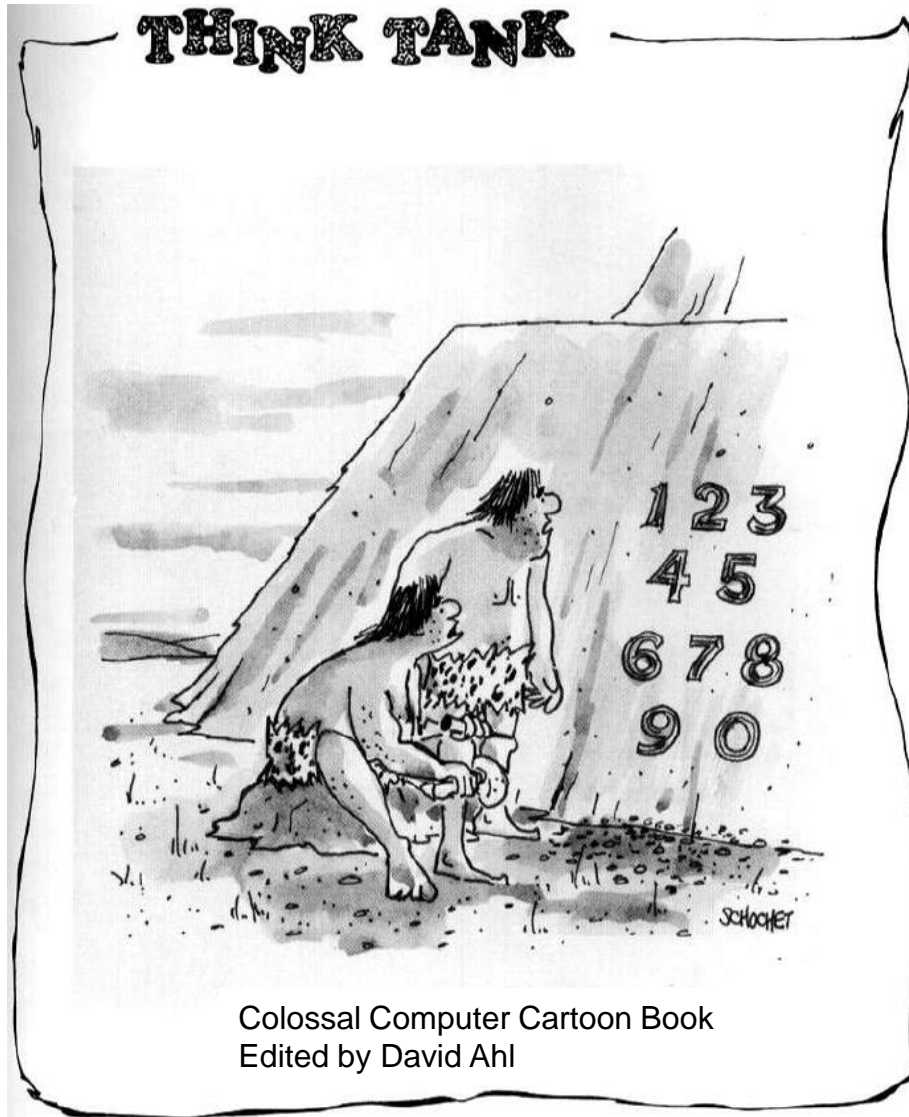


Arithmetic Operations

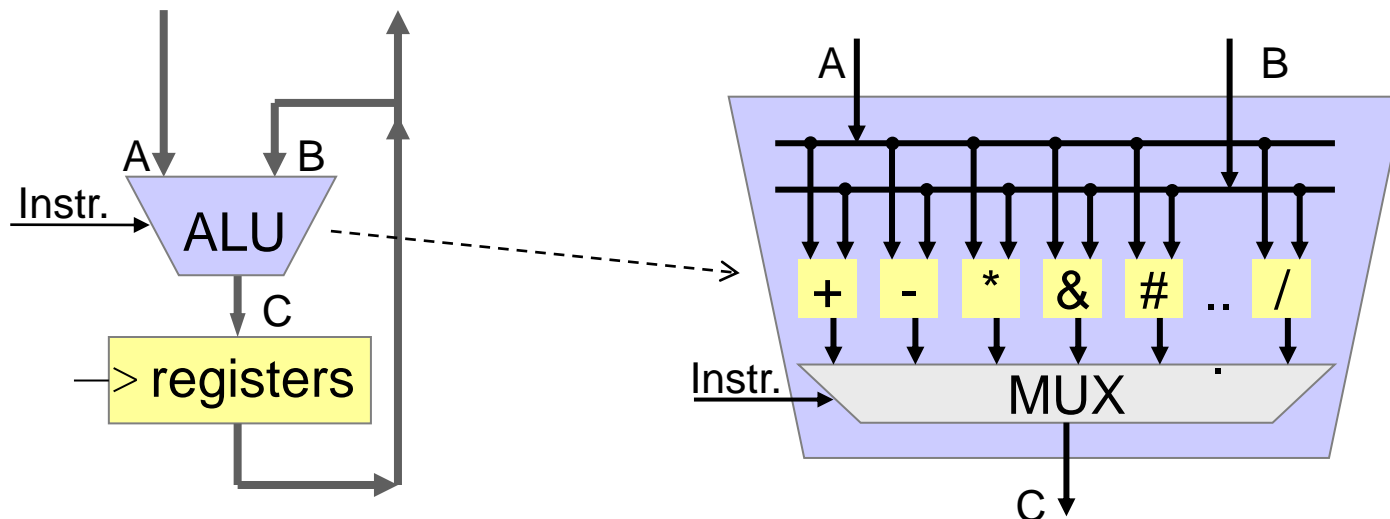
Computer Engineering 1

Motivation



"I call them numbers, you can add them, subtract them, multiply them, divide them ... find their square root ..."

- Instructions to process data in the ALU
 - **arithmetic** Addition, Subtraction, Multiplication, Division
 - **logic** NOT, AND, OR, XOR
 - **shift** Shift left/right. Fill with 0 or MSB
 - **rotate** Cyclic shift left/right: What drops out enters on the other side



■ How often is the for loop executed?

- For `#define INCREMENT 1`
- For `#define INCREMENT 10`

```
...  
int main(void) {  
    uint8_t uc;  
    uint32_t count = 0;  
  
    for (uc = 0; uc < 255; uc += INCREMENT) {  
        printf("hello again %d \n",uc);  
        count++;  
    }  
    printf("Loop executed %d times\n",count);  
}
```

Motivation

■ How often is the for loop executed?

- For `#define INCREMENT 1` → **255 times**
- For `#define INCREMENT 10` → **infinite times**
>= 255 never reached

```
...  
int main(void) {  
    uint8_t uc;  
    uint32_t count = 0;  
  
    for (uc = 0; uc < 255; uc += INCREMENT) {  
        printf("hello again %d \n",uc);  
        count++;  
    }  
    printf("Loop executed %d times\n",count);  
}
```

Why?

```
.....  
hello again 210  
hello again 220  
hello again 230  
hello again 240  
hello again 250  
hello again 4  
hello again 14  
hello again 24  
hello again 34  
hello again 44  
.....
```

Start 16.10.2023

Recap/Übung zu Data-Transfer

- Die Area `myData` liegt im Memory ab Adresse `0x20000004`.
- Ergänzen Sie die Instruktionen und Registerinhalte im Listing unten
- Zeichnen Sie Memory-Map und aktualisieren Sie diese nach jeder Store-Instruktion.

Label	Mnemonic and operands		Comments	Results
	AREA	myData, DATA, READWRITE		
my32BitVar	DCD	0x22446688		
	AREA	myCode, CODE, READONLY		
start	MOVS	R5, #3	; Initialize R5 with 3	R5 =
	LDR	R7,	; load address of my32BitVar into R7	R7 =
			; load my32BitVar as unsigned 32-Bit into R0	R0 =
			; load my32BitVar as unsigned 16-Bit into R1	R1 =
			; load my32BitVar as unsigned 8-Bit into R2	R2 =
	MOVS	R5, #3		
	STRB	R0, [R7, R5]	; Memory map on next page	
	STR	R2, [R7]	; Memory map on next page	

Recap zu Data-Transfer

Label	Menmonic and operands		Comments	Results
	AREA	myData, DATA, READWRITE		
my32BitVar	DCD	0x22446688		
	AREA	myCode, CODE, READONLY		
start	MOVS	R6, #0	; Initialize R6 with 0	R6 = 0x00000000
	LDR	R7, =my32BitVar	; load address of my32BitVar into R7	R7 = 0x20000004
	LDR	R0, [R7, #0]	; load my32BitVar as unsigned 32-Bit into R0	R0 = 0x22446688
Explicit offset	LDRH	R1, [R7, #0]	; load my32BitVar as unsigned 16-Bit into R1	R1 = 0x00006688
optional if zero	LDRB	R2, [R7]	; load my32BitVar as unsigned 8-Bit into R2	R2 = 0x00000088
	MOVS	R5, #3		
	STRB	R0, [R7, R5]	; Memory map on next page	
	STR	R2, [R7]	; Memory map on next page	

Memory Map

Address	Label	Nach Initialisierung	Nach Task 3a (STRB)	Nach Task 3b (STR)
0x2000'0003	My32BitVar			
0x2000'0004		0x88	0x88	0x88
0x2000'0005		0x66	0x66	0x00
0x2000'0006		0x44	0x44	0x00
0x2000'0007		0x22	0x88	0x00
0x2000'0008				

