Chapter 3: Independent Events

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Learning objectives

1. Define independence of 2-3 events given probability notation
2. Calculate whether two or more events are independent

Revisiting our coin toss

Question: Which of the following sequences of coin tosses of heads (H) and tails (T) is more likely to happen, assuming the coin is fair?

$$\mathrm{HTTHHHTHTHHTTTH}$$

or

$$\mathrm{HTTTTTTTTTHTTTTT}$$

Independent Events

Definition: Independence

Events A and B are **independent** if

$$\mathbb{P}(\mathrm{A} \cap \mathrm{B}) = \mathbb{P}(\mathrm{A}) \cdot \mathbb{P}(\mathrm{B}).$$

Notation: For shorthand, we sometimes write

$$\mathrm{A} \perp\!\!\!\perp \mathrm{B},$$

to denote that A and B are independent events.

Example of two dice

Example 1

Two dice (red and blue) are rolled. Let A = event a total of 7 appears, and B = event first die is a six. Are events A and B independent?

Independence of 3 Events

Definition: Independence of 3 Events

Events A, B, and C are **independent** if

1.
   - $\mathbb{P}(\mathrm{A} \cap \mathrm{B}) = \mathbb{P}(\mathrm{A}) \cdot \mathbb{P}(\mathrm{B})$
   - $\mathbb{P}(\mathrm{A} \cap \mathrm{C}) = \mathbb{P}(\mathrm{A}) \cdot \mathbb{P}(\mathrm{C})$
   - $\mathbb{P}(\mathrm{B} \cap \mathrm{C}) = \mathbb{P}(\mathrm{B}) \cdot \mathbb{P}(\mathrm{C})$
2. $\mathbb{P}(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = \mathbb{P}(\mathrm{A}) \cdot \mathbb{P}(\mathrm{B}) \cdot \mathbb{P}(\mathrm{C})$

Remark:

On your homework you will show that $(1) \nRightarrow (2)$ and $(2) \nRightarrow (1)$.

Probability at least one smoker

Example 2

Suppose you take a random sample of n people, of which people are smokers and non-smokers independently of each other. Let

- A_i = event person i is a smoker, for $i = 1, \ldots, n$, and
- p_i = probability person i is a smoker, for $i = 1, \ldots, n$.

Find the probability that at least one person in the random sample is a smoker.