Interactions

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Learning Objectives

1. Define confounders and effect modifiers, and how they interact with the main relationship we model.
2. Interpret the interaction component of a model with **a binary categorical covariate and continuous covariate**, and how the main variable's effect changes.
3. Interpret the interaction component of a model with **a multi-level categorical covariate and continuous covariate**, and how the main variable's effect changes.
4. Interpret the interaction component of a model with **two categorical covariates**, and how the main variable's effect changes.

Next time:

5. Interpret the interaction component of a model with **two continuous covariates**, and how the main variable's effect changes.
6. When there are only two covariates in the model, test whether one is a confounder or effect modifier.

Let's map that to our regression analysis process

Model Selection

- Building a model
- Selecting variables
- Prediction vs interpretation
- Comparing potential models

Model Fitting

- Find best fit line
- Using OLS in this class
- Parameter estimation
- Categorical covariates
- Interactions

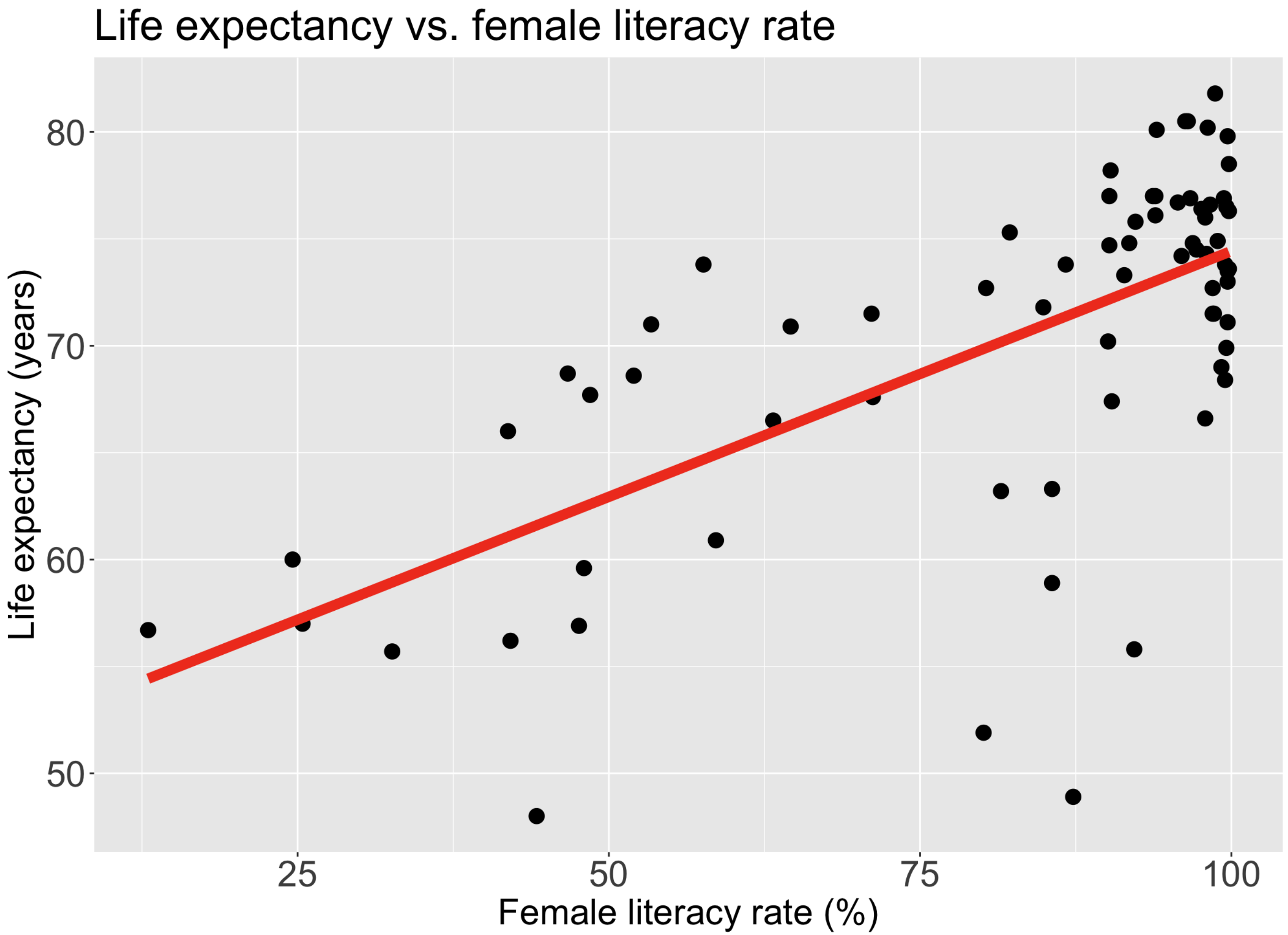
Model Evaluation

- Evaluation of model fit
- Testing model assumptions
- Residuals
- Transformations
- Influential points
- Multicollinearity

Model Use (Inference)

- Inference for coefficients
- Hypothesis testing for coefficients
- Inference for expected Y given X

Recall our data and the main relationship



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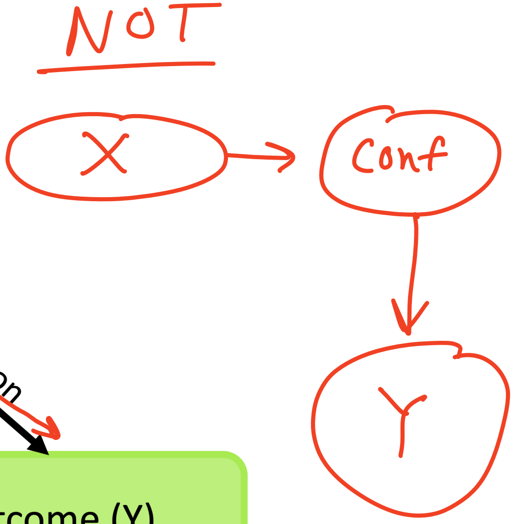
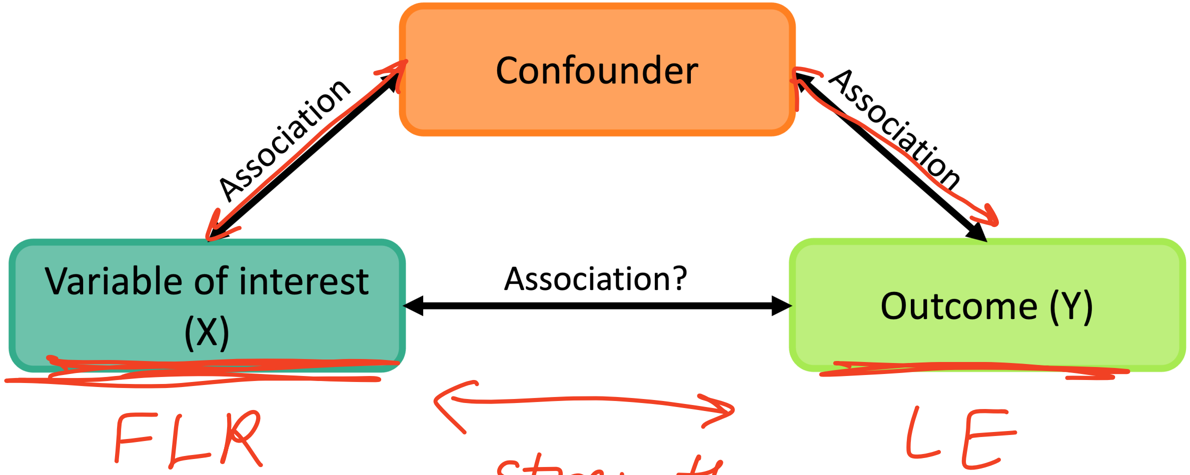
What is a confounder?

- A confounding variable, or **confounder**, is a factor/variable that wholly or partially accounts for the observed effect of the risk factor on the outcome
- A confounder must be…
  - Related to the outcome Y, but not a consequence of Y
  - Related to the explanatory variable X, but not a consequence of X





strength of this assoc.

