Nathan Wemmer

Code

This is an R Markdown (http://rmarkdown.rstudio.com) Notebook. When you execute code within the notebook, the results appear beneath the code.

Try executing this chunk by clicking the *Run* button within the chunk or by placing your cursor inside it and pressing *Ctrl+Shift+Enter*.

Add a new chunk by clicking the *Insert Chunk* button on the toolbar or by pressing *Ctrl+Alt+I*.

When you save the notebook, an HTML file containing the code and output will be saved alongside it (click the *Preview* button or press *Ctrl+Shift+K* to preview the HTML file).

The preview shows you a rendered HTML copy of the contents of the editor. Consequently, unlike *Knit*, *Preview* does not run any R code chunks. Instead, the output of the chunk when it was last run in the editor is displayed.

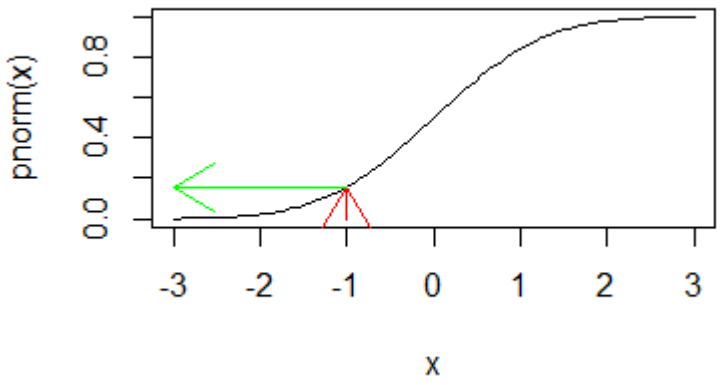
Section 7.3

Hide

```
curve(pnorm(x),-3,3)
arrows(-1,0,-1,pnorm(-1),col="red")
```

Hide

```
arrows(-1,pnorm(-1),-3,pnorm(-1),col="green")
```



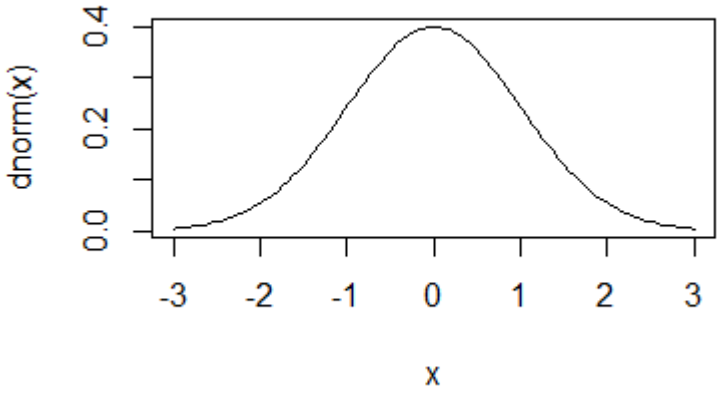
Hide

```
pnorm(-1)
```

```
[1] 0.1586553
```

Hide

```
curve(dnorm(x),-3,3)
```



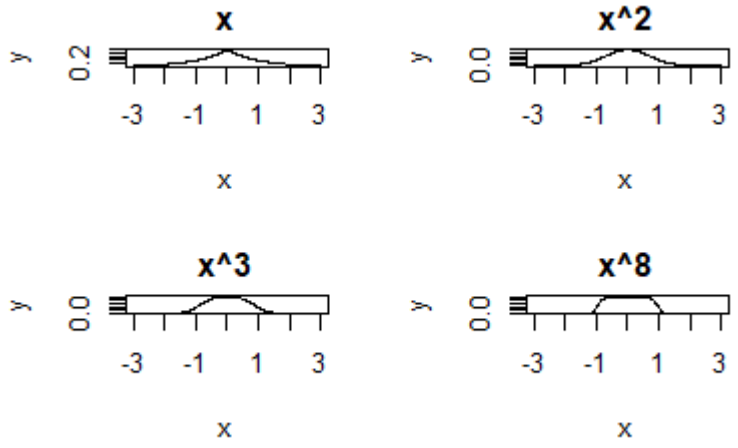
section 7.3.1

Hide

```
par(mfrow=c(2,2))
x <- seq(-3,3,0.01)
y <- exp(-abs(x))
plot(x,y,type="l",main= "x")
y <- exp(-abs(x)^2)
plot(x,y,type="l",main= "x^2")
```

Hide

```
y <- exp(-abs(x)^3)
plot(x,y,type="l",main= "x^3")
y <- exp(-abs(x)^8)
plot(x,y,type="l",main= "x^8")
```



