

Chapter 11: Expected Values of Sums of Discrete RVs

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Learning Objectives

1. Calculate the mean (expected value) of *sums of* discrete random variables

Where are we?

Basics of probability

- Outcomes and events
- Sample space
- Probability axioms
- Probability properties
- Counting
- Independence
- Conditional probability
- Bayes' Theorem
- Random Variables

Probability for discrete random variables

- Functions: pmfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

Probability for continuous random variables

- Calculus
- Functions: pdfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

Advanced probability

- Central limit theorem
- Functions: moment generating functions

Revisiting our two card draw

Example 1

Suppose you draw 2 cards from a standard deck of cards *with* replacement. Let X be the number of hearts you draw. Find $\mathbb{E}[X]$.

Recall Binomial RV with $n = 2$:

$$p_X(x) = \binom{2}{x} p^x (1-p)^{2-x} \text{ for } x = 0, 1, 2$$

What if we draw A LOT of cards?

Example 2

What is the expected number of hearts in Example 1 if you draw 200 cards?

Recall Binomial RV with $n = 200$:

$$p_X(x) = \binom{200}{x} p^x (1-p)^{200-x}$$

for $x = 0, 1, 2, \dots, 200$

Sum of discrete RVs

Theorem 11.1: Sum of discrete RVs

For discrete r.v.'s X_i and constants $a_i, i = 1, 2, \dots, n$,

$$\mathbb{E} \left[\sum_{i=1}^n a_i X_i \right] = \sum_{i=1}^n a_i \mathbb{E}[X_i].$$

Remark: The theorem holds for infinitely r.v.'s X_i as well.

- For two RVs, X and Y :
 - We can say $E[X + Y] = E[X] + E[Y]$
 - ... and constant numbers a and b , we can also say $E[aX + bY] = aE[X] + bE[Y]$
 - We can also also say $E[X - Y] = E[X] - E[Y]$, since $b = -1$

Corollaries from Thm 11.1

Corollary 11.1.1

For a discrete r.v. X , and constants a and b ,

$$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b.$$

Corollary 11.1.2

If $X_i, i = 1, 2, \dots, n$, are identically distributed r.v.'s, then

$$\mathbb{E}\left[\sum_{i=1}^n X_i\right] = n\mathbb{E}[X_1].$$

Cost of hotel rooms

Example 4

A tour group is planning a visit to the city of Minneapolis and needs to book 30 hotel rooms. The average price of a room is \$200. In addition, there is a 10% tourism tax for each room. What is the expected cost for the 30 hotel rooms?