Including a confounder in the model

confounder

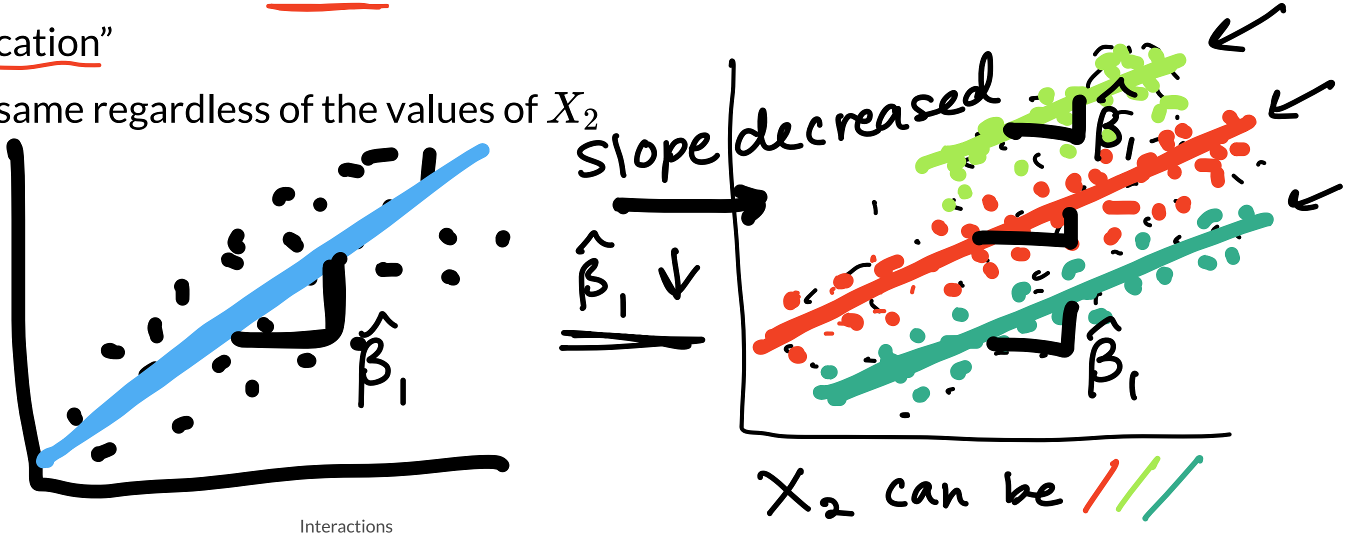
- In the following model we have two variables, X_1 and X_2

$Y = \beta_0 + \beta X_i$

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$$

$LE = \beta_0 + \beta_1 FLR + \beta_2 \underbrace{FS}_{\text{food supply}} + \varepsilon$

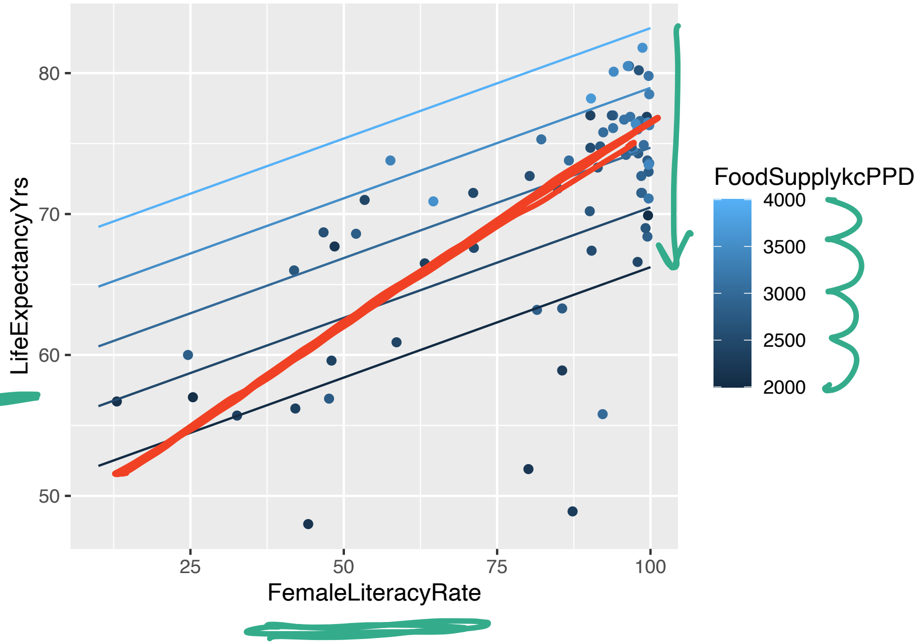
- And we assume that every level of the confounder, there is parallel slopes
- Note: to interpret β_1, we did not specify any value of X_2; only specified that it be held constant
  - Implicit assumption: effect of X_1 is equal across all values of X_2
- The above model assumes that X_1 and X_2 do not *interact* (with respect to their effect on Y)
  - epidemiology: no "effect modification"
  - meaning the effect of X_1 is the same regardless of the values of X_2



Where have we modeled a confounder before?

- We have seen a plot of Life expectancy vs. female literacy rate with different levels of food supply colored (Lesson 8)
- In our plot and the model, we treat food supply as a **confounder**
- If food supply is a confounder in the relationship between life expectancy and female literacy rate, then we only use main effects in the model:

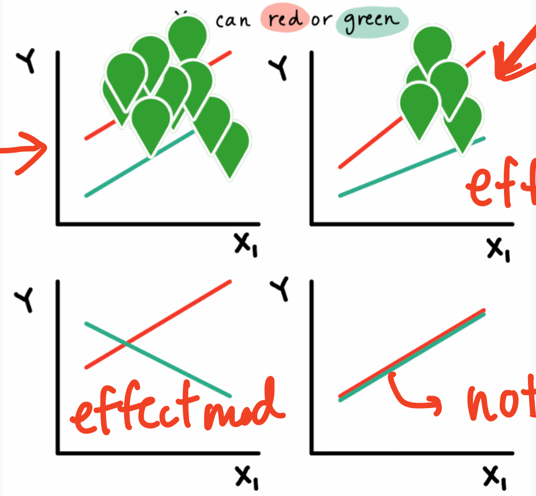
$$\text{LE} = \beta_0 + \beta_1 \text{FLR} + \beta_2 \text{FS} + \epsilon$$



Poll everywhere question 1

Join by Web **PollEv.com/nickywakim275**

If X_1 is a continuous variable, and we are interested in the relationship between Y and X_1, which of the following pictures shows X_2 as a



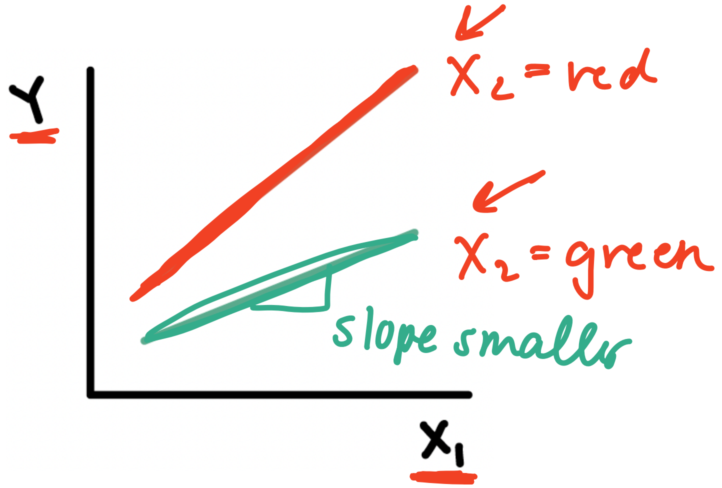
effect modifier (interaction b/w X_1 & X_2)

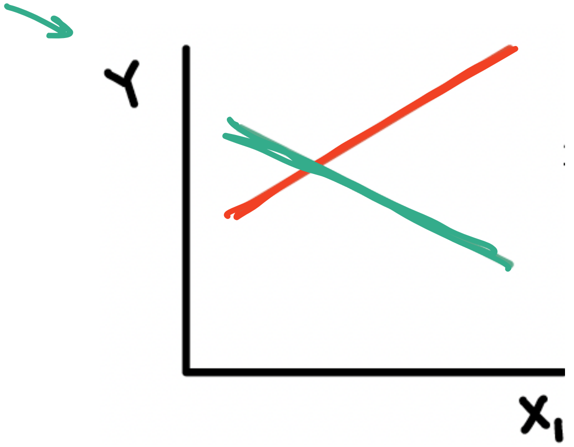
effect mod

not confounder nor eff. mod

What is an effect modifier?