■ Cortex-M0

- Data flow
- Flags
- Overview of arithmetic instructions



■ Add Instructions Cortex-M0

■ Negative Numbers

■ Addition

■ Subtraction

■ Subtract Instructions Cortex-M0

■ Multi-Word Arithmetic

■ Multiplication

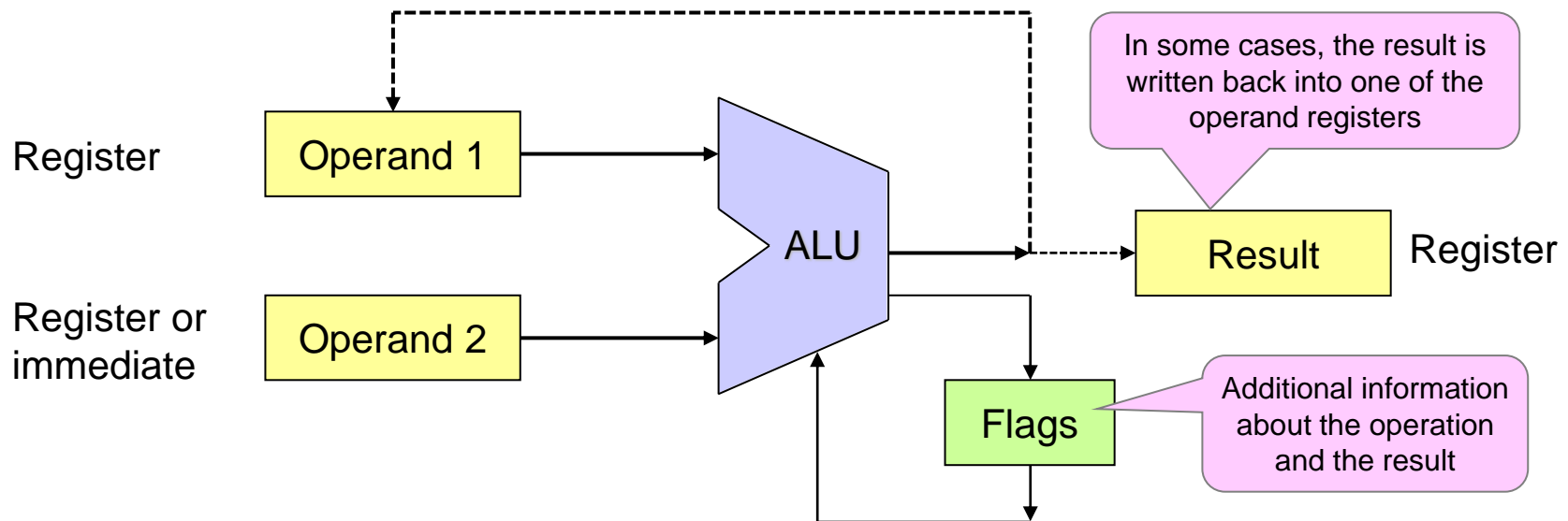
Learning Objectives

At the end of this lesson, you will be able

- to enumerate and apply the Cortex-M0 arithmetic instructions
- to interpret Cortex-M0 assembly programs with arithmetic instructions
- to enumerate and explain the meaning of the Cortex-M0 Flags (N, Z, C, V)
- to carry out additions and subtractions of signed and unsigned integers and to explain the operations with the circle of numbers
- to calculate and interpret carry/borrow and overflow/underflow
- to determine (with the help of documents) the state of Cortex-M0 Flags (N, Z, C, V) after an arithmetic instruction
- to describe how addition and subtraction are done in hardware (in the ALU)
- to program integer calculations with operands that exceed the number of bits available in the ALU
- to explain how numbers in two's complements representation are multiplied

■ Operands and results stored in registers

- Exception: Immediate operations with data in OP-code
- Load/store architecture



Examples

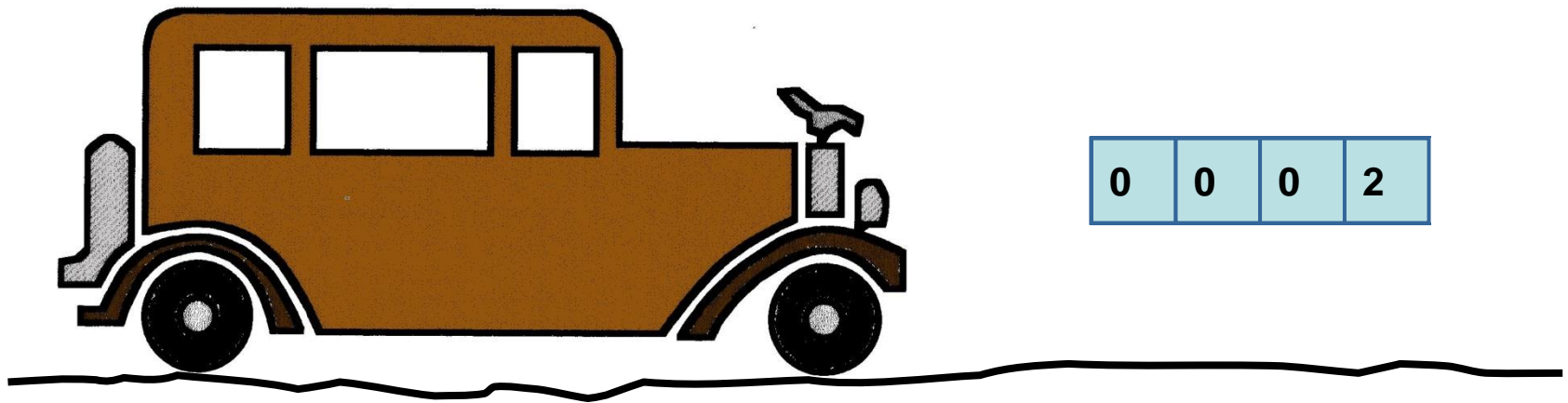
ADDS	R0,R0,R1	; R0 = R0 + R1	result back in R0
ADDS	R0,#0x34	; R0 = R0 + 0x34	
SUBS	R3,R4,#5	; R3 = R4 - 0x05	result in other reg

Wir springen vor...

- **In den nächsten Folien nehmen wir 4-Bit-Zahlen als Referenz, also die Zahlen von 0b0000 – 0b1111**

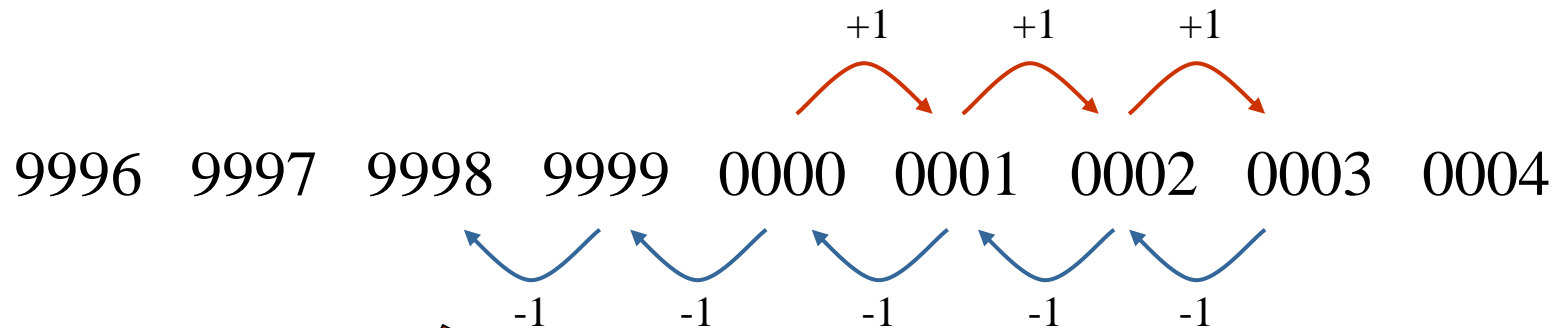
Negative Numbers

■ Lord Ardry's new Rolls Royce



Negative Numbers

■ What happened?



