Notes Template

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Contents

1 Single Value Decomposition

A reading on Medium can be found here[1]. Now we will get into the SVD itself. Since the columns of V are orthonormal...

- $V^T = eigenvector(A^T A)^T$
- $U = \begin{bmatrix} \frac{1}{\sigma 1 Av} 1 & \frac{1}{\sigma_2} & \frac{N(A^T)}{|N(A^T)|} \end{bmatrix}$

$$VV^T = I$$

$$A = U \sum V^T$$

Where do we get the columns of V and U from?

$$A^T = (U \sum U^T)^T = V \sum^2 V^T$$

diagnolization of $A^T A$, so the $\sigma = \sqrt{\text{eigenvalues of} A^T A}$

After we find the v's there is a shortcut for the u's.

$$Av_i = \sigma_i \vec{u}_i$$

Example 1: Find the SVD for $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$

- We can find $\sigma = \sqrt{5}, 3\sqrt{5}$ and eigenvalues of $5, 45$
- The full worked example can be found in your notes.

Example 2: Using the left SVD to find SVD of:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$
$$A^T A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

- Since this matrix is diagonal, the eigenvalues are $0, 1, 4, 9$. Then $\sigma = 0, 1, 2, 3$ respectively.

[1] <https://blog.statsbot.co/singular-value-decomposition-tutorial-52c695315254>

Example 3 (from MIT OCW):

Find the singular value decomposition of

$$A = \begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix}$$

Step 1: We calculate

$$A^T A = \begin{bmatrix} 25 & 7 \\ 7 & 25 \end{bmatrix}$$

Step 2: Find the eigenvectors of this matrix to get the vectors v_i, and the eigenvalues for σ_i.

For our example, the two orthogonal eigenvectors are $(1, 1)$ and $(1, -1)$. Our orthonormal basis, then is

$$v_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), v_2 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$