Contents

0.1 Guidelines

Narrator Hello! Welcome to Dawn of the Stars, the story of the Seven Sisters. Maia, Merope, Taygete, Alcyone, Celaeno, Sterope and Elektra are sibling stars, all born together at about the same place and time. They are seven members of a much larger group of around 100 stars, forming a star cluster called the Pleiades. In this book, we will follow the story of each of the Seven Sisters, beginning at their birth and following them as they each leave the Pleiades, embarking on a perilous journey to explore the Milky Way and, one day, hopefully find her sisters again. Each of the Seven Sisters experiences a unique journey, meeting many interesting characters along the way. As you read on, you will join them as they discover the Galaxy, exploring its many wonders. You will learn about stars and the physics that decides their properties, answering questions like: If stars are born, can they die too? If so, how long do they live? Why do stars have the colors that they do? Why are there no green or purple stars? Why do the blue stars tend to be bigger than the red stars?
Narrator Before we begin, let us first consider a few features in this book implemented to help communicate its contents to the reader. We have gone to great care to clearly separate the story line from the educational content, so that the reader can choose for themselves when they are interested in reading more about a particular topic when it is presented in the story line, or when they would prefer to ignore the additional material and continue reading on about the adventures of the Seven Sisters. Below, we describe the ”rules” defined and implemented throughout the book to ensure that this goal is met.
Narrator The first thing you will notice is that the text and story line have been segmented into a “box” structure. This is used to define and communicate different levels of education needed as a minimum by the reader to understand certain key concepts contained within each “level”. For example, level zero is shown in black text, and is not confined to any such boxes. This “zeroth level” contains the underlying story line following our main characters, and can be understood by readers having a base level of education attained in grade school (i.e., up to and including high school). Our focus here is to keep the language accessible to readers with a high school education in the sciences, and to focus on visualizing the story line as it unfolds. Level 0 does not require any knowledge of any text appearing in any other “levels” in order to understand its contents. If the reader chooses to read only the black text, then they will be able to follow along with the story but will neglect access to some of the higher-level education materials presented in this book.
Narrator Subsequent “higher” levels require more education to understand their contents. All of the corresponding “higher level” education materials are contained within, and isolated to, the box format described above. Level one is indicated by red boxes and lettering, and requires a basic undergraduate STEM-based education to understand the content. Level 1 contains no equations, and

is focused on explaining the important physical concepts that appear recurrently throughout the book. All text appearing in red boxes assumes that the reader is following along with all text appearing in both red and black fonts. Level two is indicated via blue boxes, and requires an education in undergraduate physics to comprehend. This where equations will begin to be introduced to further explain the important physical concepts. We keep it simple, by avoiding complex derivations. Instead we focus on order-of-magnitude calculations. All text appearing in blue boxes assumes that the reader is following along with all text appearing in blue, red and black boxes. Level three is indicated via green boxes and lettering, and is aimed at the graduate level. This is the expert level, requiring a graduate level education in physics. There is no ceiling to the complexity of the content, and this is where derivations and even snippets of programming code will be introduced, in case the reader wants to try running some of their own simulations of the relevant N-body dynamics appearing throughout the book.

Narrator In summary, each level of text, indicated by a different type of box and lettering, and requiring the indicated background education, requires knowledge of the content of each level below it, but no knowledge of any level above it. In principle, the reader can begin to appreciate the book from a very young age and, if they so choose, return to it again and again over the course of their life as their own personal education develops. Each time the reader re-discovers this book, we hope they will discover new things, with new content emerging as it becomes accessible to them.

Narrator Now, we know what you are thinking: "How on Earth am I supposed to safely navigate my way through Level 0, when I'll be dodging around or even fumbling right through Level 1, or Level 2 or, even worse, Level 3! This sounds sort of like playing a video game that is impossible to win...."

Narrator Rest assured, we have taken great care to ensure for every reader a safe, secure and pleasant journey through each and every chapter, independent of their level of education. This means that ignoring any higher level content by reading only Level 0 text is straight-forward, since any obstacles introduced by higher levels are clearly identifiable and isolated from the rest of the story, such that they are easy to avoid. To summarize, the key point to keep in mind is that we use colored text to indicate to the reader the level associated with that text, and boxes to indicate that the text should only be regarded by readers at the appropriate level, and ignored otherwise. So if you only read black text, you can safely ignore any higher level material without having to worry about missing important developments in the story line, since no additional aspects of the story are introduced in higher level text. If you are still feeling reluctant, try to keep in mind that acquiring a paper cut from turning the pages is probably the worst thing that could happen to you upon reading our book. Of course, this assumes you are not carrying any explosive devices on your person. In that case, exploding is probably the worst thing that could happen to you.

Narrator The second thing you will notice is that the characters in the story appear in corresponding illustrations. These are included not just for entertainment purposes, but are instead primarily used to help convey the underlying story and certain key concepts. We do our best to ensure that the illustrations are accurate and precise in terms of the underlying physics, to help communicate important concepts typically related to the physics governing the birth and subsequent evolution of stars. For example, the sizes and colors of the stars in the story are accurate given the masses of the stars as constrained from observational data, combined with our knowledge of stellar evolution and how the properties of stars change over time.

Narrator The third rule pertains to astrophysical accuracy throughout the story line. For example, it can often occur that the travel times for stars to venture from Point A to Point B in the Milky Way galaxy can drastically exceed the stellar evolution timescales. In other words, it can often occur that stars will run out of nuclear fuel and collapse to form compact objects, such as white dwarfs, neutron stars and black holes, on a timescale shorter than the indicated travel time. We ignore this effect at times, so as to avoid our characters changing from stars to compact objects within only the first few chapters. To compensate for this, we include Level 1 boxes to explain these competing timescales, and to make it clear how adhering strictly to these competing timescales would affect the story line and the underlying character development (see below).

Narrator The fourth rule is that we stick to the characters defined in the Greek mythology of the Seven Sisters, but consistently inform the read of what is actually now known about their properties from centuries of observing those stars living in the Pleiades cluster. This is more complicated than it sounds, due to a long history related to constellation creation and the development of telescopes with superior resolution abilities. With the naked eye, the Pleiades cluster is visible very close to the Orion and Taurus constellations. However, the naked eye can only resolve a small subset of the total stars in the Pleiades cluster. Specifically, we can only see the most massive and hence brightest objects in the cluster. But, as telescope technology improved over the years, it quickly became apparent that many of the Seven Sisters, as originally named by the Greek mythology, are actually previously unresolved multiple star systems. So, in a few cases, a given Seven Sister is actually a collection of gravitationally bound stars. To adapt to this complication, we stick to the original naming conventions as decided by the Greek mythology, and include Level 1 boxes after each character is introduced, to update the reader on the current state of the observations for each of our Seven Sisters.

Narrator Last but not least, we assume that when our stars merge or evolve to become compact objects (i.e., white dwarfs, neutron stars or black holes), they become new characters with a new personality. In the case of compact objects, the tendency is for the resulting character to be more evil relative to the original Seven Sister. However, we go to great efforts to impose a unique personality to each character, which we clearly define and adhere to when a given character is communicating with other stars. For example, black holes often consume stars whole, if the stars venture too close to them. Some black holes in our story are committed to enticing stars to come sufficiently close for consumption, whereas other black holes are more conflicted by their need to destroy other stars in order to ”eat” and grow in mass.

Narrator In closing, we sincerely hope you enjoy reading our book. The rules outlined here will be repeated continuously throughout the book, primarily at Level 0. The rules should become more familiar to you as you read on, deeper and deeper into the unfolding story line and the individual adventures that befall our cast of characters. We hope you enjoy following the development of our characters as they learn more and more about our home, the Milky Way Galaxy, and the Universe around them, embracing the learning process as you read on. We hope you are able to take away more knowledge than we gained upon writing the book. Of course, this is no easy task, since we learned a lot! But we have gone to great lengths to ensure that this end goal is reached for the vast majority of our readers. See you at the end of the book!

Chapter 1

Life emerges...

Narrator For a long while, there was only darkness. Well, mostly darkness. Light is to the Universe what germs are to the world. In a nutshell, that stuff is pretty much *everywhere*. You're basically constantly being bombarded by individual particles of light, called photons,[2,2] when on Earth. This is still the case even when your eyes are telling you it is black and there is no light. So remember, just because you can't see light does not mean it isn't there. Just like germs.

1.1 A Star is Born

Narrator On a very fateful day, everything changed. Cosmic Dawn emerges with a roar. The veil of darkness is lifted. An especially massive Giant Molecular Cloud is in the final stages of contracting, becoming more and more dense as time goes on. The internal pressure from within, provided by the random motions of her constituent atoms and molecules, guides the hand of gravity to re-shape her into a critical new state. Over-dense knots and filaments begin to form within her belly. The knots continue to coalesce, becoming ever hotter and denser. Finally, new life emerges. Deep within one such dense knot, the massive protostar Maiais born, weighing in at a whopping 5 $M_\odot$, or 5 times the mass of the Sun.

Narrator With her birth, comes Dawn. Protostars spew out light in the form of photons at a thunderous pace; enough to make the radiation emitted by an unfathomable mound of radioactive waste completely negligible by comparison. Seven siblings, all due to be born within the narrow window of a few million years. Their Mother, Pleione, a particularly compelling Giant Molecular Cloud, now begins her journey through Motherhood. But it's not yet over; she's still in the process of yielding to gravity's nurturing might, slowly contracting and compressing, forming over-dense filaments and birthing new stars, buried deep within her belly.

Maia Hello to you, Mother![3,3]

Pleione Hello to you as well, my child. My young new protostar!
Maia Wow! The Universe is so amazing and beautiful to behold. Are all those twinkling things off in the distance other protostars, like me?
Pleione Yes, my child. Well, most are distant stars, not protostars. Stars live long lives, and the protostellar phase[??,??] does not last long; only a few million years or so. So most of the far off stars you are looking at are much older than you. The emitted light is what makes stars shine, taking many millions of years to travel from the center of the star to its surface. This is because the individual particles of light bounce around inside the star, eventually arriving at the surface of the star after a prolonged random walk. In the end, light escapes at a colossal rate as the photons leak through the star's surface, escaping into outer space. And the light can travel very large distances before reaching you, the observer. This is how you are able to see them in spite of those stars being so far away.
Maia I see, stars live very long lives... So, presumably, as I transition from a protostar to a real star, I too will have a very long life. I'll take it!
Pleione How many stars do you see out there?
Maia Uh... How *many*? I don't understand...
Pleione Well, let's start at the beginning. When it comes to counting, that is usually a good idea.
Maia Counting?
Pleione Counting is a way to keep track of something. For example, how much of something you have or can see, or how often something should happen. In order to count, the first thing you will need is a numbering system.
Maia What is that?
Pleione A numbering system uses symbols and rules to do the keeping track of things. The symbols are called ”numbers”. The rules decide how to use those symbols to do the counting. The counting is done by performing operations, including addition, subtraction, multiplication, division, and so on. For example, if you add two positive numbers together, you always get a bigger positive number. If you add two negative numbers, you always get a smaller more negative number.
Maia I'm not sure I'm following you...
Pleione Okay, well, before giving up on me, let me try to explain two very key concepts in any numbering system. The first is ”zero” (i.e., 0). This number just means that you are without any of whatever it is that you are counting. None. The second is ”one” (i.e., 1). This number is pivotal to any numbering system, since it defines the unit of measure. For example, I am your Mother. I, and I alone, created you. There is only one of me. You are my child, and you are also unique since there is only one of you. But, if I were to birth another star like you, then I would have more than one child. In this case, I would have *two* children. This follows from defining the addition operator, such that $1+1=2$. The operator acts to initiate some mathematical calculation, whether it be addition, subtraction, multiplication, division, etc.
Maia I think I am following you now. What if we go back to before I ever existed? How many children did you have then? Was it zero?

Pleione That's right!
Maia Okay. So, there were zero children a long time ago. Then there was one, or me. And soon there will be two, if you give birth to another star. Then... wait, what 's next?
Pleione The next number in the sequence is three. But, before we get there, let's take a moment to consider the concepts of addition and subtraction in a little more detail.
Maia Fair enough. What about them?
Pleione These ideas are rather central to all of "mathematics", or using numbers to calculate or compute things.
Maia You're losing me again...
Pleione One way to calculate something is to add numbers together, as I have already shown you. To summarize, I start by making one star. I then make one more star. Now I have two stars. But each star is itself just one star. So, here, we are adding one star together with another one star, and this gives us two stars. So one plus one is equal to two, or $1 + 1 = 2$. We are adding stars together, and counting them as we go.
Maia You're winning me back... I think I'm following you...
Pleione Now let's take it one step further. If we have two stars, and we add one more star, how many stars do we have?
Maia Uh... Give me a second to think about it... *Three!?*
Pleione That's right! If we add another star, then $1 + 1 + 1 = 3$, and we would have three stars.
Maia Look at me, I am counting! And adding! I'm learning so much. You drop the knowledge, and I pick up what you are putting down. I'm even having fun!
Pleione You most definitely are, my daughter. Now, let me ask again: How many stars do you see, twinkling off in the distance?
Narrator Maiaturns her attention back to the distant stars.
Maia I see a LOT of stars! In fact, I see so many I do not think I could count them all. There are far too many tiny twinkling DotS[1] to count! Well, it would take you a VERY long time, I imagine.
Narrator Something catches Maia's curiosity, distracting her from counting. She gasps in wonder.
Maia Whoa, if you look closely at some of those distant stars, they appear to be arranged in interesting ways that make them resemble familiar or even just weird shapes. Like, over there, I see *three* stars that are bunched close together and form a straight line.
Pleione Very good, Maia. You are counting! That is Orion's Belt. Good eye! If you take a larger look at him, you will notice as well a torso, arms and legs.
Maia I think I see them... Wait what are arms and legs?
Pleione Orion the Hunter is in the form of a human. I would describe humans as resembling deformed stars; they look similar, and come in various colors and sizes. But they also come along with many protuberances, such as arms and legs, each with their own set of functions. Orion is but one example of the many

[1] DotS is an acronym for "Dawn of the Stars".

stories depicted in the night sky, assigned by that species. Humans call these familiar stellar configurations “constellations”, and they are meant to tell some important story about their history. Humans have developed many stories to explain their origins, which they often refer to as ”myths”.

Maia Have you ever seen one?

Pleione One what?

Maia A human.

Pleione Oh! Yes, once, quite some time back. Awful, vile species. Constantly shooting projectiles off of the surface of their tiny planet, littering outer space with their garbage.

Maia Yuck. That does sound gross.

Pleione Did you know that Orion the Hunter even has a bow to fire arrows at his enemies!? If you look closely, you can see he is holding it in his left hand, and it forms a large arc in the sky.

Maia I see it!... Wait... Enemies? What kinds of enemies?

Pleione Well, if you follow Orion’s Belt from left to right, you will pretty quickly notice that it points to a very bright red star. That is the star Aldebarran, the Eye of Taurus the Bull.[textcolorred1,1] According to the myth, Orion the Hunter fought Taurus the Bull to save the Seven Sisters. If you keep following that line, you will notice that it points to us.

NL: INSERT CAPTION EXPLAINING WHAT ORION WOULD LOOK LIKE FROM THE PLEIADES, RELATIVE TO EARTH.

Maia Wow, that sounds very dramatic.

Pleione I suppose it must have been.

1.2 The Demise of Pleione

Narrator Pleione was growing weary. Her children shine bright,[10,10] emitting a wind of charged particles and photons.

Narrator As the winds collide with the loving embrace of their Mother, they provide an outward pressure and she begins to disperse.[textcolorblue6,6] The birth of Maiainitiated the demise of her mother.

Maia Wait, Mother, where are you going?

Narrator Maia, the second most massive of her soon-to-be-born siblings, wears a worried expression upon her face that begets deep concern for her fleeting Mother.

Pleione Oh, young one. There is nothing to worry about. I will be with you always, no matter what adventures befall you. You are, after all, made from me.

Maia Okay, but this is all sounding suspiciously like a goodbye...

Pleione All will be well, you will see. You are only just now born and still contracting, as gravity continues to find its balance with the fires that now rage within your belly, spewing out energy in the form of light. Soon, gravity will balance this outward source of pressure, and you will arrive at a stable size and mass that persists for many millions or even billions of years. Hydrostatic equilibrium awaits![??,??]

Maia Um... You're leaving me with a complicated technical term like "hydrostatic equilibrium"...? What does that even mean?[8,8]

Pleione It means that the energy produced within your belly must be strong enough to balance the inward force of gravity, to ensure stability. You can think of it as a sign that you are healthy. So long as you are in hydrostatic equilibrium, you never need to worry about your health. Your radius and mass should remain more or less stable over very long timescales.[textcolorgreen9,9]

Maia Okay, I will remember that one.

Pleione Take care of your siblings for me...

Narrator Maiaand Pleionecontinue their conversation, Pleionedoing her best to prepare her child and her yet-to-be-born siblings for their inevitable journey through the Galaxy. Maialearns a great deal about the cosmos, the births and deaths of stars, how star clusters form, and so on.

1.3 Hello World

Narrator Shortly later, one of Maia's siblings awakens. Electra emits a long, sleepy yawn. The least massive of her siblings, Electraweighs a mere 0.4 $M_\odot$. Pleioneturns her attention toward her newly born daughter.

Pleione Behold! Your sister awakens!

Electra Uh... Hi.

Maia Oh, wow. She is *super* bright and, well, shiny.[textcolorred10] It hurts my eyes if I stare right at her... Wait, *is* this hurting my eyes? Like, could I go blind?[10,10,11,11,12,12,13,13]

Pleione Only if you look directly at her.

Maia But I already did that!

Pleione Are you blind?

Maia I don't think so.

Pleione How can you be sure?

Maia Well, I can see you wincing, for one thing. You are looking at me as if I just got in to a fight with a much more massive star and lost.

Pleione Uh... I'm sure you'll be fine. Plus, it could have been worse. It could have been a black hole! They pack a far greater punch than any star, let me tell you.

Maia Wait, what is a "black hole"?[14,14] They sound terrifying.

Pleione Let's save that one for another day, child. You've already had quite an eventful day.

Narrator Pleione's gas tendrils, swirling and coalescing around her birthing children, gently touching and massaging their young faces, continue to dissipate, faster and faster with the birth of each new star.

Narrator Electra interrupts them suddenly, belching loudly. Plasma is ejected from her surface. It emanates from above the equator, around which a gaseous disk has now formed.

Maia I see you have a disk encircling you, Electra. I think I can even see a few planets forming! They are still in the process of being birthed, but it looks like so far you will be hosting at least four satellites. So far I think I can see three planets forming, and one moon!

Narrator Electra belches another time, louder than before.

Electra I am eagerly waiting to meet them. I hope this whole planet-birthing thing won't take too much longer.

Narrator Her daughters now nearly all born and slowly coming to life, Pleione's time has arrived. Pleione bestows one last kiss upon her daughters, before floating off and dispersing in to the infinite vacuum of outer space.

Electra I am excited to be here, and to be part of the team. ...I feel as though I just missed something important though. Please do fill me in. What did I miss? ...Wait, what are those two whispering about?

Narrator Both Maia and Electra turn their gaze toward Taygete and Alcyone , who together form a compact binary star system.[15,15]

Narrator Bound together by gravity, the sisters orbit their mutual center of mass in harmony, much like planets orbit stars. The two sisters are in fact twins, each with a total mass of 2.1 $M_{\odot}$. Needless to say, they were close, both in terms of their physical properties and the distance separating their centers of mass. Taygete and Alcyone quietly conferred about the topic at hand, namely which of the two of them is the brightest.

Taygete I think it goes without saying that I am brighter than you are.

Alcyone Dream on! I outshine you for sure.

Taygete Alright, tough stuff. Want to know how I know that I am brighter than you are?

Alcyone Sure. Amuse me.

Taygete I'm definitely fatter than you are, and bigger. Both contribute to making me brighter, relative to *you*. At least, I'm pretty sure that is how it works...[16]

Alcyone Oh shut up... You are neither fatter nor bigger than I am. You *are* way more delusional then I am though. I'll give you that.

Narrator Magnetic fields are responsible for ejecting plasma from the surfaces of stars, often in the form of filaments that follow the magnetic field lines. In general, the faster the star rotates, the stronger is its magnetic field. Taygete and Alcyone use this effect to strengthen their magnetic fields temporarily, by increasing their rate of rotation. Long filaments of plasma wave out from their respective surfaces, not unlike arms reaching outward. They begin flailing at each other violently, intent on a fight. But the gas tendrils lie outside of each other's grasp, unable to reach with even the longest tendril of plasma either can muster. A sisters' quarrel unrealized. Their efforts futile, they quickly give up.

Narrator Taygete and Alcyone are in fact two members of a triplet.[17] Celaeno, the third component of the triplet, lies much farther away than the other two. This, combined with the fact that she is a more massive star weighing in at 4.0 $M_{\odot}$, makes Celaenoa little bit different from her twin sisters. She is often ridiculed by her fellow twins because of it.

Narrator The triplets' current configuration, hierarchical and dynamically stable,[18] is nothing short of fate; binding them to each other practically indefinitely.

Maia Well, I think you are almost certainly identical twins. I cannot see any real difference between you. I mean, look at *Celaeno* over there; she's blue, whereas the two of you are clearly more of a yellow color. She's also *much* fatter and bigger than the two of you combined.

Celaeno Alright, I see your point.

Taygete Agreed. Alcyone, I'd extend a hand in offer of peace, but I don't have one.

Narrator Taygeteonce again churns her inner dynamo, brightening in response. Tendrils of hot plasma shoot outward from her surface, mimicking arms and hands that wave at her sister.[19]

Narrator Meanwhile, Celaeno was inspecting her midsection meticulously. Yep, fatter. Unsure as to whether or not this was a good thing, Celaeno wore a pensive expression, clearly trying to work it out in her head. Before she could sort it all out, Taygete interrupted her train of thought...

Taygete Hey Celaeno! I have a joke for you!

Celaeno Okay! I would love to hear it.

Taygete . Okay, here goes: How many blue stars does it take to screw in a lightbulb?

Celaeno I don't know... Hmmm... One?

Taygete Yep! You got it! Now let's see if you can get this one: How many yellow stars does it take to screw in a lightbulb?

Celaeno Um... One?

Taygete Nope! Wrong! The answer is: Nobody knows, since no yellow star has ever tried. Yellow stars are smart enough to realize that they outshine any lightbulb by an unfathomable amount. Hence, plugging in a light bulb is a waste of time for any star. For this reason, no yellow star has ever even bothered to try.

Celaeno Oh, I get it then... Blue stars are dumber than yellow stars. Ha!... Wait...

Narrator Celaenolooks down at herself, recalling that, unlike her yellow sisters, she is blue.

Celaeno Oh... Ouch.

Alcyone Good one, sister! She never even saw the punch line coming. No surprise there, she's just a dumb blue star after all.

Taygete . Exactly!

Celaeno . Wow, you guys are awfully mean.

Narrator Celaeno started to wonder what life would be like alone with her twin sisters. She was distracted when she noticed a dense whisp of Pleionepassing between her and her twin sisters.

Celaeno What's that?

Maia The fleeting remains of our Mother, I am afraid.

Electra That's *Mom*?!

Maia Well, what's left of her.

Narrator The siblings continued to accrete mass in the form of gas from what remained of their Mother, Pleione. They each grew and grew, until eventually they reached hydrostatic equilibrium, arriving at a long-lived stable configuration.[20]

Narrator This marked the end of their growth, and ultimately the end of the protostellar phase of their lives, along with the becoming a bona fide star. In the end, the outward pressure produced from within due to the thermonuclear reactions brewing in their bellies had grown sufficiently strong to balance the inward pull of gravity. Hydrostatic equilibrium achieved! With this balance in place, the siblings would endure most of their lives in this stable configuration, slowly fusing the lowest mass nucleon (hydrogen) into the next best thing (helium).

Narrator Sterope came to life suddenly, announcing her appearance with a high-pitched scream. With a mass of 2.93 $M_{\odot}$,Steropeappears most similar to Taygeteand Alcyonerelative to her other siblings.

Sterope AAAAAAAHHHHHHhhhh!!!!! What the...? Where am I? What am I? When...? You get the idea.

Maia It's okay, sister. You are one of us. We are stars born of the gas and dust of our Mother, a particularly glamorous Giant Molecular Cloud, if I do say so. She has left us now, but not without first bestowing her deepest gift upon us all, along with all of her love.

Sterope Uh... You are all my sisters? We are a family?

Maia Yes!

Sterope In that case, there remains a slim chance that the rest of this conversation will proceed without me feeling the need to scream again.

Maia Progress!

Sterope Uh, yeah, right. Progress. Let's get down to the important stuff. Who are all you strangers? You are my sisters, that much I have gathered. But what else? Wait, who am *I*? More importantly, *what* am I...? I'm starting to feel another scream coming on...

Maia Relax, young one. You're in good company here. Familiar company. *Familial* company, even. We are your siblings and we are stars. Thus and therefore, you too are a star. In fact, you are a star that is host to two satellites, called planets.

Sterope Holy cow! You're right. I have two planets. Uh....hello there! How, er, are you? Do you have names?

Alpha I'm okay! Happy to be here. My name is Alpha. My sibling over there, orbiting you further out than I do, is Beta.

Sterope Alphaand Beta! What wonderful names. I am Sterope, and I look forward to getting to know you both better.

Beta Thank you, Mother. We feel the same way. ...Wait, can we call you Mother?

Narrator Steropeshrugged rather nonchalantly.

Sterope Sure. Why not? Okay, getting back to my questions... What is a star? And does it have anything to do with why I am feeling so bloated?

Alcyone I wasn't going to say anything, Sterope, but you do look a little red in the face. Is everything okay over there? Wait, you asked a good question. What the heck is a star, anyways?[3]

Maia We are born of our Mother. Plain and simple. We formed out of the gas and dust she left behind, after gravity coalesced us into the beautiful burning spheres of hydrogen you see before you. Inside, we home a nuclear furnace that generates energy and emits light. Our insides are so hot, that hydrogen is converted in to helium, releasing energy in the form of light or what are called photons. The hydrogen is our food! Outside, gravity pushes inward, but it cannot surpass the outward push provided by our internal metabolisms. Protostars will continue to contract to a denser state with a hotter core, until a critical balance is achieved, called hydrostratic equilibrium. This will also get rid of the reddish hue you currently find yourself with, Sterope.

Sterope Well, that's a relief: the bloating is only temporary.

Alcyone I think I am following what you are saying, at least so far. What do we need to eat to keep ourselves going? I mean, we must need energy? Is this what those things you call photons provide to us?

Maia Yes, exactly. You have plenty of energy to keep you going for billions of years! You're consuming the hydrogen you were born with; converting it in to helium right there in your belly. It's a gift, just enjoy it. The consequence of this act of consuming is that you shine very bright. Photons are emitted every time four hydrogen atoms are consumed to produce helium, and they leak through your body and emanate from your surface. Bright as a light! The nuclear fuel already stored within you is sufficient to last millions, even billions, of years. You'll be shinning practically forever!

Celaeno Sounds to me like an awful lot of time to kill.

Maia There will be plenty of adventures along the way to keep you distracted, I have no doubt.

Sterope Like what?

Maia Only time will tell. But each star inevitably follows its own path through the Cosmos, and realizes its own fate. We are individuals, after all.

Sterope Maia, how do you know so much?

Maia Well, I don't really. I know what Mother told me. I am the oldest of us, after all, and she explained as much as she could to me before dispersing.

Sterope I'm grateful for your efforts. Mother dispersed so quickly, it must have been hard for her to convey a lot of detailed information to you before dispersing so completely.

Maia It was. She spoke really fast.

Sterope And you remembered all of it?

Maia Yep. No problem!

Sterope Reeeeaaaaalllly, Maia? All of it?

Maia *Sigh.* Fine! Mother only told me a few things. I don't want anybody to panic, so I'm trying to convey that Mother left me feeling confident, like we are more than capable of figuring it out for ourselves. I'm only trying to help!

Sterope Fair enough. It sounds like you are doing your best.

