AP Physics C Notes

NEO WANG
LECTURER: MS. FAVARON

Last Updated: September 18, 2021

Contents

0.1 Unit 01

0.1.1 August 24, 2021 - Derivatives

1 **Example** (Parabolic)

Assume the graph is something generally parabolic, such as $f(t) = t^2$.

First, recall the second kinematic

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

Recall that for our tangent graphs (without the use of much calculs), we would find the slope of a secant line. We would define the slope as

$$v = \frac{\Delta x}{\Delta t}$$

which would differ from the instantaneous velocity for the vast majority of the time.

As a result, we take

$$v = \lim_{\Delta x \to 0} \frac{x_f - x_i}{t_f - t_i}$$

for

$$\lim_{\Delta x \to 0} \frac{5(t + \Delta t)^2 - 5t^2}{t + \Delta t - t} = \lim_{\Delta x \to 0} 5\Delta t + 10t = 10t$$

notice how this equation looks similar to $v = v_0 + at$

2 **Remark** (Historical Context - Leibniz and Newton)

Derivative - coined by "Leibniz."

Issues with naming it this way: derivative implies derivation, although the process itself of taking a derivative is differentation.

Δx was renamed from Δx to dx. The only real velocity equation according to most univeristy professors is

$$v = \frac{dy}{dx} = \frac{dx}{dt}$$

Then, the acceleration equation becomes

$$a = \frac{dv}{dt} = \frac{d}{dt}v = \frac{d^2y}{dx^2}$$

3 **Remark** (Rules of Differentiation)

A derivative is an expression that represents the rate of change of a function with respect to an independent variable.

1. Constant Rule. Example: if $x = 6$, then $\frac{dy}{dx} = 0$.
2. Power rule:

$$\frac{d}{dx}Cdt^n = (C \cdot n)t^{n-1}$$

4 **Example** (Example)

What is the squirrel's acceleration at $t = 1$ second if its position is given by the equation $x = 2t^5 - 3t^2 + 2t - 4$?

$$\frac{d}{dx} = 10t^4 - 6t + 2 = 40t^3 - 6$$
$$40t^3 - 6\Big|_{t=1} = 34m/s$$

0.1.2 August 25, 2021

Setting up the TI-89

5 **Remark** (TI-89 Setup)

1. Hit option, and notice that there is an apps desktop.
2. Toggle over, and select off. Then, it will take you directly to the screen to do calculations.
3. If you type in $\frac{3}{7}$, it will return the same thing. There are two modes: approximate and exact. You will change this to approximate.
4. Now, if you type in $\frac{3}{7}$ you should get 0.42857142857142855.
5. Toggle display digits from float 6 to float 10.
6. Make sure your calculator is in degree mode.
7. Turn on pretty print.

Calculating with the TI-89

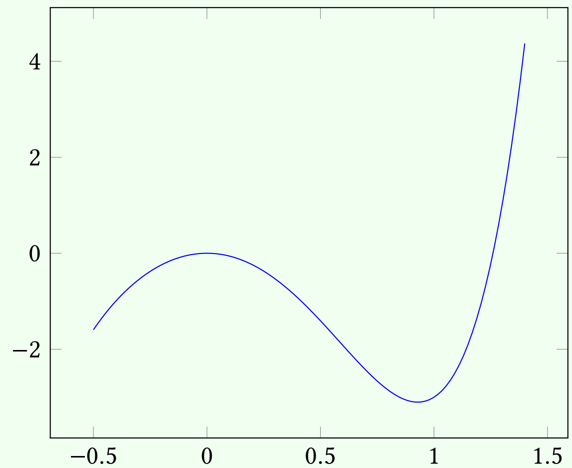
6 **Remark** (TI-89 calculations)

1. We type in our function after putting it into F3 mode (calculus).
2. If we type in '3t' the TI-89 treats that as one variable. Instead, type $3 \cdot t$.
3. $x = 3t^3$.
4. Since we want the derivative with respect to time, append a comma after your function and then type what to take it with respect to.
5. Our calculator should then output
$$\frac{d}{dx} 3t^3 = 9t^2$$
6. To clear your screen, do "F1 + 8"
7. TI-89s store variables across calculations. It's a good idea to clear your calculation before you start computing things.