$$0000 - 1 - 1 \triangleq -2 \triangleq 9998$$

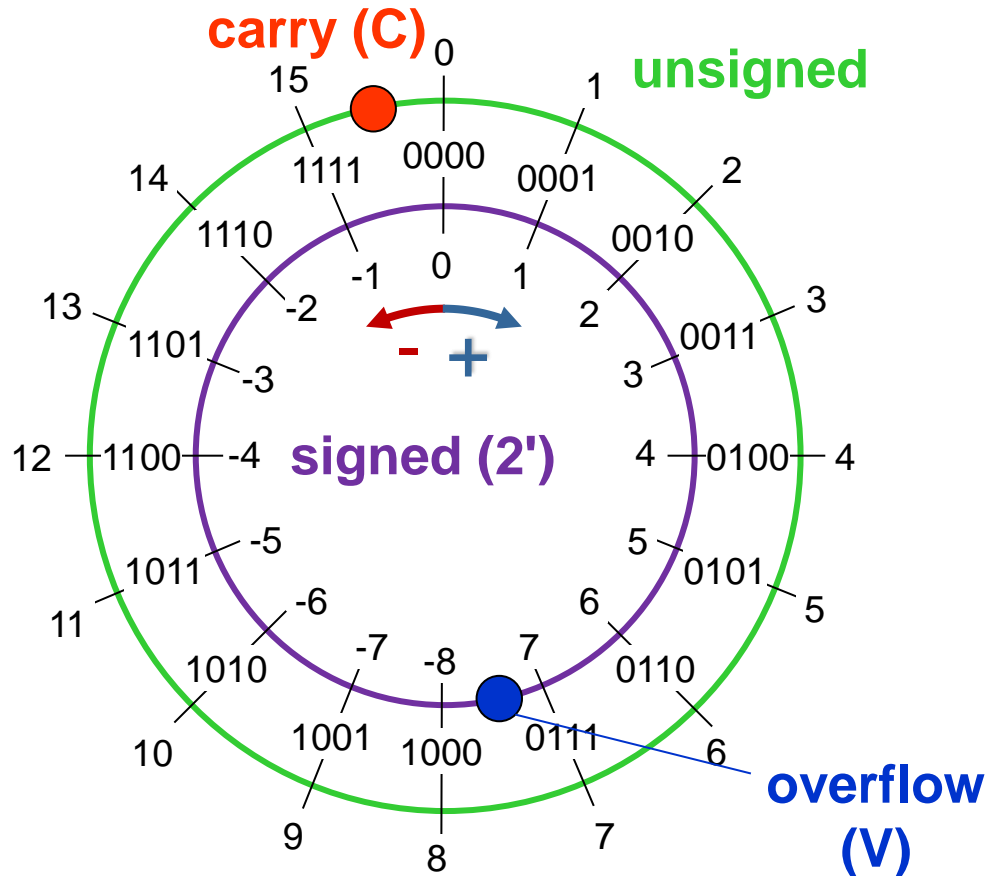
possible because

- number of digits is finite
- respectively given by the word size

Negative Numbers

signed $-b_3 \cdot 2^3 + b_2 \cdot 2^2 + b_1 \cdot 2^1 + b_0 \cdot 2^0$

unsigned $+b_3 \cdot 2^3 + b_2 \cdot 2^2 + b_1 \cdot 2^1 + b_0 \cdot 2^0$

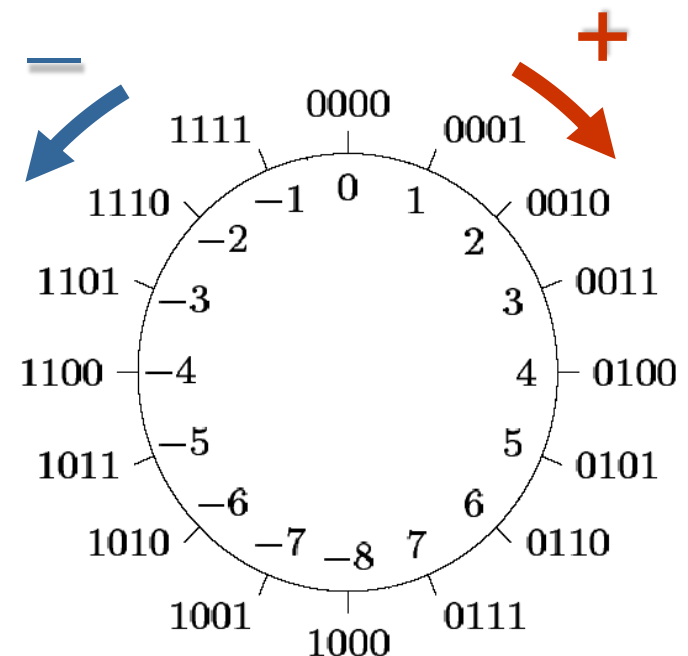


binary	unsigned	signed 2'-Compl.
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

Negative Numbers

■ Numbers with finite number of digits

- can be represented on a circle
- Addition
 - Clockwise
- Subtraction
 - Counter-clockwise

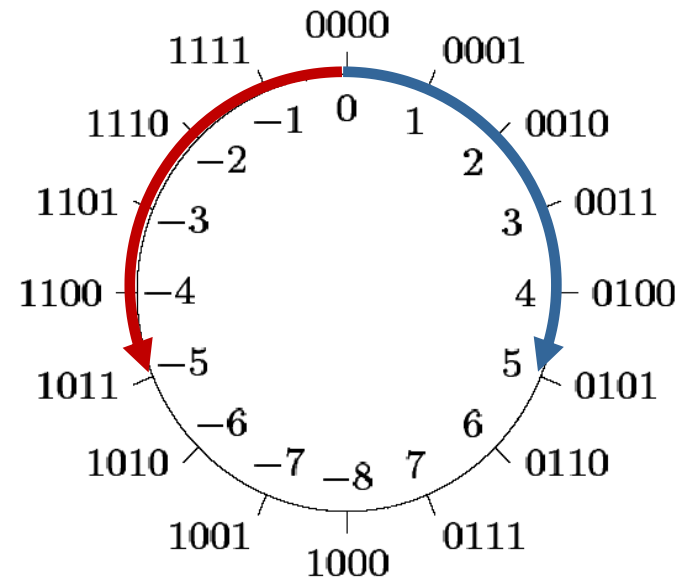


■ Negation of a number

- **Idea:** $-a = 0 - a$
- Starting at 0000 enter the absolute value of **a** counter-clockwise
- Example: $5d = 0101 \rightarrow -5d = 1011$

written calculation

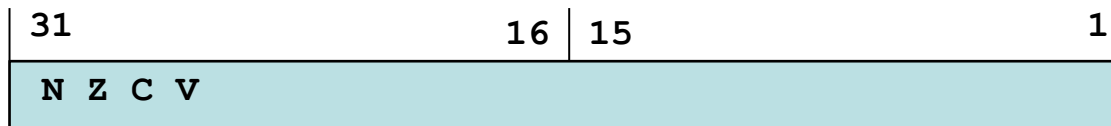
$$\begin{array}{r} 0000 \\ - 0101 \\ \hline 1\ 1\ 1\ 1 \\ \hline 1011 \\ \hline \end{array}$$



Back on Track...

■ Cortex-M0

- APSR: Application Program Status Register

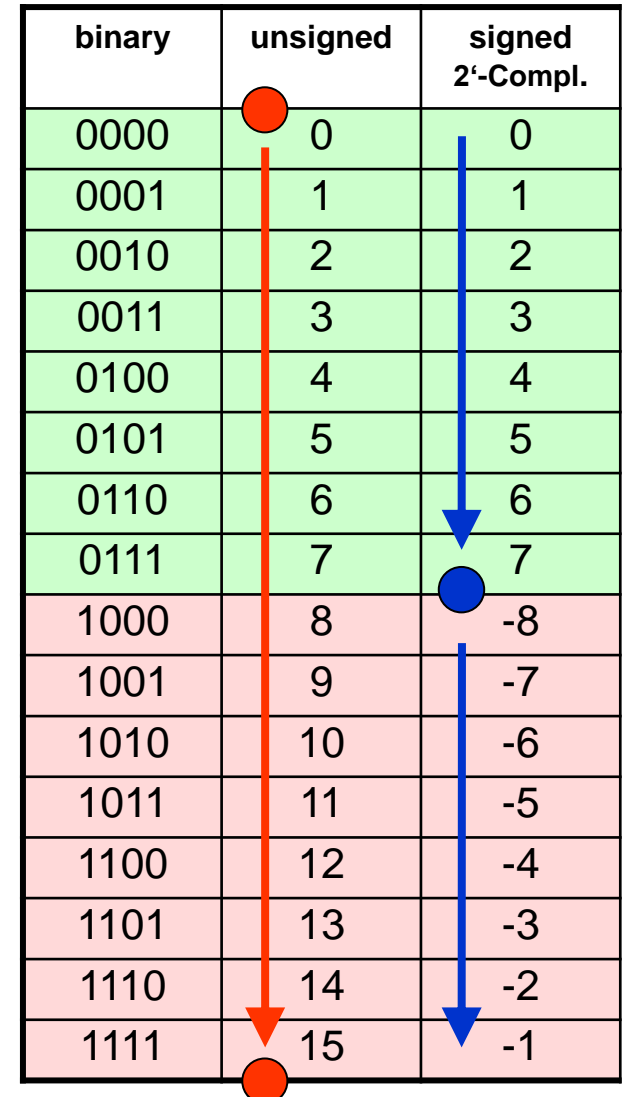


Flag	Meaning	Action	Operands
Negative	MSB = 1	N = 1	signed
Zero	Result = 0	Z = 1	signed , unsigned
Carry	Carry	C = 1	unsigned
Overflow	Overflow	V = 1	signed

- Processor does not know whether the user is working with **unsigned (C)** or with **signed (V)** numbers
→ Processor therefore always calculates C and V!

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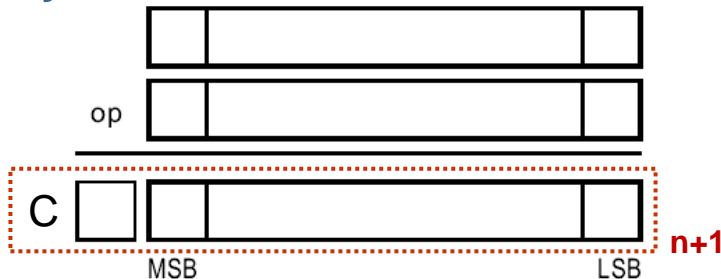
unsigned $+b_3 \cdot 2^3 + b_2 \cdot 2^2 + b_1 \cdot 2^1 + b_0 \cdot 2^0$



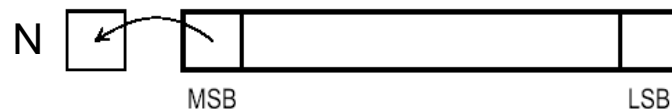
■ Cortex-M0

- Instructions ending with ,S' allow modification of flags
- Examples: **MOVS**, **ADDS**, **SUBS**, ...

