standalone chez

0.0.1 August 24, 2021 - Derivatives

1 **Example** (Parabolic)

Assume the graph is something generally parabolic, such as $f(t) = t^2$.
First, recall the second kinematic

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

Recall that for our tangent graphs (without the use of much calculs), we would find the slope of a secant line. We would define the slope as

$$v = \frac{\Delta x}{\Delta t}$$

which would differ from the instantaneous velocity for the vast majority of the time.
As a result, we take

$$v = \lim_{\Delta x \to 0} \frac{x_f - x_i}{t_f - t_i}$$

for

$$\lim_{\Delta x \to 0} \frac{5(t + \Delta t)^2 - 5t^2}{t + \Delta t - t} = \lim_{\Delta x \to 0} 5\Delta t + 10t = 10t$$

notice how this equation looks similar to $v = v_0 + at$

2 **Remark** (Historical Context - Leibniz and Newton)

Derivative - coined by "Leibniz."
Issues with naming it this way: derivative implies derivation, although the process itself of taking a derivative is differentation.
Δx was renamed from Δx to dx. The only real velocity equation according to most univeristy professors is

$$v = \frac{dy}{dx} = \frac{dx}{dt}$$

Then, the acceleration equation becomes

$$a = \frac{dv}{dt} = \frac{d}{dt}v = \frac{d^2y}{dx^2}$$

3 **Remark** (Rules of Differentiation)

A derivative is an expression that represents the rate of change of a function with respect to an independent variable.

1. Constant Rule. Example: if $x = 6$, then $\frac{dy}{dx} = 0$.
2. Power rule:

$$\frac{d}{dx}Cdt^n = (C \cdot n)t^{n-1}$$

4 **Example** (Example)

What is the squirrel's acceleration at $t = 1$ second if its position is given by the equation $x = 2t^5 - 3t^2 + 2t - 4$?

$$\frac{d}{dx} = 10t^4 - 6t + 2 = 40t^3 - 6$$
$$40t^3 - 6t = 1 = 34m/s$$