

Notes Template

Neo Wang

April 25, 2021

Contents

1 Single Value Decomposition

1

1 Single Value Decomposition

A reading on Medium can be found here¹. Now we will get into the SVD itself. Since the columns of V are orthonormal...

$$VV^T = I$$

$$A = U \sum V^T$$

Where do we get the columns of V and U from?

$$A^T = (U \sum U^T)^T = V \sum^2 V^T$$

diagonalization of $A^T A$, so the $\sigma = \sqrt{\text{eigenvalues of } A^T A}$

After we find the v 's there is a shortcut for the u 's.

$$Av_i = \sigma_i \vec{u}_i$$

Example 1: Find the SVD for $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$

- We can find $\sigma = \sqrt{5}, 3\sqrt{5}$ and eigenvalues of 5, 45
- The full worked example can be found in your notes.

Example 2: Using the left SVD to find SVD of:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$
$$A^T A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

- Since this matrix is diagonal, the eigenvalues are 0, 1, 4, 9. Then $\sigma = 0, 1, 2, 3$ respectively.

¹<https://blog.statsbot.co/singular-value-decomposition-tutorial-52c695315254>

Example 3 (from MIT OCW):

Find the singular value decomposition of

$$A = \begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix}$$

Step 1: We calculate

$$A^T A = \begin{bmatrix} 25 & 7 \\ 7 & 25 \end{bmatrix}$$

Step 2: Find the eigenvectors of this matrix to get the vectors v_i , and the eigenvalues for σ_i .

For our example, the two orthogonal eigenvectors are $(1, 1)$ and $(1, -1)$. Our orthonormal basis, then is

$$v_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right), v_2 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$$