Chapter 28: Revisiting Expected Values for Joint Distributions

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2024-11-20

Learning Objectives

1. Calculate the mean (expected value) of a joint distribution of continuous RV

Remark on expected value of one RV from joint pdf

If you are given $f_{X,Y}(x,y)$ and want to calculate $\mathbb{E}[X]$, you have two options:

1. Find $f_X(x)$ and use it to calculate $\mathbb{E}[X]$.
2. Or, calculate $\mathbb{E}[X]$ using the joint density:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{X,Y}(x,y)dydx.$$

$$E(X) = \int_{-\infty}^{\infty} x \underbrace{f_X(x)}_{\int_{-\infty}^{\infty} f_{X,Y}(x,y)dy} dx$$

Option 1: Expected value from a joint distribution

Example 3

Let $f_{X,Y}(x,y) = 2e^{-(x+y)}$, for $0 \le x \le y$. Find $\mathbb{E}[X]$.

do @ home

find $f_X(x)$ then

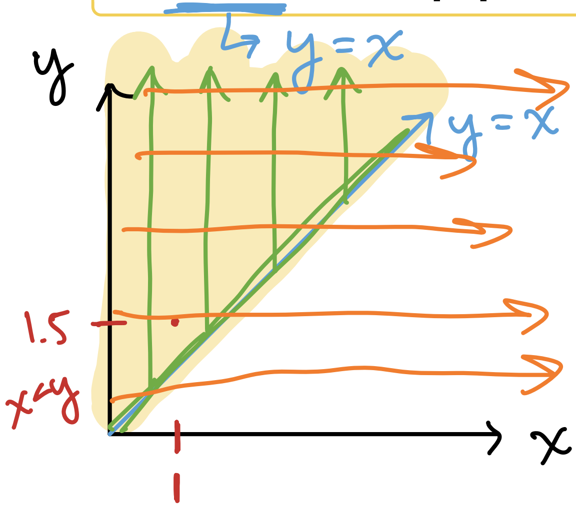
$$E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

try this way and see if it matches next page's ans!

Option 2: Expected value from a joint distribution

Example 1

Let $f_{X,Y}(x,y) = 2e^{-(x+y)}$, for $0 \le x \le y$. Find $\mathbb{E}[X]$.



$$E(X) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f_{X,Y}(x,y)\, dy\, dx$$

$$= \int_{x=0}^{x=\infty}\int_{y=x}^{y=\infty} x 2 e^{-(x+y)}\, dy\, dx$$

$$= \int_0^{\infty}\int_x^{\infty} x 2 e^{-x} e^{-y}\, dy\, dx$$

$$= \int_0^{\infty} x 2 e^{-x} \int_x^{\infty} e^{-y}\, dy\, dx$$

$$= \int_0^{\infty} x 2 e^{-x} \left[-e^{-y} \Big|_{y=x}^{y=\infty} \right] dx$$

$$= \int_0^{\infty} x 2 e^{-x} \left[-\cancelto{0}{e^{-\infty}} + e^{-x} \right] dx$$

$$= \int_0^{\infty} \underbrace{x 2 e^{-2x}}\, dx = \ldots = \frac{1}{2}$$

int by parts

can recognize that this is the pdf of an expon. dist'n! $E(X) = \frac{1}{\lambda}$ $\lambda = 2$