Hide

```
pnorm(-1.25)
```

```
[1] 0.1056498
```

Hide

```
pnorm(1.875)
```

```
[1] 0.9696036
```

Hide

```
1-pnorm(1.875)
```

```
[1] 0.03039636
```

Hide

```
pnorm(1.25)-pnorm(-0.625)
```

```
[1] 0.6283647
```

Hide

```
x <- seq(-3,3,0.01)
z <- seq(-3,-1.25,0.01)
p <- dnorm(z)
z <- c(z,-1.25,-3)
p <- c(p,min(p),min(p))
plot(x,dnorm(x),type="l",xaxt="n",ylab="probability density",xlab="height")
axis(1,at=-3:3,labels=c("146","154","162","170","178","186","192"))
```

Hide

```
polygon(z,p,col="red")
z <- seq(1.875,3,0.01)
p <- dnorm(z)
z <- c(z,3,1.875)
p <- c(p,min(p),min(p))
plot(x,dnorm(x),type="l",xaxt="n",ylab="probability density",xlab="height")
```

Hide

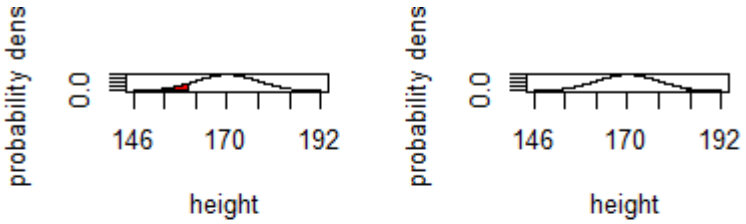
```
axis(1,at=-3:3,labels=c("146","154","162","170","178","186","192"))
polygon(z,p,col="red")
```

Hide

```
z <- seq(-0.635,1.25,0.01)
p <- dnorm(z)
z <- c(z,1.25,-0.635)
p <- c(p,0,0)
plot(x,dnorm(x),type="l",xaxt="n",ylab="probability density",xlab="height")
axis(1,at=-3:3,labels=c("146","154","162","170","178","186","192"))
```

Hide

```
polygon(z,p,col="red")
```



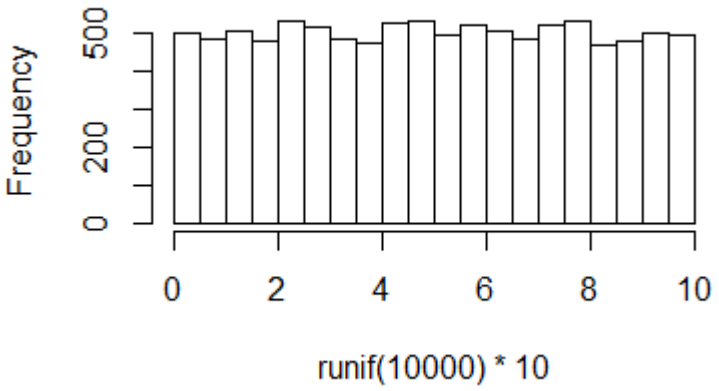
Hide

```
NA
NA
NA
NA
```

Section 7.3.2

Hide

```
par(mfrow=c(1,1))
hist(runif(10000)*10,main="")
```



Hide

```
means <- numeric(10000)
for (i in 1:10000){
  means[i] <- mean(runif(5)*10)
}
hist(means,ylim=c(0,1600),main="")

mean(means)
```

```
[1] 5.017455
```

Hide

```
sd(means)
```

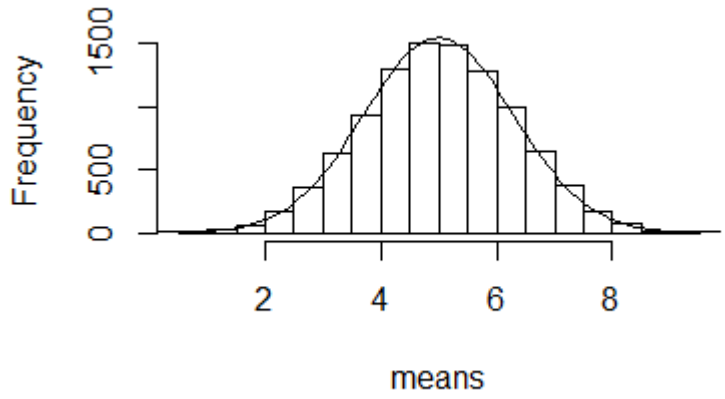
```
[1] 1.29213
```

Hide

```
xv <- seq(0,10,0.1)

yv <- dnorm(xv,mean=4.998581,sd=1.28996)*5000
lines(xv,yv)
```

Hide

```
par(mfrow=c(2,2))
```



Hide

```
hist(sample(1:6,replace=T,10000),breaks=0.5:6.5,main="",xlab="one die")

a <- sample(1:6,replace=T,10000)
b <- sample(1:6,replace=T,10000)
hist(a+b,breaks=1.5:12.5,main="", xlab="two dice")
```