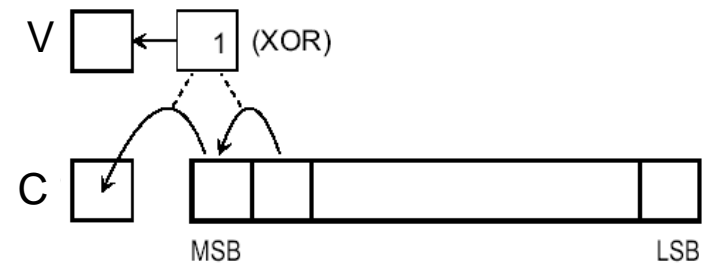
Carry



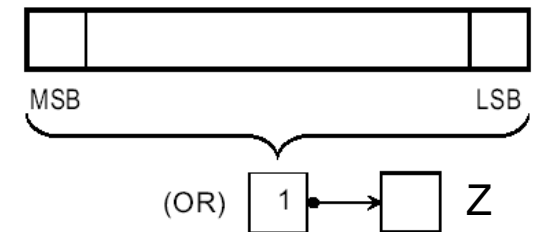
Negative



Overflow



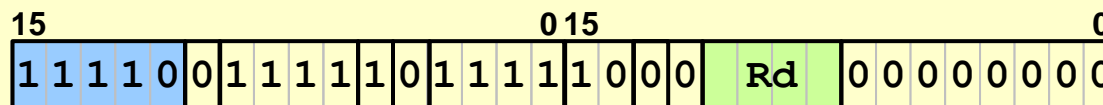
Zero



■ Instructions to access APSR

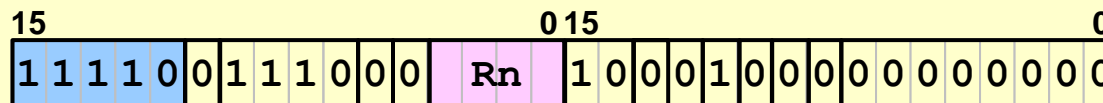
- **MRS** Move to register from special register (APSR)
- **MSR** Move to special register (APSR) from register
- 32-bit opcode

MRS <Rd>, APSR



Rd = APSR

MSR APSR, <Rn>



APSR = Rn

■ Overview Cortex-M0

Mnemonic	Instruction	Function
ADD / ADDS	Addition	$A + B$
ADCS	Addition with carry	$A + B + c$
ADR	Address to Register	$PC + A$
SUB / SUBS	Subtraction	$A - B$
SBCS	Subtraction with carry (borrow)	$A - B - \text{NOT}(c)$ ¹⁾
RSBS	Reverse Subtract (negative)	$-1 \cdot A$
MULS	Multiplication	$A \cdot B$

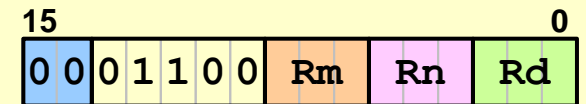
¹⁾ borrow = NOT(c)

Add Instructions Cortex-M0

■ ADDS (register)

- Update of flags
- Result and two operands
- Only low register

ADDS <Rd>, <Rn>, <Rm>



$Rd = Rn + Rm$

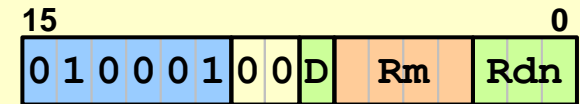
00000002	18D1	ADDS	R1, R2, R3	
00000004	1889	ADDS	R1, R1, R2	
00000006	1889	ADDS	R1, R2	; the same (dest = R1)
00000008		;ADDS	R9, R2	; not possible (high reg)
00000008		;ADDS	R1, R10	; not possible (high reg)

Add Instructions Cortex-M0

■ ADD (register)

- No update of Flags
- High or low register
- $\langle Rdn \rangle \rightarrow$ result and operand
 - I.e. same register for operand and result!

ADD $\langle Rdn \rangle, \langle Rm \rangle$



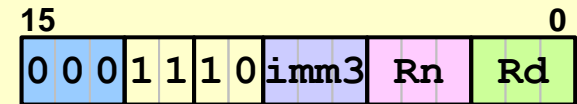
$Rdn = Rdn + Rm$

```
00000008 4411    ADD    R1,R1,R2    ; low regs
0000000A 44D1    ADD    R9,R9,R10   ; high regs
0000000C 44D1    ADD    R9,R10      ; the same (dest = R9)
0000000E 4411    ADD    R1,R1,R2
0000000E          ;ADD    R1,R2,R3    ; not possible
```

■ ADDS (immediate) – T1

- Update of flags
- Two different¹⁾ low registers and immediate value 0 - 7

ADDS <Rd>,<Rn>,#<imm3>



$Rd = Rn + \langle imm3 \rangle$

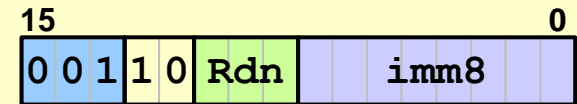
00000010 1D63	ADDS	R3,R4,#5	
00000012	;ADDS	R3,R4,#8	; out of range immediate
00000012	;ADDS	R10,R11,#5	; not possible (high reg)

¹⁾ If the same register is used for Rd and Rn, the assembler will choose the encoding T2 on the next slide

■ ADDS (immediate) – T2

- Update of flags
- Low register with immediate value 0 - 255d
- <Rdn> → Result and operand in same register

ADDS <Rdn>, #<imm8>



$Rdn = Rdn + \langle imm8 \rangle$

```
00000012 33F0  ADDS    R3,R3,#240
00000014 33F0  ADDS    R3,#240      ; the same (dest = R3)
00000016      ;ADDS    R8,R8,#240 ; not possible (high reg)
00000016      ;ADDS    R3,#260  ; out of range immediate
```

■ ADD / ADDS (Summary)

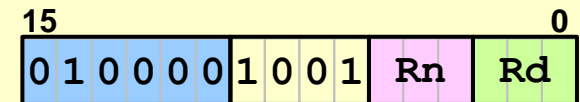
- ADD Flags not changed
- ADDS Flags changed according to Operation Result

Instr	Rd	Rn	Rm	imm	Restrictions
ADD	R0-R15	R0-R15	R0-R15	-	Rd and Rn must specify the same register. Rn and Rm must not both specify the PC (R15)
ADDS	R0-R7	R0-R7	-	0 - 7	-
ADDS	R0-R7	R0-R7	-	0 - 255	Rd and Rn must specify the same register
ADDS	R0-R7	R0-R7	R0-R7	-	-

■ Instruction RSBS

- *Reverse Subtract*
- Generates 2' complement
- Updates flags
- Only low register possible

RSBS <Rd>, <Rn>, #0



$Rd = 0 - Rn$

00000022	4251	RSBS	R1,R2,#0	
00000024	427F	RSBS	R7,R7,#0	
00000026	427F	RSBS	R7,#0	; the same (dest = R7)
00000028		;RSBS	R8,R1,#0	;not possible (high reg)
00000028		;RSBS	R1,R8,#0	;not possible (high reg)

■ Example: Instruction RSBS

C-Code

```
int32_t intA = 3;
int32_t intB;

int32_t invertSign(void)
{
    ...
    intB = -intA;
    ...
}
```

Assembler-code

```
AREA MyCode, CODE, READONLY
```

```
...
```

```
LDR    R0,=intA    ①
```

```
LDR    R0,[R0,#0]
```

```
RSBS   R0,R0,#0
```

```
LDR    R1,=intB    ①
```

```
STR    R0,[R1,#0]
```

```
...
```

```
AREA MyData, DATA, READWRITE
```