- An additional variable in the model
  - Outside of the main relationship between Y and X_1 that we are studying
- An effect modifier will change the effect of X_1 on Y depending on its value
  - Aka: as the effect modifier's values change, so does the association between Y and X_1
  - So the coefficient estimating the relationship between Y and X_1 changes with another variable





How do we include an effect modifier in the model?

- Interactions!!
- We can incorporate interactions into our model through product terms:

$$Y = \beta_0 + \underbrace{\beta_1 X_1 + \beta_2 X_2}_{\text{main effects}} + \underbrace{\beta_3 X_1 X_2}_{\text{Interaction}} + \epsilon$$

- Terminology:
  - main effect parameters: β_1, β_2
    - The main effect models estimate the *average* X_1 and X_2 effects → good example of this
  - interaction parameter: β_3

Types of interactions / non-interactions

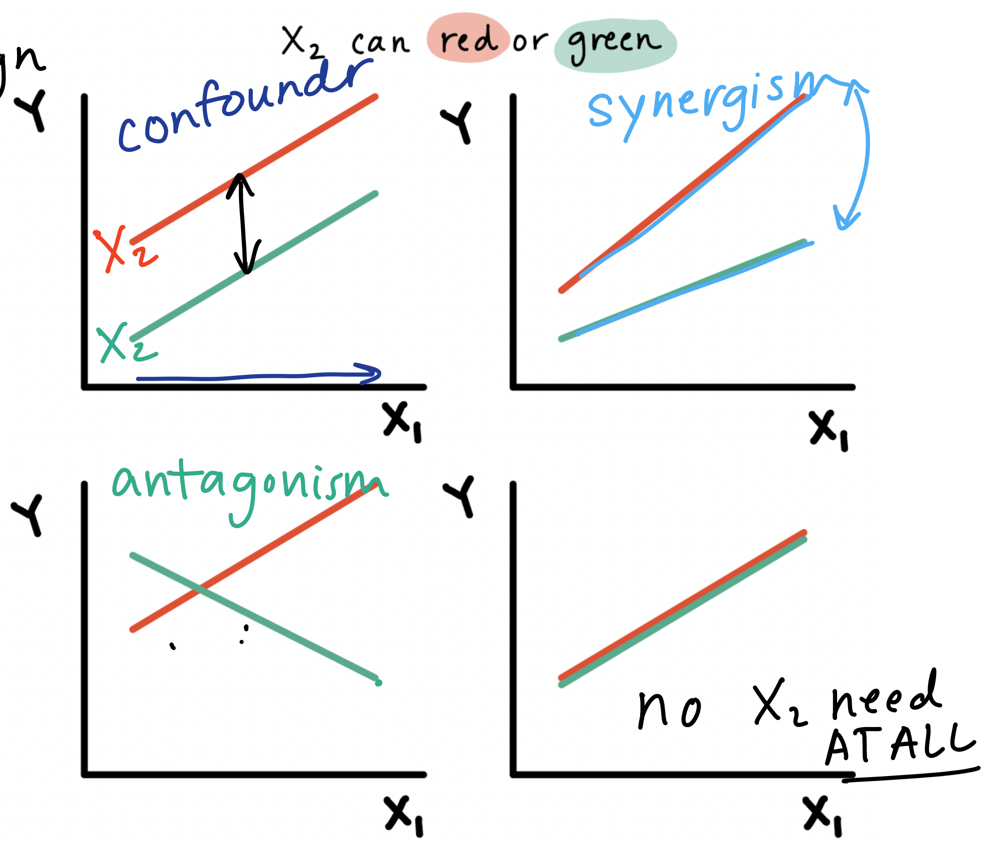
- Common types of interactions: → inc magnitude same sign
  - Synergism: X_2 strengthens the X_1 effect
  - Antagonism: X_2 weakens the X_1 effect

    flip of sign

- If the interaction coefficient is not significant
  - No evidence of effect modification, i.e., the effect of X_1 does not vary with X_2

    $\hat{\beta}_3$ is 0

- If the main effect of X_2 is also not significant
  - No evidence that X_2 is a confounder



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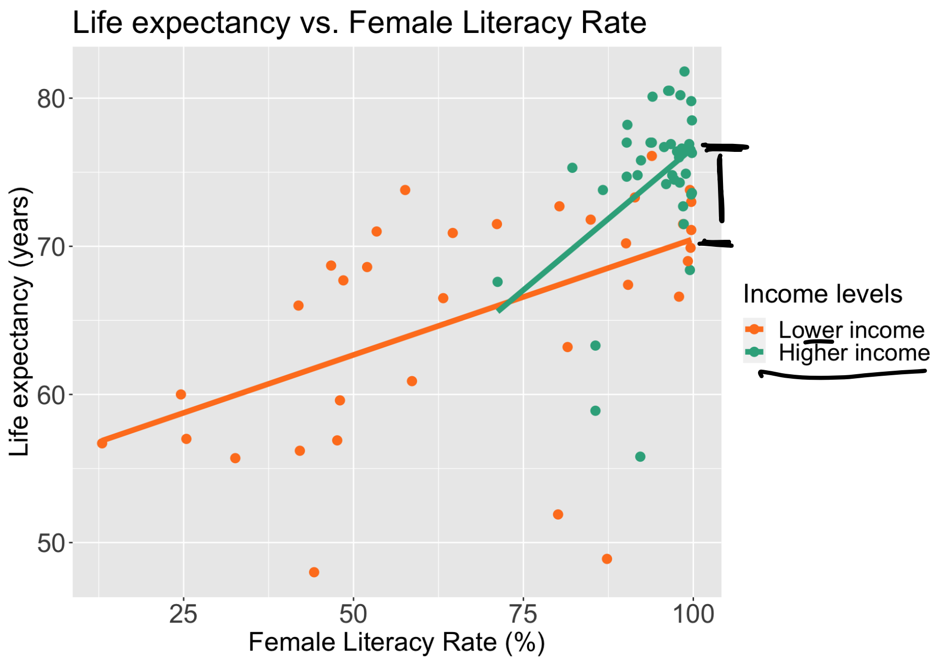
4. Interpret the interaction component of a model with **two categorical covariates**, and how the main variable's effect changes.

Do we think income level is an effect modifier for female literacy rate?

- Let's say we only have two income groups: low income and high income
- We can start by visualizing the relationship between life expectancy and female literacy rate *by income level*
- Questions of interest: Is the effect of female literacy rate on life expectancy differ depending on income level?
  - This is the same as: Is income level is an effect modifier for female literacy rate?
- Let's run an interaction model to see!



