

Chapter 11: Expected Values of Sums of Discrete RVs

Meike Niederhausen and Nicky Wakim

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Learning Objectives

1. Calculate the mean (expected value) of *sums of* discrete random variables

Revisiting our two card draw

Example 1

Suppose you draw 2 cards from a standard deck of cards *with* replacement. Let X be the number of hearts you draw. Find $\mathbb{E}[X]$.

Recall Binomial RV with $n = 2$:

$$p_X(x) = \binom{2}{x} p^x (1-p)^{2-x} \text{ for } x = 0, 1, 2$$

What if we draw A LOT of cards?

Example 2

What is the expected number of hearts in Example 1 if you draw 200 cards?

Recall Binomial RV with $n = 200$:

$$p_X(x) = \binom{200}{x} p^x (1-p)^{200-x}$$

for $x = 0, 1, 2, \dots, 200$

Sum of discrete RVs

Theorem 11.1: Sum of discrete RVs

For discrete r.v.'s X_i and constants $a_i, i = 1, 2, \dots, n$,

$$\mathbb{E} \left[\sum_{i=1}^n a_i X_i \right] = \sum_{i=1}^n a_i \mathbb{E}[X_i].$$

Remark: The theorem holds for infinitely r.v.'s X_i as well.

- For two RVs, X and Y :
 - We can say $E[X + Y] = E[X] + E[Y]$
 - ... and constant numbers a and b , we can also say $E[aX + bY] = aE[X] + bE[Y]$
 - We can also also say $E[X - Y] = E[X] - E[Y]$, since $b = -1$

Corollaries from Thm 11.1

Corollary 11.1.1

For a discrete r.v. X , and constants a and b ,

$$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b.$$

Corollary 11.1.2

If $X_i, i = 1, 2, \dots, n$, are i.i.d. r.v.'s, then

$$\mathbb{E}\left[\sum_{i=1}^n X_i\right] = n\mathbb{E}[X_1].$$

Revisiting our ghost! 👻

Example 3

The ghost is trick-or-treating at a different house now. In this case it is known that the bag of candy has 10 chocolates, 20 lollipops, and 30 laffy taffies. The ghost takes five pieces of candy without replacement. How many pieces of chocolate do we expect the ghost to take?

Cost of hotel rooms

Example 4

A tour group is planning a visit to the city of Minneapolis and needs to book 30 hotel rooms. The average price of a room is \$200. In addition, there is a 10% tourism tax for each room. What is the expected cost for the 30 hotel rooms?