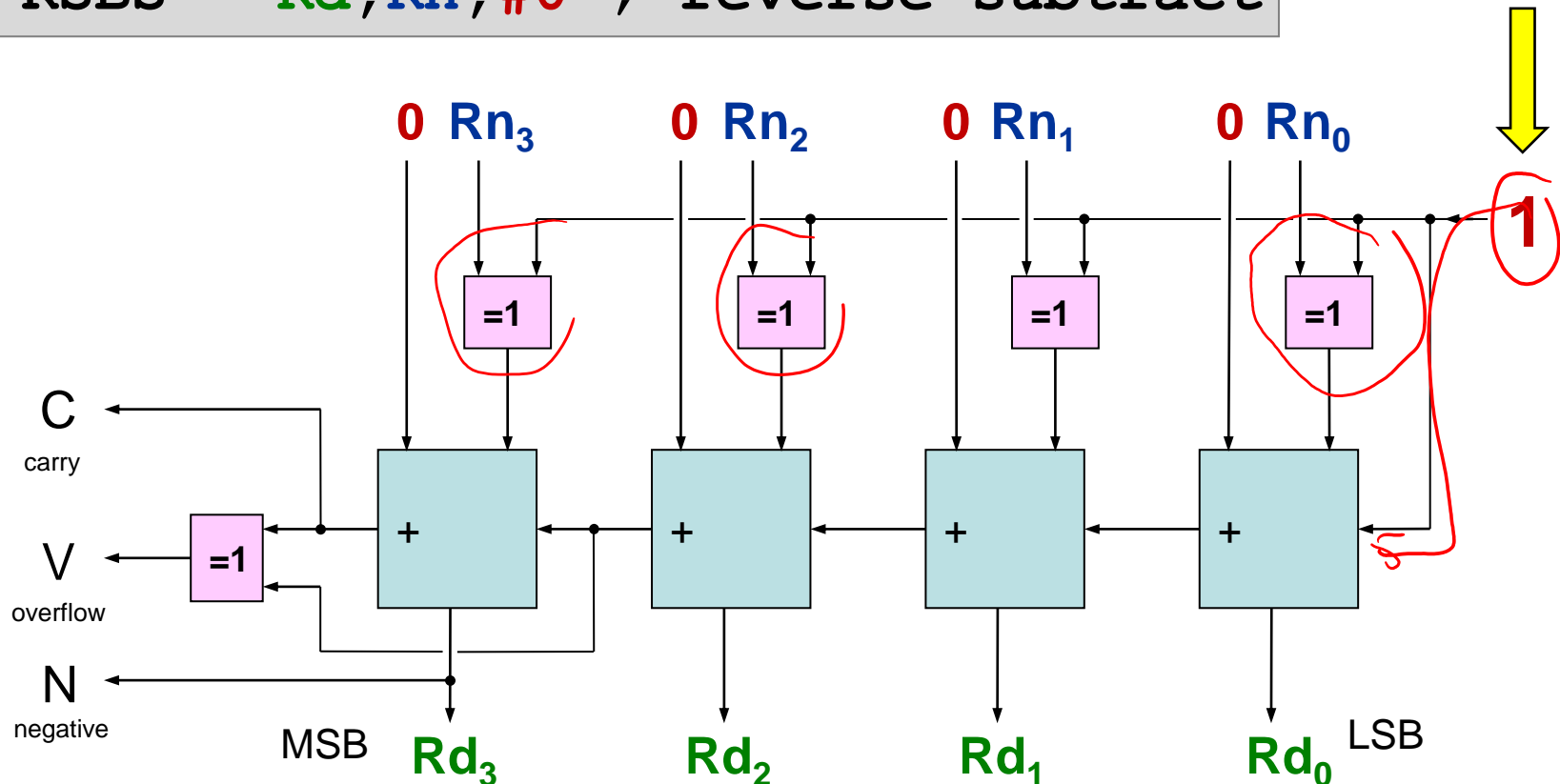
```
intA    DCD 0x00000003
```

```
intB    DCD 0x00000000
```

Negative Numbers

■ 2' complement in Hardware (ALU)

RSBS **Rd**, **Rn**, **#0** ; reverse subtract



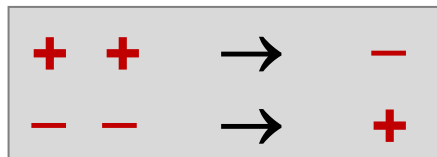
Addition

■ Unsigned

- C = 1 indicates carry
- V irrelevant
- Addition of 2 big numbers
→ can yield a small result

■ Signed

- V = 1 indicates overflow
- possible with same „sign“

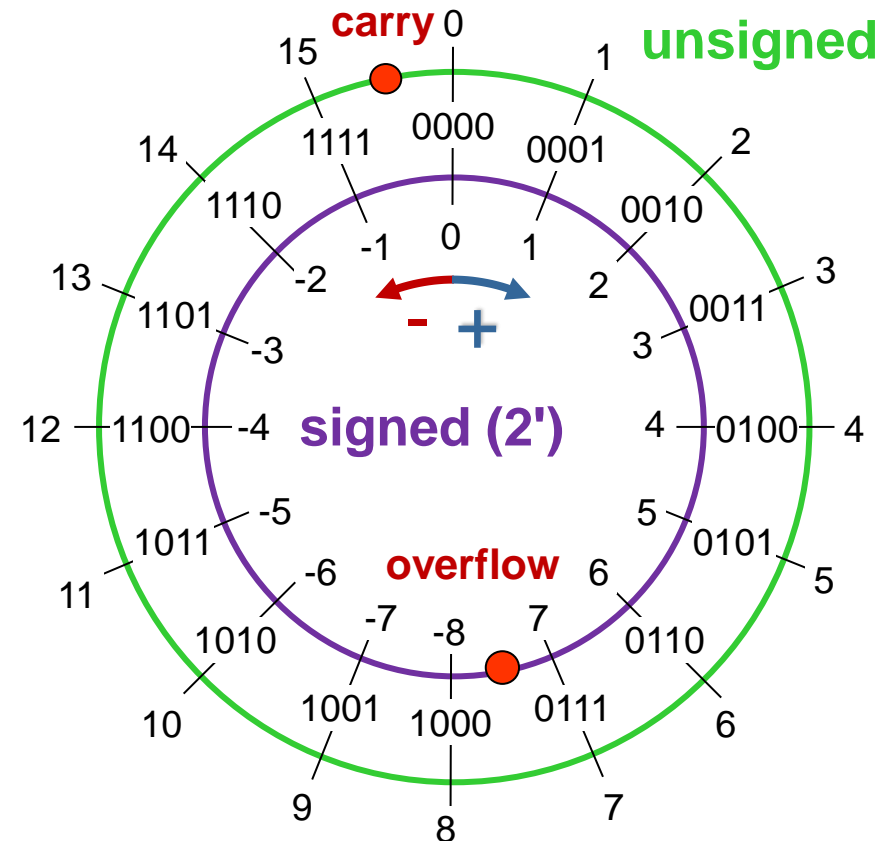


- C irrelevant

Programmer interprets register content
either as signed or as unsigned.

CPU does not care!

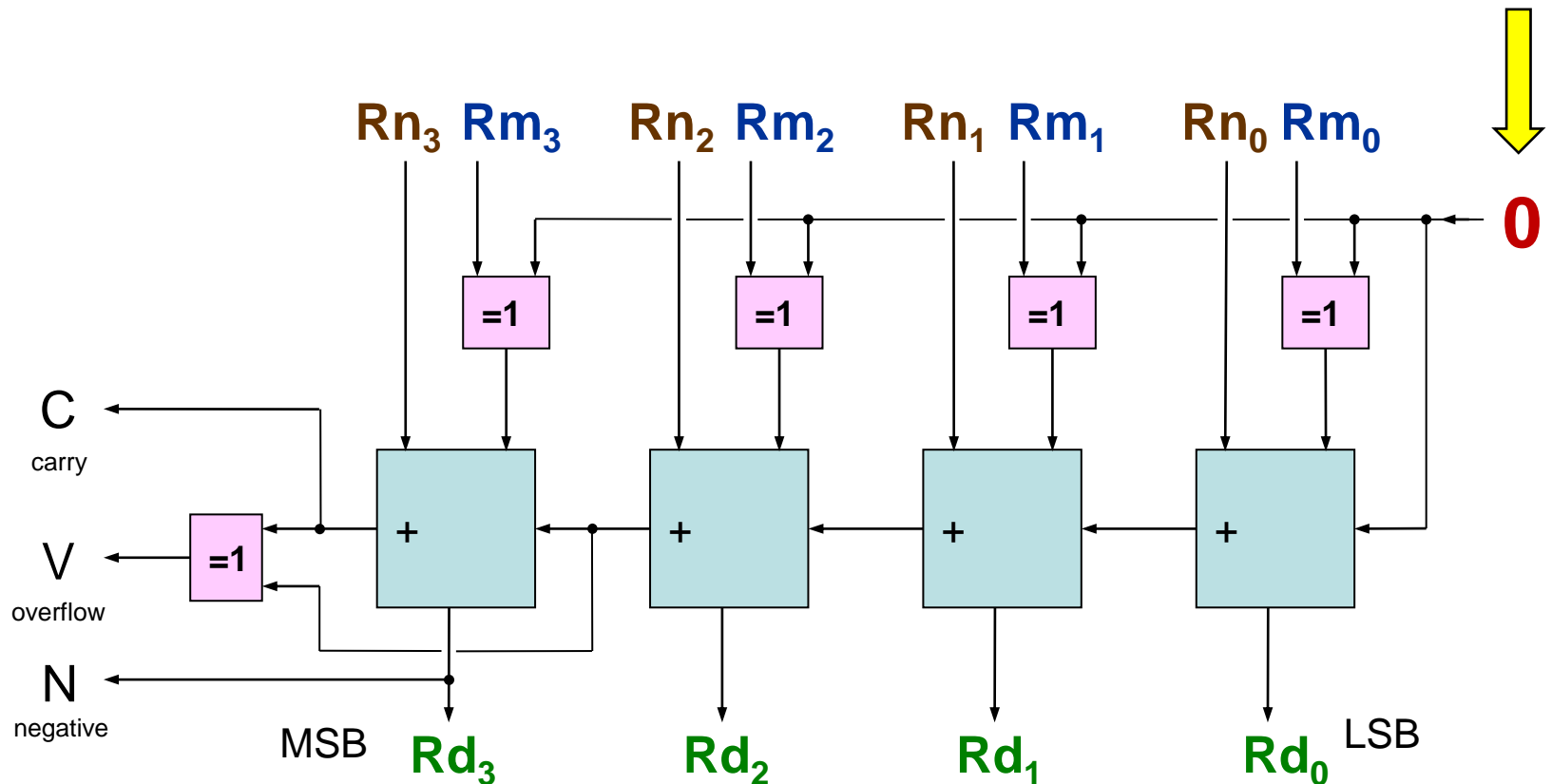
```
ADDS R1,R2,R3
```



Addition

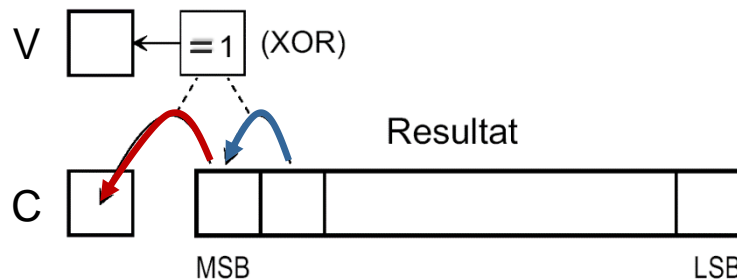
■ Addition in Hardware (ALU)

