Chapter 1: Outcomes, Events, and Sample Spaces

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Class Overview

- Outcomes, Events, and Sample Space
- Set Theory

Overview

- Outcomes, Events, and Sample Space
  - Tossing one coin
  - Tossing two coins
- Set Theory

Outcomes, Events, and Sample Space

Coin Toss Example: 1 coin (1/3)

Suppose you toss one coin.

- What are the possible outcomes?
- What is the sample space?
- What are the possible events?

Coin Toss Example: 1 coin (2/3)

Suppose you toss one coin.

- What are the possible outcomes?
  - Heads (H)
  - Tails (T)

  Note: When something happens at random, such as a coin toss, there are several possible outcomes, and *exactly one* of the outcomes will occur.

Coin Toss Example: 1 coin (3/3)

Definition: Sample Space

The **sample space** S is the set of *all* possible outcomes.

Definition: Event

An **event** is a collection of *some* possible outcomes.

Tossing two coins

Coin Toss Example: 2 coins

Suppose you toss two coins.

- What is the sample space? *Assume the coins are distinguishable*
  - $S =$

$$S = \{HH, TT, HT, TH\}$$

- What are some possible events?
  - A = exactly one H = $\{HT, TH\}$
  - B = at least one H = $\{HH, HT, TH\}$
  -
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More info on events and sample spaces

- We usually use capital letters from the beginning of the alphabet to denote events. However, other letters might be chosen to be more descriptive.
- We use the notation $|S|$ to denote the **size** of the sample space.
- The total number of possible events is $2^{|S|}$, which is the total number of possible subsets of S. We will prove this later in the course.
- The **empty set**, denoted by $\emptyset$, is the set containing no outcomes.

Example: Keep sampling until...

Suppose you keep sampling people until you have someone with high blood pressure (BP). What is the sample space?

- Let H = denote someone with high BP.
- Let H^{C} = denote someone with not high blood pressure, such as low or regular BP.
- Then, S =

$$\mathrm{S} = \{\mathrm{H}, (\mathrm{H}, \mathrm{H}^{\mathrm{C}}), (\mathrm{H}, \mathrm{H}, \mathrm{H}^{\mathrm{C}}), (\mathrm{H}, \mathrm{H}, \mathrm{H}, \mathrm{H}^{\mathrm{C}}), \ldots$$

Set Theory

Set Theory (1/2)

Definition: Union

The **union** of events A and B, denoted by $A \cup B$, contains all outcomes that are in A *or* B.

Definition: Intersection

The **intersection** of events A and B, denoted by $A \cap B$, contains all outcomes that are both in A *and* B.

Venn diagrams

Set Theory (2/2)

Definition: Complement

The **complement** of event A, denoted by A^C or A', contains all outcomes in the sample space S that are *not* in A .

Definition: Mutually Exclusive

Events A and B are **mutually exclusive**, or disjoint, if they have no outcomes in common. In this case $A \cap B = \emptyset$, where $\emptyset$ is the empty set.

Venn diagrams

BP example variation (1/3)

- Suppose you have n subjects in a study.
- Let H_i be the event that person i has high BP, for $i = 1 \ldots n$.

Use set theory notation to denote the following events:

1. Event subject i does not have high BP
2. Event all n subjects have high BP
3. Event at least one subject has high BP
4. Event all of them do not have high BP
5. Event at least one subject does not have high BP

BP example variation (2/3)

- Suppose you have n subjects in a study.
- Let $\mathrm{H_i}$ be the event that person i has high BP, for $\mathrm{i} = 1 \ldots \mathrm{n}$.

Use set theory notation to denote the following events:

1. Event subject i does not have high BP

$$\mathrm{H_i^C}$$

2. Event all n subjects have high BP

$$\mathrm{H_1 \text{ and } H_2 \text{ and } \ldots = \bigcap_{i=1}^{n} H_i}$$

3. Event at least one subject has high BP

$$\mathrm{H_1 \text{ or } H_2 \text{ or } \ldots = \bigcup_{i=1}^{n} H_i}$$

BP example variation (3/3)

4. Event all of them do not have high BP

   H_1^C and H_2^C and...

$$\bigcap_{i=1}^{n} H_i^C = \left(\bigcup_{i=1}^{n} H_i\right)^C$$

   = complement of at least one person having high BP

5. Event at least one subject does not have high BP

   H_1^C or H_2^C or...

$$\bigcup_{i=1}^{n} H_i^C = \left(\bigcap_{i=1}^{n} H_i\right)^C$$

   = complement of all having high BP

De Morgan's Laws

Theorem: De Morgan's 1st Law

For a collection of events (sets) $A_1, A_2, A_3, \ldots$

$$\bigcap_{i=1}^{n} A_i^C = \left(\bigcup_{i=1}^{n} A_i \right)^C$$

"all not A = (at least one event A)^C"

Theorem: De Morgan's 2nd Law

For a collection of events (sets) $A_1, A_2, A_3, \ldots$

$$\bigcup_{i=1}^{n} A_i^C = \left(\bigcap_{i=1}^{n} A_i \right)^C$$

"at least one event not A = (all A)^C"

Remarks on De Morgan’s Laws

- These laws also hold for infinite collections of events.
- Draw Venn diagrams to convince yourself that these are true!
- These laws are *very* useful when calculating probabilities.
  - This is because calculating the probability of the intersection of events is often much easier than the union of events.
  - This is not obvious right now, but we will see in the coming chapters why.