Hide

```


c <- sample(1:6,replace=T,10000)
hist(a+b+c,breaks=2.5:18.5,main="", xlab="three dice")

d <- sample(1:6,replace=T,10000)
e <- sample(1:6,replace=T,10000)
hist(a+b+c+d+e,breaks=4.5:30.5,main="", xlab="five dice")
```

Hide

```


mean(a+b+c+d+e)
```

```
[1] 17.5817
```

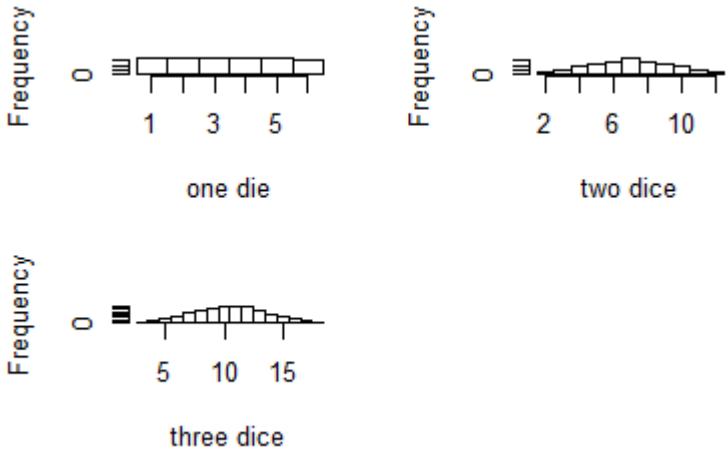
Hide

```
sd(a+b+c+d+e)
```

```
[1] 3.860468
```

Hide

```
lines(seq(1,30,0.1),dnorm(seq(1,30,0.1),17.5937,3.837668)*10000)
```



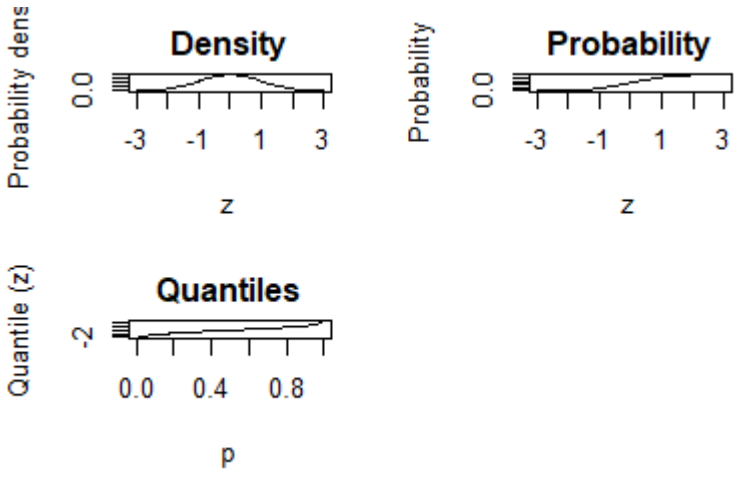
Section 7.3.3

Hide

```
par(mfrow=c(2,2))
curve(dnorm,-3,3,xlab="z",ylab="Probability density",main="Density")
curve(pnorm,-3,3,xlab="z",ylab="Probability",main="Probability")
```

Hide

```
curve(qnorm,0,1,xlab="p",ylab="Quantile (z)",main="Quantiles")
y <- rnorm(1000)
hist(y,xlab="z",ylab="frequency",main="Random numbers")
```



Section 7.3.4

Hide

```
yvals <- rnorm(100,24,4)
mean(yvals)
```

```
[1] 23.65688
```

Hide

```
sd(yvals)
```

```
[1] 4.068051
```

Hide

```
ydevs <- rnorm(100,0,1)
ydevs <- (ydevs-mean(ydevs))/sd(ydevs)
mean(ydevs)
```

```
[1] -4.963478e-18
```

Hide

```
sd(ydevs)
```

```
[1] 1
```

Hide

```
yvals <- 24 + ydevs*4
mean(yvals)
```

```
[1] 24
```

Hide

```
sd(yvals)
```

```
[1] 4
```

Section 7.3.5

Hide

```
setwd("C:/Users/Nathan/Desktop/school/statistical data management/therbook")
par(mfrow=c(1,1))
fishes <- read.table("fishes.txt",header=T)
attach(fishes)
```

```
The following object is masked from fishes (pos = 3):

    mass

The following object is masked from fishes (pos = 4):

    mass

The following object is masked from fishes (pos = 5):

    mass
```