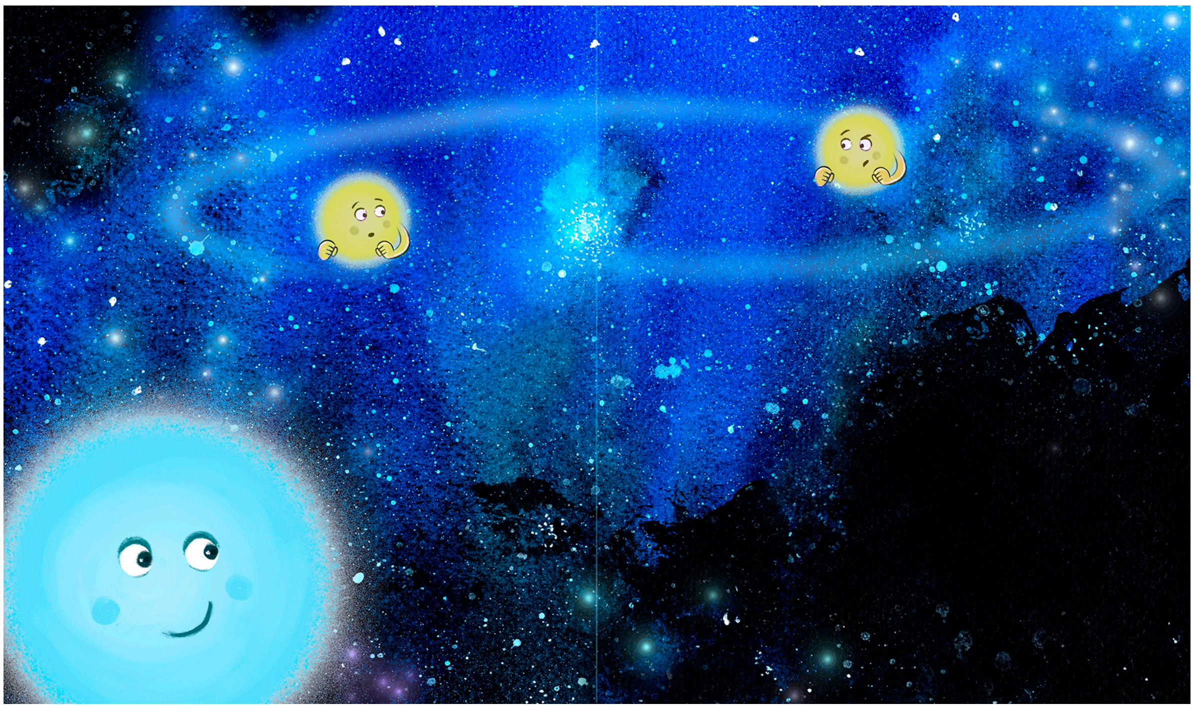

Figure 1.1: Taygete, Alcyone and Celaeno together. Illustration by Andre Pipe Oliva.

Maia I'm trying to motivate you, make you feel strong, loved and important! ...Well, you get the idea. With confidence, you can overcome any challenge that might befall you on your adventures.
Narrator Maia's shoulders slumped as she let out a prolonged sigh.
Sterope It's okay, Maia. We love you too.
Narrator Sterope flashes a warm smile at Maia. Maia smiles back, relieved. Alcyone belches loudly.
Sterope Alcyone!
Alcyone I'm sorry! It was an accident. I think it was that magnetic field thingy I have inside of me.[21] I'm learning that spontaneous emissions are, unfortunately, inevitable. Way out of *my* control, at least.
Narrator Synchronized to the microsecond, Maia and Sterope both roll their eyes.
Maia Just do your best to keep your spontaneous emissions to yourself.
Alcyone Will do.
Electra Uh...Maia, I definitely don't mean to startle you, but some freaky, ominous stuff is going on right behind you.
Maia Your goal there was to *avoid* startling me?
Electra Yep. How'd I do?
Maia Not very well at all. I'm currently terrified of what might be lurking behind me. Okay, I am turning around now...
Narrator Maia turns to see gas and dust had coalesced into a dense knot behind her. She recognized right away the familiar dynamical dance of the gas choreographed by gravity; the final stages of the birth of yet another star, another sibling.
Maia Oh, how wonderful! We are witnessing the birth of our seventh sibling. It would seem that Mother is not yet finished.
Narrator Merope came to life with a sudden jolt. And the hiccups.
Merope **Hiccup!** Excuse me. That whole being born thing was a little weird, and **Hiccup!** kind of uncomfortable. It left with me extra gas in my belly, or **Hiccup!** something else that has given me the hiccups.
Alcyone It is the magnetic field inside of you! It's nothing to be scared of though. I'm just glad I'm not the only one spontaneously ejecting plasma!
Narrator Merope takes a minute to relax and compose herself.
Merope Okay. I'm feeling better now.
Sterope Super! I'll try to find solace in your comfort as I struggle to ignore the lingering stench of your quasi-belches... Wait, who are you?
Merope Oh right. Introductions! I knew I was forgetting something. Hi! I'm Merope!
Narrator Merope was the second most massive of her siblings, weighing in at 4.5 solar masses. Gaseous emissions aside, her presence was hard to ignore amidst the seven sisters.
Maia It's wonderful to meet you, sister. It would seem that you and I form a bound pair. A binary star system! How fortunate that gravity is an attractive force. Our mutual gravitational attraction will keep us in this configuration practically forever. Well, at least until one of us explodes or something.[22]

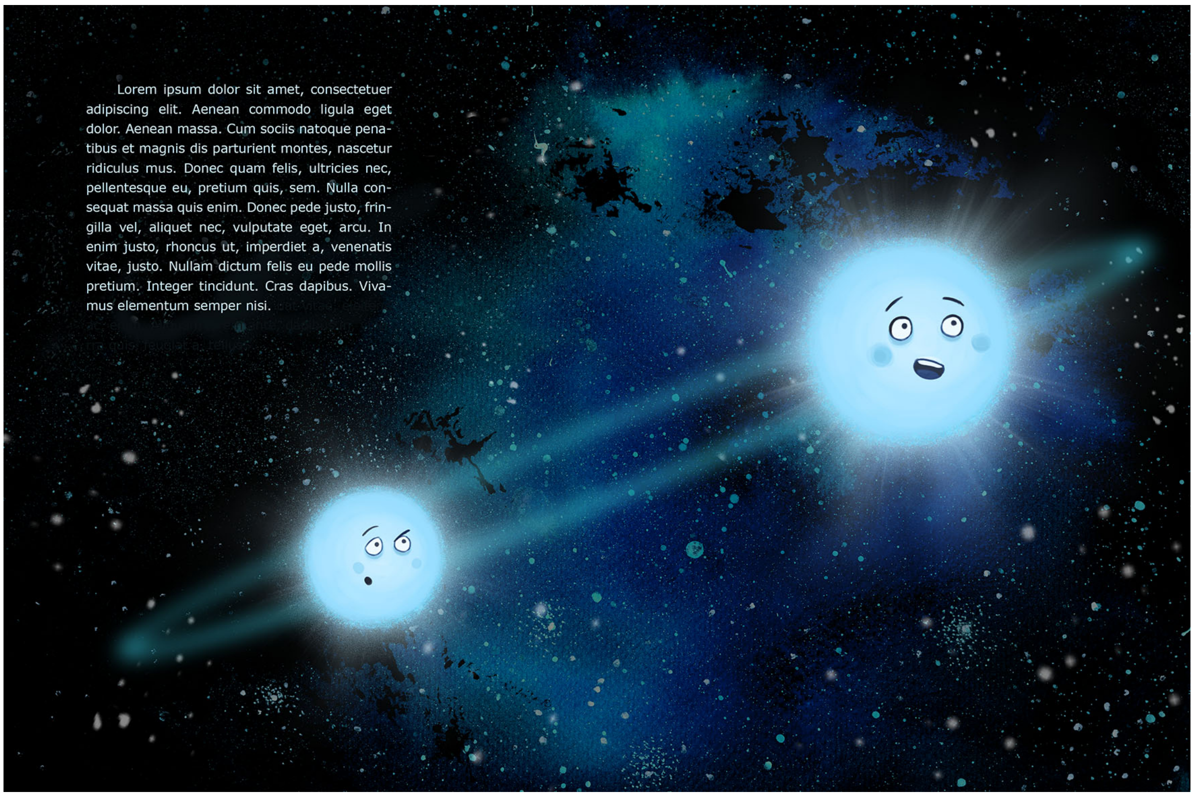

Figure 1.2: Maia and Merope, who together form a gravitationally bound binary star system. Illustration by Andre Pipe Oliva.

Merope Wait, what!? Who's exploding!? Is it me!? I don't want to explode!
Maia Shhhh.... Relax, sister. Nobody is exploding today.
Merope Today!? What about tomorrow?
Maia Nobody will be exploding tomorrow either.
Merope And the day after that?
Maia Nobody.
Merope And the day after that?
Maia Certainly not.
Merope And the day after...?
Narrator Maia interjected before Merope could finish.
Maia Nobody will be exploding for a very long time, if ever.
Merope Okay. It doesn't seem immediately urgent, I guess. But we are *definitely* circling back around to this exploding business at some point...
Narrator Just then, a tiny voice made itself heard...
Delta I, for one, am very glad you will not be exploding, Merope!
Narrator The sisters turned in unison to behold the source of the voice, a small blue-green planet orbiting Merope.
Merope Oh my! I am hosting a planet! Uh... Hello there! It's a pleasure to meet you.
Maia Indeed, it is a pleasure for me as well! We have one thing at least in common: we both orbit Merope!
Delta It's a pleasure to meet you both. A great pleasure, in fact. Our mutual journey together through space and time will be nothing short of epic!
Maia It will, I am sure. I hate to change the subject, but at the moment it seems we have a family to become acquainted with. I think that, together, we now form a young star cluster, our home.[23]
Narrator Maia turned to address her siblings.
Maia Greetings to you all! I cannot express how happy I am on this day, the day of our mutual births. The matter that forms our bodies comes from the same Mother, and to her we owe homage! Our existence is blessed by her great sacrifice, having spent herself to birth us few. Seven stellar siblings, and countless more familial satellites in the form of planets, moons, comets and even asteroids.[24] I see Electrahas a number of planets!
Electra I'm as surprised as you are! But I am eagerly looking forward to getting to know them better.
Maia All born of the same stuff, in the same place, and at about the same time. It is truly a time to celebrate. But we are all weary of a prolonged dawn, and should now rest. When we awake, we will celebrate properly!
Merope Count me in!
Electra A party sounds great. *Yawn.* Just after I get a little shut eye.
Sterope I could go for a nap. Then a party. I'm in too.
Narrator Meanwhile, Taygete, Alcyone and Celaeno had already fallen asleep, and were snoring loudly. Seven siblings, all born within the narrow window of a million years. The future looks bright for the Seven Sisters.

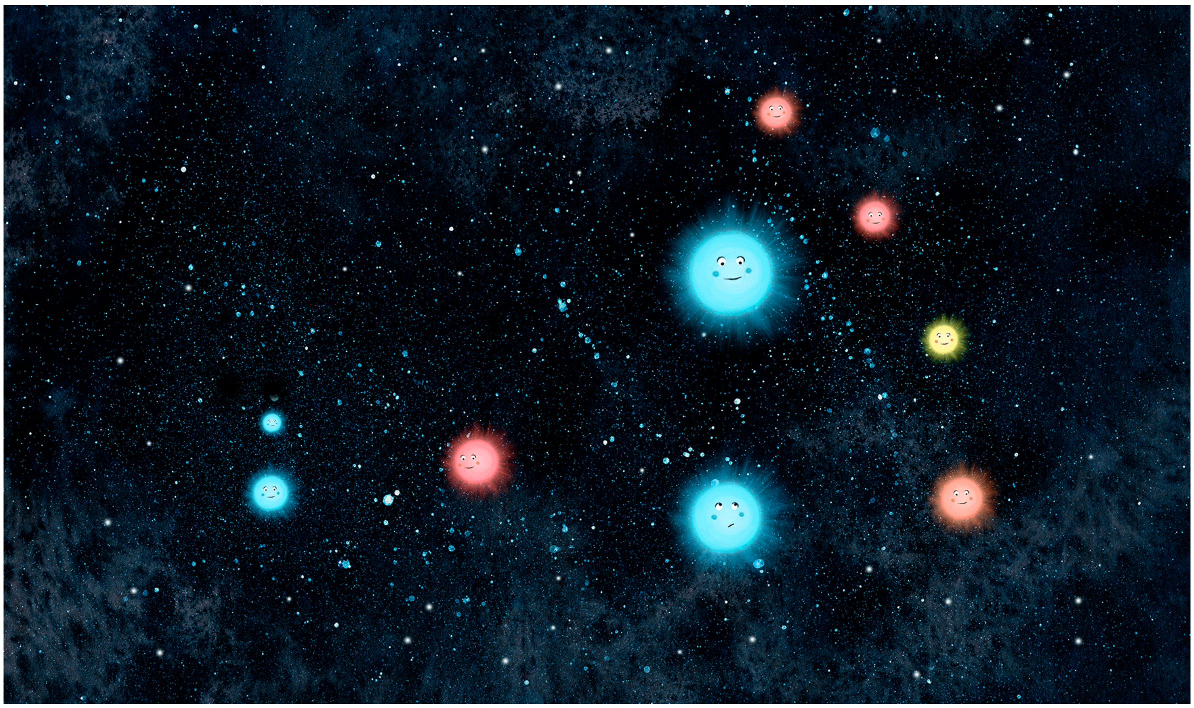

Figure 1.3: The seven sisters together, including Atlas and Lacedaemon to the left in the background. Illustration by Andre Pipe Oliva.

Figure 1.4: The Pleiades viewed from a distance, perhaps using a telescope on Earth. Illustration by Andre Pipe Oliva.

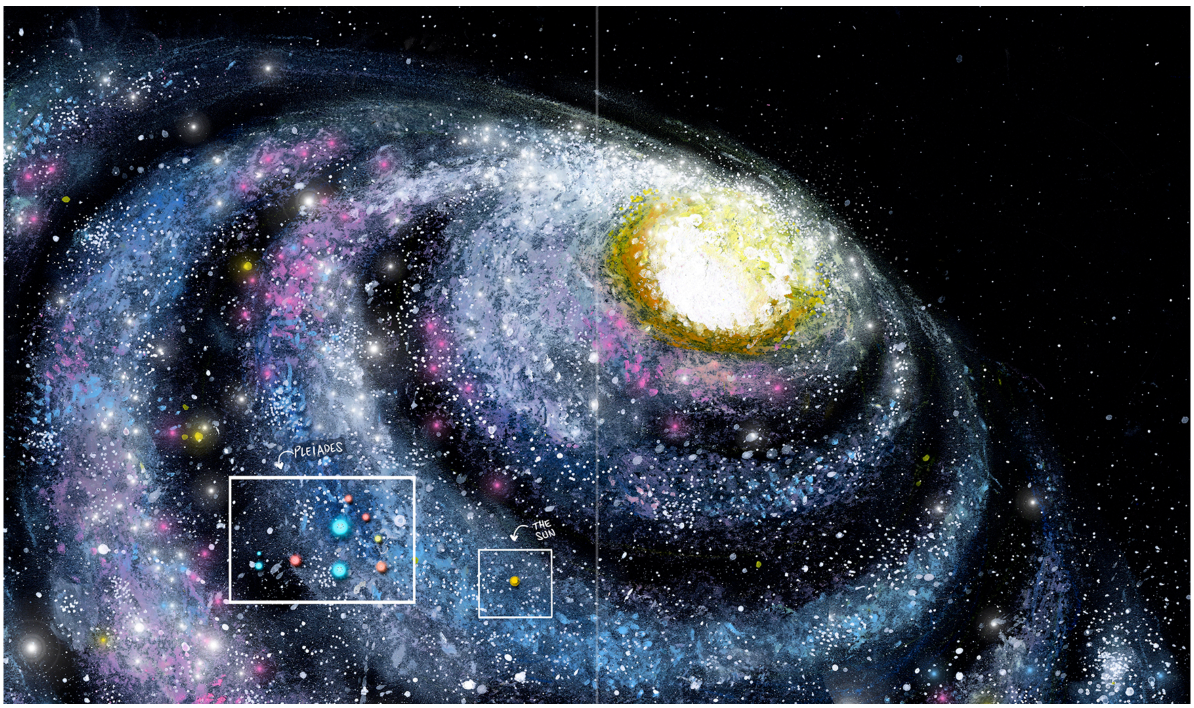

Figure 1.5: The location of the Pleiades star cluster in the Milky Way, relative to the location of the Sun. Both the Pleiades and our Sun reside in the disk of our Galaxy, where star formation is still occurring. Illustration by Andre Pipe Oliva.

1.4 Educational Material

Box 1 – Aldebaran

Aldebaran is the brightest star in the constellation Taurus. At a mass of 1.7 $M_{\odot}$, Aldebaran is more evolved than the Sun, and is thus intrinsically brighter and redder. Since it is in the process of ascending the giant branch of stellar evolution, it is preparing to fuse the hydrogen and helium in its core into more massive elements. This phase of stellar evolution comes after the main-sequence phase, during which time hydrogen is being fused into helium in the stellar core, providing a net source of energy that eventually emerges from its surface in the form of photons. As stars leave the main-sequence phase of evolution to ascend the giant branch, they begin to expand. By the end of this phase of evolution, the radii of the stars can be inflated by several orders of magnitude relative to the main-sequence phase. As the surface of the star is pushed farther and farther away from the core, where energy is being generated, its surface temperature drops. As will explained in more detail as the story develops, this changes the star's color from yellow to red. Hence, Aldebaran shines red, since its surface temperature exceeds 3,700 K, but is much lower than the 5770 K characteristic of the surface of our Sun.

Box 2 – Photons

Photons are particles of light. They travel freely through vacuum at a speed of about 299792458 meters per second or 299792 kilometers per second or, if you prefer, 671000000 miles per hour. Basically, photons travel an unfathomable distance each and every second. Photons are defined according to their energy or, equivalently, their wavelength or frequency. The spectrum of energies characterizing photons is called the electromagnetic (EM) spectrum. Photons are produced in the cores of stars, eventually working their way up to escape from the surface. These photons, in particular those in the visible portion of the EM spectrum, are transparent to the Earth's atmosphere. Photons from the visible portion of the EM spectrum are detectable by the human eye, revealing a wonderfully brilliant and colorful night sky on Earth. But the EM spectrum is vast and the visible portion, namely the light humans can detect with their eyes, is only a small component of it. So, as with germs, most of the photons landing on your skin cannot be seen with the naked eye.

Box 3 – Stellar Communication

Stars of course cannot speak. But they can communicate with each other, even over very large distances. They communicate by modulating their luminosities on short timescales, brightening and dimming, brightening and dimming, in whatever cadence properly communicates their intended message. Humans are unable to speak, write or even read the language of the stars. Throughout this book, all communications between stars will be expressed in English.

Box 4 – From Protostar to Bona Fide Star

The transition from protostar to bona fide star can take several million years to complete. During this time, the protostar is contracting, which it does by radiating away its excess energy via photons. Upon losing this energy, the star will contract and become more compact. This process continues until eventually the protostar becomes attains a stable state of equilibrium: the inward force of gravity becomes balanced with the outward pressure provided by the energy leaking out of the star's core. The star stops contracting and maintains a stable size or radius. It is at this point that the protostar becomes a real star, entering the main-sequence phase of its evolution, during which time its energy is supplied by nuclear burning in the core, via the conversion of hydrogen into helium.
EXPLAIN P-P CHAIN?

Box 5 – Virial Theorem

Over 10^{38} photons are emitted every second. Any tiny patch on the surface of a star is like the narrow entrance to a dark cave, with a swarm of bats flocking out from it. Or germs, if you prefer. In order to calculate the total energy emitted per second by the Sun, we can assume that every photon escaping from its surface has an energy of 12.86 Mev, corresponding to the highest energy photons produced at the end of the proton-proton-chain (thus our estimate here for the total number of photons should be regarded as a strict lower limit), which is the nuclear reaction process responsible for converting hydrogen in to helium. Assuming 1 J = 1.602×10^{-13} MeV, we then calculate a solar luminosity of about 2.1×10^{26} J s^{-1}. This is very close to the total value of 3.828 $\times$ 10^{26} J s^{-1} computed for the luminosity of the Sun, calculated from integrated observations of the solar spectrum (REF). Finally, we note that [J s^{-1}] = [Watts].

Box 6 – Radiation Pressure

Although photons do not have mass, they can still impart momentum by colliding with particles, including electrons, protons, atoms, molecules, and so on. More stars means more photons, and hence more overall radiation pressure applied to dust particles, in this case the dust that constitutes the body of Pleione. This accelerates the rate of dispersal of a surrounding cloud of gas and dust, since stars tend to be born near the centers of Giant Molecular Clouds where the gas densities are at their highest. Hence, on average, radiation provides an outward source of pressure, moving away from the source of radiation responsible for producing the photons (e.g., nuclear burning in the cores of stars).
EXPLAIN MOMENTUM EXCHANGE USING IMPULSE ARGUMENTS, AND THAT LIGTH CARRIES MOMENTUM BUT NO MASS.

Box ?? – Hydrostatic Equilibrium I

Hydrostatic equilibrium is what ultimately decides the size or radius of a star. The term refers to the balance between the outward radiation pressure supplied by the energy released in the core via nuclear reactions producing photons (e.g., the proton-proton-chain, which is what burns hydrogen into helium) and the inward pull of gravity.
INCLUDE A DESCRIPTION OF CHANDRASEKHAR'S AND SPITZER'S CONTRIBUTIONS TO OUR UNDERSTANDING OF STELLAR STRUCTURE, AND EXPLAIN THE BASIC CONCEPT AGAIN USING THEIR VIRIAL THEOREM APPROACH.

Box 8 – Hdrostatic Equilibrium II

DERIVE THE EQUATIONS FOR HYDROSTATIC EQUILIBRIUM.

Box 9 – What determines the radius of a star?

Stars out of hydrostatic equilibrium will either expand or contract. If the pressure exceeds the gravity, then they expand. If the gravity exceeds the pressure, then they contract. This re-adjustment of the stellar radius is needed to set the right balance between pressure and gravity. The radius of a star can only really be defined for stars in hydrostatic equilibrium, since the radius remain approximately constant in this state. Conversely, for stars out of equilibrium, the radius is dynamically changing as the star re-adjusts itself to find a new stable balance between the outward pressure generated in the stellar core and the inward pull due to gravity. But what supplies the outward source of pressure? Sure, hydrogen is being burned into helium in the central cores of main-sequence stars, and this releases energy in the form of photons, but how exactly does this work? INCLUDE DESCRIPTION OF THE P-P CHAIN.

Box 10 – Stellar Emission

Stars emit light spanning a wide range of energies. The shielding effects of the Earth's atmosphere protect human eyes from very high-energy photons that would otherwise contribute to the degradation of the human eye. From the surface of the Earth, we only see those photons within the visible portion of the electromagnetic spectrum. But from space, our eyes would not be protected. If stars' eyes are also sensitive to high-energy photons, then looking directly at other stars, especially very close ones, is anything but a good idea.

Box 11 – Blackbody

A blackbody is an idealized object that absorbs all the incident radiation that falls on it at all frequencies. Hence, incident visible light will be absorbed and not reflected, causing the surface of the object to appear black. An ideal blackbody in thermal equilibrium adheres to two important properties. First, it is an ideal emitter. This means that, at every frequency, it emits as much or more thermal radiation compared to any other body at the same temperature. Second, it is a diffuse emitter. This means that the radiation is emitted isotropically or, equivalently, the emitted radiation is the same in all directions, when the radiation is measured per unit area perpendicular to a specified direction.

Box 12 – Why is a star observed to have a given color?

The color of a star, since star's are blackbodies, is largely decided by its surface temperature. Stars emit at all wavelengths since they are blackbodies, but their temperature decides the portion of the EM spectrum where most of the radiation is produced. In general, the hotter the star, the more short wavelength light emits. The hottest ones are blue or blue-white, corresponding to the dominant emission being at shorter wavelengths. Cooler stars are red or red-brown, corresponding to longer wavelengths. But why do we not observe green or purple stars? Human eyes have evolved to view predominantly yellow and green radiation, most likely because our sun emits radiation primarily in this wavelength regime. A green star is mostly radiating in the center of the visible portion of the EM spectrum. The star should therefore appear white, due to a combination of the distribution of light that is emitted over the visible portion of the EM spectrum and the functionality of the human eyes, which have evolved to be more or less sensitive to certain wavelengths. The human eye does not see purple stars for analogous reasons, namely because our eyes are more sensitive to blue light. Hence, if stars are emitting similar amounts of purple and blue radiation, the human eye is sufficiently more sensitive to the blue contribution that this is what we tend to observe as the overall color of the star.

Box 13 – Stefan-Boltzmann Law

DERIVE THE EQUATIONS FOR HYDROSTATIC EQUILIBRIUM.

Box 14 – Black Holes

A good question. We will learn a great deal about black holes over the course of this book. For now, let us suffice it to say that many black holes are simply dead stars. Their progenitor stars ran out of nuclear fuel, which was providing the star with the outward pressure it needed to resist gravity's inward pull. With no source of outward-directed pressure, gravity wins and the progenitor star collapses to form a new, much denser object. If the progenitor star is sufficiently massive when it dies, it will collapse to form a black hole. Black holes are so dense that the strength of gravity forbids the escape of light from their interiors. Thus, they are black, and do not emit light. Until very recently and the invention of gravitational wave observatories, they have only been detectable by humans indirectly, via their gravitational influence on surrounding matter and stars. But, with the introduction of gravitational wave measurements, we are now able to directly detect bound pairs of these objects just before they merge. To date, this mainly applies to low-mass black holes, comparable in total mass to massive stars. The origins of super-massive black holes, on the other hand, are thought to be much more complicated, and to this day remain shrouded in mystery. This is because such massive black holes (with masses $\gtrsim 10^5$ $M_\odot$) cannot have formed directly from stellar evolution and require additional physical processes to grow them to their currently observed masses. This can occur by, for example, accreting matter from surrounding gas and stars. But this is a story for another day.

Box 15 – Gravitationally Bound Pairs of Objects and Binary Star Systems

Two objects are said to be gravitationally bound if their total relative energy (i.e., the sum of their kinetic and potential energies) is negative. In this case, the objects orbit their mutual center of mass, carving out circular or elliptic trajectories in a plane. If both components are stars, such two-body systems are often referred to as binary stars. The Earth is gravitationally bound to the Sun, as is the moon to the Earth. Technically, the moon is also gravitationally bound to the Sun. But gravity gets weaker with increasing distance, and the moon is close enough to the Earth and far enough away from the Sun that it orbits the former instead of the latter.

Box ?? – Dependence of luminosity on Mass and Radius

The luminosity of a star increases steeply with both increasing mass and radius. This is the case during the main-sequence phase of a star's lifetime, during which time stars are burning hydrogen into helium in their cores. All stars, once finished with the protostellar phase, become main-sequence stars. During later stages of stellar evolution, such as the red giant branch and asymptotic giant branch phases of evolution, the helium core sits at the center of the star, with hydrogen-burning occurring in the shell immediately outside the core. Moving inward from the outer parts of the star and into the inner regions of the core, heavier and heavier elements are being produced, with each shell activating nuclear burning at a later time and corresponding to a different phase of stellar evolution. Those shells forming the heaviest nuclei via nuclear burning are closest to the center of the star. Moving outward from the center, each successive shell is burning lighter elements. Hence, in very massive stars, hydrogen burning and helium formation occurs near the outer most shell, closest to the surface of the star.

Box 17 – Dynamical Stability

Stable triple star systems are composed of three stars, and hence two orbits. One of the orbits is very compact, and the other is very wide. The outer orbit is so much wider than the inner orbit, that it is a good approximation for the outer tertiary to approximate the inner binary as a single object. This is absolutely necessary to ensure the long-term dynamical stability of the triple. If the inner pair becomes too wide, the gravity exerted by the outer object will pull the inner pair apart. Chaos ensues. This chaos can mediate the ejection of one or more stars from the triple, and even direct collisions.

