Chapter 22: Introduction to Counting

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Class Overview

- Basic Counting Examples
- Permutations and Combinations
- More Examples: order matters vs. not

Learning Objectives

1. Define permutations and combinations
2. Characterize difference between sampling with and without replacement
3. Characterize difference between sampling when order matters and when order does not matter
4. Calculate the probability of sampling any combination of the following: *with or without replacement* and *order does or does not matter*

Basic Counting Examples

Basic Counting Examples (1/3)

Example 1

Suppose we have 10 (distinguishable) subjects for study.

1. How many possible ways are there to order them?
2. How many ways to order them if we can reuse the same subject and
   - need 10 total?
   - need 6 total?
3. How many ways to order them *without replacement* and only need 6?
4. How many ways to choose 6 subjects without replacement if the order doesn't matter?

Basic Counting Examples (2/3)

Suppose we have 10 (distinguishable) subjects for study.

Example 1.1

How many possible ways are there to order them?

Example 1.2

How many ways to order them if we can reuse the same subject and

- need 10 total?
- need 6 total?

Basic Counting Examples (3/3)

Suppose we have 10 (distinguishable) subjects for study.

Example 1.3

How many ways to order them without replacement and only need 6?

Example 1.4

How many ways to choose 6 subjects without replacement if the order doesn't matter?

Permutations and Combinations

Permutations and Combinations

Definition: Permutations

Permutations are the number of ways to **arrange in order** r distinct objects when there are n total.

$$nPr = \frac{n!}{(n-r)!}$$

Definition: Combinations

Combinations are the number of ways to choose (**order doesn't matter**) r objects from n without replacement.

$$nCr = \text{"n choose r"} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Some combinations properties

- $\binom{n}{r} = \binom{n}{n-r}$

- $\binom{n}{1} = n$

- $\binom{n}{0} = 1$

More Examples: order matters vs. not

More examples: order matters vs. not (1/2)

Example 2

Suppose we draw 2 cards from a standard deck without replacement. What is the probability that both are spades when

1. order matters?
2. order doesn’t matter?

Table of different cases

See table on pg. 277 of textbook

- n = total number of objects
- r = number objects needed

	with replacement	**without replacement**
order matters	n^r	$nPr = \dfrac{n!}{(n-r)!}$
order doesn't matter	$\dbinom{n+r-1}{r}$	$nCr = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$