Hide

```
names(fishes)
```

```
[1] "mass"
```

Hide

```
mean(mass)
```

```
[1] 4.194275
```

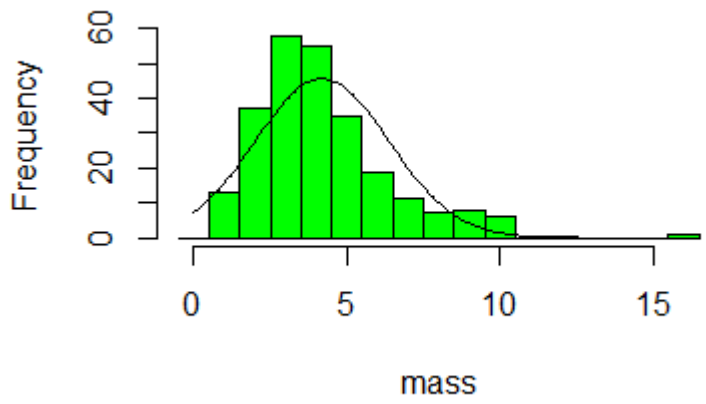
Hide

```
max(mass)
```

```
[1] 15.53216
```

Hide

```
hist(mass,breaks=-0.5:16.5,col="green",main="")
lines(seq(0,16,0.1),length(mass)*dnorm(seq(0,16,0.1),mean(mass),sqrt(var(mass))))
```



Section 7.3.6

Hide

```
1-pchisq(14.3,9)
```

```
[1] 0.1120467
```

Hide

```
qchisq(0.95,9)
```

```
[1] 16.91898
```

Hide

```
1-pf(2.85,8,12)
```

```
[1] 0.04992133
```

Hide

```
qt(0.975,10)
```

```
[1] 2.228139
```

Hide

```
windows(7,4)
par(mfrow=c(1,2))
x <- seq(0,30,.25)
plot(x,pchisq(x,3,7.25),type="l",ylab="p(x)",xlab="x")
plot(x,pchisq(x,5,10),type="l",ylab="p(x)",xlab="x")
```

Hide

```
8*10.2/qchisq(.975,8)
```

```
[1] 4.65367
```

Hide

```
8*10.2/qchisq(.025,8)
```

```
[1] 37.43582
```

Section 7.3.8

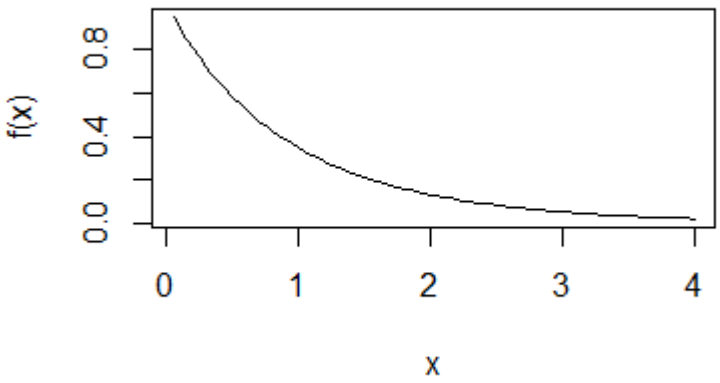
Hide

```
qf(.95,2,18)
```

```
[1] 3.554557
```

Hide

```
x <- seq(0.05,4,0.05)
plot(x,df(x,2,18),type="l",ylab="f(x)",xlab="x")
```



Hide

```
plot(x,df(x,6,18),type="l",ylab="f(x)",xlab="x")

windows(7,7)
```

Hide

```
par(mfrow=c(1,1))
df <- seq(1,30,.1)
plot(df,qf(.95,df,30),type="l",ylab="Critical F")
```

Hide

```
lines(df,qf(.95,df,10),lty=2)

x <- seq(0.01,3,0.01)
plot(x,df(x,1,10),type="l",ylim=c(0,1),ylab="f(x)")
```

Hide

```
lines(x,df(x,2,10),lty=6,col="red")
lines(x,df(x,5,10),lty=2,col="green")
```

Hide

```
lines(x,df(x,30,10),lty=3,col="blue")
legend(2,0.9,c("1","2","5","30"),col=(1:4),lty=c(1,6,2,3),
       title="numerator d.f.")
```

Section 7.3.9

Hide

```
curve( (1+x^12)^(-0.5), -3, 3, ylab="t(x)",col="red")

plot(1:30,qt(0.975,1:30), ylim=c(0,12),type="l",
     ylab="Students t value",xlab="d.f.",col="red")
```

Hide

```
abline(h=2,lty=2,col="green")

xvs <- seq(-4,4,0.01)
plot(xvs,dnorm(xvs),type="l",lty=2,
     ylab="Probability density",xlab="Deviates")
```

Hide

```


lines(xvs,dt(xvs,df=5),col="red")

qt(0.975,5)
```

```
[1] 2.570582
```

