Chapter 22: Counting

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Basic Counting Examples

Basic Counting Examples (1/3)

Example 1

Suppose we have 10 (distinguishable) subjects for study.

1. How many possible ways are there to order them?
2. How many ways to order them if we can reuse the same subject and
   - need 10 total?
   - need 6 total?
3. How many ways to order them if without replacements and only need 6?
4. How many ways to choose 6 subjects without replacement if the order doesn't matter?

Basic Counting Examples (2/3)

Suppose we have 10 (distinguishable) subjects for study.

Example 1.1

How many possible ways are there to order them?

$10!$

Example 1.2

How many ways to order them if we can reuse the same subject and

- need 10 total?
- need 6 total?

Basic Counting Examples (3/3)

Suppose we have 10 (distinguishable) subjects for study.

Example 1.3

How many ways to order them if without replacements and only need 6?

$$\frac{10!}{4!} = 151,200$$

Example 1.4

How many ways to choose 6 subjects without replacement if the order doesn't matter?

$$\frac{\frac{10!}{4!}}{6!} = \frac{10!}{4!6!} = 210$$

There are $6!$ ways to order the 6 subjects.

Permutations and Combinations

Permutations and Combinations

Definition: Permutations

Permutations are the number of ways to arrange in order r distinct objects when there are n total.

$$nPr = \frac{n!}{(n-r)!}$$

Definition: Combinations

Combinations are the number of ways to choose (order doesn't matter) r objects from n without replacement.

$$nCr = \text{"n choose r"} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Some combinations properties

- $\binom{n}{r} = \binom{n}{n-r}$
- $\binom{n}{1} = n$
- $\binom{n}{0} = 1$

More Examples: order matters vs. not

More examples: order matters vs. not (1/2)

Example 2

Suppose we draw 2 cards from a standard deck without replacement. What is the probability that both are spades when

1. order matters?
2. order doesn't matter?

More examples: order matters vs. not (2/2)

Suppose we draw 2 cards from a standard deck without replacement. What is the probability that both are spades when

1. order matters?

- $$\frac{13P2}{52P2} = \frac{\frac{13!}{11!}}{\frac{52!}{50!}} = \frac{13 \cdot 12}{52 \cdot 51}$$

2. order doesn’t matter?

- $$\frac{\binom{13}{2}}{\binom{52}{2}} = \frac{\frac{13!}{2!11!}}{\frac{52!}{2!50!}} = \frac{13 \cdot 12}{52 \cdot 51}$$

Table of different cases

See table on pg. 277 of textbook

- n = total number of objects
- r = number objects needed

	with replacement	**without replacement**
order matters	n^r	$nPr = \frac{n!}{(n-r)!}$
order doesn't matter	$\binom{n+r-1}{r}$	$nCr = \binom{n}{r} = \frac{n!}{r!(n-r)!}$