Homework 5

BSTA 550

2023-11-02

Directions

Please turn in this homework on Sakai. Please submit your homework in pdf format. You can type your work on your computer or submit a photo of your written work or any other method that can be turned into a pdf. Please let me know if you greatly prefer to submit a physical copy. We can work out another way for you to turn in homework.

Try to complete all of the problems listed below at some point this quarter! You may want to save some of them for studying later! Only turn in the ones listed in the "Turn In" column. Please submit problems in the order they are listed.

The more work you include that shows your thought process, the more I can give you feedback.

Chapter	Turn In	Extra Problems
12	TB # 2, NTB # 1-3	# 1, 7, 11, 12, 15, 19, 25, 27
13 (review)		# 3, 4, 5, 6, 8, 9, 10, 17, 25
20		# 2, 3, 4
14		# 3, 7
15	NTB # 4	# 1, 5, 11, 18, 23, NTB # 5
16	TB # 8	# 3a-g, 11, 21
17	TB # 12a-c	# 3a-g, 6, 9, 11, NTB # 6

Non-textbook problems (NTB)

1. Prove that for a r.v. X and constants a and b, that

$$\text{Var}[aX + b] = a^2\text{Var}[X].$$

Note: you will not earn credit for citing this as a special case of a more general result.

2. Let $\bar{X}$ be the random variable for the sample mean, $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$, where the X_i are i.i.d. random variables with common mean μ and variance σ^2.

   a. Find $\mathbb{E}[\bar{X}]$.

   b. Find $Var[\bar{X}]$.

3. Let $\hat{p}$ be the random variable for the sample proportion, $\hat{p} = \frac{X}{n}$, where X is the number of successes in a random sample of size n. Assume the probability of success is p.

   a. Find $\mathbb{E}[\hat{p}]$.

   b. Find $Var[\hat{p}]$.

4. Let $X_i \sim$ Binomial(n_i, p) be independent r.v.'s for $i = 1, \ldots, m$. What does the r.v. $X = \sum_{i=1}^{m} X_i$ count, and what is the distribution of X? Make sure to specify the parameters of X's distribution.Find $\mathbb{E}[X]$. *Make sure to show your work for (b) and (c). However, you may use without proof what you know about the mean and variance of each* X_i.Find $Var[X]$.

5. Read the Washington Post article *The amazing woman who can smell Parkinson's disease - before symptoms appear* (http://www.washingtonpost.com/news/morning-mix/wp/2015/10/23/scottish-woman-detects-a-musky-smell-that-could-radically-improve-how-parkinsons-disease-is-diagnosed/)

   Assuming Joy Milne does not have the ability to detect Parkinson's disease via smell, answer the following questions:

   a. What is the probability of her correctly detecting Parkinson's by smelling one t-shirt?

   b. What is the probability of her correctly detecting Parkinson's in 12 out of 12 t-shirts?

6. Let $X_i \sim$ Negative Binomial(r_i, p) be independent r.v.'s for $i = 1, \ldots, m$.

   a. What does the r.v. $X = \sum_{i=1}^{m} X_i$ count, and what is the distribution of X? Make sure to specify the parameters of X's distribution.

   b. Find $\mathbb{E}[X]$. *Make sure to show your work for (b) and (c). However, you may use without proof what you know about the mean and variance of each* X_i.

   c. Find $Var[X]$.

Some select answers

Selected answers (or hints) not provided at the end the book:

- Chapter 12
  - # 2: 64.8
  - # 12: 1,096,357
- Chapter 13
  - # 4: (a) 260/9 (b) 2.833 (c) 2.679×10^{-5} (d) Same idea as (c) Replace 10's with 100.
  - # 6: (a) $p_X(x) = \binom{4}{x}.3^x.7^{4-x}$, for $x = 0, 1, \ldots, 4$ (d) 0.3483 (e) 0.9163 (f) 0.0233 (g) 1
  - # 8: (a) T (b) F (c) F (d) F (e) T (f) T (g) T
  - # 10: (a) T (b) T (c) F (d) T (e) T (f) F (g) T (h) T (nonnegative instead of positive) (i) F
- Chapter 20
  - # 2: (a) 0.0001 (b) Discrete since X has a finite number of possible values. Uniform since each outcome is equally likely. (c) X = randomly selected 4-digit ID#; $X = 0000, 0001, \ldots, 9999$ (d) 5000.5 (e) 8,333,333.25
- Chapter 15
  - # 18 (a) Bin(21,0.65) (b) 4.78
- Chapter 16
  - # 8 (c) 1.03×10^{-6} (d) 10 questions: 91.43 minutes
- Chapter 17
  - # 6 (a) 400, 87.18 (b) No
  - # 12 (c) 0.8000