Section 7.3.10

Hide

```
x <- seq(0.01,4,.01)
par(mfrow=c(2,2))
y <- dgamma(x,.5,.5)
plot(x,y,type="l",col="red",main="alpha = 0.5")
y <- dgamma(x,.8,.8)
plot(x,y,type="l",col="red", main="alpha = 0.8")
```

Hide

```
y <- dgamma(x,2,2)
plot(x,y,type="l",col="red", main="alpha = 2")
y <- dgamma(x,10,10)
plot(x,y,type="l",col="red", main="alpha = 10")
```

Hide

```
qgamma(0.95,2/3,4/3)
```

```
[1] 1.732096
```

Hide

```
#fishes <- read.table("c:\\temp\\fishes.txt",header=T)
fishes <- read.table("fishes.txt",header=T)
attach(fishes)
```

```
The following object is masked from fishes (pos = 3):

    mass

The following object is masked from fishes (pos = 4):

    mass

The following object is masked from fishes (pos = 5):

    mass

The following object is masked from fishes (pos = 6):

    mass
```

Hide

```
names(fishes)
```

```
[1] "mass"
```

Hide

```
rate <- mean(mass)/var(mass)
shape <- rate*mean(mass)
rate
```

```
[1] 0.8775119
```

Hide

```
shape
```

```
[1] 3.680526
```

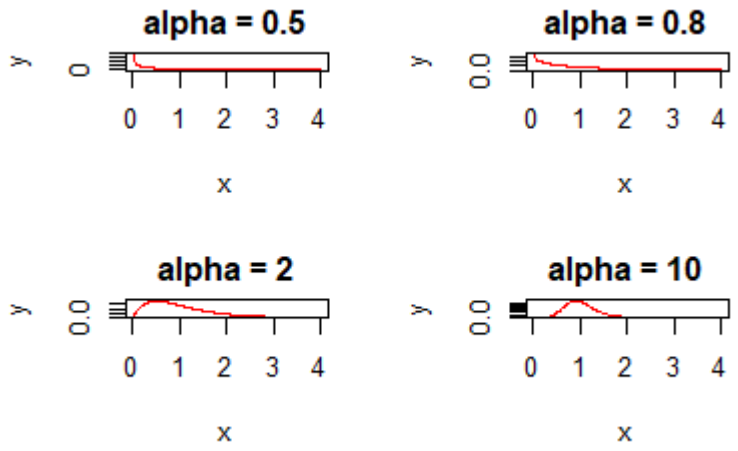
Hide

```
max(mass)
```

```
[1] 15.53216
```

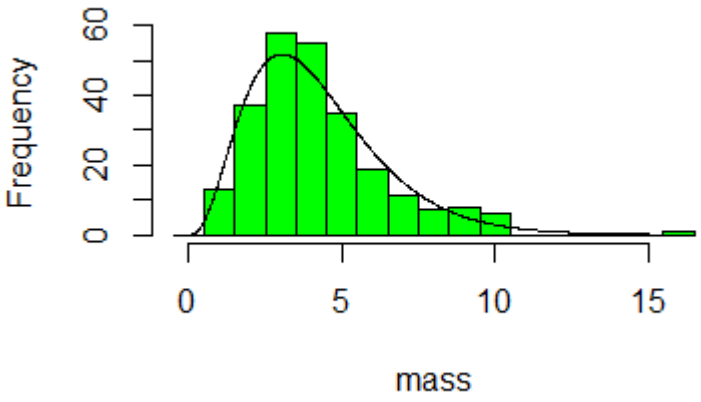
Hide

```
par(mfrow=c(1,1))
```



Hide

```
hist(mass,breaks=-0.5:16.5,col="green",main="")
lines(seq(0.01,15,0.01),length(mass)*dgamma(seq(0.01,15,0.01),shape,rate))
```



Section 7.3.11

Hide

```
rexp(15,0.1)
```

```
 [1]  5.3962126  2.4586411  0.3992737  2.2746802 10.0733188
 [6]  9.4962178 36.4320632  6.2151733 10.4916958 29.0158027
[11]  9.4627875  4.5787264  5.4972032 41.0596733  0.6546447
```