Model with interaction between a *binary categorical and continuous variables*

Model we are fitting:

$$LE = \beta_0 + \beta_1 FLR + \beta_2 I(\text{high income}) + \beta_3 FLR \cdot I(\text{high income}) + \epsilon$$

main effects

interaction

- LE as life expectancy
- FLR as female literacy rate (continuous variable)
- $I(\text{high income})$ as the indicator that income level is "high income" (binary categorical variable)

In R:

```
m_int_inc2 = lm(LifeExpectancyYrs ~ FemaleLiteracyRate + income_levels2 +
                FemaleLiteracyRate*income_levels2, data = gapm_sub)
```

OR

```
m_int_inc2 = lm(LifeExpectancyYrs ~ FemaleLiteracyRate*income_levels2,
              data = gapm_sub)
```

R will include main effects

Displaying the regression table and writing fitted regression equation

```
1 tidy(m_int_inc2, conf.int=T) %>% gt() %>% tab_options(table.font.size = 35) %>% fmt_
```

	term	estimate	std.error	statistic	p.value	conf.low	conf.high
intercept $\hat\beta_0$	(Intercept)	54.849	2.846	19.270	0.000	49.169	60.529
main $\hat\beta_1$	FemaleLiteracyRate	0.156	0.039	3.990	0.000	0.078	0.235
main $\hat\beta_2$	income_levels2Higher income	−16.649	15.364	−1.084	0.282	−47.308	14.011
interaction $\hat\beta_3$	FemaleLiteracyRate:income_levels2Higher income	0.228	0.164	1.392	0.168	−0.099	0.555

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 I(\text{high income}) + \widehat{\beta}_3 FLR \cdot I(\text{high income})$$

$$\widehat{LE} = 54.85 + 0.156 \cdot FLR - 16.65 \cdot I(\text{high income}) + 0.228 \cdot FLR \cdot I(\text{high income})$$

Poll Everywhere Question 2

β_3 FLR $\cdot$ I(high inc)

$\hat\beta_3$ is the step up from I = 0 to I = 1

Based only on the coefficient estimate for the interaction term: $\beta_3 = 0.228$.. What can we say about female literacy rate's effect?

16

b/c $\hat\beta_3$ is pos

- Female literacy rate's effect is strengthened for countries with high income (compared to low income).
- Female literacy rate's effect is ~~weakened~~ for countries with high income (compared to low income).
- Female literacy rate's effect is strengthened for countries with low income (compared to high income).
- Female literacy rate's effect is weakened for countries with low income (compared to high income).

removing $\hat\beta_3$ inverse sign = weakened

when comp I=0 to I=1

Comparing fitted regression lines for each income level

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 I(\text{high income}) + \widehat{\beta}_3 FLR \cdot I(\text{high income})$$ → equation

$$\widehat{LE} = 54.85 + 0.156 \cdot FLR - 16.65 \cdot I(\text{high income}) + 0.228 \cdot FLR \cdot I(\text{high income})$$

lines

For lower income countries: $I(\text{high income}) = 0$

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 FLR \cdot 0$$

$$\widehat{LE} = 54.85 + 0.156 \cdot FLR - 16.65 \cdot 0 + 0.228 \cdot FLR \cdot 0$$

$$\widehat{LE} = 54.85 + 0.156 \cdot FLR$$

lines

For higher income countries: $I(\text{high income}) = 1$

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 \cdot 1 + \widehat{\beta}_3 FLR \cdot 1$$

$$\widehat{LE} = 54.85 + 0.156 \cdot FLR - 16.65 \cdot 1 + 0.228 \cdot FLR \cdot 1$$

$$\widehat{LE} = (54.85 - 16.65 \cdot 1) + (0.156 \cdot FLR + 0.228 \cdot FLR \cdot 1)$$

$$\widehat{LE} = [54.85 - 16.65] + [0.156 + 0.228] \cdot FLR$$

$$\widehat{LE} = 38.2 + 0.384 \cdot FLR$$

Let's take a look back at the plot

For lower income countries: $I(\text{high income}) = 0$

$\hat{\beta}_0$

$$\widehat{LE} = \hat{\beta}_0 + \hat{\beta}_1 FLR$$

$$\widehat{LE} = 54.85 + 0.156 \cdot FLR$$

For higher income countries: $I(\text{high income}) = 1$

intercept slope

$$\widehat{LE} = [(\hat{\beta}_0 + \hat{\beta}_2)] + [(\hat{\beta}_1 + \hat{\beta}_3)] FLR$$

$$\widehat{LE} = (54.85 - 16.65) + (0.156 + 0.228) \cdot FLR$$

$$\widehat{LE} = 38.2 + 0.384 \cdot FLR$$

high

$\hat{\beta}_0 + \hat{\beta}_2$

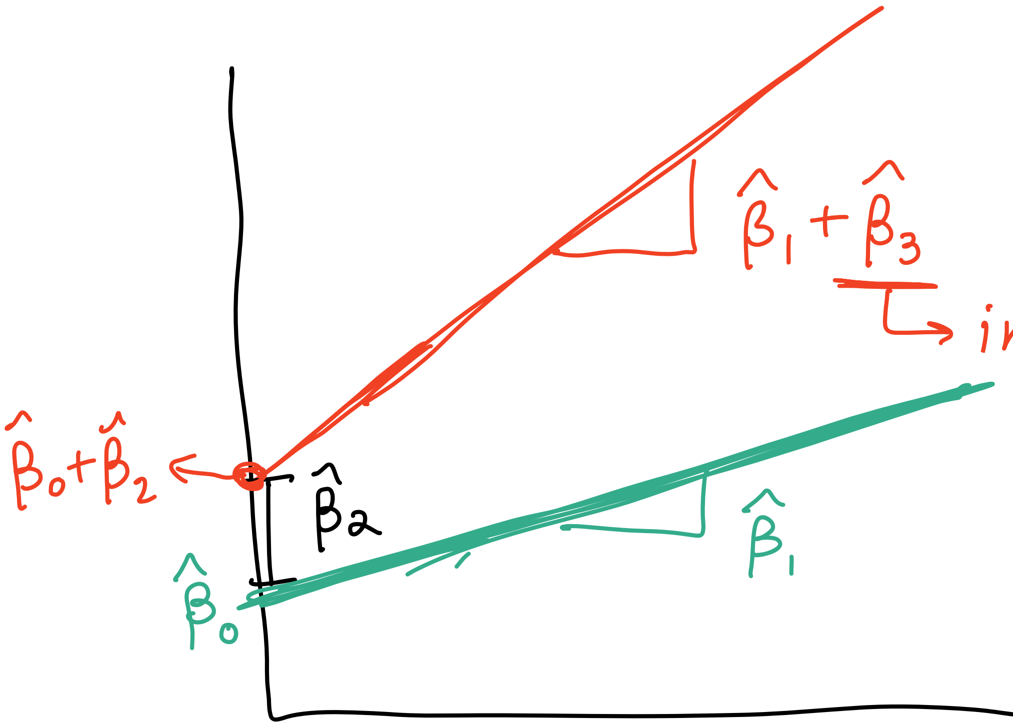
0 FLR

$\hat{\beta}_2$ (main effect for inc level)

$I(\underline{X_2} = \text{red})$

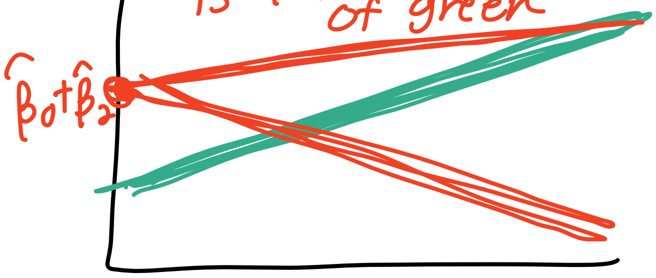
$I = 1$ red

$I = 0$ green



$\hat\beta_3$ → interaction coefficient

$\hat\beta_3$ is negative
slope of red line is less than slope of green



Interpretation for interaction between binary categorical and continuous variables

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 I(\text{high income}) + \widehat{\beta}_3 FLR \cdot I(\text{high income})$$

$$\widehat{LE} = \left[\widehat{\beta}_0 + \widehat{\beta}_2 \cdot I(\text{high income})\right] + \underbrace{\left[\widehat{\beta}_1 + \widehat{\beta}_3 \cdot I(\text{high income})\right] FLR}_{\text{FLR's effect}}$$

whole effect of FLR

- Interpretation:
  - β_3 = mean change in female literacy rate's effect, comparing higher income to lower income levels
  - where the "female literacy rate effect" equals the change in mean life expectancy per percent increase in female literacy with income level held constant, i.e. "adjusted female literacy rate effect"
- In summary, the interaction term can be interpreted as "difference in adjusted female literacy rate effect comparing higher income to lower income levels"
- It will be helpful to test the interaction to round out this interpretation!!

