Chapter 1: Outcomes, Events, and Sample Spaces

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2023-09-25

Class Overview

- Tossing One Coin (Outcomes, Events, and Sample Space)
- Tossing Two Coins (Outcomes, Events, and Sample Space)
- Set Theory

Tossing One Coin (Outcomes, Events, and Sample Space)

Coin Toss Example: 1 coin (1/3)

Suppose you toss one coin.

- What are the possible outcomes?
- What is the sample space?
- What are the possible events?

Coin Toss Example: 1 coin (2/3)

Suppose you toss one coin.

- What are the possible outcomes?
  - Heads (H)
  - Tails (T)

Note this is not necessarily an event!

When something happens at random, such as a coin toss, there are several possible outcomes, and *exactly one* of the outcomes will occur.

possible outcomes

$\{H, T\} \rightarrow \{\ldots\}$

{H, T}

math not.

Coin Toss Example: 1 coin (3/3)

Definition: Sample Space

The **sample space** S is the set of *all* outcomes

Definition: Event

↳ not necessarily the physical outcome here

An **event** is a *collection of* some outcomes. An event can include multiple outcomes or no outcomes.

- What is the sample space?
  - H
  - T
  - $S =$

{H,T} that I get H or T

neither H nor T : $\emptyset$

- What are the possible events?
  -
  -
  -
  - $2^{|S|} = 2^2 = 4 \rightarrow \boxed{4 \text{ events}}$

collection of

from combo of SS

\# of outcomes in sample space

When thinking about events, think about outcomes that you might be asking the probability of.

Tossing Two Coins (Outcomes, Events, and Sample Space)

for each $S = \{H, T\}$

↳ what combo of H & T's can we get

$\{HH, HT, TT, TH\}$

Coin Toss Example: 2 ~~coins~~ $2^{|S|} = 2^{4} = \boxed{16}$

can we get from 2 coins

Suppose you toss two coins. {HT, TH}

{HH, HT, TH}

- What is the ~~sample~~ space? *Assume the coins are distinguishable*
  - $S =$ S = @ least one T =

- What are some possible events?
  - A = exactly one H =
  - B = at least one H =
  -
  -

if you are giving the event a letter

More info on events and sample spaces

↳ total # of outcomes in SS

- We usually use capital letters from the beginning of the alphabet to denote events. However, other letters might be chosen to be more descriptive.

- We use the notation $|S|$ to denote the **size** of the sample space.

- The total number of possible events is $2^{|S|}$, which is the total number of possible subsets of S. We will prove this later in the course.

- The **empty set**, denoted by $\emptyset$, is the set containing no outcomes.

so basically $H^c \ldots$ until H

Example: Keep sampling until…

$\{H, H^cH, H^cH^cH, H^cH^cH^cH, \ldots\}$

Suppose you keep sampling people until you have someone with high blood pressure (BP)

5th, 6th …

What is the sample space?

- Let H = denote someone with high BP.
- Let H^{C} = denote someone with not high blood pressure, such as low or regular BP.

if 1st person has H

2nd person has H

3rd …

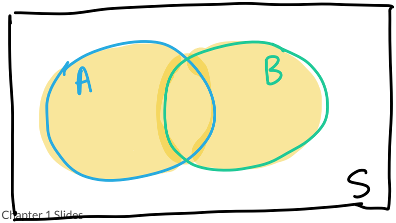
4th …

$|S| = \infty$

- Then, S =

Set Theory

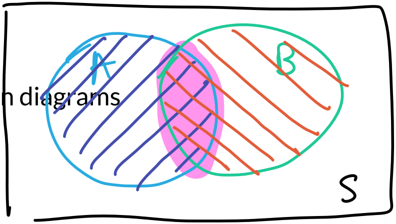
use square to denote SS



circles for events

Set Theory (1/2)

Venn diagrams



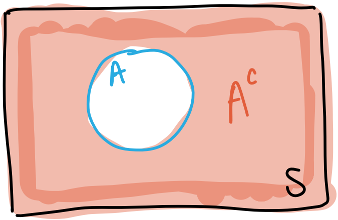
Definition: Union

The **union** of events A and B, denoted by $\mathrm{A} \cup \mathrm{B}$, contains all outcomes that are in A *or* B or both

Definition: Intersection

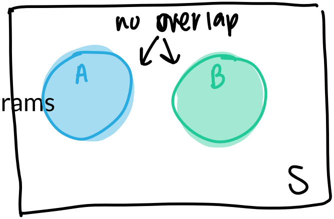
The **intersection** of events A and B, denoted by $\mathrm{A} \cap \mathrm{B}$, contains all outcomes that are both in A *and* B.