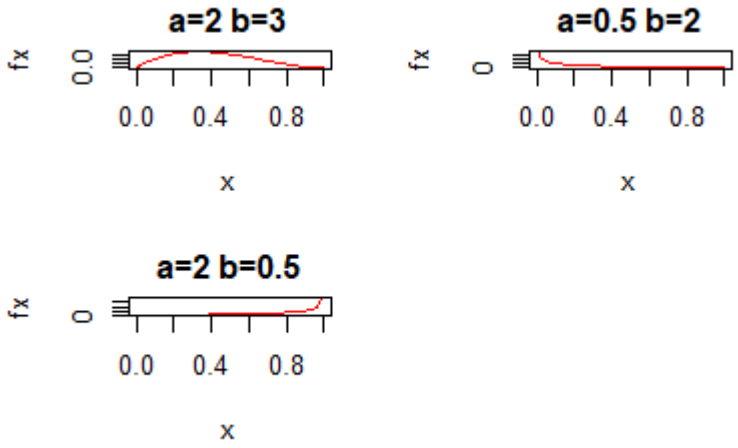
Section 7.3.12

Hide

```
par(mfrow=c(2,2))
x <- seq(0,1,0.01)
fx <- dbeta(x,2,3)
plot(x,fx,type="l",main="a=2 b=3",col="red")
fx <- dbeta(x,0.5,2)
plot(x,fx,type="l",main="a=0.5 b=2",col="red")
```

Hide

```
fx <- dbeta(x,2,0.5)
plot(x,fx,type="l",main="a=2 b=0.5",col="red")
fx <- dbeta(x,0.5,0.5)
plot(x,fx,type="l",main="a=0.5 b=0.5",col="red")
```



Hide

```
rbeta(10,2,3)
```

```
 [1] 0.22159542 0.16564147 0.45311388 0.69509337 0.58380236
 [6] 0.66297670 0.26326729 0.67570674 0.42511016 0.07817391
```

Section 7.3.13

Hide

```
windows(7,4)
par(mfrow=c(1,2))
plot(-200:200,dcauchy(-200:200,0,10),type="l",ylab="p(x)",xlab="x",
     col="red")
plot(-200:200,dcauchy(-200:200,0,50),type="l",ylab="p(x)",xlab="x",
     col="red")
```

Section 7.3.14

Hide

```
windows(7,7)
plot(seq(0,10,0.05),dlnorm(seq(0,10,0.05)),
     type="l",xlab="x",ylab="LogNormal f(x)",col="red")
```

Section 7.3.15

Hide

```
windows(7,4)
par(mfrow=c(1,2))
plot(seq(-5,5,0.02),dlogis(seq(-5,5,.02)),
     type="l",main="Logistic",col="red",xlab="x",ylab="p(x)")
plot(seq(-5,5,0.02),dnorm(seq(-5,5,.02)),
     type="l",main="Normal",col="red",xlab="x",ylab="p(x)")
```

Hide

```
NA
NA
```

Section 7.3.16

Hide

```
windows(7,4)
par(mfrow=c(1,2))
x <- seq(0.1,1,0.01)
y <- -1.4+2.1*(exp(-1.59*log(x)-1.53)/(1+exp(-1.59*log(x)-1.53)))
plot(log(x),y,type="l", main="c = -1.59", col="red")

y <- 0.1+2.1*(exp(1.59*log(x)-1.53)/(1+exp(1.59*log(x)-1.53)))
plot(log(x),y,type="l",main="c = 1.59",col="red")
```

Section 7.3.17

Hide

```
windows(7,7)
a <- 3
l <- 1
t <- seq(0,1.8,.05)
ft <- a*l*t^(a-1)*exp(-l*t^a)
plot(t,ft,type="l",col="blue",ylab="f(t) ")
a <- 1
ft <- a*l*t^(a-1)*exp(-l*t^a)
lines(t,ft,type="l",col="red")
```

Hide

```
a <- 2
ft <- a*l*t^(a-1)*exp(-l*t^a)
lines(t,ft,type="l",col="green")
legend(1.4,1.1,c("1","2","3"),title="alpha",lty=c(1,1,1),col=c(2,3,4))
```

Hide

```


# 7.3.18 Multivariate normal distribution
```

Section 7.3.18

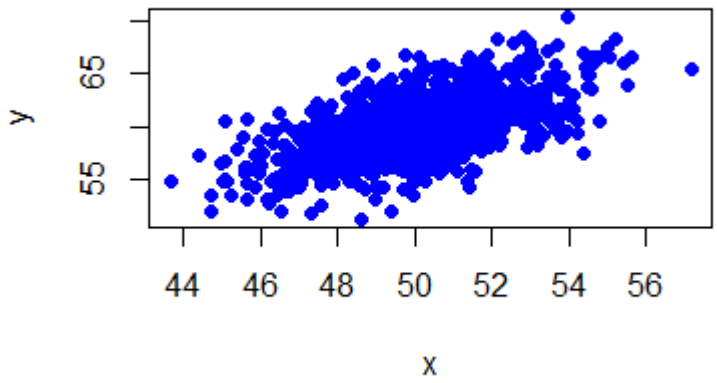
Hide

```


library(MASS)
xy <- mvrnorm(1000,mu=c(50,60),matrix(c(4,3.7,3.7,9),2))

var(xy)
```

```
         [,1]     [,2]
[1,] 3.920389 3.486102
[2,] 3.486102 8.810326
```

