0.0.1 August 24, 2021 - Derivatives

1 **Example** (Parabolic)

Assume the graph is something generally parabolic, such as $f(t) = t^2$.
First, recall the second kinematic

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

Recall that for our tangent graphs (without the use of much calculs), we would find the slope of a secant line. We would define the slope as

$$v = \frac{\Delta x}{\Delta t}$$

which would differ from the instantaneous velocity for the vast majority of the time.
As a result, we take

$$v = \lim_{\Delta x \to 0} \frac{x_f - x_i}{t_f - t_i}$$

for

$$\lim_{\Delta x \to 0} \frac{5(t + \Delta t)^2 - 5t^2}{t + \Delta t - t} = \lim_{\Delta x \to 0} 5\Delta t + 10t = 10t$$

notice how this equation looks similar to $v = v_0 + at$

2 **Remark** (Historical Context - Leibniz and Newton)

Derivative - coined by "Leibniz."
Issues with naming it this way: derivative implies derivation, although the process itself of taking a derivative is differentation.
Δx was renamed from Δx to dx. The only real velocity equation according to most univeristy professors is

$$v = \frac{dy}{dx} = \frac{dx}{dt}$$

Then, the acceleration equation becomes

$$a = \frac{dv}{dt} = \frac{d}{dt}v = \frac{d^2y}{dx^2}$$

3 **Remark** (Rules of Differentiation)

A derivative is an expression that represents the rate of change of a function with respect to an independent variable.

1. Constant Rule. Example: if $x = 6$, then $\frac{dy}{dx} = 0$.
2. Power rule:

$$\frac{d}{dx}Cdt^n = (C \cdot n)t^{n-1}$$

4 **Example** (Example)

What is the squirrel's acceleration at $t = 1$ second if its position is given by the equation $x = 2t^5 - 3t^2 + 2t - 4$?

$$\frac{d}{dx} = 10t^4 - 6t + 2 = 40t^3 - 6$$

$$40t^3 - 6\Big|_{t=1} = 34m/s$$

0.0.2 August 25, 2021

Setting up the TI-89

5 **Remark** (TI-89 Setup)

1. Hit option, and notice that there is an apps desktop.
2. Toggle over, and select off. Then, it will take you directly to the screen to do calculations.
3. If you type in $\frac{3}{7}$, it will return the same thing. There are two modes: approximate and exact. You will change this to approximate.
4. Now, if you type in $\frac{3}{7}$ you should get 0.42857142857142855.
5. Toggle display digits from float 6 to float 10.
6. Make sure your calculator is in degree mode.
7. Turn on pretty print.

Calculating with the TI-89

6 **Remark** (TI-89 calculations)

1. We type in our function after putting it into F3 mode (calculus).
2. If we type in '3t' the TI-89 treats that as one variable. Instead, type $3 \cdot t$.
3. $x = 3t^3$.
4. Since we want the derivative with respect to time, append a comma after your function and then type what to take it with respect to.
5. Our calculator should then output
$$\frac{d}{dx}3t^3 = 9t^2$$
6. To clear your screen, do "F1 + 8"
7. TI-89s store variables across calculations. It's a good idea to clear your calculation before you start computing things.

7 **Example** (Sketching graphs)