we still have main effect for inc. level

Test interaction between binary categorical and continuous variables

- We run an F-test for a single coefficient (β_3) in the below model (see lesson 9, MLR: Inference / F-test)

$$\rightarrow LE = \beta_0 + \beta_1 FLR + \beta_2 I(\text{high income}) + \beta_3 FLR \cdot I(\text{high income}) + \epsilon$$

Null H_0	Alternative H_1
$\beta_3 = 0$	$\beta_3 \neq 0$

Null / Smaller / Reduced model	Alternative / Larger / Full model
$LE = \beta_0 + \beta_1 FLR + \beta_2 I(\text{high income}) + \epsilon$? $\hat{\beta}_3$ gone	$LE = \beta_0 + \beta_1 FLR + \beta_2 I(\text{high income}) + \beta_3 FLR \cdot I(\text{high income}) + \epsilon$

- I'm going to be skipping steps so please look back at Lesson 9 for full steps (required in HW 4)

Test interaction between binary categorical and continuous variables

- Fit the reduced and full model

main effects

```
m_int_inc_red = lm(LifeExpectancyYrs ~ FemaleLiteracyRate + income_levels2,
                   data = gapm_sub)
m_int_inc_full = lm(LifeExpectancyYrs ~ FemaleLiteracyRate + income_levels2 +
                   FemaleLiteracyRate*income_levels2, data = gapm_sub)
```

→ interaction

- Display the ANOVA table with F-statistic and p-value

→ code lesson 9 → qmd file

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ FemaleLiteracyRate + income_levels2	69.000	2,407.667	NA	NA	NA	NA
LifeExpectancyYrs ~ FemaleLiteracyRate + income_levels2 + FemaleLiteracyRate * income_levels2	68.000	2,340.948	1.000	66.719	1.938	0.168

- Conclusion: There is not a significant interaction between female literacy rate and income level (p = 0.168).
  - If significant, we say more: For higher income levels, for every one percent increase in female literacy rate, the mean life expectancy increases 0.384 years. For lower income levels, for every one percent increase in female literacy rate, the mean life expectancy increases 0.156 years. Thus, the female literacy rate almost doubles comparing high income to low income levels.

X

Evid that inc level is not an effect mod.

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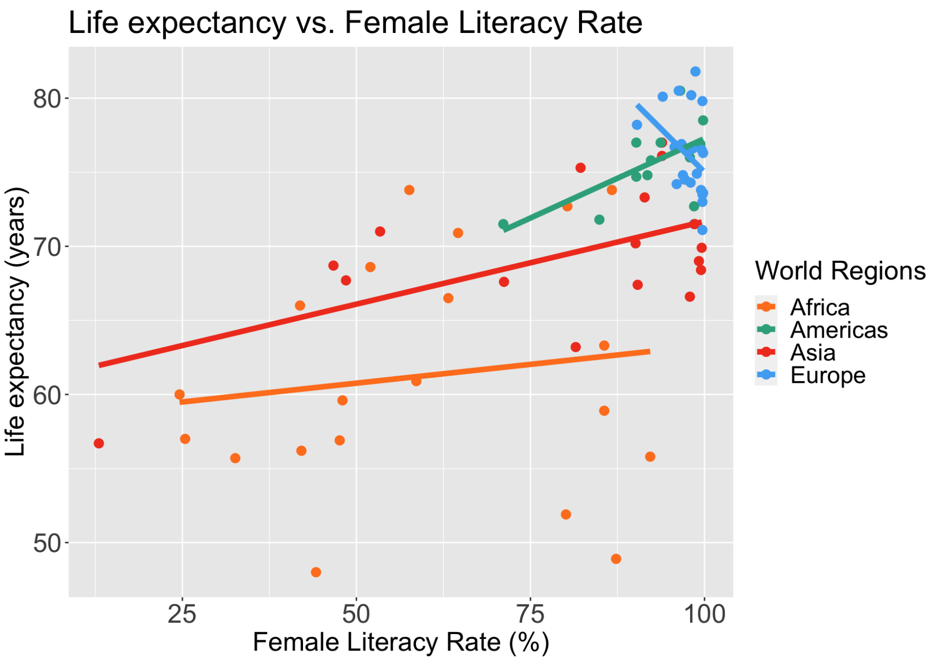
Do we think world region is an effect modifier for female literacy rate?

- We can start by visualizing the relationship between life expectancy and female literacy rate *by world region*
- Questions of interest: Does the effect of female literacy rate on life expectancy differ depending on world region?
  - This is the same as: Is world region is an effect modifier for female literacy rate?
- Let's run an interaction model to see!



Model with interaction between a *multi-level categorical and continuous variables*

Model we are fitting:

$$\begin{aligned} LE = & \beta_0 + \beta_1 FLR + \beta_2 I(\text{Americas}) + \beta_3 I(\text{Asia}) + \beta_4 I(\text{Europe}) + \\ & \beta_5 FLR \cdot I(\text{Americas}) + \beta_6 FLR \cdot I(\text{Asia}) + \beta_7 FLR \cdot I(\text{Europe}) + \epsilon \end{aligned}$$

- LE as life expectancy
- FLR as female literacy rate (continuous variable)
- $I(\text{Americas}), I(\text{Asia}), I(\text{Europe})$ as the indicator for each world region

In R:

```
m_int_wr = lm(LifeExpectancyYrs ~ FemaleLiteracyRate + four_regions +
                         FemaleLiteracyRate*four_regions, data = gapm_sub)
```

OR

```
m_int_wr = lm(LifeExpectancyYrs ~ FemaleLiteracyRate*four_regions,
                       data = gapm_sub)
```