$\boxed{S \setminus A}$



Set Theory (2/2)

Venn diagrams



Definition: Complement

The **complement** of event A, denoted by A^C or A', contains all outcomes in the sample space S that are *not* in A .

Definition: Mutually Exclusive

Events A and B are **mutually exclusive**, or disjoint, if they have no outcomes in common. In this case $A \cap B = \emptyset$, where $\emptyset$ is the empty set.

H_i^c = not high blood pressure (for person i)

BP example variation (1/3)

- Suppose you have n subjects in a study.
- Let H_i be the event that person i has high BP, for $i = 1 \ldots n$.

Use set theory notation to denote the following events:

1. Event subject i does not have high BP
2. Event all n subjects have high BP
3. Event at least one subject has high BP
4. Event all of them do not have high BP
5. Event at least one subject does not have high BP

H_i^c

H_1 & $H_2 \rightarrow H_1 \cap H_2$

$+H_3 \quad H_1 \cap H_2 \cap H_3$

$H_1 \cap H_2 \cap H_3 \cap \ldots \cap H_n$

$\bigcap^{n} H_i \qquad \sum^{n} x_i$

BP example variation (2/3)

$H_1 \,\&\, H_2 \to \{H_1 H_2^c,\ H_1 H_2,\ H_1^c H_2\}$

- Suppose you have n subjects in a study.

$H_1 \cup H_2 \cup H_3 \cup \dots \cup H_n = \bigcup_{i=1}^{n} H_i$

$\sum_{i=1}^{n} x_i$

$x_1 + x_2 + \dots + x_n$

- Let H_i be the event that person i has high BP, for $i = 1 \dots n$.

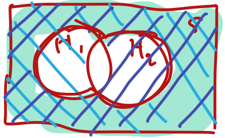
Use set theory notation to denote the following events:

1. Event subject i does not have high BP

2. Event all n subjects have high BP

$1 \& 2:\ H_1^c \,\&\, H_2^c \to H_1^c \cap H_2^c \xrightarrow{w/\,n} H_1^c \cap H_2^c \cap \dots \cap H_n^c = \bigcap_{i=1}^{n} H_i^c$

3. Event at least one subject has high BP



$(H_1 \cup H_2)^c$

person $1 \& 2:\ \{H_1^c H_2,\ H_1^c H_2^c,\ H_1 H_2^c\}$

$\to H_1^c \cup H_2^c$

$H_1^c \cup H_2^c \cup H_3^c \cup \dots \cup H_n^c = \bigcup_{i=1}^{n} H_i^c$

$\left(\bigcup_{i=1}^{n} H_i\right)^c$

$H_1 \cup H_2 \cup H_3 \cup \dots \cup H_n = \bigcup_{i=1} H_i$ ... $\left(\bigcup_{i=1} H_i\right)$

BP example variation (3/3)

@ Home: draw diagram for H_1 & H_2 if confused

4. Event all of them do not have high BP

$\left(\bigcap_{i=1}^{n} H_i\right)^c$

5. Event at least one subject does not have high BP

one H_i & all else H_i^c

De Morgan's Laws

n's can also be ∞

Theorem: De Morgan's 1st Law

For a collection of events (sets) $A_1, A_2, A_3, \ldots$

$$\bigcap_{i=1}^{n} A_i^C = \left(\bigcup_{i=1}^{n} A_i\right)^C$$

→ intersection of comp is the complement of the union

"all not A = (at least one event A)^C"

Theorem: De Morgan's 2nd Law

For a collection of events (sets) $A_1, A_2, A_3, \ldots$

$$\bigcup_{i=1}^{n} A_i^C = \left(\bigcap_{i=1}^{n} A_i\right)^C$$

union of complements is the complement of int.

"at least one event not A = (all A)^C"

Remarks on De Morgan's Laws

- These laws also hold for infinite collections of events.

- Draw Venn diagrams to convince yourself that these are true! → we did for 1 & 2 people

- These laws are *very* useful when calculating probabilities.
  - This is because calculating the probability of the **intersection of events is often much easier than the union of events**.
  - This is not obvious right now, but we will see in the coming chapters why.