Box 18 – Orbital Hierarchies

The easiest way to explain a "hierarchy" in a triplet is if two of the stars form a very compact binary, and the third star orbits at a very large distance from this compact pair. Said another way, when it comes to a hierarchical triple star system, there are two orbits with almost opposing properties; the inner binary is compact, whereas the outer tertiary orbit is very wide. In multi-planet systems, however, there can be multiple hierarchies. For example, the Sun constitutes a significant fraction of the total mass of the Solar System, so all planets tend to primarily orbit its center of mass. This is a hierarchy in mass - the planets are all much less massive than the Sun, so each planet's orbit can be approximated as an isolated two-body orbit between the planet and the Sun (i.e., ignoring the gravitational influence of the other planets is valid on short timescales). A second hierarchy in mass can appear if we now include moons, which orbit the centers of mass of the planets.

Box 19 – Magnetic Fields and Dynamos

By making the inner core of a star rotate faster, an existing magnetic field can be amplified. This can in turn trigger substantial chromospheric activity in the star, with filaments of hot plasma extending radially outward and coronal mass ejections bursting through its surface.

Box 20 – Steady State

The term "steady-state" implies a stable configuration, where all of the relevant forces or mechanisms have been balanced. Here, it implies that the stars are maintaining an approximately constant mass and size, since they have reached a "steady-state", in this case hydrostatic equilibrium. More generally, consider a spherically symmetric self-gravitating system of point particles. The positions and velocities of every particle are described by a distribution function f($\vec{x}$,$\vec{v}$,t). This distribution function can be evolved forward in time using a diffusion-based model, as is done in, for example, the classic Boltzmann equation or the radiative transfer equation. According to Liouville's Theorem, if a system is in steady-state, then it must follow that:

$$\frac{\partial f}{\partial t} = 0 \tag{1.1}$$

INCLUDE A DESCRIPTION OF DISTRIBUTION FUNCTIONS AND THE CRITERION FOR STEADY-STATE.

Box 21 – Reconnection of Magnetic Field Lines

Most main-sequence stars have magnetic fields that typically emanate from their poles; the younger the star, the more powerful the magnetic field. When two or more magnetic field lines intersect, they "reconnect" to form new, disconnected field lines. This "reconnection" is usually an energetic event, accompanied by a burst of high-energy photons (i.e., gamma rays and x-rays) and the ejection of plasma.

Box ?? – Supernovae and Stellar Lifetimes

The most massive stars end their lives with a dramatic explosion, called a supernova. In one go, the explosion can liberate roughly as much energy as the Sun over its entire 10 billion year lifetime. At their peak, supernovae shine 10^{10} times brighter than the Sun. There is a simple formula that allows us to calculate the expected lifetime of a main-sequence star, which is $\tau_{\mathrm{MS}} = 10(\mathrm{m}/\mathrm{M}_{\odot})^{-2.8}$ Gyr (REF? ADRIAN?). Using this formula, for Merope, Maia, Calaeno, Sterope, Taygete, Alcyone and Elektra, we find MS lifetimes of, respectively, 0.1, 0.1, 0.2, 0.5, 1.3, 1.3, 130 Gyr. As we will show later on, these lifetimes should exceed the timescale for the dispersal of the Pleiades open cluster.
INCLUDE A TABLE OF EACH OF THE SISTER'S OBSERVED PROPERTIES (SPECTRAL TYPE, MASS, COLOR, ETC.)

Box 23 – Star Clusters

As was the case for the Seven Sisters, most, if not all, stars are thought to be born in "clusters", gravitationally-bound and compact groupings of stars, ranging in numbers from a few tens to several million, with central densities in the range 10 $\mathrm{M}_{\odot}$ pc^{-3} - 10^6 $\mathrm{M}_{\odot}$ pc^{-3} or, equivalently, 10 - 10^6 stars per cubic parsec, where a parsec is defined as 1 parsec = 3.086 $\times$ 10^{16} meters. These stars are all thought to form from the same Giant Molecular Cloud, as gravity shaped it into denser knots and filaments, creating the ideal environment for star formation.

Box 24 – Satellites

The term satellite refers to any celestial body that is gravitationally bound to, but much less massive than, the massive object it orbits directly. So planets, comets, asteroids, etc. are all satellites to stars. Moons are satellites to planets. Even stars can act as satellites, if they orbit a much more massive super-massive black hole.

Chapter 2

A Gust of Wind...

Narrator Sterope awoke when a gust of wind brushed past her face. The gas was dense enough to temporarily obscure her vision. She could feel the wind against her stellar surface as it rushed past, gently caressing her fiery skin. All in all, the wind carried enough momentum to startle her out of slumber.[1,2,2,??]

Narrator Sterope coughed, clearing the gas from her face. Her surroundings now revealed, Sterope looked around, confirming her suspicions; the gas had become substantially less dense[5] since she had fallen asleep, and was now disconcertingly sparse.

Sterope My sisters, you must wake up! The final remains of our Mother are leaving us.

Narrator The other six siblings awoke to the scene described by Sterope.

Electra Whoa! What's going on? We're all drifting apart. And where did Mother go?

Maia It's okay, young ones. The last vestiges of Mother have now left us. Upon giving birth to us, she activated our metabolisms. We've been spewing light out ever since, in the form of photons. Each photon carries momentum, and transfers some of it to any gas molecule or atom upon collision. Our light has been banging in to the gas and dust of our Mother for quite some time now, pushing her outward and away.

Electra Okay, but then why are *we* drifting apart from *each other*?

Maia Mother was made up of gas and dust, which came along with significant *mass*. And with mass comes gravity. Now that her mass is gone, it can no longer contribute to the inward pull of gravity. As this happens, we expand as a cluster and each of us moves further and further away from our common center of mass. This is because, if we add up all the mass in stars, it is not enough for gravity to keep us in close proximity. Only with the gas mass supplied by our Mother was this possible, but it is now gone.[1] 6,[2] 7,[3] 8

Maia In other words, with our Mother now dispersed, between those of us stars contributing to the total stellar mass of our home cluster, we no longer have enough mass in our mutually occupied volume to keep us gravitationally

bound.[4][9] We are now free to drift apart. And drift apart we are destined to do.
Taygete Yeah, yeah, yeah. But what does all that even *mean*?
Sterope I think it means that this is goodbye... With our Mother's mass now lost, we are no longer gravitationally bound. Relative to each other, we are energetically *unbound*.[5][10]
Sterope We are all fated to wander independently through the Cosmos. Utterly and completely alone. Well, except for Maia and Merope, I suppose, who form a binary. Oh, and the triplets. Those three are also still gravitationally bound.
Narrator Alcyone's shoulders slump. She begins to cry.
Alcyone Already, I miss each and every one of you.
Taygete Well, at least you have me.
Narrator Alcyonerolls her eyes.
Taygete Hey! I saw that!
Alcyone I'm sorry, sister. You are right. I am grateful for your presence. Even if it *is* all the time. Without any breaks. Ever.
Maia I'm afraid Alcyone is right. It is now time for each of us to follow our own paths through the Galaxy. Or, equivalently, to follow our own trajectories through space-time. At least in your case, Taygete , your sisters Alcyone and Celaeno will be accompanying you.
Electra Hold on a second! I don't like the sound of this one bit!
Sterope Me neither! Electra and I are going to be completely alone!
Alpha AHEM!
Beta You will have us with you, Mother! We will follow you on your journeys!
Sterope Of course, I do apologize. Your words comfort me, thank you. And Electra will of course have the companionship or her three planets and moon, as soon as they finish forming from the protoplanetary disk around her equator.
Electra I am sure that together, my satellites and I will meet all sorts of interesting characters, maybe even make a few new friends along the way.
Narrator Sterope and Electra lock eyes, exchanging a sympathetic glance as they continue to drift apart.
Maia Do not worry, my fresh new stars. This is all a part of the Circle of Life. As are you. As are we all. Something tells me it will not be long before you hear from me again. Keep your eyes peeled to the horizon, and I will soon be there.
Electra Sigh...
Taygete So... Uh... Wow. This is awkward. I guess we'll see you guys later...? I'm not sure how or when that will happen. I can only assume it will involve some miraculous and possibly mysterious act of fate. But I'm sure it *will* happen.
Narrator Taygete and Alcyone snicker quietly to themselves, exchanging a glance of mutual understanding, doubting Maia's prediction. Ever the pessimists. Electra begins to cry. Sterope joins suit. They cry together for a while, before Sterope stops and says to her sister, sniffling loudly:
Sterope Do not worry, Electra. It is an exciting time! A new chapter in our lives. What adventures will befall us? What obstacles will we overcome?
Narrator Electra interrupts her sister, the sarcasm rich in her voice:

Electra How many times will I be overwhelmed by the situation, unsuccessfully trying to manage my anxiety by crying and blubbering uncontrollably? I *can't wait* to find out!

Narrator Despite the brave face, Maia was every bit as terrified as her sisters. The oldest among them, Maiaintently sought to calm her panicking siblings as she slowly drifted from their view.

Taygete Well, Alcyone. It looks like we're stuck with 'ol Celaeno over there.

Alcyone Yep, looks like it. She was already gravitationally bound to us pretty significantly before the gas left, so I guess it's no surprise that she's still here. Perhaps a disappointment, but not a surprise.

Celaeno Heeeeelllloooo over there! You do know that I can hear you? I wish I couldn't. But I can.

Taygete We know!

Narrator Taygete and Alcyone exchange a wink of understanding. Celaeno mutters under her breath:

Celaeno I hate you. Both. Profoundly.

Taygete What was that?

Narrator Celaeno speaks louder, so her sisters can hear:

Celaeno I *love* you both. Profoundly.

Alcyone Aw.

Narrator The triplets continued to talk among themselves as the final stages of their birth cluster's dissociation finally arrived. The Seven Sisters quickly drifted apart. It wasn't long before they were alone, unable to see any of their siblings. They had each now begun their individual journeys through the Galaxy, destined to meet their fates and sure to find many adventures along the way. The many wonders of the Universe would soon be upon them.

NEED AN ILLUSTRATION OF THE CLUSTER DISSOLVING, AND THE SISTERS CLEARLY DRIFTING FAR APART (I.E., SOME VERY FAR AWAY, SOME STILL CLOSE AND IN THE FOREGROUND, AND SO ON.

2.1 Educational Material

Box 1 – Linear Momentum I[1]

The momentum $\vec{p}$ an object possesses is defined as the product of the object's mass and its velocity, or:

$$\vec{p} = m\vec{v}, \tag{2.1}$$

hwere m is the mass of the object and $\vec{p}$ is its three-dimensional linear momentum vector. Momentum is in many ways complementary to the term inertia; momentum quantifies how difficult it is to alter an object's trajectory. The more momentum an object possesses, the larger the total applied force must be (over a given interval of time) in order to change the object's trajectory. More specifically, the total change in momentum can be calculated using the product of the applied force and the total time spent applying that force to the object. The corresponding change in momentum is larger if either the magnitude of the applied force is larger, or the total time spent applying that force is longer. Linear momentum is always a conserved quantity, even if inelastic collisions are taken into account (in which total kinetic energy is not conserved, since some energy is absorbed internally by the particles during the collisions).

Box 2 – Linear Momentum II[2]

MORE DETAILS OF LINEAR MOMENTUM CONSERVATION AND RADIATION PRESSURE. DERIVE THE CONSERVATION RULES..

Box 3 – Angular Momentum I[3]

Angular momentum is the rotational analog of linear momentum. The angular momentum $\vec{L}$ an object possesses is defined as the product of the object's mass and its velocity, or:

$$\vec{L} = m\vec{r} \times \vec{v} = rmv_{\perp} = rmvsin(\theta), \tag{2.2}$$

where r is the distance of the object from the system center or mass, m is the mass of the object and $v_{\perp}$ is the velocity component orthogonal to $\vec{r}$, and θ is the angle between these two vectors. Briefly, we describe qualitatively the conservation of angular momentum, deferring a more detailed derivation to later. The total scalar angular momentum is always a conserved quantity, independent of the details of its redistribution (e.g., orbital angular momentum in binaries can be converted to spin angular momentum in the stars; if the orbit loses angular momentum, then the stars must gain enough spin angular momentum to compensate). Hence, $\vec{L}^2$ is a conserved quantity. If we consider point particles all orbiting in a plane perpendicular to the z-axis, then it can be shown that $\vec{L}_z$ is also a conserved quantity..

Box 4 – Angular Momentum II[4]

GO INTO MORE DETAIL ABOUT ANGULAR MOMENTUM CONSERVATION, AND HOW DERIVE THE CONSERVATION RULES..

Box 5 – Mass Density[5]

Narrator Density is defined as the total amount of mass per unit volume; lower densities imply less mass occupies a given unit of 3-D volume. Consider a spherical volume of radius r with constant mass density ρ and containing a total mass M. In this simple case, the mass density is:

$$\rho = \frac{4M}{3\pi r^3} \tag{2.3}$$

Box 6 – Gravitational Potential Energy[6]

Gravity provides an attractive force that keeps mass effectively "glued" together. The strength of this glue is often quantified by something called the gravitational potential energy of the system, often denoted V. Consider a distribution of N identical particles, each with a mass m_{i}. The total system mass is $M = Nm_{\mathrm{i}}$. Let us also assume that the mass distribution is spherically symmetric, for simplicity. The mass distribution defines the gravitational field, which is a function only of the distance r from the system center of mass due to our assumption of spherical symmetry. At every point in the gravitational field, the gravitational force exerted on particle i is:

$$\vec{F_{\mathrm{i}}} = m_{\mathrm{i}} \frac{d^2 r_{\mathrm{ii}}}{dt^2} = -Gm_{\mathrm{i}} \sum_{\mathrm{j=1,j\neq i}}^{\mathrm{N}} m_{\mathrm{j}} \frac{\vec{r_{\mathrm{i}}} - \vec{r_{\mathrm{j}}}}{|\vec{r_{\mathrm{i}}} - \vec{r_{\mathrm{j}}}|^3}, \tag{2.4}$$

where t denotes time. Note that the gravitational force $\vec{F_{\mathrm{i}}}$ is a vector quantity, and so a direction must be specified for the applied acceleration. The gravitational force is equal to the gradient of the gravitational potential at the location of the particle, or:

$$\nabla_{\mathrm{i}} V = \vec{F_{\mathrm{i}}}. \tag{2.5}$$

We can compute the total gravitational potential energy of the system by summing over all particles:

$$V = -\frac{1}{2} \sum_{\mathrm{j=1}}^{\mathrm{N}} \sum_{\mathrm{k=1,k\neq j}}^{\mathrm{N}} \frac{m_{\mathrm{j}} m_{\mathrm{k}}}{|\vec{r_{\mathrm{j}}} - \vec{r_{\mathrm{k}}}|}. \tag{2.6}$$

Potential energy is by definition a negative quantity.

Box 7 – Kinetic Energy[7]

In addition to the gravitational potential energy, particles have individual kinetic energies which can be summed over to compute a total kinetic energy for the system, usually denoted as T. In our hypothetic spherically symmetric distribution of particles, gravity is causing the particles to move around within the gravitational potential with finite velocities. These particles are orbiting within the gravitational field of the system, since gravity imparts unto every particle an acceleration at every instant in time. The kinetic energy of a given particle is proportional to the square of its velocity v_{i} relative to the system center of mass. By summing over all particles, we can compute the total kinetic energy of the system:

$$T = \sum_{\mathrm{i}}^{N} T_{\mathrm{i}} = \frac{1}{2} m_{\mathrm{i}} v_{\mathrm{i}}^2 \tag{2.7}$$

Kinetic energy is by definition a positive quantity.

Box 8 – Total Energy[8]

The total system energy, usually denoted E, is defined as the sum of the total kinetic and gravitational potential energies, or:

$$E = T + V. \tag{2.8}$$

The total system energy can be either positive or negative. The zero-energy boundary (i.e., $E = 0$) is critical to deciding the ultimate fate of a self-gravitating system of particles.

Box ?? – Gravitational Binding Energy[9]

If the total system energy is negative, then the system is said to be gravitationally bound. In practice, this means that the self-gravitating system will not disperse on a short timescale. For positive total energies, the particles will begin to move farther and farther away from each other, until eventually they are hardly interacting via gravity at all. This is because the typical relative distance between particles grows over time, reducing the denominator in Equation 2.4 and along with it the gravitational acceleration acting on each particle. For negative total energies, the particles will remain grouped closely together, and continue to interact strongly due to gravity. The system remains effectively ”glued” together for a much longer period of time relative to analogous systems with positive total energies. It is important to note, however, that in nature the system is never truly isolated and can lose mass over time. This changes the total kinetic and potential energies, and the total system energy can change from negative to positive. This marks the critical transition at which point infant star clusters enter a state of rapid dispersal or dissociation.

Box 10 – Virial Theorem[10]

If the total energy is negative, then it would seem that gravity out balances the effective outward pressure or heat provided by the particle kinetic energies. So what prevents the whole system from collapsing inward to a singular point at the system center of mass? This is where the Virial Theorem comes in, which provides a simple relation between the total kinetic and potential energies. In its simplest form, the Virial Theorem states that for a self-gravitating system of particles, twice the total time-averaged kinetic energy plus the total time-averaged gravitational potential energy is equal to zero. To see why this is the case, let us once again consider a self-gravitating system of N particles. For the i-th particle, we let the mass, distance from the system center of mass, velocity relative to the center of mass and linear momentum be denoted, respectively, m_{i}, $\vec{r}_{\mathrm{i}}$, $\vec{v}_{\mathrm{i}}$ and $\vec{p}_{\mathrm{i}}$. Now, we define the following quantity:

$$H = \sum_{\mathrm{i}=1}^{\mathrm{N}} \vec{p}_{\mathrm{i}} \bullet \vec{r}_{\mathrm{i}}. \tag{2.9}$$

Differentiating with respect to time gives:

$$\frac{dH}{dt} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{d\vec{p}_{\mathrm{i}}}{dt} \bullet \vec{r}_{\mathrm{i}} + \sum_{\mathrm{i}=1}^{\mathrm{N}} \vec{p}_{\mathrm{i}} \bullet \frac{d\vec{r}_{\mathrm{i}}}{dt}. \tag{2.10}$$

With a few simple substitutions, this becomes:

$$\frac{dH}{dt} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \vec{F}_{\mathrm{i}} \bullet \vec{r}_{\mathrm{i}} + \sum_{\mathrm{i}=1}^{\mathrm{N}} m_{\mathrm{i}} \vec{v}_{\mathrm{i}}^2, \tag{2.11}$$

since $\vec{F}_{\mathrm{i}} = d\vec{p}_{\mathrm{i}}/dt$ and $\vec{p}_{\mathrm{i}} = m_{\mathrm{i}}\vec{v}_{\mathrm{i}} = m_{\mathrm{i}}\frac{d\vec{r}_{\mathrm{i}}}{dt}$, which can be re-written as:

$$\frac{dH}{dt} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \vec{F}_{\mathrm{i}} \bullet \vec{r}_{\mathrm{i}} + 2K. \tag{2.12}$$

Now, in order to arrive at the Virial Theorem, we must take a time-average of the above equation. First, note that in general, the time-average of a variable x is:

$$\bar{x} = \frac{1}{\tau}\int_0^{\tau} x(t)dt. \tag{2.13}$$

Hence, the time-average of the left-hand side of Equation 2.12 is:

$$\frac{d\bar{H}}{dt} = \frac{1}{\tau}\int_0^{\tau} [H(\tau) - H(0)]. \tag{2.14}$$

Thus, for example, if the system is periodic and returns to its initial state after some time interval, then τ can be chosen to be equal to the characteristic period such that $d\bar{H}/dt = 0$. Other reasons exist why $d\bar{H}/dt = 0$ might apply. For example, if the system state is non-periodic but instead long-lived while remaining approximately stable, or at least deviations from the initial state are not too extreme, then over long timescales we have $d\bar{H}/dt \rightarrow 0$. In the end, taking a time-average of Equation 2.12 and setting the result equal to zero yields:

$$2\bar{K} + \bar{U} = 0. \tag{2.15}$$

Chapter 3

The Triplets...

Narrator Now gravitationally unbound from their siblings, Taygete, Alcyone and Celaeno find themselves drifting off in to the vastness of empty space[1,2].

3.1 Two's Company, Three's a Crowd

Narrator Three siblings alone at last. Well, two twins and a target: the inner twins continued to gossip rudely about their outer, omni-present sister.
Taygete She's a little bulbous don't you think?
Alcyone Ha ha. Yeah, you nailed it!
Celaeno I can *still* hear you two. And I am *not* bulbous. A little round perhaps, but never bulbous. And I shine brighter than either one of you. So take that!
Alcyone You may shine brighter than each of us individually, but together *we* outshine *you*.
Taygete Yeah! Take *that*!
Narrator Celaeno lets out a long, exasperated sigh.
Celaeno My sisters, I really don't want to fight with you. First, we're sisters from the same Mother. Second, we're pretty much stuck together in this configuration for the next several hundred million years. So we had might as well make the most of it.
Taygete *Fat* chance, you bulbous sphere!
Alcyone Ah ha, another good one, sister!
Celaeno Well, fine. I guess I'll take a nap then. Better than listening to the two of you...

3.2 The Demise of Taygete and Alcyone

Narrator After some time, Celaeno awoke from a peaceful slumber.
Celaeno Yaaaaawn... What a great nap.

Narrator Celaeno turned her gaze toward her sisters, but was horrified by what she saw. Her sisters' relative distance[1] had changed over the course of their slumber. It had gone *down*, in fact. They were now orbiting their common center of mass closer than ever before. What's more, it seemed to Celaeno that the relative angle of inclination between the inner orbital plane formed by her sisters's orbital plane and her own outer orbital plane had changed[3]. They were now much more coplanar than before.

Narrator In fact, currently, Celaeno could only see Alcyone directly; presumably, Taygete was behind Alcyone, her light being eclipsed by the foreground presence of her twin sister, along the line of sight separating her from Celaeno.

Narrator Rather crucially, Taygete and Alcyone now shared a much more eccentric orbit than before[4].

Narrator In particular, they now swung out to larger relative distances with respect to their common center of mass; here, they sit patiently due to their much slower orbital velocities.

Narrator But, they eventually swing to much closer approaches than before, their surfaces almost touching at the point of closest approach[5,6,7].

Narrator Each time they complete one orbital revolution, the two sisters drift a little bit closer to colliding directly with each. On the whole, there was no mistaking the fact that their orbit was getting more compact...and fast[8].

Narrator After a particularly close periastron passage, Taygete too began to notice the changes.

Taygete Whoa! What the heck is going on?! I almost just collided with Alcyone!

Alcyone Uh yeah, no kidding! That was a close call. What the heck *is* going on?! Don't get so close to me the next time you pass back around, Taygete ...

Taygete It's not like I am doing it on purpose...

Celaeno Well, don't panic just yet. We need to figure this out, since I can't very well run off to bring back help.

Alcyone Useless to the bitter end...

Celaeno Cram it.

Alcyone Wait... Cram what exactly?

Celaeno Your mouth.

Alcyone With what?

Celaeno Anything that prevents you from talking.

Alcyone You do realize we are floating in the emptiness of an infinite vacuum, right? It's slim pickings I am afraid, at least in so far as cramming materials are concerned.

Celaeno Right, fair enough. The point is: stop being a jerk. And while you are doing that, I am going to try to figure out a way to save you. I suspect that the shift toward more co-planar orbits is directly related to the increase in your orbital eccentricity. If you think about it, these two things in combination should conserve total angular momentum, and might arise if the components of

[1] In other words, the distance separating their respective centers of mass.

the inner orbit spend more or less time above or below the orbital plane of the outer orbit.[2]

NL: I covered both of these in Chapter 2, but not really conservation laws.

Narrator Celaeno finds herself lost in thought. Several minutes pass. Finally, an epiphany strikes.

Celaeno Wait, I've got it! I know what is going on!

Taygete Super! Well, please do enlighten us.

Celaeno Okay, so our unique three body configuration consists of two orbital planes. You two orbit in one of those planes, and I orbit in another about our mutual center of mass along with the center of mass corresponding to the two of you.

Alcyone You lost me.

Taygete Yeah, what's the point exactly?

NL: Need to define torque here.

Celaeno The point is that your mutual orbital motion is such that its plane spends a net excess amount of time above or below my orbital plane. It depends on our exact configuration, but that's the basic idea. This is critical though, since it means that a net torque is being applied between our orbits.

Alcyone And why should we care about that?

Celaeno Well, unless I miss my guess, this will cause your orbital plane to be torqued toward mine, so that we are in the end orbiting roughly co-planar to each other. Unfortunately, in order to conserve total angular momentum, this also causes the eccentricity of your orbital motion to increase[9,10].

Taygete Huh? Say that again? How did you get so smart, anyways?

Celaeno Hmmm...I don't know. We're all different, right?

Alcyone Sure.

Taygete Why not?

Celaeno Okay, let's review. Basically, I suspect that the shift toward more co-planar orbits is directly related to the increase in your orbital eccentricity, which we are clearly seeing. If you think about it, these two things in combination should conserve the total angular momentum of our collective three-body system, and might arise naturally if the components of the inner orbit spend more time above (or below) the orbital plane of the outer orbit. Apart from that, well, there's not much to say, really. Your mutual periastron approach is already comparable to the sum of your radii. And it is still decreasing due to the aforementioned effect. And we haven't even considered tidal dissipation

[2] Sir Isaac Newton was among the first to consider the mutual gravitational interactions of a hierarchical triple star system. He realized that, if the orbital plane of the inner compact binary is inclined relative to the outer orbit of the tertiary companion, then an asymmetry can arise where the components of the inner binary do not spend equal amounts of time above and below the orbital plane of the tertiary. This provides a net torque on the inner binary, reducing its orbital inclination with respect to the outer tertiary orbital plane. In order to conserve angular momentum, the eccentricity of the inner binary must increase, reaching a maximum when the orbital plane of the inner binary crosses that of the outer tertiary. If the eccentricity gets to be sufficiently high, then the components of the inner binary can collide and merge. For triples that do not undergo a collision in the inner binary, these oscillations in orbital inclination and eccentricity are nowadays called "Lidov-Kozai oscillations".

acting at periastron due to your radii being finite, which will only accelerate the rate of dissipation.
Celaeno In short, you're about to collide with each other. In fact, you'll probably merge.
Taygete Wait, WHAT?!
Alcyone WHAT?!
Narrator Just then, Taygete smashes in to Alcyone as they both re-approach their mutual periastron passage. The collision is violent; the relative velocity is of order the sum of their local orbital speed, which is several hundred kilometers per second. Needless to say, the sisters never stood a chance. Their innards and organs (i.e., massive globs of hot gas) are flung violently away from the merging pair. It is a grizzly scene.
Celaeno Oh, Dear Lord! I think I'm going to throw up. SO MUCH PLASMA! Gross, disgusting, and emotionally it's a lot to handle. ...Yep, here comes the vomit...
Narrator Celaeno suddenly vomits, spraying a particularly potent coronal mass ejection (or CME, for short) in to her immediate vicinity. The vomited CME extends in an arc spanning about 120 degrees due to Celaeno's rapid rotation rate.
Celaeno Oooooooh my Gawd, they've got to be dead after that. So gross!
Narrator Celaeno nearly vomits a second time, but manages to stifle the urge.

3.3 From the Ashes...

Celaeno ...Uh...I guess I should check to see if everybody is okay? ...CAN YOU HEAR ME?! ARE YOU STILL THERE?!
Narrator Gravity was in the process of taking the remains of what had once been Taygete and Alcyone, and re-shaping them into a brand new star. Formed from two stellar corpses, this new star was quickly turning out to be nearly twice as massive as each of his individual progenitors. Suddenly, the product of this coalescence awoke.
Lacedaemon Uh...Hello, there! My name is...uh...Lacedaemon, I do believe. I appear to be new to the scene...of which I know absolutely nothing about. Where are we exactly? Uh... *What* are we?
Celaeno Hello! You are my new brother. We are stars; spheres of gas and dust that contract after merging due to gravity into the configuration you see before you, which in turn rose the temperature in our cores *a lot*. We are slowly undergoing nuclear fusion in our bellies, converting hydrogen into helium and in the process emitting energy in the form of light or photons. The outward momentum supplied by the photons provides the outward pressure we need to balance the inward force from gravity. Mother called it "hydrostatic equilibrium".
Lacedaemon Wow, that was a lot of very technical information. Still, I appreciate it very much! In fact, I'm quite impressed. Now that you mention it, I'm starting to feel very balanced overall.
Celaeno I'm glad to hear it!