ADDS Rd, Rn, Rm



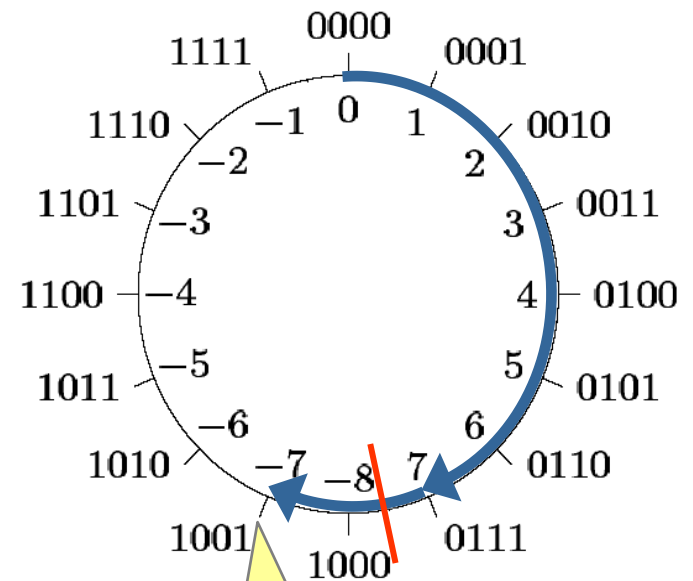
■ Signed

- Overflow carry to MSB has different value than carry from MSB



- Examples for 4 cases

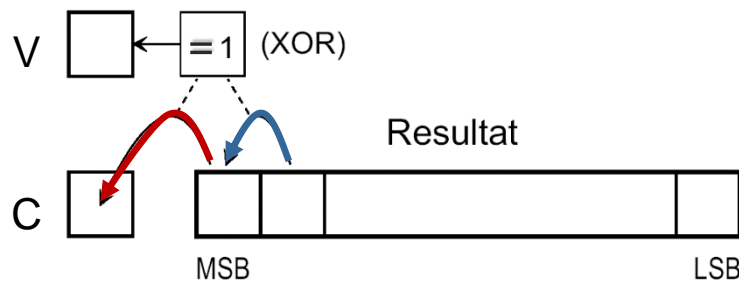
0 1	1 0	0 0	1 1
$\begin{array}{r} 0111 \\ +0010 \\ \hline 01110 \\ \hline 1001 \end{array}$	$\begin{array}{r} 1010 \\ +1101 \\ \hline 10101 \\ \hline 0111 \end{array}$	$\begin{array}{r} 0001 \\ +1110 \\ \hline 00110 \\ \hline 1111 \end{array}$	$\begin{array}{r} 0011 \\ +1111 \\ \hline 01111 \\ \hline 0010 \end{array}$
V = 1	V = 1	V = 0	V = 0



Number 9 cannot be represented

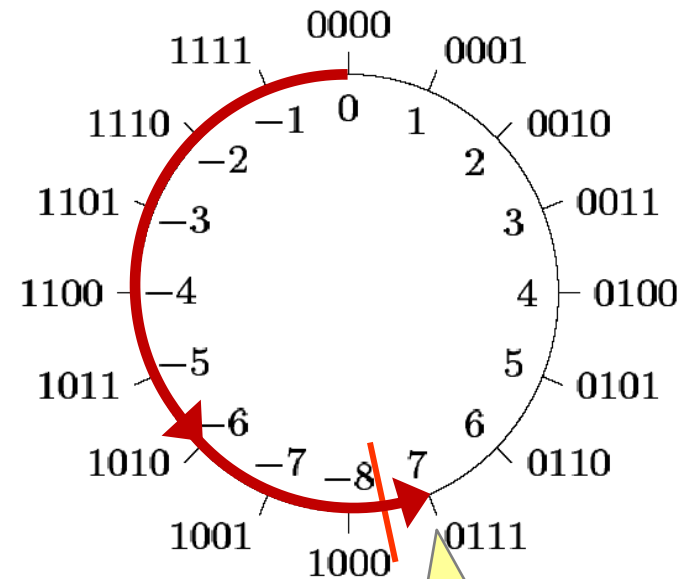
■ Signed

- Overflow carry to MSB has different value than carry from MSB



- Examples for 4 cases

0 1	1 0	0 0	1 1
$\begin{array}{r} 0111 \\ +0010 \\ \hline 1001 \\ \hline \end{array}$	$\begin{array}{r} 1010 \\ +1101 \\ \hline 1000 \\ \hline 0111 \\ \hline \end{array}$	$\begin{array}{r} 0001 \\ +1110 \\ \hline 0000 \\ \hline 1111 \\ \hline \end{array}$	$\begin{array}{r} 0011 \\ +1111 \\ \hline 1111 \\ \hline 0010 \\ \hline \end{array}$
V = 1	V = 1	V = 0	V = 0



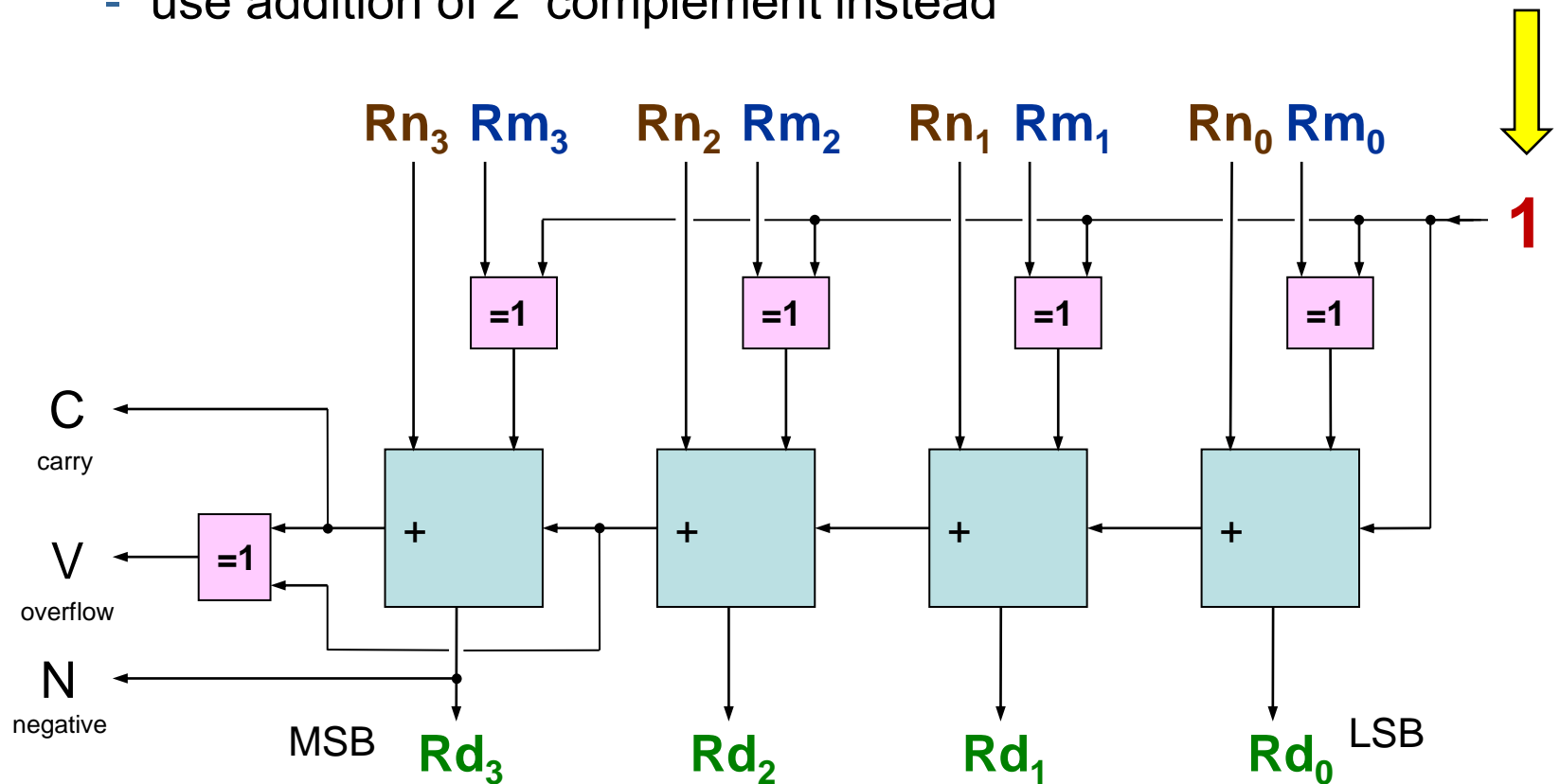
Number -9 cannot be represented

Subtraction

■ Subtraction in Hardware (ALU)