Displaying the regression table and writing fitted regression equation

```
1 tidy(m_int_wr, conf.int=T) %>% gt() %>% tab_options(table.font.size = 35) %>% fmt_n
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	58.225	3.377	17.240	0.000	51.478	64.972
FemaleLiteracyRate	0.051	0.053	0.957	0.342	−0.055	0.157
four_regionsAmericas	−2.406	17.913	−0.134	0.894	−38.191	33.379
four_regionsAsia	2.283	5.410	0.422	0.674	−8.525	13.091
four_regionsEurope	63.628	46.414	1.371	0.175	−29.095	156.350
FemaleLiteracyRate:four_regionsAmericas	0.164	0.197	0.830	0.410	−0.231	0.558
FemaleLiteracyRate:four_regionsAsia	0.061	0.073	0.830	0.410	−0.086	0.208
FemaleLiteracyRate:four_regionsEurope	−0.519	0.476	−1.090	0.280	−1.471	0.432

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\ & \widehat{\beta}_5 FLR \cdot I(\text{Americas}) + \widehat{\beta}_6 FLR \cdot I(\text{Asia}) + \widehat{\beta}_7 FLR \cdot I(\text{Europe}) \\ \widehat{LE} =& 58.23 + 0.051 \cdot FLR - 2.41 \cdot I(\text{Americas}) + 2.28 \cdot I(\text{Asia}) + 63.63 \cdot I(\text{Europe}) + \\ & 0.164 \cdot FLR \cdot I(\text{Americas}) + 0.061 \cdot FLR \cdot I(\text{Asia}) - 0.519 \cdot FLR \cdot I(\text{Europe})\end{aligned}$$

Comparing fitted regression lines for each world region

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\ & \widehat{\beta}_5 FLR \cdot I(\text{Americas}) + \widehat{\beta}_6 FLR \cdot I(\text{Asia}) + \widehat{\beta}_7 FLR \cdot I(\text{Europe})\end{aligned}$$

$$\begin{aligned}\widehat{LE} =& 58.23 + 0.051 \cdot FLR - 2.41 \cdot I(\text{Americas}) + 2.28 \cdot I(\text{Asia}) + 63.63 \cdot I(\text{Europe}) + \\ & 0.164 \cdot FLR \cdot I(\text{Americas}) + 0.061 \cdot FLR \cdot I(\text{Asia}) - 0.519 \cdot FLR \cdot I(\text{Europe})\end{aligned}$$

Africa

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \\ & \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 0 + \\ & \widehat{\beta}_4 \cdot 0 + \widehat{\beta}_5 FLR \cdot 0 + \\ & \widehat{\beta}_6 FLR \cdot 0 + \widehat{\beta}_7 FLR \cdot 0 \\ \widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR\end{aligned}$$

The Americas

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \\ & \widehat{\beta}_2 \cdot 1 + \widehat{\beta}_3 \cdot 0 + \\ & \widehat{\beta}_4 \cdot 0 + \widehat{\beta}_5 FLR \cdot 1 + \\ & \widehat{\beta}_6 FLR \cdot 0 + \widehat{\beta}_7 FLR \cdot 0 \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_2) + \\ & (\widehat{\beta}_1 + \widehat{\beta}_5) FLR\end{aligned}$$

Asia

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \\ & \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 1 + \\ & \widehat{\beta}_4 \cdot 0 + \widehat{\beta}_5 FLR \cdot 0 + \\ & \widehat{\beta}_6 FLR \cdot 1 + \widehat{\beta}_7 FLR \cdot 0 \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_3) + \\ & (\widehat{\beta}_1 + \widehat{\beta}_6) FLR\end{aligned}$$

Europe

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \\ & \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 0 + \\ & \widehat{\beta}_4 \cdot 1 + \widehat{\beta}_5 FLR \cdot 0 + \\ & \widehat{\beta}_6 FLR \cdot 0 + \widehat{\beta}_7 FLR \cdot 1 \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_4) + \\ & (\widehat{\beta}_1 + \widehat{\beta}_7) FLR\end{aligned}$$

Poll Everywhere Question 3

Centering continuous variables when we are including interactions

- For Europe, the mean life expectancy had a regression line with a large intercept

$$\widehat{LE} = (\widehat{\beta}_0 + \widehat{\beta}_4) + (\widehat{\beta}_1 + \widehat{\beta}_7) FLR$$
$$\widehat{LE} = (58.23 + 63.63) + (0.051 - 0.519) FLR$$
$$\widehat{LE} = 121.86 - 0.468 FLR$$

- Centering the continuous variables in a model (when they are involved in interactions) helps with:
  - Interpretations of the coefficient estimates
  - Correlation between the main effect for the variable and the interaction that it is involved with
    - To be discussed in future lecture: leads to multicollinearity issues
- Other online sources about when and when not to center:
  - The why and when of centering continuous predictors in regression modeling
  - When not to center a predictor variable in regression

It'll be helpful to center female literacy rate

- Centering female literacy rate:

$$FLR^c = FLR - \overline{FLR}$$

- Centering in R:

```
gapm_sub = gapm_sub %>%
  mutate(FLR_c = FemaleLiteracyRate - mean(FemaleLiteracyRate))
```

- I'm going to print the mean so I can use it for my interpretations

```
(mean_FLR = mean(gapm_sub$FemaleLiteracyRate))
```

```
[1] 82.03056
```

- Now all intercept values (in each respective world region) will be the mean life expectancy when female literacy rate is 82.03%
- We will used center FLR for the rest of the lecture

Now we refit the model with the centered FLR

```
m_int_wr_flrc = lm(LifeExpectancyYrs ~ FLR_c*four_regions,
                   data = gapm_sub)
tidy(m_int_wr_flrc, conf.int=T) %>% gt() %>% tab_options(table.font.size = 35) %>%
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	62.387	1.626	38.358	0.000	59.138	65.637
FLR_c	0.051	0.053	0.957	0.342	−0.055	0.157
four_regionsAmericas	11.032	2.918	3.781	0.000	5.203	16.862
four_regionsAsia	7.287	2.042	3.568	0.001	3.207	11.367
four_regionsEurope	21.038	7.698	2.733	0.008	5.659	36.417
FLR_c:four_regionsAmericas	0.164	0.197	0.830	0.410	−0.231	0.558
FLR_c:four_regionsAsia	0.061	0.073	0.830	0.410	−0.086	0.208
FLR_c:four_regionsEurope	−0.519	0.476	−1.090	0.280	−1.471	0.432

- What changed? What stayed the same? What’s the new intercept for Europe?

Interpretation for interaction between multi-level categorical and continuous variables

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\ & \widehat{\beta}_5 FLR \cdot I(\text{Americas}) + \widehat{\beta}_6 FLR \cdot I(\text{Asia}) + \widehat{\beta}_7 FLR \cdot I(\text{Europe})\end{aligned}$$

$$\begin{aligned}\widehat{LE} =& \left[\widehat{\beta}_0 + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe})\right] + \\ & \underbrace{\left[\widehat{\beta}_1 + \widehat{\beta}_5 \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot I(\text{Asia}) + \widehat{\beta}_7 \cdot I(\text{Europe})\right]}_{\text{FLR's effect}} FLR\end{aligned}$$

- Interpretation:
  - β_5 = mean change in female literacy rate's effect, comparing countries in the Americas to countries in Africa
  - β_6 = mean change in female literacy rate's effect, comparing countries in Asia to countries in Africa
  - β_7 = mean change in female literacy rate's effect, comparing countries in Europe to countries in Africa
- It will be helpful to test the interaction to round out this interpretation!!

Test interaction between multi-level categorical & continuous variables

- We run an F-test for a group of coefficients (β_5, β_6, β_7) in the below model (see lesson 9)

$$\begin{aligned} LE =& \beta_0 + \beta_1 FLR + \beta_2 I(\text{Americas}) + \beta_3 I(\text{Asia}) + \beta_4 I(\text{Europe}) + \\ & \beta_5 FLR \cdot I(\text{Americas}) + \beta_6 FLR \cdot I(\text{Asia}) + \beta_7 FLR \cdot I(\text{Europe}) + \epsilon \end{aligned}$$

Null H_0

$\beta_5 = \beta_6 = \beta_7 = 0$

Alternative H_1

$\beta_5 \neq 0$ and/or $\beta_6 \neq 0$ and/or $\beta_7 \neq 0$

Null / Smaller / Reduced model

$$\begin{aligned} LE =& \beta_0 + \beta_1 FLR + \beta_2 I(\text{Americas}) + \\ & \beta_3 I(\text{Asia}) + \beta_4 I(\text{Europe}) + \epsilon \end{aligned}$$

Alternative / Larger / Full model

$$\begin{aligned} LE =& \beta_0 + \beta_1 FLR + \beta_2 I(\text{Americas}) + \beta_3 I(\text{Asia}) + \\ & \beta_4 I(\text{Europe}) + \beta_5 FLR \cdot I(\text{Americas}) + \\ & \beta_6 FLR \cdot I(\text{Asia}) + \beta_7 FLR \cdot I(\text{Europe}) + \epsilon \end{aligned}$$

Test interaction between multi-level categorical & continuous variables

- Fit the reduced and full model

```
m_int_wr_red = lm(LifeExpectancyYrs ~ FLR_c + four_regions,
                   data = gapm_sub)
m_int_wr_full = lm(LifeExpectancyYrs ~ FLR_c + four_regions+
                   FLR_c*four_regions, data = gapm_sub)
```

- Display the ANOVA table with F-statistic and p-value

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ FLR_c + four_regions	67.000	1,705.881	NA	NA	NA	NA
LifeExpectancyYrs ~ FLR_c + four_regions + FLR_c * four_regions	64.000	1,641.151	3.000	64.731	0.841	0.476