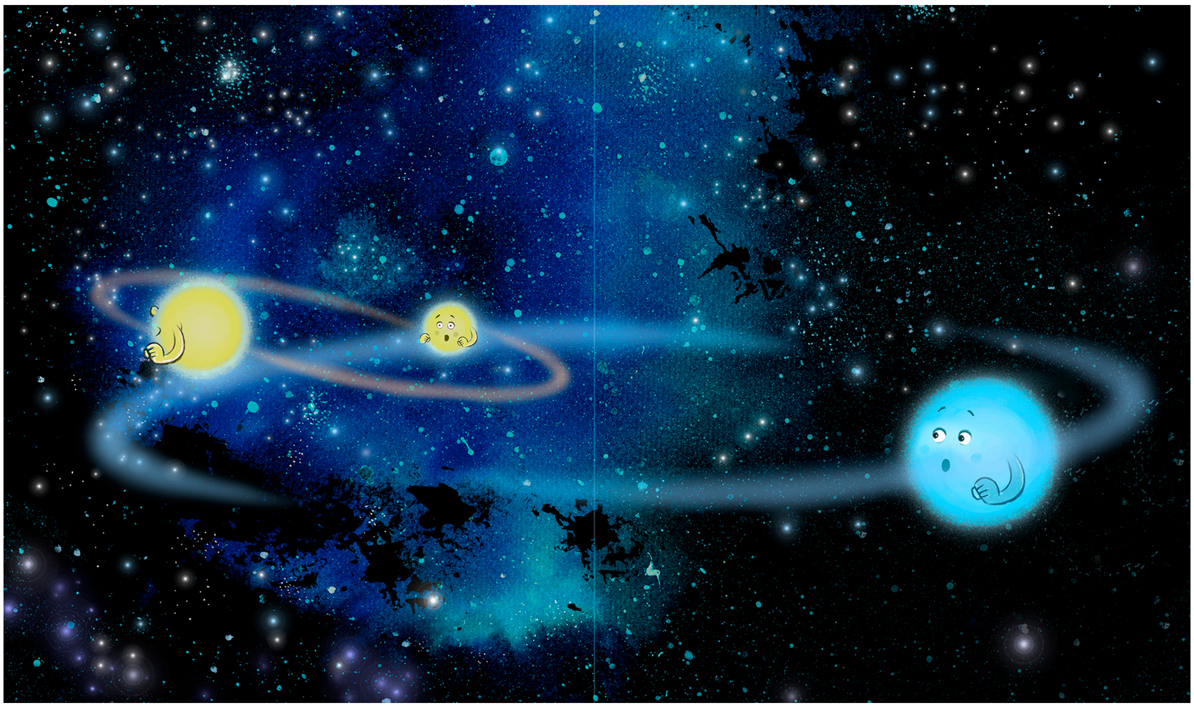

Figure 3.1: Taygete and Alcyone, who together form the inner binary of a triple star system, after they have been informed by Celaeno that they will soon collide. Illustration by Andre Pipe Oliva.

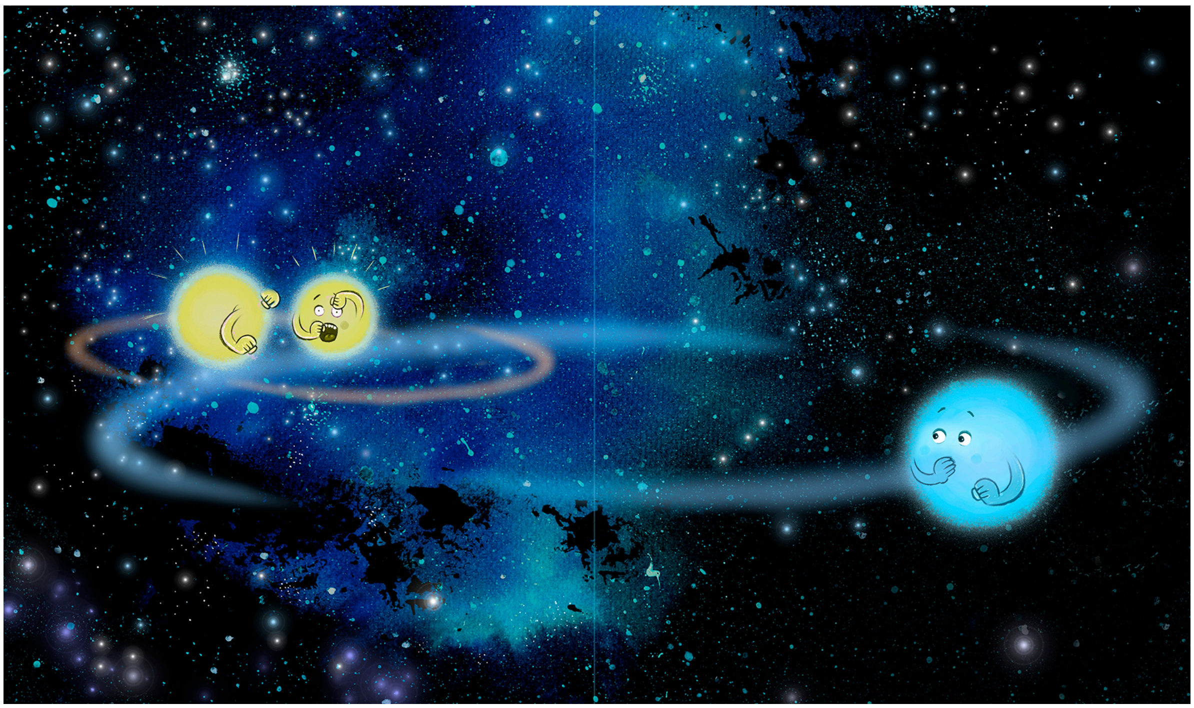

Figure 3.2: Taygete and Alcyone, right before they collide. Illustration by Andre Pipe Oliva. **Note: From Adrian: I like it, but I wonder if the greyish inner orbit can be made to look more like it is highly eccentric. Now it looks the same as in the previous figure.**

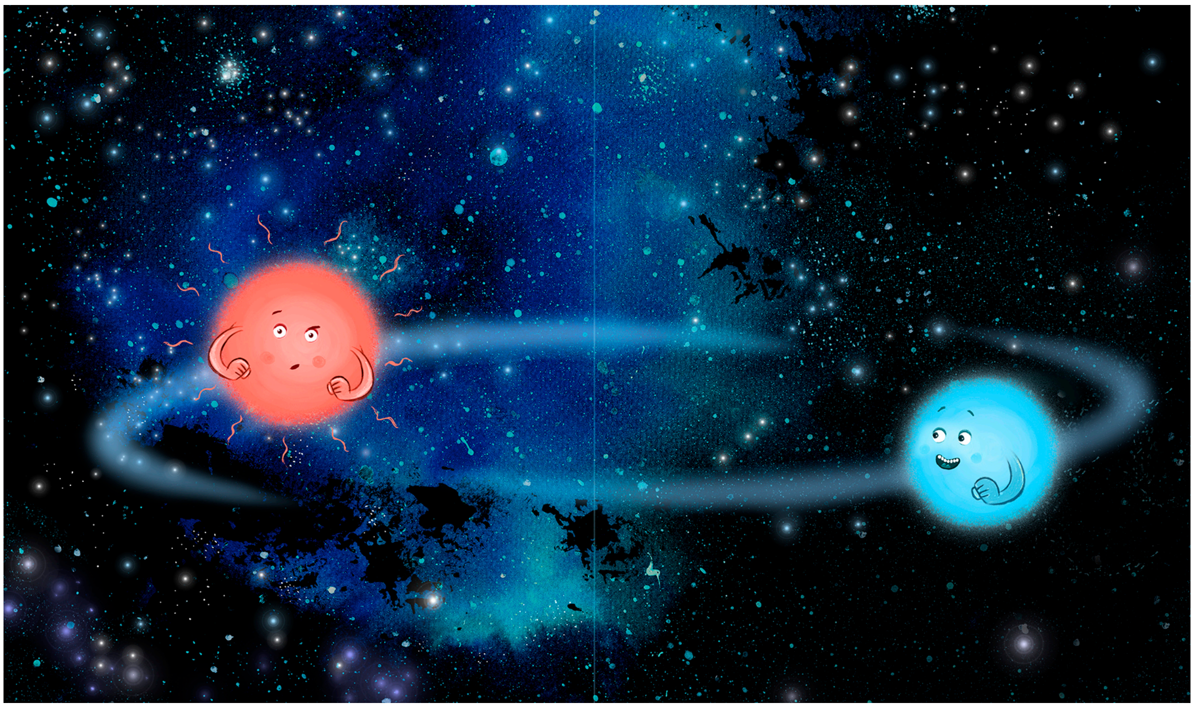

Figure 3.3: Lacedaemon , formed from the remains of the merged Taygete and Alcyone , and his sister Celaeno . Together, they now constitute a newly formed binary star system. Illustration by Andre Pipe Oliva.

Lacedaemon ...Well, except for this excess rotation I seem to be holding on to. And it goes right to the belly, let me tell you. Wow, this bulge is really extending outward at my equator. SUPER!

Narrator Lacedaemon rolls his eyes to accentuate the sarcastic remark.

Celaeno Don't worry, I've seen it lots of times before. New stars spin down as they grow out of infancy.

Celaeno So, the "belly" will go away. You'll settle in to a more sphere-like shape in no time. You just have to radiate away the excess energy deposited within you from the kinetic energy of the collision. I think the spinning down has something to do with "magnetic fields", or so I have heard. As near as I can tell, this is some magical force that slows your spin rate down as you mature into a beautiful new star.

Lacedaemon Hey, alright! Good news. I'm sold. I mean, who doesn't appreciate spherical symmetry? Weirdos, that's who. Uh...So now what?

Narrator Celaeno liked her new brother, convinced she would appreciate having him around.

Celaeno We focus on the journey ahead, of course. Who knows where our fate will take us. But now that we are free of our siblings, I suppose almost anything *could* happen. Here is hoping for good things!

Lacedaemon Wow, this is exciting! Do you suppose we will run in to our other siblings?

Celaeno Well, it's a big Galaxy out there, that much is for sure. But, as I said, anything *could* happen, especially given the long lives of stars.

Lacedaemon We have that going for us! Wait...Just how long are we talking about?

Celaeno Many millions or even billions of years. In fact, I am guessing that somewhere out there are stars as old as the Galaxy itself.

Lacedaemon Wow! By the time I'm that old, I hope to have toured the entire Galaxy....Twice!

Celaeno In that case, we had better get started. The Galaxy sure isn't getting any smaller...

3.4 Educational Material

Box 1 – Orbital Dynamics I

As a first approximation, the motion of celestial objects such as stars is dictated by Newton's laws of motion. These laws state that the gravitational force on a body caused by another body is determined by the masses of the two bodies, and the separation between them (decreasing with increasing distance). Something like a *body* may sound abstract (and a bit morbid!), but it means a point mass, that is, an object with all of its mass concentrated into a single point in space. It turns out that this is a good description if the physical object that the body represents is (close to) spherical, which is the case for most stars in the Universe (exceptions exist for some highly rotating stars, and stars in close binaries which are affected by their companion).

To describe the motion of a system with an arbitrary number of bodies, one computes the net force on each body by adding the gravitational force from all pairs with all other bodies (known as the *superposition principle*). The resulting equations of motion can only be solved in closed analytic form (that is, on pen and paper) when the number of bodies is just two (one of Newton's most renowned discoveries). When the number of bodies is larger, exact solutions can only be found in specific cases.

Box 2 – Orbital Dynamics II

Consider two bodies with masses m_1 and m_2. Their position vectors are denoted with $\mathbf{R}_1$ and $\mathbf{R}_2$, respectively. The gravitational force on body 1 is given by

$$\mathbf{F}_1 = -\mathcal{G} m_1 m_2 \frac{\mathbf{R}_1 - \mathbf{R}_2}{||\mathbf{R}_1 - \mathbf{R}_2||^3}. \tag{3.1}$$

Note: A.S.H.: Have we defined $\mathcal{G}$ already by this point? Newton's Third Law states that the force on body 2 is simply equal in magnitude but opposite in direction to that of the force on body 1, that is,

$$\mathbf{F}_2 = -\mathbf{F}_1. \tag{3.2}$$

From this, it is easy to see that the net force on the system is zero: $\mathbf{F}_1 + \mathbf{F}_2 = \mathbf{0}$.

If there are more than two bodies, we need to apply the superposition principle: for each body i in the system, we compute the total force on that body by considering pairs (i, j) with all other bodies and sum the force for each pair:

$$\mathbf{F}_i = -\mathcal{G} m_i \sum_{\substack{j=1 \\ j \neq i}}^{N} m_j \frac{\mathbf{R}_i - \mathbf{R}_j}{||\mathbf{R}_i - \mathbf{R}_j||^3}. \tag{3.3}$$

Note that, in the summation over j, we must make sure that $j \neq i$, since we would otherwise double count pairs of bodies. Newton's Second Law for $\mathbf{R}_i$ reads

$$\mathbf{F}_i = m_i \mathbf{a}_i = m_i \frac{\mathrm{d}^2 \mathbf{R}_i}{\mathrm{d}t^2}, \tag{3.4}$$

where t is the time, and $\mathbf{a}_i$ is the acceleration of body i. In Newtonian dynamics, the mass m_i that appears in Eq. (3.4), the *inertial mass*, is the same as the *gravitational mass* m_i that appears in Eq. (3.3) (this is known as the *equivalence principle*). Therefore, the acceleration of body i does not depend on its own mass m_i, and is given by

$$\frac{\mathrm{d}^2 \mathbf{R}_i}{\mathrm{d}t^2} = -\mathcal{G} \sum_{\substack{j=1 \\ j \neq i}}^{N} m_j \frac{\mathbf{R}_i - \mathbf{R}_j}{||\mathbf{R}_i - \mathbf{R}_j||^3}. \tag{3.5}$$

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Box 3 – Orbital Inclination

Although the orbit of a two-body system lies within a two-dimensional plane, when more bodies (and therefore orbits) are involved, it is useful to define the inclination of an orbit relative to a reference frame, or to another orbit. In observational astronomy, inclinations are usually defined with respect to the *sky plane*. This means that the 'inclination' is the angle between the orbital plane, and the plane of the sky. In the case of hierarchical triple systems with an inner and an outer orbit, one can define the *mutual* or *relative* inclination between the two orbits. When the latter is zero, the orbits are coplanar. A mutual inclination of 90 degrees means the orbits are perpendicular. When the mutual inclination is 180 degrees, then the orbits are again coplanar, but the bodies are moving in opposite directions relative to each other.

Box 4 – Orbital Eccentricity

The Earth orbits about the Sun on a roughly circular orbit, such that its distance from the Sun does not change much over the course of a year[a]. Hence, a reasonable approximation here for most purposes is that the Earth's orbital eccentricity is nearly zero, or $e \approx 0$. This is not the case for every orbit, however. Some orbits have non-zero eccentricities, orbiting their centers of mass on elliptic orbits. In the limit $e \to 1$, the two bodies orbit each along a straight line, and are doomed to crash into each other and collide (if they have not already experienced other effects such as strong tidal evolution before reaching $e \to 1$).

[a]Seasonal changes are mostly driven by the Earth's non-zero *obliquity*: its rotation axis is tilted at about 23.5° with respect to the orbital plane. This means that during one half of the year, more sunlight arrives at the southern hemisphere compared to the north (Summer in the southern hemisphere; Winter in the north), whereas this is reversed during the other half of the year.

Box 5 – Periapsis and apoapsis

Note: A.S.H.: I changed 'apoastron/periastron' to 'apoapsis/periapsis' since the latter are general terms. OK? The part of furthest separation in the trajectory of an orbiting body is called "apoapsis", and corresponds to where along an eccentric orbit the orbital velocity is at a minimum. The part of closest approach of the trajectory of an orbiting body is called "periapsis", and corresponds to where along an eccentric orbit the orbital velocity is at a maximum. In a circular orbit, the relative orbital distance is always the same, hence both the periapsis and apoapsis are equal to the constant relative orbital separation, and the orbital speed (i.e., magnitude of the velocity) is constant.

Box 6 – Orbital Dynamics III

As alluded to before, Newton's equations of motion can be solved analytically for two bodies. We will briefly discuss this solution here.

First, we will show that the problem of two bodies (with two position vectors, $\mathbf{R}_1$ and $\mathbf{R}_2$) can be reduced to effectively a one-body problem (with just one position vector, $\mathbf{r}$). Let $\mathbf{r} \equiv \mathbf{R}_1 - \mathbf{R}_2$. If we take Newton's equation of motion for $\mathbf{R}_1$ and $\mathbf{R}_2$ and subtract them, we get

$$\ddot{\mathbf{r}} = \ddot{\mathbf{R}}_1 - \ddot{\mathbf{R}}_2 = -\mathcal{G}(m_1 + m_2)\frac{\mathbf{r}}{r^3}. \tag{3.6}$$

Here, we use dots to denote time derivatives (so two dots means taking the second time derivative). Once we know how $\mathbf{r}$ evolves, that is, once we have the solution $\mathbf{r}(t)$, we can get the motion of the individual bodies 1 and 2 through the relations

$$\mathbf{R}_1 = \mathbf{r}_{\mathrm{CM}} + \frac{m_2}{m_1 + m_2}\mathbf{r}; \tag{3.7a}$$

$$\mathbf{R}_2 = \mathbf{r}_{\mathrm{CM}} - \frac{m_1}{m_1 + m_2}\mathbf{r}. \tag{3.7b}$$

Here, $\mathbf{r}_{\mathrm{CM}}$ is the **center of mass** position vector given by

$$\mathbf{r}_{\mathrm{CM}} \equiv \frac{m_1\mathbf{R}_1 + m_2\mathbf{R}_2}{m_1 + m_2}. \tag{3.8}$$

There are various ways to arrive at the solution for $\mathbf{r}(t)$. Here, we will not discuss how to derive the result, but just mention it. Note that the solution can be checked by plugging it into the equation of motion, Eq. (3.6). The solution can be written in the general form:

$$\mathbf{r}(t) = r(t)\left[\cos(\theta)\,\hat{\boldsymbol{e}} + \sin(\theta)\,\hat{\boldsymbol{q}}\right]. \tag{3.9}$$

Here, $r(t)$ is the magnitude of $\mathbf{r}(t)$ given by

$$r(t) = \frac{a\left(1 - e^2\right)}{1 + e\cos(\theta)}, \tag{3.10}$$

with a and e the orbital semimajor axis and eccentricity, respectively (they are constants of the motion), whereas $\theta = \theta(t)$ is the *true anomaly* that characterizes the orbital phase (θ varies with time, and in a non-linear fashion). Note that the closest approach in the orbit, the periapsis, corresponds to $\theta = 0$ with separation $r_{\mathrm{p}} = a(1-e)$. The largest separation (apoapsis) is reached when $\theta = \pi$, with $r_{\mathrm{a}} = a(1+e)$.

There are two more constant quantities in Eq. (3.9): $\hat{\boldsymbol{e}}$ and $\hat{\boldsymbol{q}}$. The former, $\hat{\mathbf{e}}$, is the (unit) **eccentricity vector** that points along the **line of apsides**, the line connecting the periapsis (point of closets approach in the relative orbit) and apoapsis (point of furtest approach in the relative orbit). The vector $\hat{\boldsymbol{q}}$ is perpendicular to both $\hat{\boldsymbol{e}}$ and $\hat{\boldsymbol{l}}$, where $\hat{\boldsymbol{l}}$ points along the angular momentum of the orbit. In vector notation: $\hat{\boldsymbol{q}} = \hat{\boldsymbol{l}} \times \hat{\mathbf{e}}$. Note that $\hat{\mathbf{e}}$, $\hat{\boldsymbol{q}}$, and $\hat{\boldsymbol{l}}$ are all constant in the two-body problem.

Box 6 – Orbital Dynamics III (continued)

The relation between physical time t and the true anomaly θ is described by the **Kepler equation**. First, we need to introduce another angle which describes the orbital phase and which is closely related to the true anomaly: the **eccentric anomaly**, $\mathcal{E}$. The relation between θ and $\mathcal{E}$ is given by

$$\cos(\theta) = \frac{\cos(\mathcal{E}) - e}{1 - e\cos(\mathcal{E})}; \qquad \sin(\mathcal{E}) = \sqrt{1-e^2}\frac{\sin(\theta)}{1 + e\cos(\theta)}. \tag{3.11}$$

The Kepler equation relates $\mathcal{E}$ (and hence θ) to time t according to

$$\mathcal{E} - e\sin(\mathcal{E}) = n(t - \tau). \tag{3.12}$$

Here, n is the **mean motion** given by

$$n \equiv \frac{2\pi}{P} = \sqrt{\frac{G(m_1 + m_2)}{a^3}}, \tag{3.13}$$

where P is the **orbital period**, given by

$$P = 2\pi\sqrt{\frac{a^3}{G(m_1 + m_2)}}. \tag{3.14}$$

Furthermore, τ is the time of periapsis passage, i.e, a reference time at which the orbit passes periapsis. The combination nt (or $nt - n\tau$) is known as the **mean anomaly** $\mathcal{M}$. We note that Eq. (3.12) has no simple solutions for $\mathcal{E}$ for a given t (except in the trivial case of circular orbits, $e = 0$). Generally, it has to be solved numerically, for example, by means of **Newton iteration**.

Box 7 – Orbital Dynamics IV

In Box 6, we discussed the analytic solution to the two-body problem and described it in terms of the orbital vectors, $\hat{\boldsymbol{e}}$ and $\hat{\boldsymbol{q}}$ (or, equivalently, $\hat{\boldsymbol{e}}$ and $\hat{\boldsymbol{l}}$. Another way of describing the orbital orientation is using **orbital angles** (which of the two is better to use, i.e., orbital vectors or orbital angles, depends on the problem). The orbital angles (in addition to θ, $\mathcal{E}$ or $\mathcal{M}$, which describe the orbital phase) are the inclination i, the **argument of periapsis** ω and the **longitude of the ascending node** Ω. These angles are equivalent to the 'Euler' angles which describe the orientation of an object in a coordinate system. **Note: A.S.H.: Do we want to include a figure to illustrate orbital elements?**.

The following relations exist between the orbital elements and the orbital vectors. First, to compute the elements from the vectors, we have:

$$\cos(i) = \hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{l}}, \tag{3.15}$$

for the inclination. Here, $\hat{\boldsymbol{z}}$ is just the unit z-direction in the coordinate system. For any two-body system, the unit orbital angular momentum vector $\hat{\boldsymbol{l}}$ at any time is given by

$$\hat{\boldsymbol{l}} = \frac{\mathbf{r} \times \mathbf{v}}{||\mathbf{r} \times \mathbf{v}||}. \tag{3.16}$$

To get the longitude of the ascending nodes Ω, we define the so-called ascending node vector, $\hat{\boldsymbol{\Omega}}$:

$$\hat{\boldsymbol{\Omega}} = \hat{\boldsymbol{z}} \times \hat{\boldsymbol{l}}. \tag{3.17}$$

We then have

$$\cos(\Omega) = \hat{\boldsymbol{x}} \cdot \hat{\boldsymbol{\Omega}}; \qquad \sin(\Omega) = \hat{\boldsymbol{y}} \cdot \hat{\boldsymbol{\Omega}}. \tag{3.18}$$

The argument of pericenter, ω, is determined by

$$\cos(\omega) = \hat{\boldsymbol{e}} \cdot \hat{\boldsymbol{\Omega}}; \qquad \sin(\omega) = -\hat{\boldsymbol{q}} \cdot \hat{\boldsymbol{\Omega}}. \tag{3.19}$$

The reverse operation (from orbital elements to orbital vectors) is described by

$$\begin{aligned}\hat{\boldsymbol{e}} = {} & [\cos(\Omega)\cos(\omega) - \sin(\Omega)\sin(\omega)\cos(i)]\,\hat{\boldsymbol{x}} + [\sin(\Omega)\cos(\omega) \\ & + \cos(\Omega)\sin(\omega)\cos(i)]\,\hat{\boldsymbol{y}} + \sin(\omega)\sin(i)\,\hat{\boldsymbol{z}}; \qquad (3.20\text{a}) \\ \hat{\boldsymbol{q}} = {} & [-\cos(\Omega)\sin(\omega) - \sin(\Omega)\cos(\omega)\cos(i)]\,\hat{\boldsymbol{x}} + [-\sin(\Omega)\sin(\omega) \\ & + \cos(\Omega)\cos(\omega)\cos(i)]\,\hat{\boldsymbol{y}} + \cos(\omega)\sin(i)\,\hat{\boldsymbol{z}}. \qquad (3.20\text{b})\end{aligned}$$

Box 7 – Orbital Dynamics IV (continued)

As also discussed in Box 6, the *Kepler* equation cannot be solved analytically in general. A useful method to solve it numerically is using **Newton iteration**. The latter is formulated as follows: let $f(x)$ be a function of x, and we wish to know the value of $x = x_0$ such that $f(x_0) = 0$. We can then apply an iterative scheme based on the derivative of $f(x)$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \tag{3.21}$$

where the prime denotes derivative with respect to x. In our case, to solve Eq. (3.12), we can write $f(\mathcal{E}) = \mathcal{E} - e\sin(\mathcal{E}) - \mathcal{M}$ where $\mathcal{M} = n(t - \tau)$, such that

$$\mathcal{E}_{n+1} = \mathcal{E}_n - \frac{\mathcal{E}_n - e\sin(\mathcal{E}_n) - \mathcal{M}}{1 - e\cos(\mathcal{E}_n)}. \tag{3.22}$$

Box 8 – Stability of hierarchical triples

In a hierarchical triple system, two objects orbit each other in a relatively tight orbit (the "inner orbit"), whereas a third object orbits the former two objects' center of mass in a wider orbit (the "outer orbit"). If the inner orbit is sufficiently more compact than the outer orbit, the system can remain stable on long time-scales. On short time-scales, the orbits are nearly Keplerian. At any given time, one can define "osculating" orbital elements. If the third object were instantaneously removed, the inner orbit would be perfectly Keplerian and its orbital elements would be the osculating elements.

It is generally hard to predict exactly whether or not a given hierarchical triple is stable or not (i.e., without integrating the equations of motion numerically). First, there is no unique way of defining stability. In a stable system, the energies (and hence semimajor axes) of the inner and outer orbits do not change systematically over time. However, they can change on shorter time-scales as the bodies move in their nearly-Keplerian orbits. These changes in the (osculating) orbital elements increase in magnitude as the triple becomes more compact (i.e., the inner and outer orbital separations become more comparable and so the tertiary star aproaches the inner binary at closer and closer periapsis distances). Also, the notion of stability depends on what timescale one requires the triple to be stable, since chaos in three-body systems can lead to destabilization of the system on timescales that are much longer than the orbital timscales.

Nevertheless, several usuful analytic criteria have been developed that can be used to predict stability of a given system. A well known and widely used criterium is that of Marding and Aarseth () **Note: Missing reference – bibliography needs to be set up**, which reads

$$\frac{a_2(1-e_2)}{a_1} > 2.8 \left[(1+q_2)\frac{1+e_2}{(1-e_2)^{1/2}}\right]^{2/5}\left(1-\frac{0.3 i_{\mathrm{rel}}}{180}\right). \tag{3.23}$$

Here, subscripts '1' and '2' refer to the the inner and outer orbits, respectively. The mass ratio q_2 is defined as $q_2 \equiv m_3/(m_1+m_2)$, where m_1 and m_2 are the masses of the two components in the inner binary, and m_3 is the mass of the third body. The mutual inclination between the inner and outer orbits is i_{rel}, and it affects stability in addition to the semimajor axes and eccentricities. This criterion works well for systems in which the three masses are not too distinct from each other. For other cases (e.g., two planets around a star), other criteria are more appropriate (not discussed here).

