Notes Template

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April 21, 2021

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1 Single Value Decomposition

Since the columns of V are orthonormal...

$$VV^T = I$$

$$A = U \sum V^T$$

Where do we get the columns of V and U from?

$$A^T = (U \sum U^T)^T$$

$= V \sum^2 V^T$, diagnolization of $A^T A$, so the $\sigma = \sqrt{\text{eigenvalues of} A^T A}$
After we find the v's there is a shortcut for the u's.

$$Av_i = \sigma_i \vec{u}_i$$

Example 1: Find the SVD for $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$

- We can find $\sigma = \sqrt{5}, 3\sqrt{5}$ and eigenvalues of $5, 45$
- The full worked example can be found in your notes.

Example 2: Using the left SVD to find SVD of:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$
$$A^T A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

- Since this matrix is diagonal, the eigenvalues are $0, 1, 4, 9$. Then $\sigma = 0, 1, 2, 3$ respectively.