

Homework 5

BSTA 550

Directions

Please turn in this homework on Sakai. Please submit your homework in pdf format. You can type your work on your computer or submit a photo of your written work or any other method that can be turned into a pdf. The Adobe Scan phone app is an easy way to scan photos and compile into a PDF. Please let me know if you greatly prefer to submit a physical copy. We can work out another way for you to turn in homework.

Try to complete all of the problems listed below at some point this quarter! You may want to save some of them for studying later! Only turn in the ones listed in the “Turn In” column. Please submit problems in the order they are listed.

You must show all of your work to receive credit.

| Chapter | Turn In | Extra Problems |
|---------|---------|-------------------------------|
| 14 | | # 3, 7 |
| 15 | NTB # 3 | # 1, 5, 11, 18, 23, NTB # 5 |
| 16 | TB # 7 | # 3a-g, 8, 11, 21 |
| 17 | TB # 9 | # 3a-g, 6, 11, 12a-c, NTB # 6 |
| 18 | TB # 20 | # 1, 24, 26, 27 |
| 19 | TB # 6 | # 1, 18, 19 |
| 20 | | # 2, 3, 4 |

Non-textbook problems (NTB)

1. Prove that for a r.v. X and constants a and b , that

$$\text{Var}[aX + b] = a^2\text{Var}[X].$$

2. Let \bar{X} be the random variable for the sample mean, $\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$, where the X_i are i.i.d. random variables with common mean μ and variance σ^2 .

- a. Find $\mathbb{E}[\bar{X}]$.
 - b. Find $\text{Var}[\bar{X}]$.
3. Let $X_i \sim \text{Binomial}(n_i, p)$ be independent r.v.'s for $i = 1, \dots, m$.
- a. What does the r.v. $X = \sum_{i=1}^m X_i$ count, and what is the distribution of X ? Make sure to specify the parameters of X 's distribution.
 - b. Find $\mathbb{E}[X]$. *Make sure to show your work for (b) and (c). However, you may use without proof what you know about the mean and variance of each X_i .*
 - c. Find $\text{Var}[X]$.

Extra Problems

4. Let \hat{p} be the random variable for the sample proportion, $\hat{p} = \frac{X}{n}$, where X is the number of successes in a random sample of size n . Assume the probability of success is p .
- a. Find $\mathbb{E}[\hat{p}]$.
 - b. Find $\text{Var}[\hat{p}]$.
5. Read the Washington Post article *The amazing woman who can smell Parkinson's disease - before symptoms appear* (<http://www.washingtonpost.com/news/morning-mix/wp/2015/10/23/scottish-woman-detects-a-musky-smell-that-could-radically-improve-how-parkinsons-disease-is-diagnosed/>)

Assuming Joy Milne does not have the ability to detect Parkinson's disease via smell, answer the following questions:

- a. What is the probability of her correctly detecting Parkinson's by smelling one t-shirt?
 - b. What is the probability of her correctly detecting Parkinson's in 12 out of 12 t-shirts?
6. Let $X_i \sim \text{Negative Binomial}(r_i, p)$ be independent r.v.'s for $i = 1, \dots, m$.
- a. What does the r.v. $X = \sum_{i=1}^m X_i$ count, and what is the distribution of X ? Make sure to specify the parameters of X 's distribution.
 - b. Find $\mathbb{E}[X]$. *Make sure to show your work for (b) and (c). However, you may use without proof what you know about the mean and variance of each X_i .*
 - c. Find $\text{Var}[X]$.