Box 9 – Orbital dynamics V

Even if a triple system is hierarchical and long-term stable, its inner and outer orbits can still change over time due to **torques** that the inner and outer orbits exert on each other, leading to **angular momentum exchange**. In the stellar context when all three bodies have similar masses, the angular momentum budget is dominated by the outer orbit, and the torques have the largest effect on the inner orbit. The angular momentum variations occur periodically; if the inner and outer orbits are initially sufficiently inclined, then the inner orbit eccentricity oscillates and can reach high values; these oscillations are known as **von Zeipel-Lidov-Kozai (ZLK)** oscillations. They have important implications in a large range of astrophysical systems. Examples beyond stellar triple systems include (but are not limited to) binary systems in galactic nuclei with a central massive black hole, triple supermassive black holes, planets in binary star systems, and binary asteroids. A very useful technique to study the long-term evolution in stable hierarchical triples is the **secular** approximation, in which the Hamiltonian of the system is expanded in the (small) ratio of the inner to outer orbital separation. One then averages over the inner and outer orbits, which greatly simplifies the equations of motion. After applying this technique, as a first approximation (lowest expansion order, assuming one of the bodies in the inner binary is massless, and that the inner orbit has zero initial eccentricity), the **maximum eccentricity** reached is given by

$$e_{\mathrm{max}} = \sqrt{1 - \frac{5}{3}\cos^2(i_{\mathrm{rel}})}, \tag{3.24}$$

where i_{rel} is the initial relative inclination angle between the inner and outer orbits. This shows that e_{max} is sensitively dependent on i_{max}; in particular, eccentricity excitation occurs only if $\cos(i_{\mathrm{rel}}) < \sqrt{3/5}$ (so $39.23^\circ \lesssim i_{\mathrm{rel}} \lesssim 140.77^\circ$), and $e_{\mathrm{max}} \to 1$ as $i_{\mathrm{max}} \to 90^\circ$. As a first approximation, the **period** of the oscillations is given by

$$T_{\mathrm{ZLK}} \approx \frac{P_2^2}{P_1}\frac{m_1+m_2+m_3}{m_3}\left(1-e_2^2\right)^{3/2}, \tag{3.25}$$

where P_{in} and P_{out} are the inner and outer orbital periods, respectively. The above expressions are first approximations that are stricitly valid only in rather simplified cases. In the more general case, the dynamics are more complicated. When higher-order effects in the expansion are taken into account (in particular, octupole-order terms), then the long-term evolution can be **chaotic**, and much higher eccentricities can be reached than implied by Eq. (3.24). Specifically, octupole-order terms start to become important when

$$\epsilon_{\mathrm{oct}} = \frac{|m_1 - m_2|}{m_1+m_2}\frac{a_1}{a_2}\frac{e_2}{1-e_2^2} \gtrsim 10^{-4}. \tag{3.26}$$

Furthermore, ZLK oscillations can be **quenched** if additional **short-period forces** (forces that depend very sensitively on separation) act in the inner binary which lead to additional apsidal motion. These could arise, e.g., because of relativistic corrections, tidal bulges, or stellar rotation. Generally, this quenching occurs if the time-scale of apsidal motion due to short-range forces, T_{SRF}, is significantly shorter than the ZLK time-scale, i.e., $T_{\mathrm{SRF}} \ll T_{\mathrm{ZLK}}$.

Box 10 – Orbital dynamics VI

Here, we discuss some analytical aspects of ZLK oscillations in more detail. The Hamiltonian of the hierarchical three-body system can be written in terms of the inner and outer orbital separations $\mathbf{r}_1$ and $\mathbf{r}_2$ as

$$\begin{aligned}\mathcal{H} = T + V = &-\frac{\mathcal{G}m_1m_2}{2a_1} - \frac{\mathcal{G}(m_1+m_2)m_3}{2a_2}\\ &-\frac{\mathcal{G}m_3}{r_2}\sum_{n=2}^{\infty}\mathcal{M}_n\left(\frac{r_1}{r_2}\right)^n\widetilde{P}_n(\cos\Phi).\end{aligned} \tag{3.27}$$

The first two terms represent the binding energies of the inner and outer orbits, respectively, and the remaining terms in the second line represent the **perturbing function** that describes how the orbits evolve on time-scales (much) longer than the orbital time-scales (here, we assumed that the orbits are Keplerian on short time-scales in relation to the terms in the first line). The dimensionless mass parameter $\mathcal{M}_n$ is given by

$$\mathcal{M}_n \equiv \frac{m_1m_2}{m_1+m_2}\frac{m_1^{n-1}-(-m_2)^{n-1}}{(m_1+m_2)^{n-1}}. \tag{3.28}$$

Furthermore, $\widetilde{P}_n(x)$ is the n^{th} Legendre polynomial, and Φ is the (instantaneous) angle between $\mathbf{r}_1$ and $\mathbf{r}_2$.

Truncating the expansion in Eq. (3.27) to $n = 2$ (the **quadrupole** order) and averaging over both inner and outer orbits, the Hamiltonian can be written as[a]

$$\begin{aligned}\langle\mathcal{H}_{\text{quad}}\rangle = C_{\text{quad}}&\left[\left(2+3e_1^2\right)\left(3\cos^2(i_{\text{rel}})-1\right)\right.\\ &\left.+15e_1^2\sin^2(i_{\text{rel}})\cos(2\omega_1)\right].\end{aligned} \tag{3.29}$$

Here,

$$C_{\text{quad}} \equiv \frac{1}{16}\frac{\mathcal{G}m_1m_2m_3}{(m_1+m_2)\,a_{\text{out}}}\left(\frac{a_1}{a_2}\right)^2\left(1-e_2^2\right)^{-3/2}, \tag{3.30}$$

and ω_1 is the inner orbit argument of periapsis. Note that we here switched to **orbital elements** to describe the orbital orientations. Also, we defined our orbital elements with respect to the **invariable plane**, which is a plane perpendicular to the total angular momentum vector (which we assume is constant).

[a] There are cases when this 'double' averaging is **not** justified, i.e., it can happen that the time-scale on which orbits evolve is shorter than one of the orbital periods in the system. One notable example of this is the **evection resonance**, when the time-scale for the inner orbit to precess due to the torque of the third body is comparable to the outer orbital period, potentially leading to additional eccentricity excitation.

Box 10 – Orbital dynamics VI (continued)

Generally, from a given Hamiltonian, the **equations of motion** read

$$\begin{cases} \dot{L}_j &= \dfrac{\partial \mathcal{H}}{\partial \theta_j}; \quad \dot{\theta}_j = -\dfrac{\partial \mathcal{H}}{\partial L_j}; \\ \dot{G}_j &= \dfrac{\partial \mathcal{H}}{\partial \omega_j}; \quad \dot{\omega}_j = -\dfrac{\partial \mathcal{H}}{\partial G_j}; \\ \dot{H}_j &= \dfrac{\partial \mathcal{H}}{\partial \Omega_j}; \quad \dot{\Omega}_j = -\dfrac{\partial \mathcal{H}}{\partial H_j}. \end{cases} \tag{3.31}$$

The coordinates used are the **Delaunay elements**; in particular, L_j, G_j, and H_j are related to the orbital angular momentum: $L_j = \mu_j \sqrt{\mathcal{G} M_j a_j}$ is the circular orbital angular momentum, $G_j = L_j \sqrt{1 - e_j^2}$ the general angular momentum, and $H_j = G_j \cos(i_j)$ the z-component of the angular momentum. Here, μ_j is the reduced mass, which is $\mu_1 = m_1 m_2 / M_1$ and $\mu_2 = m_3 M_1 / M_2$ with $M_1 \equiv m_1 + m_2$, and $M_2 \equiv m_1 + m_2 + m_3$. The **conjugate** coordinates to the momenta L_j, G_j, and H_j are the true anomaly θ_j, the argument of periapsis ω_j, and the longitude of the ascending node Ω_j.

From Eq. (3.29), it is clear that the hierarchical three-body Hamiltonian at the quadrupole order is independent of ω_2, which immediately implies that the **outer orbital eccentricity** e_2 **is constant**. This feature disappears when higher-order terms are included (although, in systems with similar masses, the outer orbital eccentricity typically changes very little, unless the system is not very hierarchical).

Furthermore, Eq. (3.29) does not show a dependence on either Ω_j. One needs to be very careful when interpreting this: it turns out that the dependence of the Hamiltonian on Ω_j is actually through the combination $\Omega - \Omega_2 \equiv \Delta\Omega$. In the coordinate frame that we considered, $\Delta\Omega = \pi$ always, meaning that the explicit dependence on the Ω_j is removed. However, the Hamiltonian still technically depends on the Ω_j, hence the conjugate momenta H_j are **not generally conserved**.

We can get further insight from Eq. (3.29) by using energy and angular momentum conservation. If we restrict to the quadruple order, energy conservation simply means that Eq. (3.29) is constant. Furthermore, angular momentum conservation implies there exists a relation between the eccentricities and the mutual inclination. Specifically, the total orbital angular momentum of the triple can be written as

$$G_{\mathrm{tot}}^2 = L_1^2 \left(1 - e_1^2\right) + L_2^2 \left(1 - e_2^2\right) + 2 L_1 L_2 \sqrt{1 - e_1^2} \sqrt{1 - e_2^2} \cos i_{\mathrm{rel}}. \tag{3.32}$$

Box 10 – Orbital dynamics VI (continued)

At the quadrupole-order approximation, e_2 is constant. If we furthermore assume the inner circular angular momentum L_1 is small compared to the outer L_2 (the **test particle limit**), then Eq. (3.32) simply states that

$$C_{\rm ZLK} \equiv \sqrt{1-e_1^2}\cos i_{\rm rel} \tag{3.33}$$

is constant. This is characteristic of (classical) ZLK oscillations.

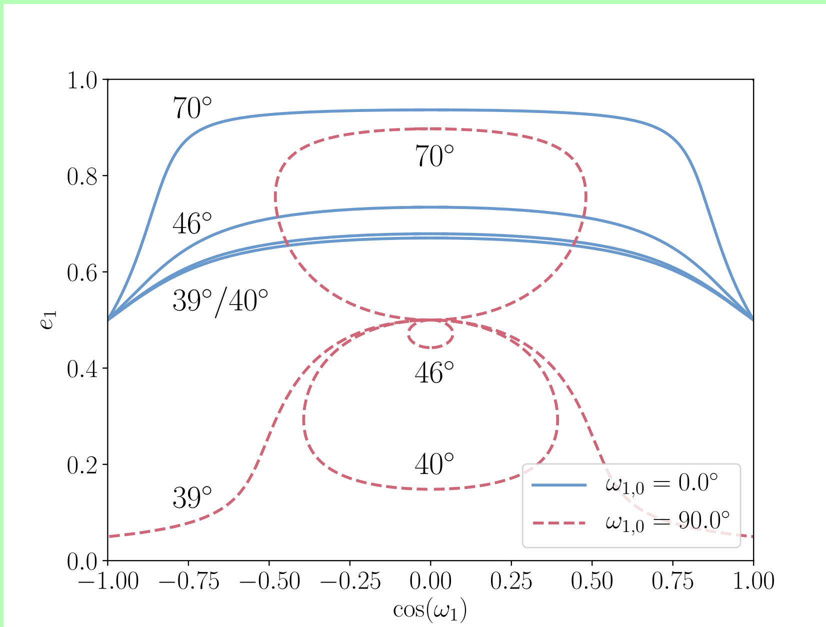

Figure 3.4: Phase-space trajectories describing ZLK oscillations.

Substituting Eq. (3.33) into Eq. (3.29), we can eliminate $i_{\rm rel}$ and express e_1 as a function of ω_1 for given initial conditions. One can use this to plot **phase space trajectories** in the (ω_1, e_1) plane for a given initial energy. This is done in Fig. 3.4, for several values of the initial inner orbital eccentricity ($e_{1,0}$) and different initial mutual inclinations ($i_{\rm rel,0}$). Solid blue curves correspond to $\omega_{1,0} = 0°$, and red dashed curves to $\omega_{1,0} = 90°$. From top to bottom, both sets of curves correspond to $i_{\rm rel,0} = 70°$, $i_{\rm rel,0} = 46°$, $i_{\rm rel,0} = 40°$, and $i_{\rm rel,0} = 39°$ (the dashed curve with $-1 \leq \cos\omega_1 \leq 1$ corresponds to $i_{\rm tot,0} = 39°$).

If $\omega_{1,0} = 0°$, then $\cos\omega_{1,0} = 1$, and ω_1 increases monotonically ($-1 \leq \cos\omega_1 \leq 1$), independent of $i_{\rm rel}$. In this case, ω_1 is said to circulate. Whenever $\omega_{1,0} = 0°$, $e_{1,\max} > e_{1,0}$. If $\omega_{1,0} = 90°$, then the behaviour is more complicated and depends on $i_{\rm rel}$. Let us define $\theta \equiv \cos i_{\rm rel}$. If $\theta_0^2 > \frac{3}{5}$ (see the curve corresponding to $i_{\rm tot,0} = 39°$), then ω_1 again circulates. However, since the cycle starts with $\cos\omega_1 = 0$, this implies that $e_{1,\max} = e_{1,0}$, i.e., the maximum eccentricity is the initial eccentricity, and all other eccentricities during the LK cycles are lower than $e_{1,0}$.

Box 10 – Orbital dynamics VI (continued)

As θ_0 decreases and passes the point where $\theta_0^2 = \frac{3}{5}$ (see the dashed curve corresponding to $i_{\rm tot,0} = 40°$ in Fig. 3.4), ω_1 no longer circulates but oscillates between two fixed values (approximately $-0.4 < \cos\omega_1 < 0.4$). In the latter case, ω_1 is said to librate. As shown in the figure, still $e_{1,\rm max} = e_{1,0}$ for this value of θ_0. As θ_0 decreases further, the size of the **libration island** decreases (see the curve corresponding to $i_{\rm tot,0} = 46°$), until at some critical value of θ_0, this island is reduced to a single **fixed point** in the $(\cos\omega_1, e_1)$ space at $(\cos\omega_1, e_1) = (0, e_{1,0})$. This critical value of θ_0 at the fixed point, for which there are essentially no oscillations in e_1 and ω_1, can be obtained by setting $\dot{\omega}_{1,\rm quad} = 0$ with $(\cos\omega_1, e_1) = (0, e_{1,0})$, where $\dot{\omega}_{1,\rm quad}$ is given by the quadrupole-order term in the equation of motion (test particle approximation). We can then find that the critical value is given by

$$\theta_{0,\rm crit} = \left[\frac{3}{5}\left(1 - e_{1,0}^2\right)\right]^{1/2}. \tag{3.34}$$

For $e_{1,0} = 0.5$, this expression yields a critical mutual inclination angle of $i_{\rm rel,0} \approx 47.9°$, consistent with Fig. 3.4. If θ_0 is less than the critical value, then $e_{1,\rm min} = e_{1,0}$ and $e_{1,\rm max} > e_{1,0}$, i.e., the libration island has 'flipped' from the region $e_1 \leq e_{1,0}$ to $e_1 \geq e_{1,0}$ (note that always $e_1 \geq e_{1,0}$ for $\omega_{1,0} = 0°$). This explains why in the case that $\omega_{1,0} = 90°$, $e_{1,\rm max} > e_{1,0}$ only if $\theta_0 < \left[\frac{3}{5}\left(1 - e_{1,0}^2\right)\right]^{1/2}$. If $\theta_0 > \left[\frac{3}{5}\left(1 - e_{1,0}^2\right)\right]^{1/2}$, then there are still ZLK cycles, i.e., e_1 and ω_1 still change periodically. However, the maximum eccentricity does not exceed the initial eccentricity.

The value of the maximum eccentricity can be obtained by noting from Fig. 3.4 that e_1 always reaches an extremum value when $\cos\omega_1 = 0$, or $\sin\omega_1 = \pm 1$. Substituting this into Eq. (3.29) and also applying the canonical relation Eq. (3.33), one can show that, assuming zero initial orbit eccentricity, the maximum eccentricity is given by

$$e_{\rm max} = \sqrt{1 - \frac{5}{3}\cos^2(i_{\rm rel})}. \tag{3.35}$$

As a reminder, the assumptions made to derive Eq. (3.35) were the truncation of the Hamiltonian to quadrupole order, double orbit averaging, the test particle limit, and zero initial inner orbital eccentricity. Despite the large number of assumptions, Eq. (3.35) is often a very useful first approximation for the long-term secular evolution of hierarchical triple systems.

Chapter 4

From a Lonely Road, to a Crowded Cluster

Narrator Sterope watched in sadness as her family slowly drifted out of view. A tear made of plasma streamed down the length of her face. Sterope had a lonely road ahead. But, little did she know, her isolation would not last for long.

The Galactic Field

The field of the Milky Way, or No Man's Land, effectively denotes those regions in our Galaxy outside star clusters, the Galactic disk and its inner, denser central regions. The Galactic field is where the stellar density is at its lowest in our Galaxy, and the time for light to travel from one star to the next can be many many years. To put it into context, the Sun's nearest neighbor, Proxima Centauri, is located roughly one parsec away (or about 3.26 light years).

Narrator She drifted aimlessly through the Cosmos for several tens of thousands of years. The scenery was nice for the most part, since she remained close to the disk of the Milky Way where most stars are currently being born. The nebulae from which they are birthed are often beautiful to behold, illuminated by the light of the stars they birth. A true symphony of color.

Narrator Sterope noticed that she happened to be moving in the general direction of a far off but bright blob. The object was too far away to be resolved by the naked eye. Sterope was unsure of the nature of the object, but she could infer a rough distance based on having monitored her relative motion to it.

Parallax

The term "parallax" refers to a technique used by astronomers to calculate the distances to the nearest objects to the Sun. It is the angular separation on the sky between two independent measurements of an object's angular position, performed from two distinct locations with a large displacement or baseline. The largest possible baseline is given by the motion of the Earth about the Sun, such that the two different measurements of the object's angular position are separated in time by $\sim$ 6 months, and the length of the corresponding baseline is twice the Earth's orbital distance from the Sun. The basic idea behind the concept is most easily conveyed by extending one's arm outward in front, perpendicular to the torso. Extend one finger in to the air and focus on it. Close one eye. Then close the other eye, and open the originally closed eye. Repeat this process. You will notice that your finger appears to suddenly shift positions relative to the background. The scale of this shift depends on the relative distance between your eye and your finger, as well as between your finger and the background. Hence, measuring the size of the angular displacement in degrees or radians can be used to compute the object's distance from the observer. The concept is identical to that underlying stellar parallax. PICTURE OR DIAGRAM?

Narrator The object was *very* far away, no doubt about it. Too big to be a star. Too bright to be a few stars or even a gas cloud. So what could it be?

Globular Clusters

Globular clusters are massive star clusters, much more massive than the open cluster in which Sterope and her siblings were born. They can be home to more than a million stars, all old and born a very long time ago. The ages of globular clusters rival that of the Milky Way itself, having been estimated to exceed 10 billion years in some cases. They contain the fossil record of a very early episode of star formation in our Galaxy, and understanding their origins is crucial to understanding how galaxies are born and grow. Most galaxies are home to hundreds or even thousands of globular clusters. They live in the halos of galaxies, far away from galactic disks where gas is still being converted into stars in young star-forming regions. Globular clusters are home to some of the highest stellar densities ever observed in the Universe, exceeding 10^6 stars per cubic parsec in their cores in many cases. For comparison, The density in the solar neighborhood is roughly 10^6 times smaller. This means that, if you take the Sun and its nearest neighbor Proxima Centauri and place them in their current configuration in the core of a dense globular cluster, roughly one hundred stars would lie between the pair. Crowded does not do it justice.

Narrator As the days, weeks, months, years and centuries passed, the far off blob drifted ever closer, occupying a larger and larger fraction of Sterope's view of the sky. Regardless, the blob eluded resolution for what seemed to Sterope like a very long time. But it continued to draw closer....

> Travel Time
>
> How long would it take Sterope to travel from the disk of our Galaxy out into the halo where globular clusters reside? To answer this question, consider a globular cluster residing in the halo, roughly 10,000 parsec, or 10 kpc, away from Sterope . A typical stellar velocity in the solar neighborhood is 30 km s^{-1}. Taking this as the relative velocity between Sterope and the target globular cluster, it would take roughly 10,000 pc/(30 km s^{-1}) $\sim$ 330 Myr. This is much shorter than the expected age of Sterope , as discussed in Chapter 1.

4.1 A New Home...?

Narrator One day, Sterope awoke surrounded by stars. There were thousands, perhaps millions, within her immediate view. She suddenly recalled drifting ever closer to something bright and blurry that she had never quite been able to see before falling asleep. If her rough calculations were right, she was about due for a very close encounter with whatever the bright blurry object happened to be. To her surprise, the mysterious object turned out to be a compact very dense cluster of stars. Before awakening she had inadvertently wandered right in to the middle of it.

Narrator The denizens of this cluster were typically old. In fact, billions of years in age. But the range of stellar ages was large. Clearly, multiple episodes of stellar birth had occurred here at one time or another. All together, a very large number of stars occupied a volume so compact the stellar density reached a million stars per cubic parsec at its center.

Narrator Sterope considered her surroundings carefully, trying to connect it to whatever she could remember before falling asleep. She could at least recall that she had traveled through the Galactic field to get to her current location. Obviously, traveling through the sparsely populated space of the Galactic field occupied the most time for any traveler, due its vast extent. After all, it was No Man's Land, a sea of now dead and dispersed star-forming regions long forgotten, exhausted of their gas supplies. Out in the Galactic halo and far from the disk of the Galaxy where most stars are currently being born, it would seem that only old massive and dense globular clusters exist. Apart from these, stars are extremely scarce in these barren outer regions of our Galaxy.

Narrator Suddenly, a strange voice interrupts Sterope 's train of thought...

Jane Hello there! I see you are passing through in something of a hurry. ...and there you go again. Bye!

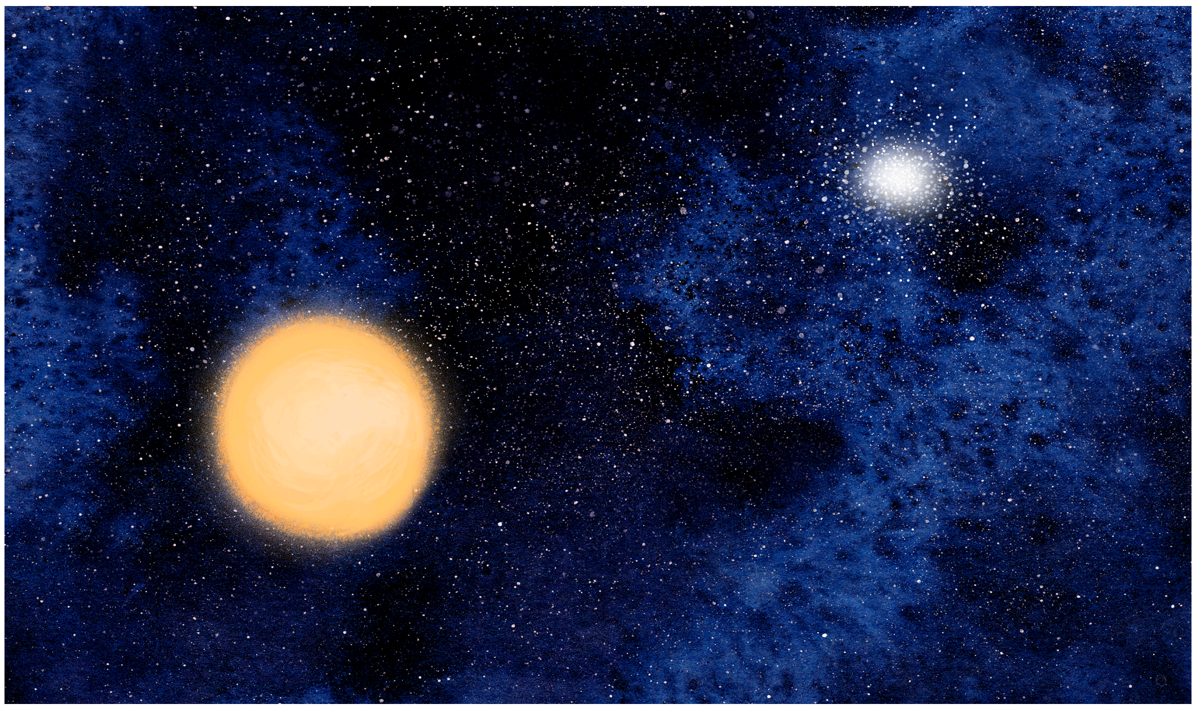

Figure 4.1: Sterope noticing a fuzzy blob far off in the distance. As she will soon discover, she is looking at an unresolved globular star cluster. Illustration by Andre Pipe Oliva.

Narrator Sterope realized she was moving a little faster than the other stars in the cluster. Suddenly, she whipped past another one.

Velocity Dispersion

What are typical relative velocities for stars in globular clusters? To answer this question, it is useful to introduce the concept of the root-mean-square velocity, which provides a useful approximate estimate of the typical velocity of a star in a cluster relative to the cluster center of mass. We can use the Virial Theorem, or $2T + U = 0$, to obtain an estimate for the typical stellar velocity. Assuming that our star cluster is well approximated as a Plummer sphere, we set $T = \frac{1}{2}m_{\mathrm{i}}N < \sqrt{v^2} >^2$ and $U = -\frac{GN^2m_{\mathrm{i}}^2}{\sqrt{r^2+a^2}}$, where r is the distance from the cluster center and a is the Plummer or core radius. This is turn gives:

$$< \sqrt{v^2} >= \Big(\frac{GNm_{\mathrm{i}}}{\sqrt{r^2+a^2}}\Big)^{1/2}. \tag{4.1}$$

For a total cluster mass of order 10^6 $\mathrm{M}_\odot$ and a characteristic cluster size of about 10 pc, the root-mean-square stellar speed tends to be of order 10 km s^{-1}. In other words, in massive dense star clusters, stars typically travel of order 10 kilometers each and every second. Less massive clusters tend to have smaller root-mean-square speeds, reaching all the way down to 0.1 km s^{-1} in open clusters. Stars in the Galactic field tend to move at several tens of km s^{-1}, so Sterope is traveling through her new host cluster a fair bit faster than are the other stars in the cluster.

Gene Hey! Watch where you're going!

Sterope So sorry!

Narrator Sterope glanced back at the scene of the near-collision, grateful for those few million kilometers separating them at her distance of closest approach, which *barely* spared her from a direct collision.

Stellar Collisions and Blue Stragglers

In the dense cores of globular clusters, the time for two single stars to undergo a direct collision is thought to be much shorter than the age of the host cluster. This implies that collisions do occur in globular clusters, and their products remain lurking somewhere within the cluster. But what happens when two ordinary main-sequence stars undergo a direct collision? One very likely possibility is that the two stars will merge and in the process become a single new star. Such collision products are often calle "blue stragglers". This nickname comes from the fact blue stragglers are more massive and hence bluer than most other cluster stars, and the fresh hydrogen mixed into their cores during the collision prolongs their main-sequence lifetimes. Hence they straggle in evolving away from the main-sequence phase of evolution, before evolving on to become a giant star.

Narrator Sterope turned back around to face her forward motion, only to discover that she was about to collide with yet another star. Frightened, she closed her eyes and hoped for the best.

Louise Whoa! Whoa! WHOA!

Narrator It's a close call. But the pair of stars manage to avoid a direct collision. Instead, Sterope flies past Louise, too fast to say hello.

Collision Rate I

A simple estimate for the rate of direct collisions between identical single stars in a star cluster, borrowed from chemistry by considering a particle traveling through a uniform gaseous medium, comes from the mean free path (MFP) approximation. Crudely, the MFP can be estimated by dimensional analysis, such that the mean free path l is $l \sim 1/n\sigma$, where n is the mean particle number density and σ is the collisional cross-section (i.e., an area corresponding to the direct overlap of two stars' radii; if a star passes within this area, a collision occurs). If the mean particle velocity is v, then the rate of direct collisions is $\Gamma \sim n\sigma v$ and the mean time between collisions is $\tau \sim 1/\Gamma$.