SUBS **Rd**, **Rn**, **Rm**

- There is no subtraction!
 - use addition of 2's complement instead



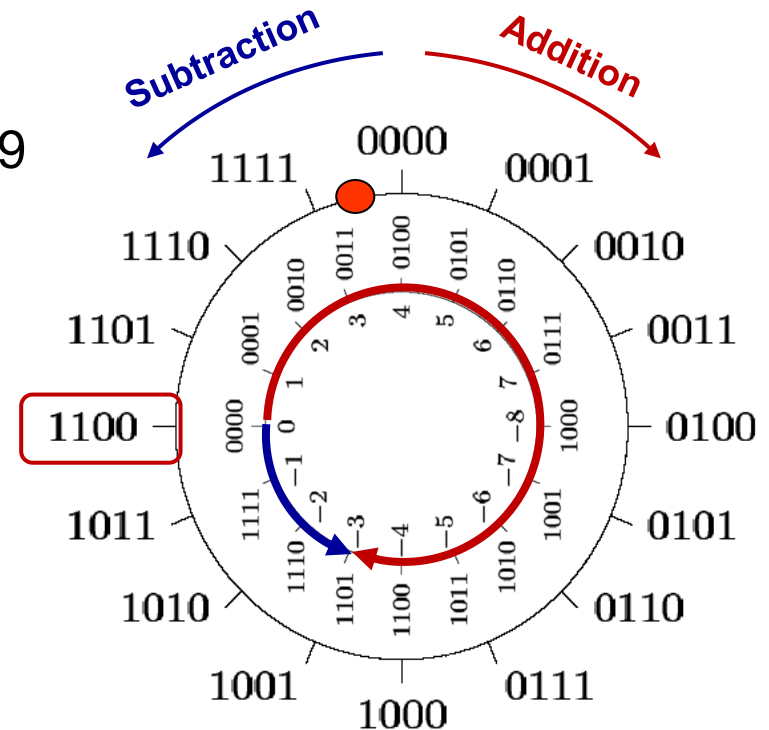
Subtraction

■ unsigned

- Use 2' complement as well
- Example 4-Bit unsigned: $12 - 3 = 9$

12	1100	1100
- 3	- 0011	+ 1101
<hr/>		
9	1001	1 1001
	human	computer

- Attention
 - $C = 1 \rightarrow$ NO borrow
 - $C = 0 \rightarrow$ borrow
- Borrow
 - operation yields negative result \rightarrow cannot be represented in unsigned
 - in multi-word operations missing digits are borrowed from more significant word



Subtraction

Programmer interprets register content
either as signed or as unsigned.

CPU does not care!

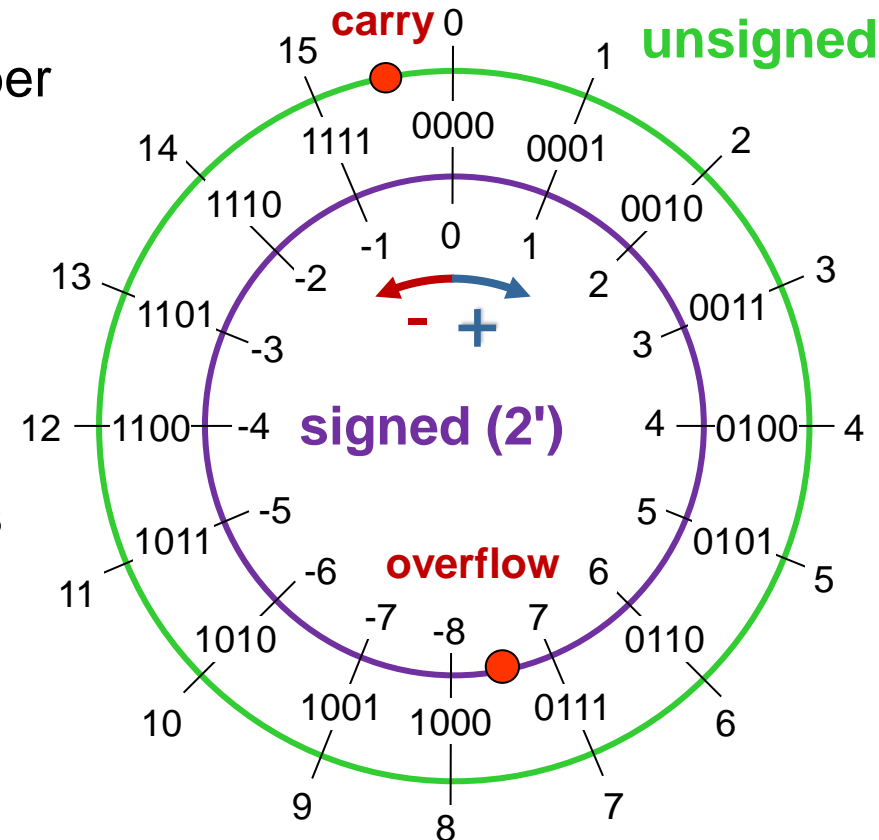
```
SUBS R1,R2,R3
```

■ Unsigned

- C = 0 indicates borrow
- V irrelevant
- Subtraction from a small number
→ can yield a big result

■ Signed

- V = 1 indicates overflow or underflow
 - Possible with opposite signs
 - NOT possible when operands have same sign
- C irrelevant

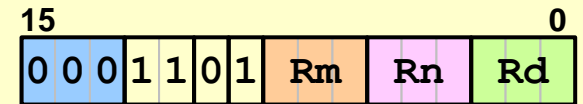


Subtraction Operations Cortex-M0

■ SUBS(register)

- Updates flags
- Result and 2 operands
- Only low register

SUBS <Rd>, <Rn>, <Rm>



$$\begin{aligned} \text{Rd} &= \text{Rn} - \text{Rm} \\ &= \text{Rn} + \text{NOT}(\text{Rm}) + 1 \end{aligned}$$

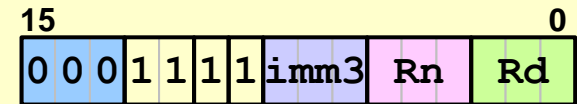
```
00000016 1AD1      SUBS      R1,R2,R3
00000018 1B64      SUBS      R4,R4,R5
0000001A 1B64      SUBS      R4,R5          ; the same (dest = R4)
0000001C
0000001C          ;SUBS      R8,R4,R5      ; not possible (high reg)
0000001C          ;SUBS      R4,R8,R5      ; not possible (high reg)
0000001C          ;SUBS      R4,R5,R8      ; not possible (high reg)
```


Subtraction Operations Cortex-M0

■ SUBS (immediate) – T1

- Updates flags
- 2 different low registers and immediate value 0 - 7d

SUBS <Rd>, <Rn>, #<imm3>



$Rd = Rn - \langle imm3 \rangle$

$= Rn + NOT\langle imm3 \rangle + 1$

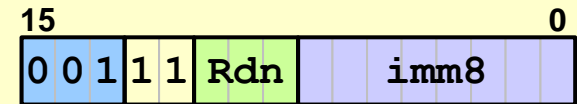
0000001C 1F63	SUBS	R3, R4, #5	
0000001E	; SUBS	R3, R4, #8	; out of range immediate
0000001E	; SUBS	R10, R11, #5	; not possible (high reg)

Subtraction Operations Cortex-M0

■ SUBS (immediate) – T2

- Updates flags
- One low register and immediate value 0 - 255d
- <Rdn> → Result and operand
→ same register for result and operand

SUBS <Rdn>, #<imm8>



$Rdn = Rdn - \langle imm8 \rangle$
 $= Rdn + NOT\langle imm8 \rangle + 1$

```
0000001E 3BF0 SUBS R3, R3, #240
00000020 3BF0 SUBS R3, #240 ; the same (dest = R3)
00000022 ;SUBS R8, R8, #240 ; not possible (high reg)
00000022 ;SUBS R3, #260 ; out of range immediate
```

Addition / Subtraction

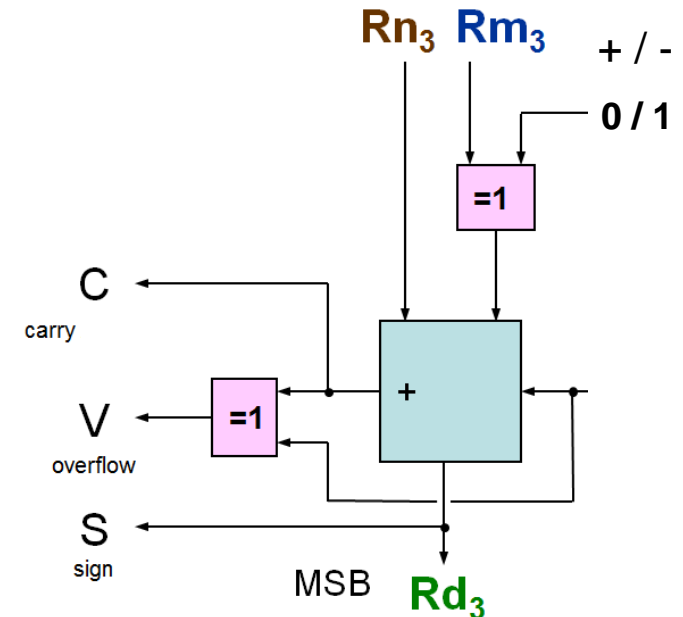
■ Interpretation of carry flag

- Addition $C = 1 \rightarrow \text{Carry}$

1	1	0	1	13d	
0	1	1	1	7d	
1	1	1	1		
1	0	1	0	0	20d \rightarrow 16d + 4d

- Subtraction $C = 0 \rightarrow \text{Borrow}$

6d - 14d = 0110b - 1110b = 0110b + 0010b					
0	1	1	0	6d	
0	0	1	0	2d = TC(14d)	
0	1	1	0		
0	1	0	0	0	8d \rightarrow - 16d + 8d