7 **Example** (Sketching graphs)

$$y = 3t^5 - 6t^2 = 3t^2(t^3 - 2)$$

1. Find intercepts of the function.
   a) Hit F2 to get the calculator to do algebra. Hit SOLVE.
   b) Directly type the equation above into the the calculator.
   c) Hit SOLVE, and include your implicit operator t.
   d) Ask it to solve for t
   e) The resulting roots are $0, \sqrt{2}, -\sqrt{2}$.
   f) These should be easy to graph now.
2. Find critical points:
   a)
   $$\frac{dy}{dt} = 15t^4 - 13t^3 = 0$$
   b) Our resulting points are: $(-1.1, 3.15), (0, 0), (-1.1, -3.15)$
3. Find whether they are maximum or minimum or neither.
   - We can use the first derivative or second derivative test.
4. Find points of inflection
   - $(0.774, -1.9)$
   - $(-0.774, 1.9)$



0.1.3 August 26, 2021 - Integration

8 **Example**

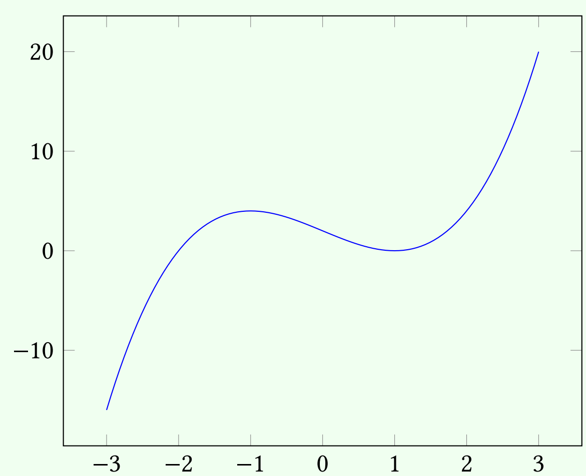
$$x = t^3 - 3t + 2$$

1. Find intercepts, $0 = t^3 - 3t + 2$
2. Find critical points. $\frac{dx}{dt} = 3t^2 - 3 = 0$. This returns $(-1, 4)$, $(1, 0)$
3. Check Concavity

$$\frac{d^2x}{dt^2} = 6t$$

4. And set the resulting points to the critical point values. Evaluate, and determine their resulting sign.
5. Find points of inflection.

$$0 = \frac{d^2x}{dt^2} = 6t, t = 0$$



9 **Example** (Integration 1)
Let $a = 6t$, then

$$\int \frac{d}{dt} 6t = 3t^2 + C$$

What does this actually do? Solve for the area under the curve.

10 **Example** (Integration 2)
If $v = 10t^2$, what is the displacment of the object at time $t = 5$?

$$\int \frac{d}{dt}{}_0^5 10t^2 = \frac{10}{3}t^3\Big|_{t=5} - \frac{10}{3}t^3\Big|_{t=0} \approx 416.667$$

Some things you may have learned from Calculus AB/BC

$$\sum dx = \sum v \cdot dt$$
$$x = \int v \cdot dt$$

0.1.4 August 27 - Kinematics

11 **Definition** (Kinematics)

- 1st Kinematic =

$$\vec{v} = \vec{v}_0 + \vec{a} \cdot t$$

- 2nd Kinematic =

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

- 3rd Kinematic =

$$\vec{v}^2 = \vec{v_0}^2 + 2\vec{a}\Delta\vec{x}$$

- 4th Kinematic =

$$\Delta\vec{x} = \frac{1}{2}(v + v_0) \cdot t = \int_{t=a}^{t=b} f(x)dx$$

0.2 Unit 02

0.2.1 Notes

0.2.2 September 02, 2021 - Vectors

12 **Remark**
Let's suppose we have a vector A of length 2 meters and 40° N of E
We also have a vector B of lf length $2m$ and 8° W of S
We wish to compute $A + B$
Graph A:
Graph B:

Let's convert vectors to x and y coordinates, then

$$A = \begin{bmatrix} 1.53209 \\ 1.28558 \end{bmatrix}, B = \begin{bmatrix} -0.273846 \\ -1.98054 \end{bmatrix}$$

Then,

$$A + B = \begin{bmatrix} 1.53209 + (-0.273846) \\ 1.28558 + (-1.98054) \end{bmatrix} = \begin{bmatrix} 1.25374 \\ -0.694956 \end{bmatrix} = 1.25374\hat{i} - 0.694956\hat{j}]$$

To find the length, we can use the Pythagorean theorem:

$$1.25374^2 + (-0.694956)^2 = 1.43347$$

13 **Definition** (Unit Vectors)

- $\hat{i}$ is the unit vector in the x direction
- $\hat{j}$ is the unit vector in the y direction
- $\hat{k}$ is the unit vector in the z direction

14 **Remark**

The syntax for a vector in the TI-89 graphing calculator is $[r, \angle\theta$.

- You can also switch between polar (cylindrical) and rectangular.

15 **Remark**

You can take the derivative of a vector with respect to each of its components.

0.2.3 September 03, 2021 - Derivations

16 **Example**

$$\Delta y = v_0 t + \frac{1}{2}at^2$$

$$t = \frac{x}{v_0 cos(\theta)}$$

$$y = \frac{v_0 \sin(\theta) x}{v_0 \cos(\theta)} - \frac{gx^2}{2v_0^2 cos^2(\theta)}$$

$$y = \tan(\theta)x - \frac{gx}{2v_0^2 cos^2(\theta)}$$

0.3 Unit 02 Homework

5. A train at a constant 60.0 km/h moves east for 40.0 min, then in a direction 50.0° east of due north for 20.0 min, and then west for 50.0 min. What are the (a) magnitude and (b) angle of its average velocity during this trip?

$$= 40\hat{i} + 20cos(40)\hat{i} + 20sin(40)\hat{j} - 50\hat{i}$$

$$= 5.32089\hat{i} + 12.8558\hat{j}$$

$$= \boxed{13.9077 m/s, \theta = 67.51^\circ \text{N of E}}$$

21. A dart is thrown horizontally with an initial speed of 10 m/s toward point P, the bull's-eye on a dart board. It hits at point Q on the rim, vertically below P, 0.19 s later. (a) What is the distance PQ? (b) How far away from the dart board is the dart released?

a) Solving for $\hat{\imath}$ component we get $10 \cdot 0.19 = 1.9m$, then solving for $\hat{\jmath}$ we get

$$\Delta x = \frac{1}{2}at^2 = \frac{1}{2}(-9.8)(0.19)^2 = -0.17689$$

So the distance $\vec{PQ} = |-0.17689| = \boxed{0.17689m}$
b) Then, the distance $\vec{PQ}$ must be $\sqrt{0.17689^2 + 1.9^2} = \boxed{1.90822m}$

23. A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air? (b) At what horizontal distance from the firing point does it strike the ground? (c) What is the magnitude of the vertical component of its velocity as it strikes the ground?

a)

$$\Delta x = -45 = \frac{1}{2}at^2 = \frac{1}{2}(-9.8)t^2$$
$$t = \pm 3.0305$$

, so the projectile must remain $\boxed{3.0305}$ seconds in the air.
b) $3.0305 \cdot 250 = \boxed{757.625m}$
c) $v = v_0 + at$ and $v_0 = 0, a = 9.8, t = 3.0305$, so

$$v = -9.8 \cdot 3.0305 = \boxed{-29.6989m/s}$$

25. The current world-record motorcycle jump is 77.0 m, set by Jason Renie. Assume that he left the take-off ramp at 12.0° to the horizontal and that the take-off and landing heights are the same. Neglecting air drag, determine his take-off speed.