$$y = 3t^5 - 6t^2 = 3t^2(t^3 - 2)$$

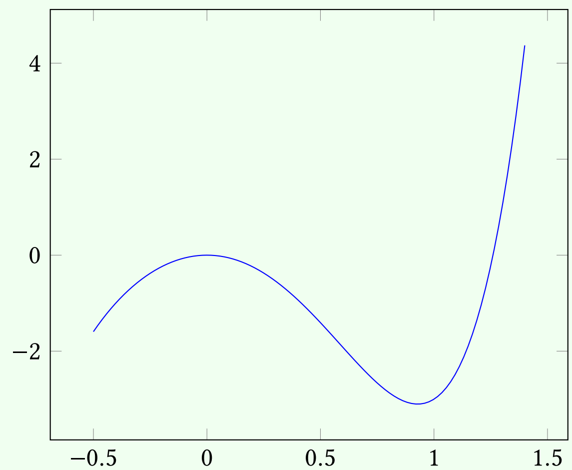
1. Find intercepts of the function.
   a) Hit F2 to get the calculator to do algebra. Hit SOLVE.
   b) Directly type the equation above into the the calculator.
   c) Hit SOLVE, and include your implicit operator t.
   d) Ask it to solve for t
   e) The resulting roots are $0, \sqrt{2}, -\sqrt{2}$.
   f) These should be easy to graph now.
2. Find critical points:

a)

$$\frac{dy}{dt} = 15t^4 - 13t^3 = 0$$

b) Our resulting points are: $(-1.1, 3.15), (0, 0), (-1.1, -3.15)$

3. Find whether they are maximum or minimum or neither.
   - We can use the first derivative or second derivative test.
4. Find points of inflection
   - $(0.774, -1.9)$
   - $(-0.774, 1.9)$



0.0.3 August 26, 2021 - Integration

8 **Example**

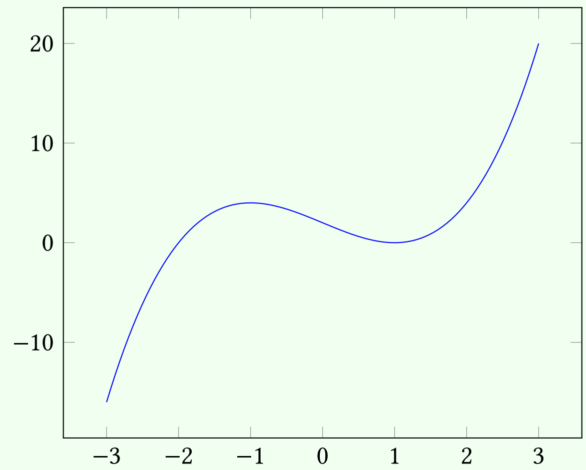
$$x = t^3 - 3t + 2$$

1. Find intercepts, $0 = t^3 - 3t + 2$
2. Find critical points. $\frac{dx}{dt} = 3t^2 - 3 = 0$. This returns $(-1, 4), (1, 0)$
3. Check Concavity

$$\frac{d^2x}{dt^2} = 6t$$

4. And set the resulting points to the critical point values. Evaluate, and determine their resulting sign.
5. Find points of inflection.

$$0 = \frac{d^2x}{dt^2} = 6t, t = 0$$



9 **Example** (Integration 1)

Let $a = 6t$, then

$$\int \frac{d}{dt} 6t = 3t^2 + C$$

What does this actually do? Solve for the area under the curve.

10 **Example** (Integration 2)

If $v = 10t^2$, what is the displacment of the object at time $t = 5$?

$$\int \frac{d}{dt}{}_0^5 10t^2 = \frac{10}{3}t^3\Big|_{t=5} - \frac{10}{3}t^3\Big|_{t=0} \approx 416.667$$

Some things you may have learned from Calculus AB/BC

$$\sum dx = \sum v \cdot dt$$
$$x = \int v \cdot dt$$

0.0.4 August 27 - Kinematics

11 **Definition** (Kinematics)

- 1st Kinematic =
$$\vec{v} = \vec{v}_0 + \vec{a} \cdot t$$
- 2nd Kinematic =
$$\Delta x = v_0 t + \frac{1}{2}at^2$$
- 3rd Kinematic =
$$\vec{v}^2 = \vec{v_0}^2 + 2\vec{a}\Delta\vec{x}$$
- 4th Kinematic =
$$\Delta\vec{x} = \frac{1}{2}(v + v_0) \cdot t = \int_{t=a}^{t=b} f(x)dx$$