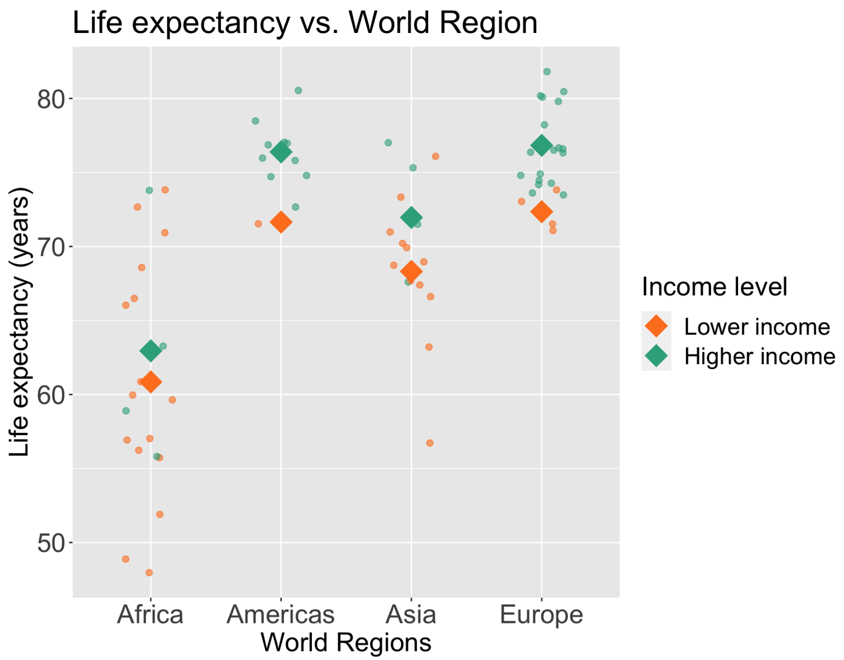
- Conclusion: There is not a significant interaction between female literacy rate and income level (p = 0.478).

Learning Objectives

1. Define confounders and effect modifiers, and how they interact with the main relationship we model.
2. Interpret the interaction component of a model with **a binary categorical covariate and continuous covariate**, and how the main variable's effect changes.
3. Interpret the interaction component of a model with **a multi-level categorical covariate and continuous covariate**, and how the main variable's effect changes.
4. Interpret the interaction component of a model with **two categorical covariates**, and how the main variable's effect changes.

Do we think income level can be an effect modifier for world region?

- Taking a break from female literacy rate to demonstrate interactions for two categorical variables
- We can start by visualizing the relationship between life expectancy and world region *by income level*
- Questions of interest: Does the effect of world region on life expectancy differ depending on income level?
  - This is the same as: Is income level an effect modifier for world region?
- Let's run an interaction model to see!



Model with interaction between a *multi-level categorical and continuous variables*

Model we are fitting:

$$\begin{aligned} LE = & \beta_0 + \beta_1 I(\text{high income}) + \beta_2 I(\text{Americas}) + \beta_3 I(\text{Asia}) + \beta_4 I(\text{Europe}) + \\ & \beta_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \beta_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & \beta_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) + \epsilon \end{aligned}$$

- LE as life expectancy
- $I(\text{high income})$ as indicator of high income
- $I(\text{Americas}), I(\text{Asia}), I(\text{Europe})$ as the indicator for each world region

In R:

```
# gapm_sub = gapm_sub %>% mutate(income_levels2 = relevel(income_levels2, ref = "Hi

m_int_wr_inc = lm(LifeExpectancyYrs ~ income_levels2 + four_regions +
                      income_levels2*four_regions, data = gapm_sub)
m_int_wr_inc = lm(LifeExpectancyYrs ~ income_levels2*four_regions,
                     data = gapm_sub)
```