Hide

```
x <- xy[,1]
y <- xy[,2]
plot(x,y,pch=16,ylab="y",xlab="x",col="blue")
```



Hide

```
var(x)
```

```
[1] 3.920389
```

Hide

```
var(y)
```

```
[1] 8.810326
```

Hide

```
var(x+y)
```

```
[1] 19.70292
```

Hide

```
var(x)+var(y)
```

```
[1] 12.73071
```

Hide

```
var(x-y)
```

```
[1] 5.758511
```

Hide

```
cor(x,y)*sqrt(var(x)*var(y))
```

```
[1] 3.486102
```

Section 7.3.19

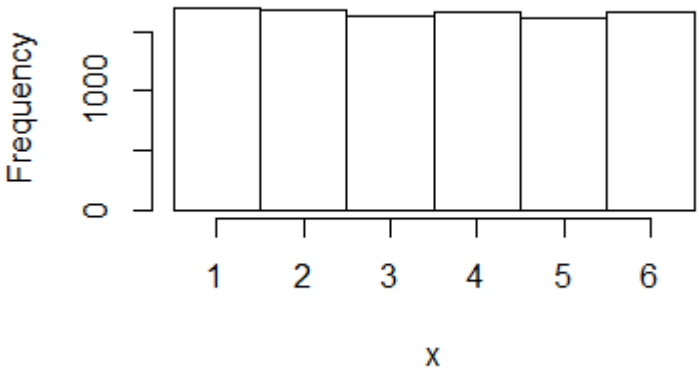
Hide

```
x <- ceiling(runif(10000)*6)
table(x)
```

```
x
   1    2    3    4    5    6
1705 1683 1633 1677 1625 1677
```

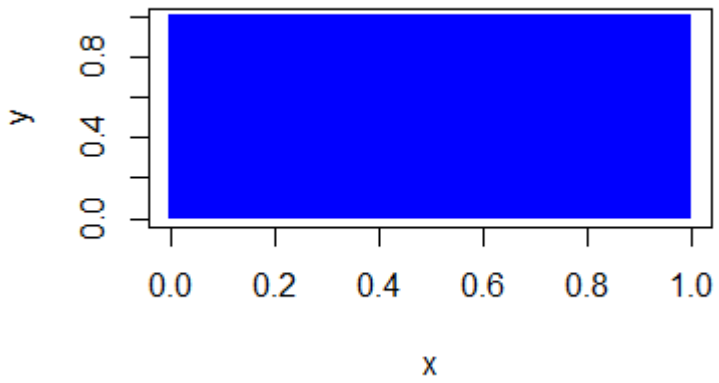
Hide

```
hist(x,breaks=0.5:6.5,main="")
```



Hide

```
x <- runif(1000000)
y <- runif(1000000)
plot(x,y,pch=".",col="blue")
```



Hide

```
table(cut(x,6),cut(y,6))
```

```
                  (-0.000999,0.167] (0.167,0.333]
  (-0.000998,0.167]             27833         27752
  (0.167,0.333]                 27530         27425
  (0.333,0.5]                   27793         27800
  (0.5,0.667]                   27716         27765
  (0.667,0.833]                 27966         27892
  (0.833,1]                     27775         27866

                  (0.333,0.5] (0.5,0.667] (0.667,0.833]
  (-0.000998,0.167]       27999       27960         27716
  (0.167,0.333]           28004       27803         27712
  (0.333,0.5]             27941       28072         27631
  (0.5,0.667]             27826       27812         27824
  (0.667,0.833]           27619       27698         27516
  (0.833,1]               27558       27783         27582

                  (0.833,1]
  (-0.000998,0.167]     27480
  (0.167,0.333]         27779
  (0.333,0.5]           28048
  (0.5,0.667]           27712
  (0.667,0.833]         27925
  (0.833,1]             27887
```

Hide

```
range(table(cut(x,6),cut(y,6)))
```

```
[1] 27425 28072
```

Section 7.3.20

Hide

```
fishes <- read.table("fishes.txt",header=T)
attach(fishes)
```

```
The following object is masked from fishes (pos = 4):

    mass

The following object is masked from fishes (pos = 5):

    mass

The following object is masked from fishes (pos = 6):

    mass

The following object is masked from fishes (pos = 7):

    mass

The following object is masked from fishes (pos = 8):

    mass
```