■ **unsigned** Interpretation

- Program must check **carry flag (C)** after operation
- **C = 1 for Addition** **C = 0 for Subtraction**
 - Result cannot be represented (not enough digits / no negative numbers)
 - Full turn on number circle must be added or subtracted
→ odometer effect
- Overflow flag (V) irrelevant

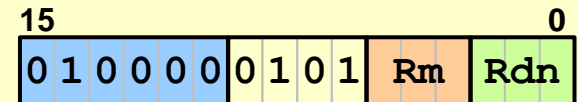
■ **signed** Interpretation

- Program must check **overflow flag (V)** after operation
- **V = 1** means
 - Not enough digits available to represent the result
 - Full turn on number circle must be added or subtracted
→ odometer effect
- Carry flag (C) irrelevant

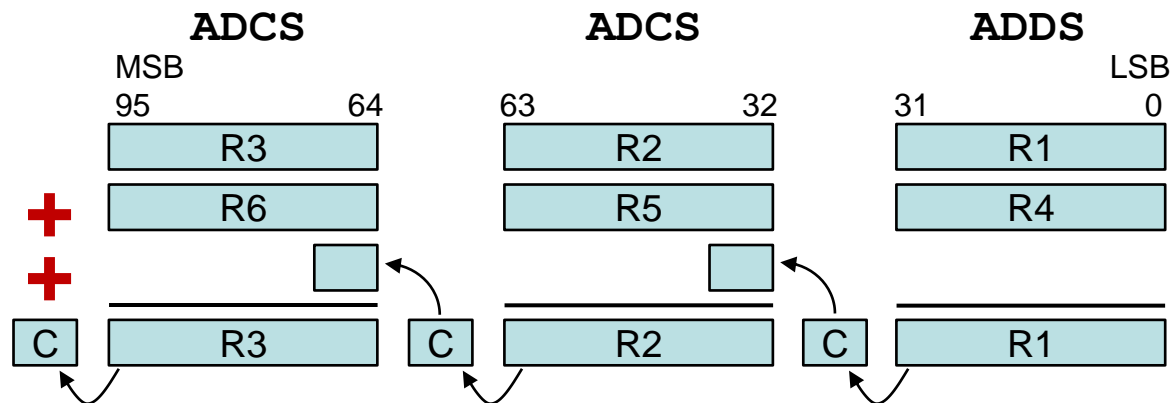
■ Multi-Word Addition ADCS

- Example: Addition of two 96-bit Operands

ADCS <Rdn>, <Rm>



$Rdn = Rdn + Rm + C$



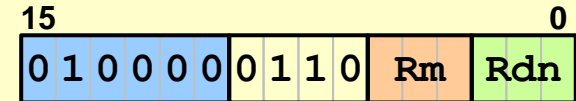
```
; Operand A:      96-bit in R3 (MSW) ,R2 ,R1 (LSW)
; Operand B:      96-bit in R6 (MSW) ,R5 ,R4 (LSW)
; Result = A + B: 96-bit in R3 (MSW) ,R2 ,R1 (LSW)

00000028 1909      ADDS      R1 ,R1 ,R4
0000002A 416A      ADCS      R2 ,R2 ,R5
0000002C 4173      ADCS      R3 ,R3 ,R6
```

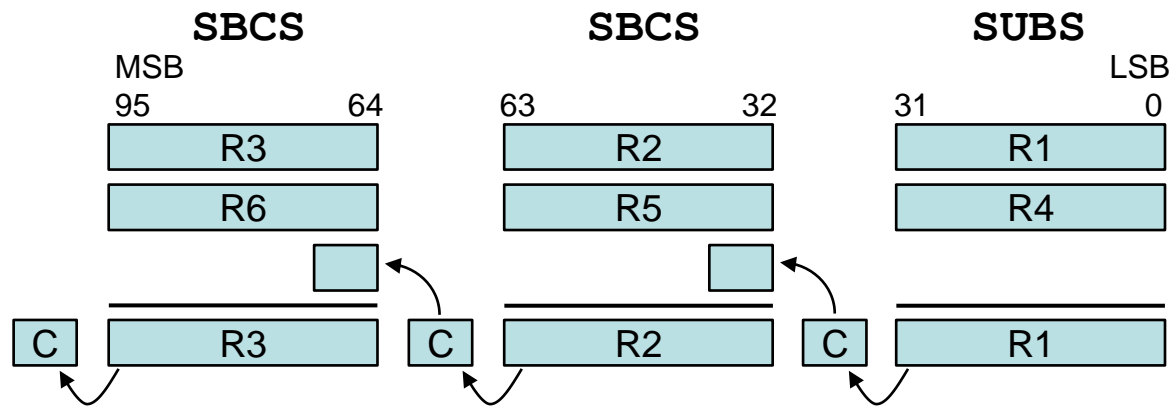
■ Multi-Word Subtraction SBCS

- Example: Subtraction of two 96-bit Operands

SBCS <Rdn>, <Rm>



$$\begin{aligned} Rdn &= Rdn - Rm - \text{NOT}(C) \\ &= Rdn + \text{NOT}(Rm) + C \end{aligned} \quad ^1)$$



```
; Operand A: 96-bit in R3 (MSW), R2, R1 (LSW)
; Operand B: 96-bit in R6 (MSW), R5, R4 (LSW)
; Result A - B: 96-bit in R3 (MSW), R2, R1 (LSW)
0000002E 1B09 SUBS R1, R1, R4
00000030 41AA SBCS R2, R2, R5
00000032 41B3 SBCS R3, R3, R6
```

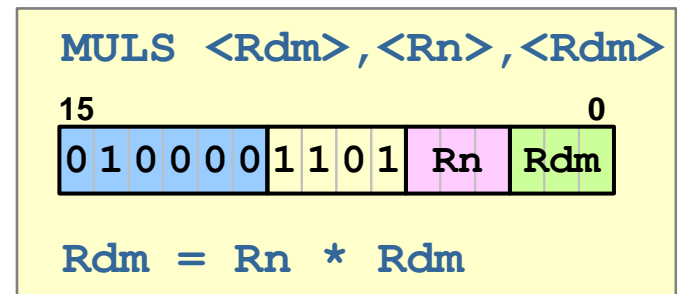
¹⁾ $-Rm - \text{NOT}(C)$
 $= (\text{NOT}(Rm) + 1) - \text{NOT}(C)$
 $= \text{NOT}(Rm) + C$

The CPU actually calculates
 $Rdn + \text{NOT}(Rm) + C$

Start 23.10.2023

■ MULS (register)

- Flags
 - N and Z updated
 - C and V unchanged
- Operands
 - Rn multiplicand
 - Rdm multiplier
 - only low registers
- Result
 - **Rdm contains only lowest 32 bits of product**



```
0000002E 4351 MULS R1,R2,R1
```

```
00000030 ;MULS R1,R1,R2 ; not possible: destination and
00000030 ; 2nd source must be same
00000030 ;MULS R1,R8,R1 ; not possible: high reg
```


- **Result requires twice as many binary digits**
 - Example 4-bit * 4-bit → 8-bit result
- **Signed and unsigned multiplication are different**

```

0101 * 0011
-----
      0011
     0000
    0011
   0000
  -----
00001111
    
```

Interpretation **unsigned**
5d * 3d = 15d → correct

Interpretation **signed**
5d * 3d = 15d → correct

unsigned

```

0101 * 1101
-----
 00001101
00000000
 00001101
00000000
  -----
0000100001
    
```

zero extension
of multiplier

Interpretation **unsigned**
5d * 13d = 65d → correct

Interpretation **signed**
5d * -3d = -15d → wrong

signed

```

0101 * 1101
-----
 11111101
00000000
 11111101
00000000
  -----
1001111001
    
```

sign extension
of multiplier

Interpretation **unsigned**
5d * 13d = 241d → wrong

Interpretation **signed**
5d * -3d = -15d → correct

■ MULS on Cortex-M0

- R_n (32-bit) * R_{dm} (32-bit) \rightarrow R_{dm} (32-bit)
 - Upper 32-bit of result are lost!
 - Lower 32-bit are the same for unsigned and signed
- unsigned \rightarrow watch out if result requires more than 32 bits
- signed \rightarrow part with sign bit is missing

■ Processor Arithmetic

- Odometer effect because of finite word length

■ Processors do **not** distinguish **signed** and **unsigned**

- User (resp. compiler) has to know, whether he is working with signed or unsigned numbers
- Processor always calculates flags for both cases
 - carry (C) unsigned
 - overflow (V) signed
negative (N)

■ unsigned

- Addition → $C = 1$ → carry
result too large for available bits
- Subtraction → $C = 0$ → borrow
result less than zero → no negative numbers

■ signed

- Addition → potential overflow in case of operands with equal signs
- Subtraction → potential overflow in case of operands with opposite signs

■ Arithmetic Instructions

- | | | |
|------------|------|------|
| • ADD/ADDS | ADCS | ADR |
| SUB/SUBS | SBCS | RSBS |
| MULS | | |