

Chapter 7: Discrete vs. Continuous Random Variables

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2023-10-09

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Learning Objectives

1. Map the sample space to the set of real numbers using a discrete and continuous random variable
2. Distinguish between discrete and continuous random variables from a written description

What is a random variable?

Definition: Random Variable

For a given sample space S , a **random variable** (r.v.) is a function whose domain is S and whose range is the set of real numbers \mathbb{R} . A random variable assigns a real number to each outcome in the sample space.

Let's demonstrate this definition with our coin toss

Example 1

Suppose we toss 3 fair coins.

1. What is the sample space?
2. What are the probabilities for each of the elements in the sample space?

Let's stretch our definition of random variables

Example 2

What are some other random variables we could consider in Example 1?

Some remarks on random variables

- A random variable's value is completely determined by the outcome ω , where $\omega \in S$
 - What is *random* is the outcome ω
- A random variable is a function from the sample space (with outcomes ω) to the set of real numbers
 - We typically write X instead of $X(\omega)$, where X is our random variable
- For example, if we roll three dice, there are $6^3 = 216$ possible outcomes (which is ω)
 - We can define a random variable as the sum of the of the three dice
 - If our outcome is the set of numbers the dice landed on ($\omega = (a, b, c)$), then

$$X(\omega) = X = a + b + c$$

Let's look at a continuous R.V.

Example 3

Let X = how many hours you slept last night.

- What is the sample space S ?
- What is the range of possible values for X ?
- What is $X(\omega)$?

Discrete vs. Continuous r.v.'s

- For a **discrete** r.v., the set of possible values is either finite or can be put into a countably infinite list
 - You could *theoretically* list the specific possible outcomes that the variable can take
 - If you sum the rolls of three dice, you must get a whole number. For example, you can't get any number between 3 and 4.
- **Continuous** r.v.'s take on values from continuous *intervals*, or unions of continuous intervals
 - Variable takes on a range of values, but there are infinitely possible values within the range
 - If you keep track of the time you sleep, you can sleep for 8 hours or 7.9 hours or 7.99 hours or 7.999 hours ...