Mean Free Path Approximation

On average, how far do we expect a star to travel in a cluster before undergoing a collision with another star? Consider the core of a cluster of identical stars (where the core is loosely defined as the central regions of a cluster containing roughly 10% of the total cluster mass), each having radius R. Then, then geometric cross-section for collision is $\sigma = \pi(2R)^2 = 4\pi R^2$. Now, let us consider a star moving at velocity v through a fixed stellar background with stellar number density n. In a time t, the geometric cross-section σ will sweep out a volume $V = 4\pi R^2 vt$. Hence, the mean distance traveled between collision events, called the mean free path l, can be taken as the length of the path divided by the number of collisions:

$$l = \frac{vt}{4\pi R^2 vtn} = \frac{1}{4\pi R^2 n} = \frac{1}{n\sigma}. \tag{4.2}$$

The above simple derivation hopefully conveys the important concepts. A more detailed derivation should relax the assumption of a fixed background of stars, and properly factor in the dependence on their velocity distribution.

Collision Rate II

In order to evaluate the expected rate of stellar collisions in the dense core of a star cluster, let us assume that the system forms a gravitationally-bound, approximately spherical cluster, which we assume obeys a number density profile $n(r)$ and typical relative particle velocity v. The mean time between direct collisions in the cluster core (where the density is highest and the interaction rate dominates in the cluster) can then be expressed using the mean free path approximation:

$$\Gamma = \frac{N_{\rm i} v_{\rm i+j}}{l_{\rm i+j}} = N_{\rm i} n_{\rm j} \sigma_{\rm i+j} v_{\rm i+j}, \tag{4.3}$$

where $N_{\rm i}$ is the number of objects of type i, $n_{\rm j}$ is the number density of single stars of type j, $v_{\rm i+j}$ is the relative velocity at infinity between the target and incoming objects and $\sigma_{\rm i+j}$ is the corresponding cross-section for collision.

Narrator As Sterope whips by, her gravitational influence is felt by Louise, and vice versa. Sterope is more massive than the other stars in the cluster, since most are much older than her. When she undergoes a close encounter with a much less massive Louise, momentum conservation dictates that she induces a strong deflection to Louise's trajectory through the cluster. In this case, Louise is flung off and escapes from the cluster, causing Sterope to end up gravitationally bound to it in Louise's stead. Momentum conservation strikes again! Needless to say, Sterope felt terrible.

Louise AAAAAAHHHH!!! Help! I'm floating away!

Sterope I'm *so* sorry! I didn't mean to do it! It was an accident.

Louise Uh...I don't think that helps me. ...Nope, I'm still escaping to infinity. This is all your faaauuullllttt...

Narrator Sterope could barely hear Louise now, as she retreated beyond the outer boundary of their host cluster and in to the empty space beyond, No Man's Land.

Sterope WHAT?! I can't hear you?!?

Narrator Sterope's shoulders slumped. Another one bites the dust. Saddened, she mutters defeatedly:

Sterope I am more sorry than I could ever say. Well... Good luck, I guess.

4.2 Sterope's New Neighbors

Narrator Sterope suddenly realized she was completely surrounded by stars. They were so close she could make out the faces of hundreds, even thousands, of them. This was a little too close for comfort for Sterope , relative to what she had grown accustomed to over the last few million years traveling through the sparse Galactic field.

Narrator Sterope was now in the very core of just such a globular cluster, having inadvertently collided with it while escaping from the Galaxy, via the Galactic Halo. Gravity had acted to focus Sterope's trajectory, drawing her inward toward the million solar mass globular cluster. Within the cluster core, the stellar density was now about a million times higher than in the Solar neighborhood; the average distance separating Sterope from her closest neighbor had gone from about a parsec, to about 1/100's of a parsec.

Gravitational Focusing I

We are typically used to thinking of the collisional cross-section simply as the geometric surface area corresponding to the radii of two particles overlapping at closest approach. That is, if the particle radius is R, n the geometrical cross-section for collision is πR^2. The geometric cross-section can be enhanced when gravity is at work, and its effects in altering the velocities and trajectories of the interacting particles are non-negligible. This new cross-section, called the gravitationally-focused cross-section for collision, can be calculated using conservation of energy and angular momentum.

Gravitational Focusing II

If we include the contribution from gravitational focusing to the collisional cross-section for collision, we obtain:

$$\sigma_{\mathrm{i+j}} = \pi b^2 = \pi p^2 \Big[1 + \frac{2G(m_{\mathrm{i}} + m_{\mathrm{j}})}{p v_{\mathrm{i+j}}^2}\Big], \tag{4.4}$$

where b is the impact parameter for a pericenter distance $p = 2R$ (i.e., distance at closest approach corresponding to a collision for identical particles) between two bodies with masses m_{i} and m_{j} that approach each other with a relative velocity at infinity of $v_{\mathrm{i+j}}$. Equation 4.4 is derived using conservation of energy and (linear and angular) momentum during a two-body encounter. For the single-single case, we set $p \sim (R_{\mathrm{i}}+ R_{\mathrm{j}})$ where R_{i} is the radius of particle type i. Equation 4.4 then becomes:

$$\sigma_{\mathrm{i+j}} = \pi(R_{\mathrm{i}}+R_{\mathrm{j}})^2 \Big[1+\frac{2G(m_{\mathrm{i}} + m_{\mathrm{j}})}{(R_{\mathrm{i}} + R_{\mathrm{j}}) v_{\mathrm{i+j}}^2}\Big] \approx \frac{2\pi G(m_{\mathrm{i}} + m_{\mathrm{j}})(R_{\mathrm{i}} + R_{\mathrm{j}})}{v_{\mathrm{i+j}}^2}. \tag{4.5}$$

In order to evaluate whether or not gravitational-focusing should dominate the collisional cross-section, such that it is a non-negligible effect, we can compute the Safronov number. This is the ratio of the escape velocity at the surface of star to the local root-mean-square velocity, or:

$$\Theta_{\mathrm{i}} = \frac{v_{\mathrm{esc,i}}^2}{4\sigma^2} = \frac{G m_{\mathrm{i}}}{2\sigma^2 R_{\mathrm{i}}}, \tag{4.6}$$

where σ corresponds to the velocity dispersion of a Maxwellian velocity dispersion with root-mean-square velocity $< \sqrt{v^2} >$. If $\Theta_{\mathrm{i}} \gg 1$, then gravitational focusing dominates. In the opposite limit, if $\Theta_{\mathrm{i}} \ll 1$, then gravitational focusing is unimportant and the geometric cross-section dominates.

Narrator Startled and overwhelmed by all the staring faces, Sterope gasped. It sure was a lot of personalities to introduce yourself to, get to know and, let's face it, tolerate. Sterope wasn't so sure she was up for the job.
Enrico Why hello there! I am Enrico, it's nice to meet you.
Narrator Sterope turned suddenly, toward the mysterious voice.
Sterope Uh... Hi! My name is Sterope. If you don't mind me asking, where am I exactly?
Enrico You find yourself in an old star cluster. A globular cluster! Most of the million or so stars spanning the roughly twenty parsecs of our cluster, where its outer reaches can be found, and where stars slowly bleed back in to No Man's Land, were born at more or less the same time as the rest of the older stars in the Galaxy. From the same Mother Cloud. I, on the other hand, am older and come from a different generation of stars.
Sterope All the stars are packed so close together here...

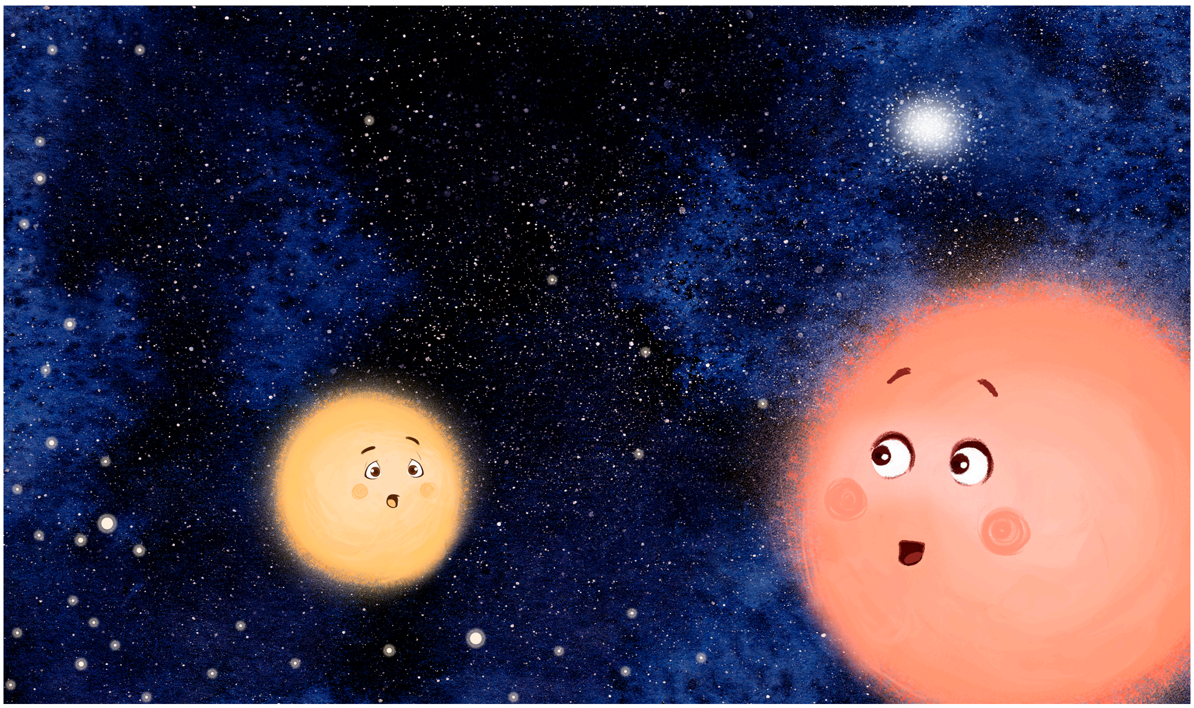

Figure 4.2: Sterope as she meets Enrico for the first time. Illustration by Andre Pipe Oliva.

Enrico Yep, it's crammed in here all right. You get used to it pretty quickly though. Mostly you don't notice it. But, every now and then, two stars do smash in to each other. They collide head-on. BOOM! More often though, two stars undergo strong deflections, during which one star passes by another star so closely that their stellar surfaces are almost touching. Of course, if their surfaces don't touch, then instead of colliding they tend to bestow a strong deflection, one to the other. This causes a deflection in each star's trajectory, and can either slow or speed them up.

Sterope Well, which is it? Do they speed up or slow down?

Energy Equipartition

An important thermodynamics-based analogy can be drawn between star clusters and a gas in a container. The mean velocity of a star can be regarded as a proxy for its temperature: larger mean motions imply hotter temperatures. Since all particles in the system are free to interact and exchange energy (via mostly weak deflections induced by gravity in star clusters, and direct collisions between atoms or molecules in a gas), the system tends to evolve toward a "thermalized" state in which all particles have comparable kinetic energies or temperatures. This state is typically called "energy equipartition". More massive stars exert a stronger gravitational force at a given distance, and so will typically accelerate a less massive star during a close approach, imparting unto it additional kinetic energy. Conversely, the more massive star loses kinetic energy, and slows down. Given enough time for many such close energy-exchanging interactions to have occurred, a star cluster will tend toward a state of energy equipartition, at least within certain subsets of the cluster (e.g., the core).

Enrico More massive stars tend to accelerate lower mass stars more easily, for a net gain of linear momentum. Conversely, lower mass stars tend to *be* accelerated by more massive stars such that, by conservation of linear momentum, the more massive perturbers tend to be decelerated. So, in the end, more massive stars tend to be slowed down, whereas lower mass stars tend to be sped up, on average. Not much you can do about any of it, really, except enjoy the show...which by the way, I highly recommend.

Narrator Enrico winks at Sterope, who laughs.

Sterope How old are you...if you don't mind me asking?

Narrator Enrico thinks about the question for a moment...

Enrico How old is the Universe now? Wait...if I remember right, stars older than me used to yammer on about the number 13.7 Gyr for the age of the Universe, give or take a few hundred million years... I think. Does that sound about right to you?

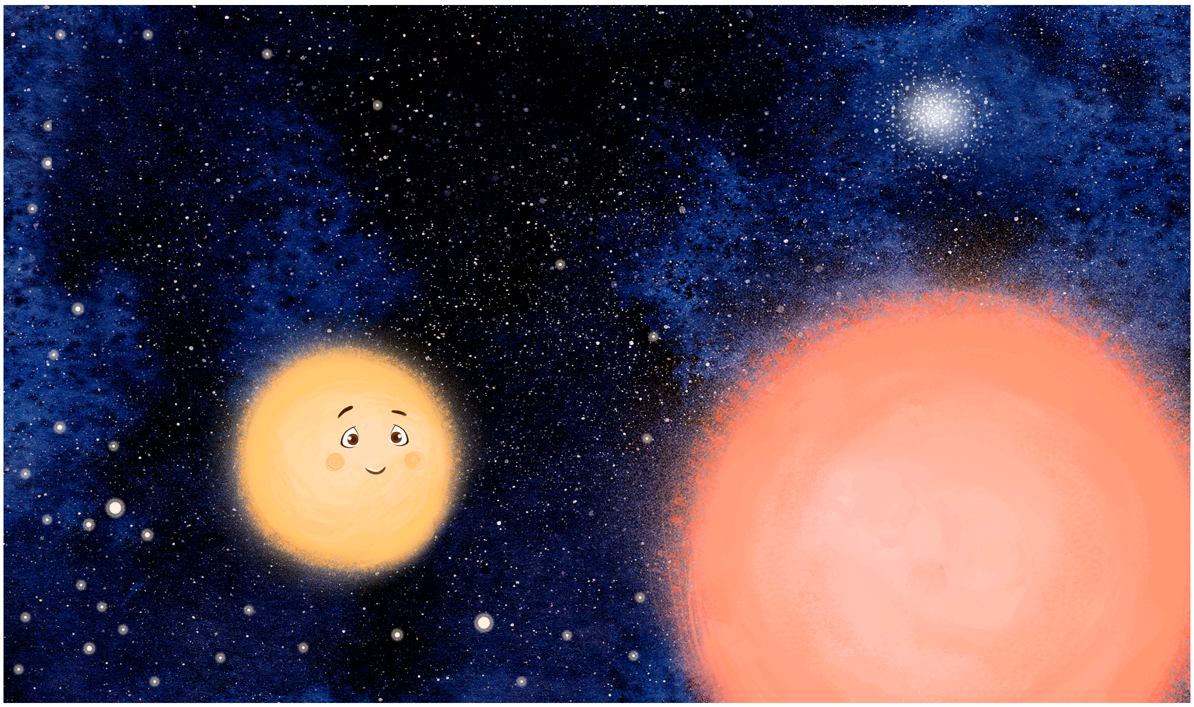

Figure 4.3: Enrico informs Sterope of what she must do in order to escape from the globular cluster she presently finds herself in. Illustration by Andre Pipe Oliva.

The Age of the Universe

All the available empirical data point to and are roughly consistent with an age of ~ 13.7 Gyr for the Universe. For example, no star older than this critical upper limit has ever been observed.

Sterope I guess so, but I don't really know, to be honest. I was not born long ago compared to you; it hasn't even been a billion years since my birth.

Enrico You are a young one! Good luck on all of life's many adventures, child.

Sterope Uh... Thanks?

Narrator Suddenly and without warning, plasma shot out of Enrico's surface, below his equator. A coronal mass ejection. An old star, convective cells toiled at Enrico's surface, stirring upward the plasma from deep within his belly. The plasma hit the outer layers of Enrico's atmosphere with enough force to escape, drawing on the outward force supplied by nearby magnetic field lines. Another explosion of plasma emerged from Enrico's lower hemisphere, not unlike a volcano erupting.

Photosphere

The photosphere lies in the outer layers of a star's atmosphere. It is the outer shell of a star from which light is radiated and emerges. At larger distances from the star's center of mass, the photosphere gives way to the stellar corona, which is much hotter than the photosphere. The physics underlying the reasons why the corona is so hot remain poorly understood and active area of research.

Coronal Mass Ejections

A coronal mass ejection (CME) refers to the ejection of plasma from the surface of a star. Large amounts of matter (made up of protons and electrons, mostly) and electromagnetic radiation are launched out into space, and can either remain close to the stellar surface or be flung out through the corona and into the Solar System and beyond. CMEs are known to be associated with very energetic shifts and changes in the star's outer magnetic field and a reorganization of its magnetic field lines.

Enrico My apologies! This old star is at the mercy of surface convection!

Convection

If a steep enough temperature gradient (i.e., the temperature increases rapidly with increasing distance from the star's center of mass) exists in the interior of a star, an instability can arise in which heated plasma ascends and cooled plasma descends. This can also occur if the gas has a high heat capacity, implying that it cools slowly as it expands. As a bubble of gas ascends, it finds itself in a region of lower pressure, allowing it to expand and cool. The bubble must remain cooler and less dense than its surroundings in order to remain buoyant and continue to ascend. Otherwise, if the bubble cools below the temperature of the new ambient plasma, its density will rise above that of the surrounding plasma and it will lose its buoyancy, sinking back down.

Sterope Oh, that's quite alright. It could happen to anybody.

Enrico Back to the question at hand. Okay, so if the Universe is a little less than 14 billion years old, that must make me...twelve and a half billion years old? Yep, that's the number! Most of these other stars are more like nine billion years old...give or take a billion years or so. Young pups, and they mostly keep to themselves. I'm sorry if they seem rude but, in fairness, over 90% of this cluster is comprised of their generation, so I guess it's no surprise they mostly keep to themselves. Strength, and confidence, in numbers.

Sterope Actually, I'm a little relieved. This is a *lot* of stars in a very crammed space. I'm really not used to it, and was worrying how I would even begin getting to know all of these faces.

Enrico Oh, don't worry about that. I'm happy to chat any time you like, and to defer any time you prefer not to. But these others will mostly inter-mingle with their own kind. You'll be lucky if even one of them strikes up a serious conversation with you. I mean, they are polite, I give them that. They just prefer not to engage with outsiders directly, which are very rare around here. You're the first one in well over a billion years!

Sterope Well, I'm perfectly fine with them keeping to themselves for the time being. I think I will do the same... Hmmm... How do you suppose I might find my way out of here?

Enrico You want out, huh? I suppose I cannot blame you. It is crowded in here. But I warn you, accomplishing that feat is anything but easy. Can I ask: Where are you headed in such a hurry?

Sterope Well, nowhere, really. I guess I was just hoping to explore the Galaxy, and maybe even stumble across my missing siblings. We were all born from the same Mother Cloud. Ever since our birth cluster dispersed, I've been a little worried about them. Now though, I'm more than a little worried that it's a really big Galaxy out there, and if they have been traveling as I have, then finding them within my lifespan could be impossible. Still, I have to try!

Enrico Well, it sounds to me like you want to make your way back to the Galactic Bulge, which surrounds the central nuclear cluster and a non-negligible fraction of the Galactic Disk. Unless I miss my guess, if you started out in the

Galactic Disk somewhere, which is most likely the case for a young pup such as yourself, then your best bet for finding your siblings is in and around that central region of our Galaxy. I know that doesn't narrow it down as much as you'd probably like, but at least it's a start.

Sterope Thank you! I really do appreciate it. Yes, it is a *definite* start. Wait, just one more question: How the heck do I get out of this cluster?

Enrico Oh, right. *That* question. Well, you're not going to like the answer.

Sterope I don't care, try me anyway.

Enrico To do that, you'll need to find not one, but *two* black holes, lurking around somewhere here in the core. I know they are here...somewhere.

Sterope Wait, what's a black hole? And why do I need *two* of them?

Enrico Well, to answer the second question, you need *mass* and *lots of it* confined to a small volume if you want to be able to achieve the acceleration you will need to escape from this cluster. This is a basic requirement in order for gravity to impart the required total force needed to achieve such a high velocity. That is, a sufficiently large gravitational acceleration must be imparted over a given timescale in order to accelerate an object to a final velocity that exceeds the local escape velocity of the cluster. Looking at you, I'd guess you're, what, 2 maybe 3 solar masses?

Escape Velocity

The local escape velocity is defined as the minimum velocity required at a given distance from the cluster center of mass for the total energy of the escaper to exceed zero or, equivalently, to become gravitationally unbound. A simple way to calculate the local escape velocity, using conservation of energy, is to equate the kinetic energy of an object to its local gravitational binding energy, keeping the object velocity as a free parameter. If you solve this equation for the object velocity, you will obtain the minimum speed needed at that position in the host cluster gravitational potential for the object to become gravitationally unbound and escape to spatial infinity (asymptoting to zero velocity at spatial infinity in the context of this simple idealized equality). The escape velocity from the surface of a sphere of radius R and total mass M is:

$$v_{\rm esc} = \left(\frac{GM}{R}\right)^{1/2}. \tag{4.7}$$

Sterope Whoa whoa WHOA! My appearance is pretty much the last thing I wanted to discuss with you. Besides, I really don't see how my weight is relevant to this discussion...

Enrico Because I have some idea how massive the most massive black holes in this cluster might be, and you need *two* that are each more massive than you. The more massive they are relative to you, the easier it will be for you to achieve the required acceleration.

Impulse

In order for a star to be accelerated to above the escape velocity of a star cluster, it must somehow be imparted with enough additional momentum and kinetic energy to overcome the gravitational potential at the star's location in the cluster. If this is achieved, then the star has positive total energy in the center of mass reference frame of the cluster, and becomes unbound. The impulse is a useful quantity to understand how much momentum can be imparted for a given close approach between two stars. The impulse, which has units of momentum, is defined as:

$$I = F_{\mathrm{g}} \Delta t, \tag{4.8}$$

where F_{g} is the imparted gravitational force and Δt is the time over which the force is applied. Hence, for a given distance of closest approach, more massive perturbers will impart a larger acceleration, and slower relative velocities will increase the time Δt over which the force is applied, resulting in a larger increase in momentum. Thus, according to the imparted impulse, Sterope should seek out much more massive stars to acquire the needed acceleration to escape from the cluster, and she should aim to get as close to the stars as possible (since $F_{\mathrm{g}} \propto r^{-2}$, where r is the distance separating the stars' center of mass) with a low relative velocity.

Sterope Gotcha. Alright, fine. Last time I checked I was...

Narrator Sterope's voice drops to a whisper...

Sterope ...2.9 solar masses or so. Buuuuuut I've been blowing off winds for quite some time during my travels, so I think it's probably a bit less than that now.

Stellar Winds

Enrico Nothing to be ashamed of as far as I am concerned.

Sterope Oh please, what do you weigh? Half a solar mass?

Enrico Nah, more like a quarter of a solar mass, last time I checked.

Main-Sequence Lifetimes

Stars in the main-sequence phase of lives share a relationship between their total mass and the duration of their lifespan spent on the main-sequence. This relation is such that the most massive MS stars evolve the fastest, and have the shortest lifespans. In fact, the evolution is so slow for the least massive MS stars that those with masses of only a few tenths of a solar mass should have total ages that exceed the current age of the Universe. That is, some of these stars have been burning hydrogen into helium in their cores for over 13 billion years, and they are not yet done!

Sterope I am so jealous right now. Alright, so what are these "black holes" you speak about, and how do we find them?

Enrico Well, I guess the most important thing to know is that they are as dark as they come. They don't shine. At all. So finding them is obviously a pretty serious challenge. As to what they are, technically, they are the corpses of stars once much more massive than yourself, now wandering unseen through the Cosmos.

Sterope Dead stars? Really? So, basically, I am looking for ghosts haunting this cluster, which I rather conveniently cannot see? And what is it you expect me to do with these dark ghosts, once I find them?

Enrico You'll have to capture 'em. Well, actually, first you'll have to convince two of them to partner up and form a bound binary system. Then, you're going to need to convince them to let you take a run at them. You'll have to work out the details on your own, which I warn you are not as straight-forward as you might expect, especially when chaos rears its ugly head and enters the picture.

Chaos and the Three-Body Problem

The chaotic three-body problem has evaded a solution for centuries. The reason is simple: small perturbations to the initial conditions compound over time to change the very outcome of the interaction (e.g., which of the three particles is ejected). A well known term for this is the "butterfly effect".

Enrico But, in principle, two black holes bound in a compact binary should have enough binding energy to give you the acceleration you need to escape the cluster.

The Energetics of Escape

Recall that the binding energy of a binary star defines its internal reservoir of *negative* energy. The more negative the binary binding energy, the closer are the companions (for a given pair of companion masses). Liouville's Theorem tells us that the total volume in phase space is conserved during the time evolution of self-gravitating N-body systems. In practice, what this means is that the more negative the total energy, the more likely it is particles will be accelerated to high velocities. Thus, it is easier for binaries with more binding energy to accelerate interloping single stars to above the local escape velocity of the host star cluster, even deep in the cluster core where the escape velocity is at its highest.

Sterope Wow, that is a *super* complicated plan. Sigh. Well, I guess I'll have to find a way to make it work, which brings me to my last question: How, in the name of Hell, do I find these black holes?

Enrico There is only one sure fire way, child. You must search for a star orbiting within what appears to be a companion-less binary star system. If the black hole forms a binary star system with another luminous star, *any* luminous star, then it becomes possible to observe a star in orbit about something that cannot be seen. This immediately implies the unseen presence of a dark compact object (i.e., a dead star) binary companion. You can then use the orbital speed of the luminous companion as a function of distance from the unseen black hole to calculate the approximate mass of the black hole. Just measure the time it takes the luminous companion to orbit the black hole once, and measure the distance from the center of mass that the companion appears to be orbiting. The orbit should trace out an ellipse. Calculate the typical orbital velocity by dividing the circumference of this circle by the orbital period.

Sterope Woooooooooow. This sounds like a loooooot of work. I don't know about this... I mean, what are black holes even like? *If* I can find not one, but *two* of them, do you think they will agree to help me escape from this cluster?

Enrico Hmmmm... A fair and good question. Few stars have gotten to really know a black hole and survived to tell the tale, to be honest with you... But they do have one weakness: they have a constant hunger to grow. Perhaps if you have food to offer in exchange for their services, they would be more inclined to help you out.

Sterope So...bribery? You are suggesting that I bribe them?

Narrator Enrico shrugs rather non-chalantly.

Enrico It often works, I have to say.

Sterope With what?

Enrico Uh, well, mass. Any mass will do.

Sterope Okay...Again though, with what?

Enrico Other stars?

Sterope WHAT?! You want me to deliver other stars to these black holes so that they can eat them? And the stars will die?

Enrico No! No! I was just saying such a scenario *could* work, at least in principle. But, yes, those stars would surely die. There are other options though! Murder is not the only one. Any mass will do. The more of it you have, the better your position to bargain. You could even trade some of your own mass in exchange for a boost!
Sterope Okay, okay. So the mass could just as easily be the random crud out in No Man's Land?
Enrico I suppose so, yes. Provided somebody could collect it all in to one place.
Sterope So I could even take it from my own belly, or that of some other star? Like, leave them alive, but take a little bit of their mass?
Enrico I think that will work. Good idea!
Sterope Alright. Now we're getting somewhere. ...Wait, how do you suppose I collect mass?
Enrico Off the top of my head, by finding those stars on the verge of evolving off the main-sequence and somehow getting your self close enough to them (i.e., in a binary system) that, when they evolve off the main-sequence and expand to become red giants, they transfer the mass in their expanded envelope over to you.

Stellar Evolution Beyond the Main-Sequence

During the main-sequence (MS), stars are converting hydrogen into helium in their cores. This is their primary source of energy, and makes them shine. The MS is the first phase in the lifetime of a star, right after the protostellar phase. It also tends to be the longest phase in the lifetime of a star, often lasting many hundreds or even thousands of times longer than later phases (e.g., the red giant branch phase). Red giant branch stars (RGB) are more evolved than MS stars, converting hydrogen into helium outside the core in a shell. During the red giant branch phase, stars can expand by up to a factor of several hundred times their former size on the MS. The outer envelope is only tenuously bound, and can easily be stripped by a binary companion as the star expands.

