Chapter 4: Conditional Probability

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Learning Objectives

1. Calculate the probability of an event occurring, given that another event occurred.
2. Define keys facts for conditional probabilities using notation.

Let's revisit our deck of cards

Example 1

Suppose we randomly draw 2 cards from a standard deck of cards. What is the probability that both cards are spades?

Solution: Let

- Let A = event 1^{st} card is spades
- Let B = event 2^{nd} card is spades

Conditional Probability facts (1/2)

Fact 1: General Multiplication Rule

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B|A)$$

Fact 2: Conditional Probability Definition

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

Conditional Probability facts (2/2)

Fact 3

If A and B are independent events ($\mathrm{A} \perp\!\!\!\perp \mathrm{B}$), then

$$\mathbb{P}(\mathrm{A}|\mathrm{B}) = \mathbb{P}(\mathrm{A})$$

Fact 4

$\mathbb{P}(\mathrm{A}|\mathrm{B})$ is a probability, meaning that it satisfies the probability axioms. In particular,

$$\mathbb{P}(\mathrm{A}|\mathrm{B}) + \mathbb{P}(\mathrm{A}^{\mathrm{C}}|\mathrm{B}) = 1$$

Conditional probability with two dice

Example 2

Two dice (red and blue) are rolled. If the dice do not show the same face, what is the probability that one of the dice is a 1?