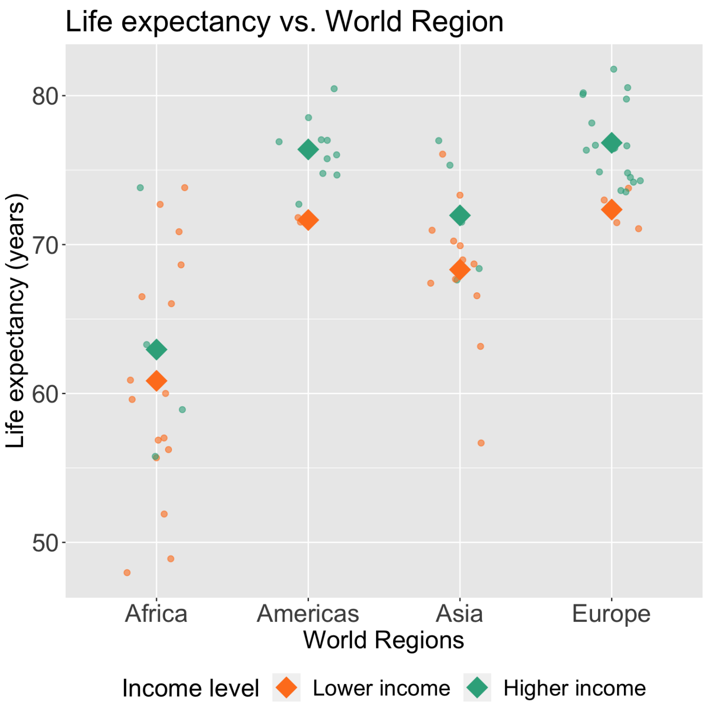
Displaying the regression table and writing fitted regression equation

```
1 tidy(m_int_wr_inc, conf.int=T) %>% gt() %>% tab_options(table.font.size = 25) %>% f
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	60.850	1.281	47.488	0.000	58.290	63.410
income_levels2Higher income	2.100	2.865	0.733	0.466	−3.624	7.824
four_regionsAmericas	10.800	3.844	2.810	0.007	3.121	18.479
four_regionsAsia	7.467	1.957	3.815	0.000	3.556	11.377
four_regionsEurope	11.500	2.865	4.014	0.000	5.776	17.224
income_levels2Higher income:four_regionsAmericas	2.640	4.896	0.539	0.592	−7.141	12.421
income_levels2Higher income:four_regionsAsia	1.543	3.956	0.390	0.698	−6.360	9.447
income_levels2Higher income:four_regionsEurope	2.382	4.020	0.592	0.556	−5.649	10.412

$$
\begin{aligned}
\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\
& \widehat{\beta}_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\
& \widehat{\beta}_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) \\
\widehat{LE} =& 60.85 + 2.10 \cdot I(\text{high income}) + 10.8 \cdot I(\text{Americas}) + 7.47 \cdot I(\text{Asia}) + 11.50 \cdot I(\text{Europe}) + \\
& 2.64 \cdot I(\text{high income}) \cdot I(\text{Americas}) + 1.54 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\
& 2.38 \cdot I(\text{high income}) \cdot I(\text{Europe})
\end{aligned}
$$

Poll Everywhere Question 4



Comparing fitted regression *means* for each world region

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\ & \widehat{\beta}_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & \widehat{\beta}_7 \cdot I(\text{high income}) \cdot I(\text{Europe})\end{aligned}$$

$$\begin{aligned}\widehat{LE} =& 60.85 + 2.10 \cdot I(\text{high income}) + 10.8 \cdot I(\text{Americas}) + 7.47 \cdot I(\text{Asia}) + 11.50 \cdot I(\text{Europe}) + \\ & 2.64 \cdot I(\text{high income}) \cdot I(\text{Americas}) + 1.54 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & 2.38 \cdot I(\text{high income}) \cdot I(\text{Europe})\end{aligned}$$

Africa

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \\ & \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 0 + \widehat{\beta}_4 \cdot 0 + \\ & \widehat{\beta}_5 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_6 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_7 I(\text{high income}) \cdot 0 \\ \widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income})\end{aligned}$$

The Americas

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \\ & \widehat{\beta}_2 \cdot 1 + \widehat{\beta}_3 \cdot 0 + \widehat{\beta}_4 \cdot 0 + \\ & \widehat{\beta}_5 I(\text{high income}) \cdot 1 + \\ & \widehat{\beta}_6 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_7 I(\text{high income}) \cdot 0 \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_2) + \\ & (\widehat{\beta}_1 + \widehat{\beta}_5) I(\text{high income})\end{aligned}$$

Asia

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \\ & \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 1 + \widehat{\beta}_4 \cdot 0 + \\ & \widehat{\beta}_5 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_6 I(\text{high income}) \cdot 1 + \\ & \widehat{\beta}_7 I(\text{high income}) \cdot 0 \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_3) + \\ & (\widehat{\beta}_1 + \widehat{\beta}_6) I(\text{high income})\end{aligned}$$

Europe

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \\ & \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 0 + \widehat{\beta}_4 \cdot 1 + \\ & \widehat{\beta}_5 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_6 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_7 I(\text{high income}) \cdot 1 \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_4) + \\ & (\widehat{\beta}_1 + \widehat{\beta}_7) I(\text{high income})\end{aligned}$$

Comparing fitted regression *means* for each income level

$$
\begin{aligned}
\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\
& \widehat{\beta}_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\
& \widehat{\beta}_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) \\
\widehat{LE} =& 60.85 + 2.10 \cdot I(\text{high income}) + 10.8 \cdot I(\text{Americas}) + 7.47 \cdot I(\text{Asia}) + 11.50 \cdot I(\text{Europe}) + \\
& 2.64 \cdot I(\text{high income}) \cdot I(\text{Americas}) + 1.54 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\
& 2.38 \cdot I(\text{high income}) \cdot I(\text{Europe})
\end{aligned}
$$

For lower income countries: $I(\text{high income}) = 0$

$$
\begin{aligned}
\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 \cdot 0 + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\
& \widehat{\beta}_5 \cdot 0 \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot 0 \cdot I(\text{Asia}) + \widehat{\beta}_7 \cdot 0 \cdot I(\text{Europe}) \\
\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe})
\end{aligned}
$$

For higher income countries: $I(\text{high income}) = 1$

$$
\begin{aligned}
\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 \cdot 1 + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\
& \widehat{\beta}_5 \cdot 1 \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot 1 \cdot I(\text{Asia}) + \widehat{\beta}_7 \cdot 1 \cdot I(\text{Europe}) \\
\widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_1) + (\widehat{\beta}_2 + \widehat{\beta}_5) I(\text{Americas}) + (\widehat{\beta}_3 + \widehat{\beta}_6) I(\text{Asia}) + \\
& (\widehat{\beta}_4 + \widehat{\beta}_7) I(\text{Europe})
\end{aligned}
$$

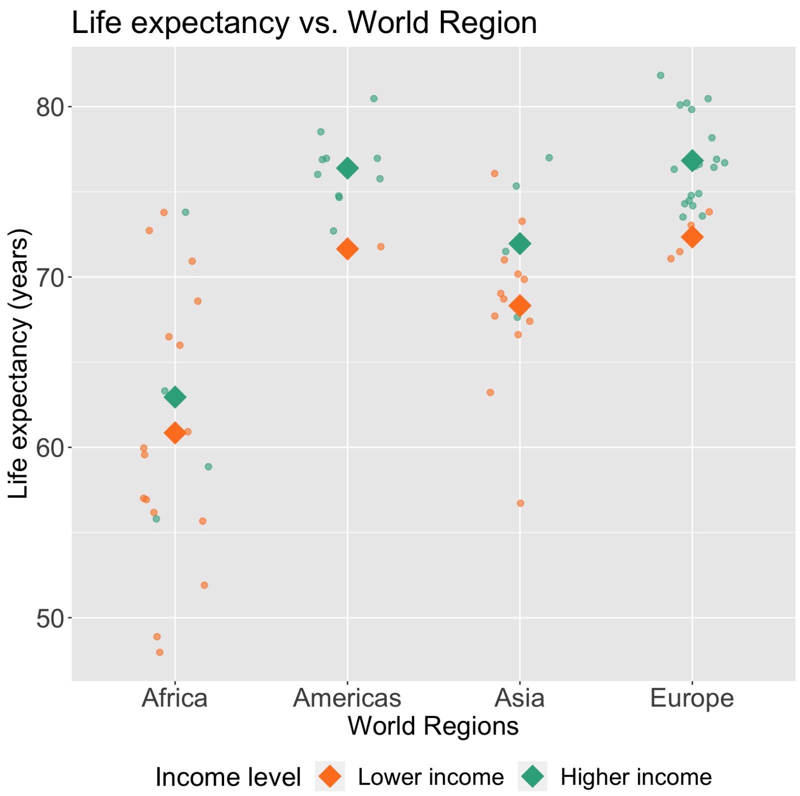
Let's take a look back at the plot

For lower income countries: $I(\text{high income}) = 0$

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe})$$

For higher income countries: $I(\text{high income}) = 1$

$$\widehat{LE} = (\widehat{\beta}_0 + \widehat{\beta}_1) + (\widehat{\beta}_2 + \widehat{\beta}_5) I(\text{Americas}) + (\widehat{\beta}_3 + \widehat{\beta}_6) I(\text{Asia}) + (\widehat{\beta}_4 + \widehat{\beta}_7) I(\text{Europe})$$



Interpretation for interaction between two categorical variables

$$\begin{aligned}
\widehat{LE} = &\widehat{\beta}_0 + \widehat{\beta}_1 \cdot I(\text{high income}) + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\
&\widehat{\beta}_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\
&\widehat{\beta}_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) \\
\widehat{LE} = &\left[\widehat{\beta}_0 + \widehat{\beta}_1 \cdot I(\text{high income})\right] + \left[\widehat{\beta}_2 + \widehat{\beta}_5 \cdot I(\text{high income})\right] I(\text{Americas}) + \\
&\left[\widehat{\beta}_3 + \widehat{\beta}_6 \cdot I(\text{high income})\right] I(\text{Asia}) + \left[\widehat{\beta}_4 + \widehat{\beta}_7 \cdot I(\text{high income})\right] I(\text{Europe})
\end{aligned}$$

- Interpretation:
  - β_1 = mean change in the Africa's life expectancy, comparing high income to low income countries
  - β_5 = mean change in the Americas' effect, comparing high income to low income countries
  - β_6 = mean change in Asia's effect, comparing high income to low income countries
  - β_7 = mean change in Europe's effect, comparing high income to low income countries

Test interaction between two categorical variables

- We run an F-test for a group of coefficients (β_5, β_6, β_7) in the below model (see lesson 9)

$$\begin{aligned} LE = & \beta_0 + \beta_1 I(\text{high income}) + \beta_2 I(\text{Americas}) + \beta_3 I(\text{Asia}) + \beta_4 I(\text{Europe}) + \\ & \beta_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \beta_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & \beta_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) + \epsilon \end{aligned}$$

Null H_0

$\beta_5 = \beta_6 = \beta_7 = 0$

Alternative H_1

$\beta_5 \neq 0$ and/or $\beta_6 \neq 0$ and/or $\beta_7 \neq 0$

Null / Smaller / Reduced model

$$\begin{aligned} LE = & \beta_0 + \beta_1 I(\text{high income}) + \beta_2 I(\text{Americas}) + \\ & \beta_3 I(\text{Asia}) + \beta_4 I(\text{Europe}) + \epsilon \end{aligned}$$

Alternative / Larger / Full model

$$\begin{aligned} LE = & \beta_0 + \beta_1 I(\text{high income}) + \beta_2 I(\text{Americas}) + \beta_3 I(\text{Asia}) + \\ & \beta_4 I(\text{Europe}) + \beta_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \\ & \beta_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \beta_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) + \epsilon \end{aligned}$$

Test interaction between multi-level categorical & continuous variables

- Fit the reduced and full model

```r
m_int_wr_inc_red = lm(LifeExpectancyYrs ~ income_levels2 + four_regions,
                      data = gapm_sub)
m_int_wr_inc_full = lm(LifeExpectancyYrs ~ income_levels2 + four_regions +
                              income_levels2*four_regions, data = gapm_sub)
```

- Display the ANOVA table with F-statistic and p-value

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ income_levels2 + four_regions	67.000	1,693.242	NA	NA	NA	NA
LifeExpectancyYrs ~ income_levels2 + four_regions + income_levels2 * four_regions	64.000	1,681.304	3.000	11.938	0.151	0.928

- Conclusion: There is not a significant interaction between female literacy rate and income level (p = 0.928).

Next time (hour before quiz)

Go back to the remaining learning objectives:

5. Interpret the interaction component of a model with **two continuous covariates**, and how the main variable's effect changes.
6. When there are only two covariates in the model, test whether one is a confounder or effect modifier.