$$77 = \frac{v_0^2 \sin(2\theta)}{g}$$

$$\boxed{v = 43.0727m/s}$$

27. A certain airplane has a speed of 290.0 km/h and is diving at an angle of 30.0° below the horizontal when the pilot releases a radar decoy (Fig. 4-33). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700m$. (a) How long is the decoy in the air? (b) How high was the release point?
a) $d = 700m$, so then the it must take the horizontal component $\frac{700}{290\cos(30°)} = \boxed{2.787}$ seconds.
b) $v_0 = -290\sin(30°) = -145$

$$\Delta y = -145t + \frac{1}{2}at^2$$

$$\Delta y = -145(2.787) + \frac{1}{2}(-9.8)(2.787)^2 = \boxed{-442.175m}$$

28. In Fig. 4-34, a stone is projected at a cliff of height h with an initial speed of 42.0 m/s directed at angle u0 60.0° above the horizontal. The stone strikes at A, 5.50 s after launching. Find (a) the height h of the cliff, (b) the speed of the stone just before impact at A, and (c) the maximum height H reached above the ground.

$$r(t) = (42\cos(60°)t)\hat{\imath} + (h - 42\sin(60°)t - \frac{1}{2}(9.8)t^2)\hat{\jmath}$$

a)

$$42\sin(60^\circ)t - \frac{1}{2}(9.8)t^2 = h$$
$$42\sin(60^\circ)(5.50) - \frac{1}{2}(9.8)(5.50)^2 = h$$
$$\boxed{h = 51.8269m}$$

b)

$$v_x(5.50) = 42\cos(60^\circ) = 21, v_y(5.50) = 42\sin(60^\circ) - 9.8(5.5) = -17.52$$
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{21^2 + 17.52^2} = \boxed{27.34m/s}$$

c)

$$0 = v_0^2 + 2a\Delta y = (42\sin(60^\circ))^2 - 2(9.8)\Delta y$$
$$\boxed{\Delta y = 67.5m}$$

29. A projectile's launch speed is five times its speed at maximum height. Find launch angle θ_0.

This problem is trivial:

$$v_0 = 5 \cdot v_{\text{max}}$$

$$v_0 = 5v_0 cos(\theta)$$

Divide both sides by $5v_0$, and we get $\frac{1}{5} = cos(\theta)$, so

$$cos^{-1}\left(\frac{1}{5}\right) = \boxed{78.463^\circ}$$

39. n Fig. 4-37, a ball is thrown leftward from the left edge of the roof, at height h above the ground. The ball hits the ground 1.50 s later, at distance d = 25.0 m from the building and at angle u = 60.0° with the horizontal. (a) Find h. (Hint: One way is to reverse the motion, as if on video.) What are the (b) magnitude and (c) angle relative to the horizontal of the velocity at which the ball is thrown? (d) Is the angle above or below the horizontal?

a) $h = \frac{1}{2}(9.8)t^2$ and $t = 1.5$, so $h = \boxed{0.5(9.8)(1.5)^2 = 11.025m}$

b) If $d = 25$ and $t = 1.5$, then $v_x = \frac{25}{1.5} = 16.66m/s$, and $r_y = \frac{1}{2}(9.8)(1.5)^2 = 11.025m/s$

$$\sqrt{v_x^2 + v_y^2} = \sqrt{16.66^2 + 11.025^2} = \boxed{19.9776m/s}$$

c)

$$\arctan(\frac{v_y}{v_x}) = \arctan(\frac{11.025}{16.66}) = \boxed{33.5^\circ}$$

d) The angle is above the horizontal.