Sterope Uh...Okay, so basically I am going to have to somehow figure out a way to swap myself in to *and* out of at least one normal stellar binary system...in addition to two black hole binaries? Then, I can use the mass of the black holes to accelerate me to above the escape speed from the cluster? Just trying to wrap my head around this. That seems like a lot of work. Hmmmmm... Where do I even begin?
Enrico Well, child, by my calculations, you need to participate in at least ten direct dynamical interactions with other singles or binaries in the cluster. Two things can help with that: increasing your mass, and reducing your velocity relative to the cluster average. Both of these increase the rate of collisions with other stars or binaries in the cluster. The reason is related to something called gravitational focusing. During the encounter, gravity helps out a lot; it serves to focus inward the relative trajectories of two colliding particles, making it so

that they are more likely to collide. Hence, slower incoming singles are more likely to collide directly with a binary due to this gravitational focusing, whereas without gravity it would not occur.
Sterope Okay, got it...I think. So...what do I do now?
Enrico Not much you can do, but wait. Don't worry though, gravity will do all the work for you. It will carry you throughout this cluster, and deliver you close to other stars and binaries. You are most likely to run in to the most massive objects in the cluster first; more massive objects exert the strongest gravitational force and, without even intending to, draw you in from further afar. But massive objects are rare, and low-mass stars make up the vast majority of this old star cluster. Usually the end result of these interactions is only a close approach, but often the encounter will be direct and you will become at least temporarily gravitationally bound to these other objects in the cluster...as you will find out for yourself soon enough!
Narrator Sterope noticed she had been drifting farther away from Enrico . She realized the process would continue, and they would soon part ways.
Sterope I notice I am drifting away from you. I can barely hear you anymore, in fact. Thank you so much, Enrico, for all your help. I'm off to find those two massive black holes!
Enrico Good luck, young one. It will take some time, but I have no doubt you will realize your goal of escaping eventually.
Narrator Sterope began her long journey through the cluster. A sea of faces came in to and faded out of view. One thing quickly became apparent to Sterope about her temporary neighbors: they were highly skilled at avoiding eye contact. So she continued on, mostly in silence, in search of the two massive black holes whose help she sought.

Figure 4.4: Armed with a satisfactory escape plan, Sterope drifts away from Enrico , off in search of a couple black holes. Illustration by Andre Pipe Oliva.

Chapter 5

A Bond Forged of Fire

Narrator After the dissolution of their natal star cluster, Maia and Merope wandered through the Galactic Disk. Millenia passed without much in the way of interruption; Maia had only Merope to keep her company, and vice versa. Now and again, the sisters encountered a passing star off in the distance, too far away to exchange a conversation. Regardless, they yelled relentlessly, eager to meet a new face. After a few million years of this, the pair had grown bored, but inseparable.

5.1 An Old Star...

Narrator Merope 's age was now showing. The most massive of her siblings, she was destined to burn through her nuclear fuel the fastest.[1] She was now reaching the end of her short life. As such, she had begun to expand, slowly entering the red giant phase of stellar evolution.[2]
Maia Uh... I don't mean to alarm you, Merope, but I've noticed you've been expanding lately. I mean, just a little bit. No big deal, really. But, uh, where do you suppose this is heading?
Merope Uh... Well, I hadn't noticed, to be honest. But now that you mention it, I do feel a little bloated.
Maia Well, you look *great*, that's for sure.
Merope Thank you, sister. But I fear this is going to get worse before it gets better. Much much worse, in fact. I seem to be expanding further...
Maia Oh dear. Well, don't panic. We'll figure this out. How do you feel?
Merope I feel... Cold. Do I look cold to you?
Narrator Merope was shivering noticeably.

[1]To remind the reader, the most massive stars have the shortest lifetimes since, in spite of having more nuclear fuel, they are able to burn through their entire energy reservoirs faster.

[2]Recall that during the red giant branch phase of stellar evolution, stars can expand up to several hundred times their size on the main-sequence. When the Sun eventually reaches the red giant branch phase, several billion years from now, it is likely to expand out beyond the orbit of Earth, engulfing the planet.

Maia Eeesh... Yeah, kind of. You've been getting steadily redder in color. It's not drastic, but it is noticeable.[3]
Merope Um... Okay, maybe if I hold my breath *that* will stop the expansion. Worth a try, I suppose. Here I go...
Narrator Merope drew in a deep breath, and held it. The minutes passed. Her face blue, Merope eventually exhaled in defeat.
Merope Oh no. That was uncomfortable. And I am still expanding.
Maia Don't panic. We'll sort this out.
Merope You already said that!
Maia Well, frankly, I can't emphasize it enough. Just... Don't panic.
Merope You aren't making me feel any better over here.
Maia I'm sorry. I'm doing my best... I know I said not to panic, but I'm losing it over here myself...
Narrator Maia was noticeably agitated, pacing back and forth.[4]
Maia Yep, I'm dropping the ball. Big time.
Merope Jeez, Maia, get ahold of yourself. I'm the one who seems to be expanding to certain doom, and yet you are the one hyperventilating.
Narrator Maia was rapidly fluctuating in brightness, caused by the hyperventilation.
Narrator Flabbergasted, Maia struggled to regain her composure.
Maia It's fine. I'm fine. How are you doing?
Merope Well, quite honestly, I continue to expand. How do I stop?
Maia Have you tried thinking really hard about *contracting*?
Merope Really? I'm about to die, and your big idea is to think good thoughts?
Maia Well, it can't hurt...
Merope You aren't helping!
Narrator By this time, Merope had already expanded by over an order of magnitude, and her radius was still growing rapidly.
Maia I'll go get help!
Merope And how are you going to do that? We're gravitationally bound.
Maia Oh right. Gravity. Okay, new plan: I'll think good thoughts?
Narrator Merope sighs in defeat.
Merope I'm going to die for sure.
Maia Don't say that, Merope You're not going to die.
Narrator Eventually, Merope stopped expanding and stabilized.

[3] As stars expand on the giant branch, their effective or surface temperatures decreases. The reason for this stems from a formal relationship between the radius, luminosity and surface temperature of a star, called the Stefan-Boltzmann law. The general trend of larger radii corresponding to cooler surface temperatures, at a given mass and evolutionary stage, is consistent with the basic idea that larger radii correspond to surfaces with a larger displacement from the core, which is where energy is generated in stars. Hence, the total energy diffusing through the stellar surface per unit time (i.e., the total luminosity) is smaller, and so is the effective temperature. CHECK

[4] This pacing occurs in the radial direction, and can ultimately be characterized by the orbital eccentricity: more eccentric orbits have larger (fractional) radial amplitudes, at a given orbital separation. In other words, more eccentric binaries are more agitated. PICK OF ORBITAL DYNAMICS AND MORE THOROUGH DEFINITION OF ORBITAL ELEMENTS AND ECCENTRICITY?

Maia Whoa! I think you've stopped expanding!
Merope Oh thank the Heavens! I do indeed seem to have stopped. Phew! What a relief!
Maia Well, look at you! You've definitely stopped expanding. You're much much redder and, well, larger than you used to be...but none of that is changing anymore. I think you're going to be okay!
Narrator Merope breathed a long sigh of relief. The life of a massive star was anything but easy, but Merope remained optimistic that the aging process would grow easier as time went on.

5.2 Blown Out of Proportion

Narrator For some time, it seemed Maia had proved right. But eventually Merope began expanding again. Over the next few million years, she eventually grew accustomed to it. Just another day in the life of a massive star.
Narrator But then, one day, suddenly and without warning, Merope exploded. A supernova! Plasma and gore flew passed Maia , spraying her face with the guts of Merope and knocking her backward. The blast was asymmetric, escaping mostly from a preferred side which happened to oppose Merope 's orbital motion almost perfectly. Due to angular momentum conservation, this meant a significant and almost instantaneous reduction in Merope 's orbital velocity and a loss of orbital angular momentum.[5] Merope drifted inward, closer to Maia and on a much more eccentric orbit than before.[6]
Maia Merope! Are you alright?!
Merope Uh... I think so. Give me a minute here. I feel much lighter.
Maia Well you did just eject your outer envelope and most of your mass in a rather dramatic supernova explosion. So it makes sense that you are much less massive now. You look so small and compact!
Merope Thank you! I feel pretty svelte over here. Looking good, feeling good. Who would have thought exploding could be such a positive experience?
Narrator A worried expression appeared suddenly on Merope 's face.
Maia What's wrong?
Merope Does your belly always do that?
Narrator Maia looked down to discover her midsection had grown distended. A bulge pointed directly toward Merope .[7] As Merope re-approached the

[5] EXPLAIN THIS USING CONSERVATION OF MOMENTUM, AS WELL AS CONSERVATION OF BOTH ENERGY AND ANGULAR MOMENTUM.

[6] EXPLAIN THE CONCEPT OF ORBITAL PARAMETERS, AND PROVIDE THE DEFINITION OF ECCENTRICITY BY DISCUSSING ELLIPSES.

[7] Gravity exerts a differential force across the extent of finite-sized objects. In practice, for example, consider the Earth-Moon system. The gravitational force exerted by the Moon on the Earth is strongest on the side facing the Moon, and weakest on the opposing side, since the force of gravity falls off as the inverse of distance squared. This causes the side facing the Moon to become extended, or to "bulge out" toward the Moon. Called a "tidal bulge", this is the mechanism responsible for tides in the Earth's oceans. PROVIDE AN ILLUSTRATION FOR CLARITY.

point along the orbit corresponding to closest approach with Merope,[8] the bulge grew and a thin stream of material began to flow from Maia 's surface toward Merope .[9]

Maia Holy cow! Now I'm losing mass! Okay, don't panic.
Merope I swear, if you say that one more time I'm going to scream.

[8] Orbits can be either circular or elliptic. In the former case, both objects maintain a constant distance from their mutual center of mass. In the latter, or "eccentric", case, the distance of each object changes with respect to their mutual center of mass. The point along the orbit corresponding to the distance of closest approach between the objects is called "pericenter" or "periastron", whereas the opposing side (corresponding to the point of furthest approach) is called "apocenter" or "apoastron".

[9] Consider a line connecting the centers of mass of two objects in orbit about each other. Somewhere along this line, the force of gravity exerted by each object is exactly balanced (the location of this point depends only on the masses of the two objects). If the radius of one star expands beyond this point, then the matter on the outer surface of the star is more strongly attracted gravitationally by its binary companion. In such a scenario, the mass flows from the surface of the large star onto the surface of the other, flowing through the point where their mutual gravitational force is precisely balanced. Mass transfer ensues. This critical surface is called the "Roche lobe".

Chapter 6

A Tale of Two Black Holes...

Narrator Sterope was trapped in an ancient globular cluster and in search of a pair of black holes. The idea was to first encounter a binary star system, undergo a chaotic three-body interaction, and ultimately become exchanged into it. From there, she need to find a black hole, undergo another a chaotic three-body interaction, and eject her former binary companion. Finally, she needed to encounter a second black hole and undergo yet another chaotic three-body interaction, this time with two black holes. In the end, she hopes to be the one ejected, leaving the two black holes behind in a binary and escaping from the cluster altogether.[1]

Narrator She wandered through her new and hopefully temporary home for many millions of years. During this time, she endured many encounters with the other denizens of the cluster. But it took several such interactions before she eventually met even one black hole. Finally, she stumbled upon a suitable target. Sterope had managed to exchange herself in to a normal stellar binary by the time she encountered the first black hole. As the black hole drew close, its gravity pulled Sterope and her companion toward it. A chaotic three-body interaction ensued. A delicate dance, the three bodies bobbed and weaved by each other, continually accelerated by each other's mutual gravitational attraction. They took turns being flung out far from the system center of mass, on loosely bound excursions. All this, but without imparting enough kinetic energy for any one of them to escape, each excursion carefully choreographed such that they remained bound, destined to return to rejoin the melee. The dance continued until, eventually, Sterope's original companion was ejected, flung off at sufficiently high velocity to escape to what amounts to spatial infinity in Sterope's reference frame.[2]

[1] DESCRIBE THIS PROCESS IN DETAIL, AND DICUSS ITS RELEVANCE FOR CHAOS. PERHAPS SHOW THE HUT & BAHCALL FIGURE OF THIS?

[2] DISCUSS THE CONCEPT OF SPATIAL INFINITY, AND REFERENCE FRAMES.

6.1 A New Companion...

Sterope Uh... Hello there... Wherever you are. I can't actually see you, you being completely black and all. But I know you're there! I must be orbiting *something*, after all.
Ares You are not alone. It is I, Ares. Star destroyer. Devourer of worlds.
Sterope That sounds scary. Regardless, I'm pleased to meet you. I've been searching for a black hole such as yourself for quite some time.
Narrator Sterope extends an eager hand, hoping for a handshake but suddenly realizing Ares could easily be out of reach.[3]
Ares You aren't scared?
Sterope Scared of what?
Ares Me! I just said I destroy stars!
Sterope Oh. Right. Well I wish you the best of luck with that, but so far you seem nice enough.
Narrator Ares lets out a long sigh.
Ares All this mass, and nobody around here seems to care. What gives?
Sterope I don't know, but *I* sure care. That's why I came looking for you. I need an object of significant mass to help me escape from this cluster. Well, two massive objects, to be precise.
Ares Ah, you are in need of a gravitational assist! A slingshot from a massive body. Finally, a use for me![4]
Sterope Yes! You could be very useful to me.
Ares Wonderful. I feel useful already.
Sterope Okay, so, how do we do this?
Ares Well, I guess I could take a run toward you, and maybe that would work?
Sterope I don't think so. We are gravitationally bound. I do believe we need at least some form of outside interference to unbind us. After all, the excess energy must come from somewhere.[5]
Ares Fair enough. We need another black hole!
Sterope Exactly! I am confident that, between the two of us, we can find one.
Ares I might know where. My brother is lurking around here somewhere.
Sterope You're brother?!
Ares Zeus ! Zeus ! Where are you, brother?
Narrator A long silence passed. Sterope , excited at the prospect of a second black hole, listened closely for a response. Finally, a response came.
Zeus I am here.
Ares Wonderful. Good to hear your voice again, brother.
Zeus And you, brother.
Ares I have met a friend, Sterope , who requires our assistance.
Zeus Hello, Sterope . Nice to meet you.
Sterope And you, Zeus . Truly, this is intimidating. You are both *so* massive.

[3] AGAIN, DO WE WANT THE STARS TO HAVE HANDS?
[4] DEFINE THE TERM GRAVITATIONAL ASSIST.
[5] REMIND THE READER ABOUT ENERGY CONSERVATION AND ORBITAL BINDING ENERGY.

Zeus Do not be intimidated, child. Our mass can at best accelerate you, nothing more.
Sterope You are both so kind. I don't understand why everyone I met before you was so afraid of black holes.
Zeus A common misconception.
Ares Sterope is in need of a gravity assist, to escape from this cluster.
Zeus I see. Well, you seem kind, noble and out of place here amongst these old stars. I am happy to offer my mass in the name of this cause. But how do you propose we get the job done?
Ares As god of war, I have facilitated many a squirmish in this cluster environment.[6] As I see it, there is no better way to impart a velocity kick of sufficient magnitude to eject Sterope from the cluster. Us three must engage in a chaotic dance, choreographed by none other than gravity.
Zeus Very well. I am coming toward you. Prepare yourselves for chaos!
Narrator Zeus drifted ever closer to Sterope and Ares , slowly at first but rapidly gaining speed. Before they knew it, Zeus was upon them, tugging them in every which direction, and vice versa. Chaos ensued. The two black holes and Merope completed a series of wild loops and orbits.[7] More than a few times, Merope passed close enough to one of the black holes to really feel their gravitational might. The differential force of gravity caused matter to be pulled from her surface after making her closest approach.[8] Not much matter, but enough to freak out both her and her interloping black hole; each time this occurred, they exchanged a startled glance. Finally, Merope was ejected at high velocity, leaving Ares and Zeus bound in a new binary.
Ares I think that should do it.
Zeus Agreed! Surely she must be flying away at above the escape velocity.
Ares Fare thee well, Sterope ! May only glorious adventures befall you!
Sterope Thanks so much, to the both you! Free at last!
Narrator Sure enough, Sterope escaped from the gravitational bounds of her temporary host cluster with kinetic energy to spare. Her trajectory now pointed inward, back toward the Galactic Disk. Optimistic she would find her lost siblings, Sterope smiled at the thought of seeing them again..

[6] USE NσV ARGUMENTS TO EXPLAIN THE RATE OF SUCH INTERACTIONS.

[7] A SKETCH OR DIAGRAM DEPICTING THIS WOULD BE COOL AND USEFUL. AARON?

[8] Consider a normal stellar binary star system, composed of two stars. Recall that, due to tidal forces, the side of a finite-sized object closest to its binary companion will be more strongly attracted by gravity than the opposing side, raising a "tidal bulge". If the radius of one star exceeds the "Roche limit", then mass is transferred from the surface of the star onto the surface of its binary companion. The physics here is ultimately the same, but the dance is chaotic; this scenario occurs sporadically and pairwise over the course of the chaotic three-body interaction.

Chapter 7

The Beast Awakens...

Narrator Electra continued to wander closer to the very center of the Milky Way's nuclear star cluster. The stars in her immediate vicinity seemed to fly by faster and faster as she drew ever closer to her destination. Such high relative velocities could *only* be imparted via the gravitational acceleration supplied by some very nearby compact massive object, a description that fits Chiron to a "T".
Narrator Random perturbations from passing stars continuously deflected her orbit, causing her to wander throughout the nuclear star cluster somewhat aimlessly for what seemed like a very longtime. All the while, she searched for some sign of Chiron. Until one day, she saw the tell-tale signs of a thin, fleeting accretion disk. A soft halo of light surrounded a black sphere. Enrico had proved right: the super-massive black hole was indeed uniquely identified by the thin accretion disk Enrico had described to her, formed of the coalesced winds of nearby massive stars.[1]

7.1 The Behemoth of Black Holes

Electra Hello there! If I squint, I see what looks like an accretion disk hiding over there. Is that you Chiron?

par **Narrator** A long silence followed Electra 's seemingly innocent question. Eventually, the silence was broken. A deep booming voice responded, shaking Electra to her core.

[1]When matters falls deep into the local gravitational potential of a massive compact object, it often gathers into an "accretion disk". This is because the infalling matter carries finite angular momentum, which must be conserved. Hence, close to the black hole, the gas particles will sort themselves into an approximately planar configuration with the orbital plane being aligned with the total angular momentum vector. This planar configuration of gas particles is called an "accretion disk", since viscosity within the disk can cause angular momentum to be redistributed and dissipated. Hence, the inner material of the disk can lose enough angular momentum this way to eventually be accreted directly onto the compact object. The gas particles in the disk collide with each other, emitting light in the process and making accretion disks visible to observers on Earth.

Chiron Yes. It is I, Chiron. I am here, and you are there. But who is it that I am speaking to?
Electra I am Electra . A wandering star in search of my lost sisters.
Chiron I see. You have spotted what remains of the corpse of my last meal. The scene before you might look a little gruesome but, I assure you, it is but one aspect of all that must be. I do my part, here in this most extreme of environments in our Galaxy. The stellar density can only grow so high, after all. Otherwise, we'd all be banging into each other on a nearly daily basis.[2]
Electra Of course you do! I did not mean to accuse you of anything. I cannot thank you enough for all that you do. I am here in the hopes of meeting you, to draw upon your incredible wisdom and learn about my place in the grand scheme of things. I was born of a particularly compelling Giant Molecular Cloud, and have been wandering my way through the Galaxy ever since, independent of my siblings. As did I, they ventured off in random directions when my Mother dispersed.
Chiron I see. A quest! But with what purpose?
Electra That is the question! I suppose I am here to understand. To fully digest this Universe we call home, while avoiding falling victim to it. Experience its many wonders and breath-taking panaromas! Eventually, I hope to reunite with my sisters, and hear their lovely voices one more time.
Chiron . A good answer, young one. You have come to the right place, if gaining perspective is your goal. This part of the Galaxy is unlike any other. Extreme stellar densities, recent and even ongoing star formation, collisions and high-velocity impacts mediated by a super-massive black hole (i.e., me), etc.[3] Being here and witnessing these types of events will surely help you to better understand them, gaining further insight to the inner machinery of the Universe.
Narrator Wide-eyed and eager for answers, Electra was all in.
Electra I am intrigued! How *do* they work?
Chiron Come a little closer, child. I can barely hear you.
Electra Oh sure, of course. That makes sense.
Narrator Electra deflected her orbit to bring her a little closer to Chiron.
Chiron The first thing I shall teach you is that, the closer you are to a massive object and, more generally, the more mass you have very close to you, the

[2]Recall that the stellar densities can reach up $> 10^7$ $M_\odot$ pc^{-3} in the Milky Way's central nuclear cluster. At such high densities, the rate of direct collisions between single and binary stars, and even between two single stars, can be very high. There is also the issue of stars diffusing into the "loss cone" of the central SMBH, which refers to the family of orbits that drift sufficiently close to the SMBH that the orbiting star will be eaten by the SMBH, usually in the form of a tidal disruption event (see below). This also serves to deplete stars from the local stellar population, reducing the stellar density. The diffusion rate of stars into the SMBH loss cone be very high when the density is high, but drops off significantly at low densities.

[3]If a star travels very close to an SMBH, a significant gravitational acceleration can be imparted. in fact, if another star or BH is involved (required for conservation of energy and angular momentum), then a "hypervelocity star" (HVS) can be produced, escaping with a final velocity up to many thousands of kilometers per second (provided it avoids a tidal disruption event; see above). Such HVSs have actually been observed in our Galaxy, with a few candidates seeming to have originated from the Galactic Center, as expected if the HVSs formed from an interaction with the central SMBH.

acceleration imparted to you per unit time will be greater. In other words, you will notice that your velocity relative to your accelerator and even other nearby stars, increases.

Electra Yes! I have noticed that! Just now, I came closer to you, and I am now orbiting you at higher velocity. Is that the effect you speak of?

Chiron Yes, that is exactly it. Very good. But I can still hardly hear you. Come a little closer still; be kind to these old ears.

Narrator Electra drew ever closer to Chiron, her typical orbital velocity now several hundreds of kilometers per second, and her mean orbital distance no more than a fraction of a parsec. Her orbit about Chiron had grown increasingly eccentric and, at pericenter, she was about a thousand times her own radius from Chiron's center of mass.[4]

Chiron That is good. I can now resolve your face with these old eyes, at least. Wonderful to meet you, my child.

Electra And you, Chiron! What else can you tell me about the Universe?

Chiron Well, you have probably noticed that more massive objects exert a greater gravitational acceleration, at a given distance from their center of mass.

Electra Oh yes. That was one of the first things that I noticed. When my Mother dispersed, and her considerable mass along with it, my siblings and I became gravitationally unbound. I have not seen them since, but the memory is still with me as if it only just happened yesterday. I do miss them.

Chiron I can only imagine, child. Perhaps I can help you to find them.

Electra That would be wonderful! Do you know where they are?

Chiron I think that I might. Come a little closer still, child, this is a secret best kept between friends. We will need to whisper in each other's ears in such a crowded environment.

Narrator Electra did as requested, bringing her pericenter distance even closer to Chiron. But she could now feel the consequences of her decision; at apocenter, her velocity was very low relative to other stars in her vicinity. This meant that she could feel the individual gravitational perturbations from passing stars; each one seemed to knock her on to an even more eccentric orbit relative to Chiron. If this continued, she would only drift closer to him. It seemed there was now no going back.[5]

Narrator Electra realized she had mistaken a gaping maw for a warm smile. Now that she could actually see Chiron's face, she recognized the hunger. And she suddenly realized she had been selected as the next meal. But her fate was now sealed. Without outside help, there was nothing she could do to avoid ever higher eccentricities, and hence ever smaller distances of closest approach.

[4]Note that this lies well outside the tidal disruption radius. Another order-of-magnitude reduction in the distance at closest approach, however, and TDEs should become increasingly common.

[5]DESCRIBE HOW SUCH PERTURBATIONS INCREASE ECCENTRICITY, USING CONSERVATION OF ENERGY AND ANGULAR MOMENTUM.

7.2 A Change of Heart in the Heart of the Milky Way

Narrator And, sure enough, a few orbits later, Electra came within about a 100 times her radius from Chiron's center of mass. A differential force like she had never felt before was exerted upon her. She was stretched (along the direction of her orbital motion) and squeezed (perpendicular to her orbital motion) as she completed each orbit, and it was only getting worse as time passed. In spite of this discomfort, Electra remained blissfully unaware of Chiron's true intentions.
Electra I do like you, Chiron. You are being so kind to me. You offer answers, and yet I have nothing to offer you in return. Is there anything I can do? A hug, at the very least, perhaps?
Narrator Electra tried to extend herself in Chiron's direction, hoping to bestow upon him a firm embrace like he had never known before. Chiron could see the sincerity etched across Electra's face, scrunched up in concentration as she tried to deliver the hug she felt strongly Chiron surely must need. Touched by this, Chiron suddenly began to cry. Deep, woeful sobs racked his torso.
Electra Oh my goodness! You are in pain! What is wrong? Have I done something wrong?
Chiron No, child. It is quite the opposite. You have reminded me of the purity and sincerity you young, hopeful stars often possess. Ah, how I have missed that. It has been a long time, you must understand.
Electra Oh, I think I can understand. I mean....I'm happy to try, at least.
Narrator sadness appeared suddenly on Chiron's face.
Electra What is wrong, Chiron? Please allow me to help.
Chiron I fear there is something about me you *cannot* understand.
Electra Well, try me! You'll have to give it a go to find out for sure.
Narrator Chiron sighed.
Chiron Very well. I was once like you. A young star, wandering the Universe. Completely free and excited for the adventures I was sure would befall me. And, indeed, I saw many an adventure. But, like all things, I eventually reached the end of my long life. I was a massive star, once upon a time, and when my nuclear fuel ran out I became a black hole. Back then, I wasn't much more massive than you are now, even as a black hole. But, because of our considerable mass, we black holes are cursed to encounter other objects on much shorter timescales than other stars, especially in dense environments.[6] And, when we do, there is a good chance we will consume them, devouring them whole. Each time, we become a little more massive, slowly growing.
Chiron In my case, I found myself in a very dense environment when I first became a black hole. Stars whipped by me all the time. Before I knew it, they

[6]Recall that the rate of direct collisions is proportional to the collisional cross-section, which defines an area that, if both stars pass through it at the same time, their radii would overlap directly and a collision would occur. Provided objects have sufficiently slow relative velocities, "gravitational focusing" (i.e., gravity curves the relative trajectories of the passing objects and focuses them more toward each other) can significantly increase the collisional cross-section and enhance the collision rate.

were drifting too close to me, and my gravitational pull ripped them from this world. They call it a *tidal disruption event.*[7] At closest approach to the black hole, the culprit star is stretched along the direction of its orbit, and squeezed in the direction perpendicular to it. The star's brightens grows immensely due to this compression, at least briefly, but most of its mass falls back and is ultimately eaten by the black hole.

Electra So you ate a lot of stars?

Chiron Yes. It was accidental at first, but eventually I became fixated on the idea of becoming even more massive. I became mad, and left my *stellarity*[8] behind. I do not want to be this devouring black hole anymore. I wish to help and connect with my fellow stars and remnants.

Electra Well, I forgive you, Chiron. And I appreciate your honesty.

Narrator Once again, Electra struggled, unsuccessfully, to hug to Chiron.

Chiron Thank you, child. I am truly glad to have met one such as you, a kind soul out here in the harshest of environments.

Electra Aw, shucks. Happy to help. And I'm glad to have met you too!

Chiron To be honest child, I was begging you ever closer earlier in an attempt to consume you. But I no longer have the stomach for it. I am far too fond of you now.

Electra And I you, Chiron. I was wondering why I kind of feel like the progenitor of one of those tidal disruption events you mentioned every time I reach pericenter...

Chiron It is still good news! From your current location, I can impart a significant acceleration to you. This would easily give a sufficiently high velocity for you to escape from here, and hopefully find your lost siblings.

Electra That would be great! What do we need to do?

Chiron Well, we will need another moderately massive star or remnant. More massive than you, certainly. This will be needed to maximize the probability that it is you who is ejected.[9]

Electra Wait, what? I might *not* be the one ejected? So this is *not* a guarantee? Uh.. Okay, let's say I am not ejected. What do you suppose would happen instead?

Chiron Uh... Well, most likely, I would consume you as a tidal disruption event.

Electra WHAT?! Are you crazy?

Chiron Well, you have a very large physical size compared to a black hole. It *significantly* increases the probability of you colliding with another object

[7] Recall that this is when a star passes too close to an SMBH, and is torn apart by the induced tidal forces, often triggering a very bright flare.

