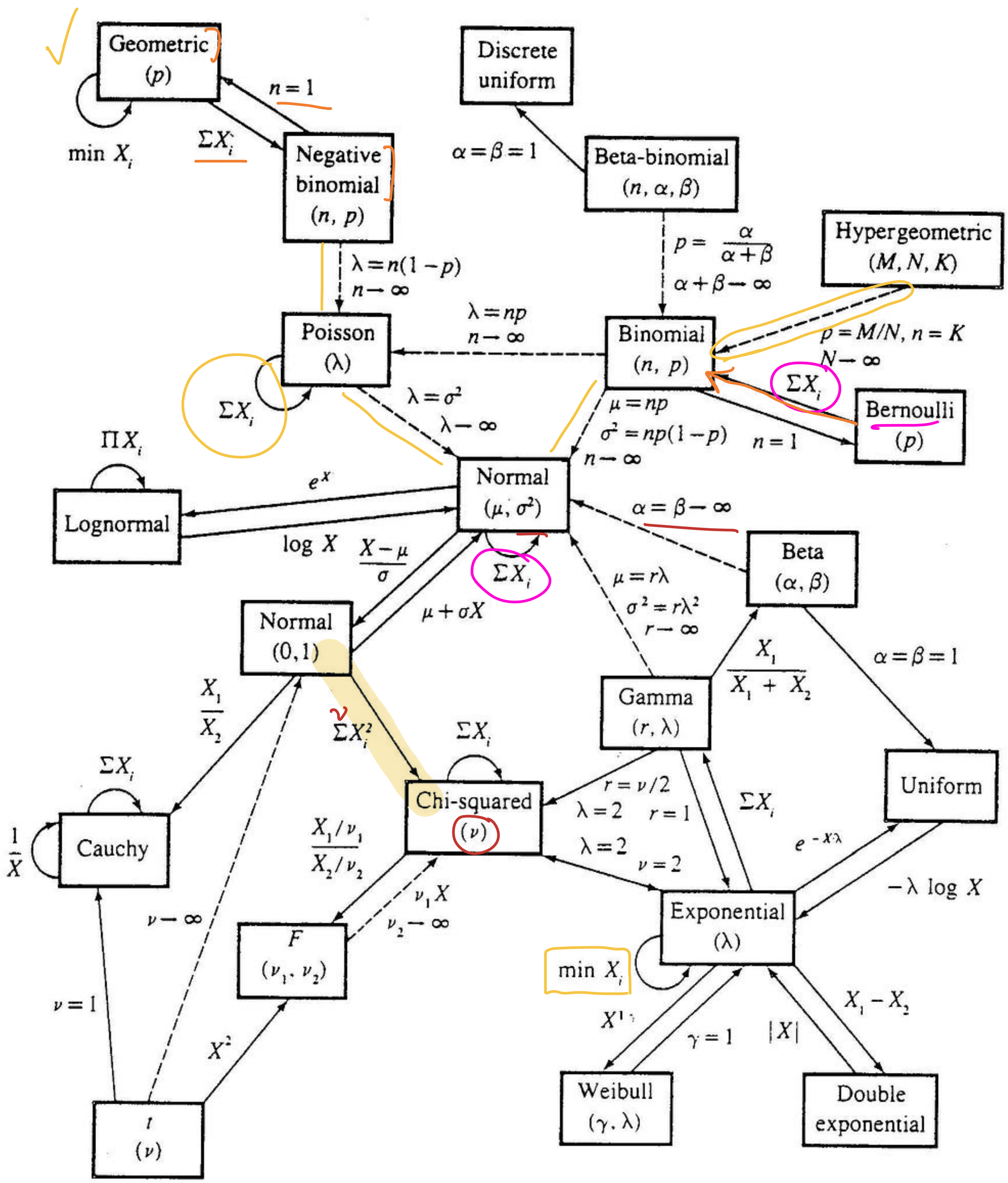
$$\sum_{i=1}^{40} X_i$$

2. Repeat the previous question for a patient with 10 IOP readings.

$$\sum_{i=1}^{10} X_i$$



χ^2 dist from the sum of squared Normals



Relationships among common distributions. Solid lines represent transformations and special cases, dashed lines represent limits. Adapted from Leemis (1986).

casella & berger →
map of distribution