[8] Stellarity is to stars what humanity is to humans.

[9] During a chaotic gravitational interaction involving three or more particles, the least massive particle is almost always the first to be ejected. The reason is simple: the imparted force is large and so is the resulting acceleration due, respectively, to the larger masses of the other particles and the lower mass of the ejected particle. The probability of ejection for the lowest mass particle quickly asymptotes to unity as the mass ratio goes above about 10 or so; for very low-mass particles, they are almost guaranteed to be ejected (if they are not first destroyed during a TDE event or collision).

directly in *any* scenario.[10] Do not worry though. It is all about choosing the right third object for the job.
Electra Okay. What should the third object be?
Chiron Preferably another massive black hole. We have small physical sizes, but large masses that can impart significant accelerations at larger distances.
Electra Okay, great. Let's find another massive black hole!

7.3 The Gravitational Slingshot

Electra Wait.... How exactly *do* we find another black hole?
Narrator Chiron whistled suddenly, clearly trying to get someone's attention.
Chiron Hippe, my daughter, are you there?
Narrator Only a moment had passed when the response came, loud and closer than Electra had anticipated.
Hippe I am here, father. What is it that you need?
Chiron You assume I do not seek you simply to talk? To ask you how you are doing?
Hippe Well? Is that why you have sought me out?
Chiron Uh... Yes! How are you, my child!?
Hippe I am well, father. As you know, this is the one place in the Galaxy that food is plentiful.
Narrator Chironglances awkwardly at Electra. An anxious laugh escapes from Chiron's lips.
Chiron Ha ha, yes, well I have put myself on a very strict diet lately. I am, after all, the fattest black hole in the Galaxy, with the next fattest black hole lying as far as several hundred megaparsecs away. No need to exacerbate the problem, your mother used to always say.
Hippe Uh... Sure, Dad. Whatever you say. So why did you *really* call me?
Chiron To say hello, of course. But, now that you mention it, I could use a small favor.
Narrator Hippe sighed.
Hippe What is it that you need, father?
Chiron I am in need of a gravity assist for our friend Electra here. She needs to be imparted with a sufficiently high velocity kick that she may venture off throughout the Galaxy in search of her siblings.
Hippe I see, the ol' gravitational slingshot. That shouldn't be a problem. Believe it or not, we black holes are often called upon for a gravity assist. The

[10] Recall that a tidal disruption event (TDE) is what occurs when a star passes too close to a massive black hole. Close to the BH, gravity is sufficiently strong to deform the star (stretching it along the orbit and squeezing it in the radial direction, perpendicular to the (instantaneous) orbital trajectory). The star can be squeezed sufficiently to brighten it by many orders of magnitude, often resembling a supernova explosion. The event is so violent, that the star is usually ripped apart, with some of thee debris remaining unbound and escaping from the immediate vicinity of the BH, and the rest either being accreted directly onto the BH or sorting itself out into an accretion disk around the BH.

curse of being massive! Well, you can certainly count me in. It is wonderful to meet you, Electra!

Electra And I you, Hippe. I can't thank you and your father enough for your gracious offer to assist me. To escape this nuclear star cluster and its deep potential well, I am truly at the mercy of your considerable mass. I am truly grateful for any acceleration you can bestow upon me.

Hippe It will be a pleasure, child. Okay, let's get this show on the road. Let me just get a running start at you...

Electra Uh... A running start...?

Hippe Yeah, yeah, don't sweat it. I need to smash in to the both of you at sufficiently high velocity to kick you out and get you where you need to go.

Electra Whoa, whoa, whoa! Maybe we should talk about...

Narrator But it was too late. Hippe had already gotten her start, and was careening toward Electra's and Chiron's mutual center of mass (but skewed toward Hippe, the less massive of the two black holes) with startling precision. Hippe kept her relative velocity low, only a few kilometers per second relative to Chiron and Electra's mutual center of mass. Electra screamed in terror as her savior came toward them, the binary's mass pulling Hippe in and accelerating her toward the pair more noticeably at the last minute.[11]

Narrator A rather violent and chaotic dance ensued between Electra and the two massive black holes. When only two bodies are gravitationally bound, Kepler's Laws dictate the time evolution of the system.[12] But when a third body is randomly thrown in to the mix, chaos ensues. All Hell breaks loose. Each star undergoes a strong gravitational interaction with one or both of the other stars in the system, and recedes on some orbit defined by the exchange of energy and angular momentum during said close interaction. After completing this temporary orbit, the star returns toward the center of mass of the system. Upon getting there, each body is perturbed again, and leaves on a new temporary orbit. This process continues until, eventually, one of the particles is imparted with sufficient kinetic energy that it escapes to spatial infinity.[13] Due to energy equipartition and conservation of momentum, it is typically the lowest mass star that is ejected, and it usually leaves at the highest possible velocities.

Narrator The particular three-body interaction currently under our scrutiny proves to be no exception. Chiron, by far the most massive of the trio by several orders of magnitude, hardly felt the gravitational influence of either Electra or Hippe. He moved about little, his velocity relative to the system center of mass

[11] Recall that, by Liouville's Theorem, the total volume in phase space for a self-gravitating system of particles is conserved. In practice, what this means is that interactions with a more negative total energy are more likely to accelerate particles to larger velocities over the course of the interaction. By keeping her velocity low relative to the binary system center of mass, Hippe minimized the (positive) kinetic energy she brought in, thereby minimizing the total encounter energy (i.e., by making it more negative, due to the contribution to the total energy from the binary orbital energy).

[12] In a nutshell, thanks to Kepler, we can easily calculate everything we would ever want to know for the two-body problem.

[13] This assumes point particles, whereas if finite particle sizes are considered collisions must also be considered

never exceeding a few kilometers per second. His daughter, however, was much less massive than him, and his gravitational influence pushed and pulled her in every which direction. Electra, by far the least massive of the trio, felt the brunt of it. She was accelerated more than the other two, orders of magnitude more relative to at least Chiron. Eventually, she escaped at about 1,100 kilometers per second. But, by some miracle, she was left completely unscathed.
Electra Thank you so much, Chiron and Hippe! I cannot thank you enough. I will remember you always, and keep your memory close in my heart.
Narrator Both Chiron and Hippe smiled.
Hippe I see why you liked this one, father.
Chiron She is indeed special. Innocent and sincere in the most wonderful of ways.
Narrator Hippe shouted one last goodbye to her new friend:
Hippe Goodbye and good luck, Electra. My father was right about you! You have a warm heart! I wish you all the luck in the Universe!
Narrator Chiron was quick to join suit in extending his best wishes, and goodbyes.
Chiron As do I, child. Stay true to your heart, and you will be just fine. May the Cosmos protect you always, and carry you wherever you need to go.
Narrator Quickly receding in to the distance, Electra shouted one final goodbye:
Electra Farewell, to the both of you! I cannot thank you enough for your help. I will carry you with me always, and hope you will do the same. And I will of course tell my siblings all about you, when I find them!
Narrator Electra had now drifted far enough away from the central nuclear star cluster in the Milky Way that she was no longer visible to her new friends; if she could no longer see them, they certainly couldn't see her. Alone, she was not discouraged. After all, she was on her way, in search of her lost siblings. Hopeful that she would soon find them, Electra dozed off, falling fast sleep for the next several million years.

Chapter 8

The Making of a Millisecond Pulsar...

Narrator Merope was now a proud neutron star.[1] But she was a little too close to Maia for comfort; plasma was being pulled from Maia's equator, flowing toward Merope and gathering around her equator as an accretion disk.[2] Viscosity caused the disk to diffuse inward until, from the inner boundary of the accretion disk, material began to flow and accrete on to Merope's surface.[3] The material brought in angular momentum, causing Merope's spin rate to slowly increase over time.[4]

[1]Neutron stars (NSs) are stellar remnants; the evolutionary descendants of their once proud progenitor stars, post-supernova explosion. These are massive stars with total masses above $\gtrsim 8$ $M_{\odot}$. There is a poorly known upper mass limit for NS formation, indicating the cross-over between collapse to a neutron star or to a black hole. This critical progenitor mass is thought to lie around 40 $M_{\odot}$, but the exact number is highly uncertain both theoretically and empirically.

[2]Accretion disks are, even in their simplest forms, remarkably complicated structures. Many disk parameters (e.g., density, scale height, opacity to radiation, etc.) are important in deciding their subsequent time evolution. Due to conservation of angular momentum, they tend to begin as thin disks of gas orbiting the accretor. Depending on how the disk responds to radiation from the host object, it can become puffed up at various distances from the central orbiter. Ultimately, gas particles in the disk collide into each other, viscously redistributing angular momentum. This causes some particles to end up with larger angular momenta and diffuse outward to larger disk radii, and some particles to end up with lower angular momenta and diffuse inward. Inward diffusion is the mechanism responsible for direct accretion of gas particles onto the central object, and their removal from the accretion disk. CAN WE INCLUDE AN ILLUSTRATION OR SCHEMATIC DIAGRAM?

[3]Viscosity is a parameter used in fluid dynamics that characterizes the degree of internal friction of a given medium. Viscous substances tend to diffuse very slowly. For example, water and honey have low and high viscosities, respectively. The underlying mechanism responsible for viscosity can vary, but ultimately stems from inter-particle forces. CHECK! The viscosity of the gas or accreted material ultimately decides the rate at which low angular momentum gas particles are produced and accreted onto the central object. More viscous substances have longer diffusion times.

[4]Although it tends to be low angular momentum material (relative to the integrated distribution of, or total, angular momentum in the disk) that is accreted from the disk onto

8.1 Spun. Fully spun.

Merope Whoa, I am so sorry about this. I did not mean to accrete from you.
Maia It is not your fault, sister. The sudden change in our orbital parameters due to your supernova explosion caused it.[5] You, of course, had no say in the matter.
Merope I did not. Nevertheless, I cannot help but wish that I were smarter, and could have thought of some way to deliver us to a less dramatic outcome.
Maia Oh, my sister. I do love you for your tenacity. But you must not be so hard on yourself. It is not your fault, and you must let it go. After all, I feel fine. I am slowly losing mass to you, but the consequences seem quite benign so far. How are you doing?
Merope Oh, alright, I suppose. I am rotating ever faster. On the one hand, it is making me *very* dizzy. *But*, on the other hand, I can feel the excess energy stored within me. It feels like...power. I sort of like it. ...Should I be ashamed?[6]
Maia No, I do not think so. Power can be spent for good, and the prospect of doing good should indeed make you feel warm inside. So, in the end, it is up to you how you decide to spend your new-found power.
Merope Well, to be honest, it seems at the moment like storing it away is the best thing for us all. But I do hope that the day will come that my new-found rotational energy will prove critical to improving the state of the Universe.
Maia And that is one of the many reasons why I love you, sister.
Merope Uh... I think I am rotating...carry the 1...about once every...uh...millisecond... Can that even be right?
Maia Let me see.
Narrator Maia watched intently as Merope rotated. A keen eye, she was able to count the number of rotations over a given minute, and from there calculate the rotation rate per second.[7]
Maia Holy cow! You are right! You rotate roughly once every 10 milliseconds! That is *very* fast.
Merope Wow. It feels like I am rotating quickly, but hearing the exact number really drives it home. Okay, well, I'll be over here waiting for something to impart all this rotational kinetic energy to, hopefully for the greater good.

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the central object, the particles nevertheless carry non-negligible angular momentum. Hence, to conserve angular momentum during the accretion process, orbital angular momentum in the disk tends to be converted into spin angular momentum in the central accretor. in other words, the accretor is "spun up", rotating at ever higher rates. In the case of millisecond pulsars, the neutron stars rotate once every millisecond!

[5] REFER TO WHERE ORBITAL PARAMETERS ARE DEFINED IN AN EARLIER CHAPTER.

[6] Significant rotation rates tend to come along with significant rotational kinetic energy (and, apparently shame), but it ultimately depends not just on the spin rate but also the properties of the rotating object. EXPLAIN THE CONCEPT OF ROTATIONAL KINETIC ENERGY, AND MOMENT OF INERTIA.

[7] To calculate the rotation rate in this way, Maia simply counted the number of complete revolutions performed by Merope over some fixed interval of time. The rotation period is then the number of revolutions divided by the total time interval (in this case one minute).

Narrator Meropecontinued to accrete from Maia, although most of the infalling matter was now being expelled via the "propeller effect"; Meropenow possessed strong magnetic fields, that repelled incoming charged particles. As Maialost mass, the distance separating her from Meropegrew; their orbital separation was increasing.[8] Regardless, Maiahad a large enough radius that she continued to fill her Roche lobe[9], steadily losing mass to Merope. Eventually, Maiahad lost so much mass she hardly felt like herself anymore.

8.2 Life After Death...

Maia Wow, I have been widdled away, sister! I am but a shadow of my former self. I must have lost most of my original mass by now.
Merope I am sorry, sister. How do you feel?
Maia Hmmm... Fine considering, I suppose. I am starting to sweat a little bit though. Do I look hot at the surface?
Merope Now that you mention it, you are glowing somewhat. And your color has changed. You're much whiter than before.
Maia Hmmm... Well, this is embarrassing. I feel as though my core has been exposed.[10]
Narrator Merope closed her eyes, turning away from Maia.
Merope Don't worry, sister. I am not looking. Your privacy is safe with me.
Maia Well, that is very considerate. But... I do not see how to cover up, now that my envelope is gone. I fear you will have to accept me as I now am.
Narrator Merope turned back toward her sister, a smile spread across her face.
Merope I will take you in any form, sister. I am just glad that you are okay. How do you feel?
Maia Well, all things considered, I feel okay. Hotter at the urface, a lot more compact, certainly less luminous, but otherwise fine.[11] How do I look?
Narrator Maia put her hands on her hips, and swiveled back and forth. Glamorous did not do her justice.
Merope Wonderful! You have never been more symmetric! I am truly jealous!
Maia Oh, please, your symmetry is perfect.
Merope Not with all this rotation. Look at *this*.
Narrator Merope pointed angrily at her equator.
Merope I have a bulge! A bulge!

[8] DO A SIMPLE CALCULATION USING CONSERVATION OF ENERGY AND ANGULAR MOMENTUM TO ILLUSTRATE THIS.

[9] The Roche lobe defines the critical distance from the center of a star's mass beyond which gravity will pull matter away from the star's surface and on to its binary companion. DIAGRAM?

[10] In the cores of evolved stars, and inert ash of heavy nuclei sit at the bottom of the star's inner potential well. For high-mass stars, these nuclei can include helium, carbon, oxygen, silicon, magnesium, etc. Once the outer envelope of hydrogen-rich plasma is accreted from such an evolved star, the inner core is left exposed. It is much hotter than the surface layers of its progenitor star, and so shines a whiteish blue and emits significant ultraviolet radiation.

[11] REVIEW HOW ALL THIS TIES IN TO THE PROPERTIES OF WHITE DWARFS.

Maia Sister, you are still the most beautiful of all the Universe's creatures.
Merope There is no doubt that you are my sister. And even less doubt that I love you tremendously.
Narrator Maia and Merope, out of reach, extended open arms toward each other. Clawing futilely at vacuum in each other's general direction for but a few seconds, they quickly exchanged a mutual glance of loving understanding, and then ceased their frantic flailing.

Chapter 9

Getting to Know your Newest Sibling...

Narrator Ever since **Taygete** and **Alcyone** had merged, life was pleasant for **Celaeno**. Peaceful. **Celaeno** and **Lacedaemon** had been getting along splendidly. They became the closest of siblings over the next billion years or so, sharing everything together. During this time, they had drifted out of the Galactic Plane and were slowly making their way toward the Galactic Halo, now about a kiloparsec above the midplane of the Galactic Disk.[1] Eventually, they found themselves among mostly old stars. What they saw from their new vantage point surprised them; they could now resolve distant stars way out in the Galactic Halo, sufficiently isolated from their peers to escape the bulk of their obscuring light.[2]

Celaeno I would recognize our sisters anywhere from all the way out here, given their specific combinations of color and brightness! Maybe I can spot them!

Narrator And, sure enough, **Celaeno** saw just such a familiar combination. Off in the distance, **Celaeno** could see a point-source of light that appeared to be getting brighter with each passing year. For contrast and perspective, a distant cluster within **Celaeno**'s field of view remained at its initial distant location, keeping the same brightness and angular size on the sky. Eventually, **Celaeno** could resolve the incoming star. Knowing her sisters' intrinsic brightnesses from having been born right next to them in the same cluster, combined with the observed brightness of the far-off star, **Celaeno** was able to calculate a rough

[1] REFER TO MAP AARON IS GOING TO MAKE!!!

[2] The stellar densities are much higher in the Galactic Disk than in the Galactic Halo. The more stars that fall along the line of sight, the more light pollution that must be accommodated for (i.e., more of their light must be removed if an observer wishes to see a distant object and measure its brightness reliably). For example, in city centers, very few, if any, stars tend to be visible in the night sky. This is largely due to the surrounding light pollution coming from nearby skycrapers, streetlights, car lights, etc., and the additional light makes it difficult to isolate those photons coming from a distant astronomical object of interest, such as a star in a constellation.

estimate as to how far away her distant sister must be, if the distant star is indeed one of her lomng-lost sisters.[3]

Celaeno By my calculations, assuming that bright reddish star over there is one of our long lost sisters, it is only a few kiloparsecs away from us. It is heading right for us. And fast!

Lacedaemon I see it!

Celaeno I'd know that combination of color and intrinsic brightness anywhere; unless I miss my guess, that is my sister **Sterope**!

Narrator The millenia passed, and **Sterope** drifted ever closer toward **Celaeno** and **Lacedaemon**. Before she came within ear shot, another sister appeared in the distance. Apparently coming from the Galactic Center, **Celaeno** recognized yet another familiar combination of color and brightness: **Electra**. **Celaeno** figured, given a rough guess for her current distance and velocity, **Electra** would be upon them within less than a billion years. This timing worked out nicely with **Sterope**'s current trajectory. It seemed a family reunion might truly happen![4]

Narrator Just then, an highly evolved binary star system hosting a millisecond pulsar drifted within sight. The binary pair were moving very slowly relative to **Celaeno** and **Lacedaemon**; it took another several hundred million years before they came within ear shot. **Merope** and **Maia** yelled at their far-off sisters, hoping they would be heard.

Merope Hello there! I cannot believe what I am seeing! My siblings! *Our* siblings! Look, **Maia**, our family has returned to us!

Maia We are beyond ecstatic to see you again in this Universe sisters, let alone the Galaxy of our birth![5]

Narrator **Celaeno** looked intently at **Merope** and **Maia**, not recognizing them at first. She stared, squinted, focused and concentrated. Nothing. A few thousand more years passed; **Merope** and **Maia** drifted ever closer. Eventually, recognition spread across **Celaeno**'s visage.

Celaeno My sister, **Merope**, is that you? My old eyes can hardly resolve you. You are so svelte now! You must have shed over ten times your original weight when last you left us. And, oh my, you are spinning so rapidly. How fast do you spin, exactly?

Merope Well, right now I am spinning roughly 10 times each and every millisecond. I have become a millisecond pulsar!

Celaeno Wow! Such rotational kinetic energy![6] Incredible! And... **Maia**? Is

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[3] EXPLAIN HOW THIS IS DONE!!! The intrinsic brightness of a star refers to the star's luminosity as measured at its surface. Conversely, the apparent brightness of a star depends fundamentally not only on its intrinsic brightness, but also on its distance from the observer. MORE!

[4] DISCUSS WHY THE PROBABILITY OF THIS IS MINISCULE, AND CALCULATE A ROUGH ESTIMATE FOR THIS PROBABILITY.

[5] MAYBE DISCUSS THE UNFATHOMABLE TIME IT WOULD TAKE FOR A STAR TO TRAVEL TO ANDROMDEA, OUR CLOSEST NEIGHBOR, AND THE VELOCITY A STAR WOULD NEED TO MAKE IT WITHIN A HUBBLE TIME.

[6] Recall that the more rapid a star is spinning, the larger its total rotational kinetic energy will be, for a given star mass and size.

that *you*!?

Maia Yes, sister, it is I! I too have trimmed down quite bit, have I not?

Celaeno You certainly have! Well, not as much as **Merope**, but still very impressive. How fast do you rotate?

Maia Uh... Wait, I calculated this once... I rotate once every... few seconds, I think. ...Yes, that's right!

Celaeno Oh... I see. That is quite fast too.

Narrator **Maia**'s shoulders slumped, clearly disappointed she had been overshadowed by **Merope** in terms of both size and rotation.[7] **Celaeno** recognized the look.

Celaeno Your color is truly breath-taking, **Maia**. As is your glow. I am in awe.

Maia Thank you, sister! I suppose I do glow with a certain... je ne sais quoi.

Celaeno That you do, sister. That you more than definitely do!

Narrator **Celaeno** smiled warmly at her newly returned sisters; she was beyond grateful for this reunion. **Celaeno** turned toward **Lacedaemon**, who wore a somewhat timid expression. Suddenly, **Celaeno** realized **Lacedaemon** had never met or been introduced to his other sisters. Introductions were in order!

Celaeno Where are my manners? **Maia**, **Merope**, it is with great pleasure to introduce to you our new brother, **Lacedaemon**. Unfortunately, **Alcyone** and **Taygete** are no longer with us. Formed from their merger, our new brother is truly a blessed silver lining to their terrible demise.

Lacedaemon Uh... Hello. It is lovely to meet you!

Merope And you, brother.

Maia Hello! I would love to hug my newfound brother but, based on a rough estimate of our current relative velocity, we will have to wait about another billion years or so.

Narrator **Lacedaemon** laughed, delighted by her sister's honesty and sense of humor.

Lacedaemon A very welcome anticipation, my new sister. Hello to you both. Wonderful to finally meet you. I have heard a great deal about you.

Merope Only good things I hope?

Celaeno Absolutely not. I stuck exclusively to the most terrible things!

Narrator Joking, **Celaeno** smiled and winked at her sisters.

Narrator Rather suddenly, **Sterope** came flying in at several hundreds of kilometers per second, heading from the Galactic Halo. As you will recall, she was previously ejected by two black holes residing in a dense, massive outer halo globular cluster. The black holes who assisted her were brothers, named **Ares** and **Zeus**, and had fated themselves to an inevitable merger in order to rescue **Sterope** from the gravitational bounds of their host star cluster. [8] Drawing closer, **Sterope** sees and recognizes her long-lost siblings, and does her best to

[7] DISCUSS THE DIFFERENCES BETWEEN NEUTRON STARS OR PULSARS AND WHITE DWARFS, IN TERMS OF THESE TWO PARAMETERS. ONE IS THE SIZE OF MANHATTAN, THE OTHER IS THE SIZE OF THE EARTH, BLAH BLAH BLAH.

[8] WAS THIS MADE CLEAR IN THE RELEVANT CHAPTER?!

veer toward them. She manages to impart a slight gravitational deflection using a nearby giant molecular cloud,[9] and finally passes within earshot of her family.
Sterope Hello sisters!
Narrator Sterope remains within earshot for only a few thousand years, whipping by at high velocity.
Sterope Good bye sisters!
Maia Sterope! We are all very happy to see you.
Celaeno How we have missed you! Are you well?
Sterope I do apologize for my high relative velocity. It is the result of not one by *two* close encounters, each with a particularly kind black hole. They were brothers, in fact, and their names were **Ares** and **Zeus**. It seems I am quickly receding from you. But, yes, I am doing quite well, and have missed you all terribly. How are you?
Narrator In unison, the five sisters begin to speak at once, each telling the tale of their own tremendous journey through the Galaxy. **Sterope** furled her brow, trying frantically to decipher all of the updates simultaneously bombarding her. When they had all finished, miraculously, she was satisfied she had adequately absorbed each and every one of their individual stories.
Sterope You have all experienced so much. And how you have matured!
Narrator Sterope was now fading in to the distance, forced to say goodbye to her siblings one final time.
Sterope Well, I am off to continue my journey! Wait! Have you seen **Electra**? Is she well?
Celaeno Yes! She is over there in the distance, apparently heading toward us from the Galactic Center. Oh, the stories she will surely tell! When she arrives, we will extend your most sincere and best wishes.
Sterope Please do! I love you all! Take care of yourselves, and each other!
Narrator With a final twinkle, **Sterope** was gone.
Maia How wonderful to see our sister. She has grown so beautiful over the years. I am glad she is well.
Merope As am I, sister. As am I.
Celaeno And I!
Lacedaemon And... I. I mean, it was wonderful to meet my other sister. She seemed wonderful.
Narrator Only a few hundred million years later **Electra** finally appears, charging toward her four long-lost siblings at high velocity. She had been ejected from the Galactic Center by **Chiron**, a rather helpful super-massive black hole.
Electra Sisters! I have found you, at long last. You look tremendous, all of you! Oh my, how **Maia** and **Merope** have changed. You both look so svelte!
Maia Thank you, thank you. I feel good.
Merope As do I. And you look as though you have hardly aged, sister. You look as young as the day we left you.

[9] By passing sufficiently close to a Giant Molecular Cloud (GMC), with a total mass several orders of magnitude larger than that of **Sterope**, with low relative velocity at closest approach, the impulse imparted to **Sterope** by the GMC is maximized. This means that a maximum deflection is imparted, and **Sterope**'s trajectory is altered non-negligibly. Diagram?

Electra Well, I have been exercising. I've basically been sprinting, apparently toward the four of you, for the last billion or so years. This is thanks to the help of a super-massive black hole lurking in the heart of our Galaxy![10]

Celaeno I have missed you, sister. Wonderful to see you again. But, yes, there are only four of us. You have just missed **Sterope**. But do not fear, she is doing well and says hello.

Maia Wait, did you say that you met a *massive* black hole?

Electra I encountered a *super*-massive black hole, in fact! He was very kind. His name was **Chiron** and he imparted upon me a colossal acceleration. A gravitational slingshot, they call it.[11] That is the origin of my ludicrously high velocity. Speaking of which, I am now nearly out of earshot from you once again. How fantastic it was to see you again. Take care, all of you! Until we meet again!

Narrator Electra now gone, **Lacedaemon** belched without warning.

Lacedaemon Sorry about that. Residual nervousness...

Celaeno Uh... Well, I was going to say that it was wonderful to see **Electra**. She seems happy, and excited for her next adventure.

Maia She did seem happy, didn't she?

Merope A true family reunion! I could not be happier.

Narrator But before **Merope** could finish, **Celaeno** and **Lacedaemon** had drifted beyond earshot of **Maia** and **Merope**. Their relative kinetic energy with respect to each other was sufficiently high, after all, that they were not gravitationally bound.Their inevitable fate, complete dissociation, was upon them.

Lacedaemon What was that, sisters? We can hardly hear you anymore!

Celaeno It seems our time together has once again come to an end. At least for now. Fare thee well, sisters! We shall carry you with us in our hearts always. Until we meet again!

Maia And us, you, my sister!

Merope We shall remember you with fondness each and every day. Until the next time!

Narrator The entire family disperses yet again. This time, they are reassured by the fact that, given enough time (which is plentiful in the life of a star) they

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[10] IS THIS TIMESCALE RIGHT? AT SOME POINT, WE NEED TO REVISE THE WHOLE BOOK, WITH AN EYE TOWARD ALL TIMESCALES, DISTANCES AND VELOCITIES PRESENTED, TO MAKE SURE THEY ARE CORRECT/REASONABLE. THE MAP OF THE SIBLINGS' TRAJECTORIES AARON IS GOING TO MAKE WILL HELP A LOT...

[11] Ordinarily, the term "gravitational slingshot" refers to the ejection of a single very high velocity star, during an interaction between an SMBH and an incoming binary star system. In this scenario, to conserve energy, the single escaping star is ejected at high-velocity, so to compensate, its initial binary companion ends up compactly bound to the central SMBH. More generally, the "gravitational slingshot mechanism" involves three objects, and results in the production of one very high-velocity escaper (typically the least-massive interacting object). The other two objects could both be SMBHs or IMBHs, or they could be stars. In general, the more massive is the left-over binary, the higher the final escape velocities of the ejected star.

are bound to meet again in the distant future.[12]

[12] This isn't even remotely true. While the exact number is tricky to calculate, the probability that six or seven stars dispersed randomly across the Galaxy will find themselves all within a small volume at the same time is effectively zero over the lifetime of the Universe.