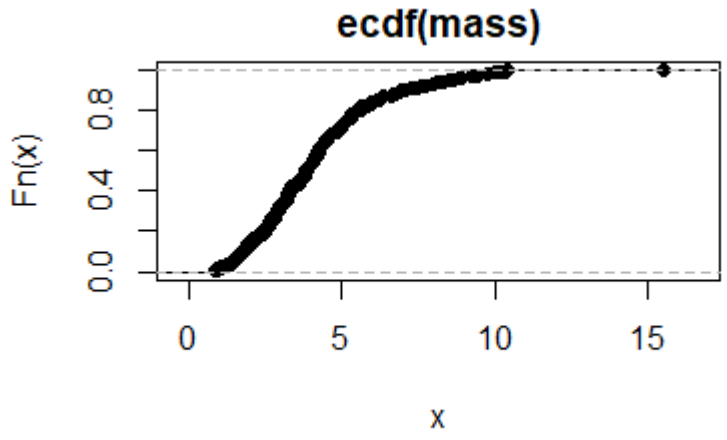
Hide

```
names(fishes)
```

```
[1] "mass"
```

Hide

```
plot(ecdf(mass))
```



Section 7.4

Section 7.4.1

Section 7.4.2

Hide

```
factorial(49)/(factorial(6)*factorial(49-6))
```

```
[1] 13983816
```

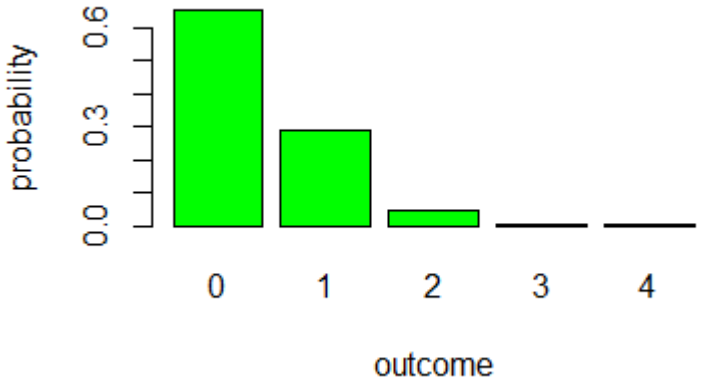
Hide

```
choose(49,6)
```

```
[1] 13983816
```

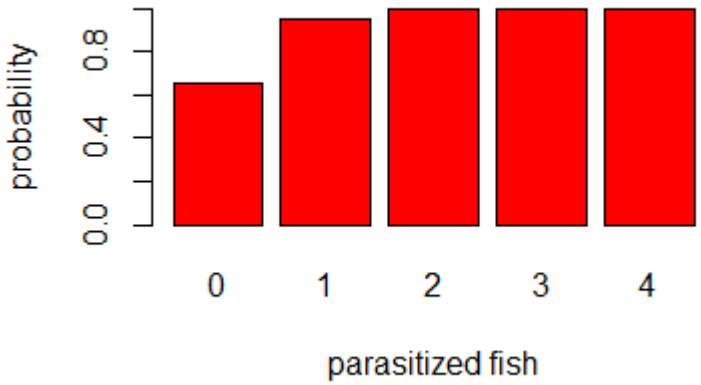
Hide

```
p <- 0.1
n <- 4
x <- 0:n
px <- choose(n,x)*p^x*(1-p)^(n-x)
barplot(px,names=x,xlab="outcome",ylab="probability",col="green")
```



Hide

```
barplot(pbinom(0:4,4,0.1),names=0:4,xlab="parasitized fish",
        ylab="probability",col="red")
```



Hide

```
qbinom(.025,4,0.1)
```

```
[1] 0
```

Hide

```
qbinom(.975,4,0.1)
```

```
[1] 2
```

Hide

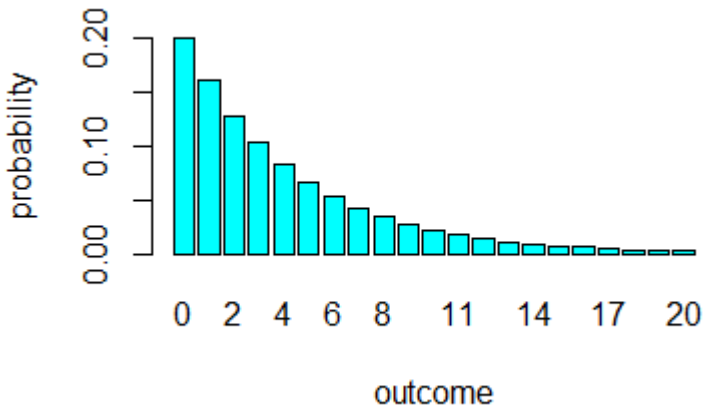
```
rbinom(10,4,0.1)
```

```
 [1] 0 0 0 1 0 0 0 0 1 1
```

Section 7.4.3

Hide

```
fx <- dgeom(0:20,0.2)
barplot(fx,names=0:20,xlab="outcome",ylab="probability",col="cyan")
```



Hide

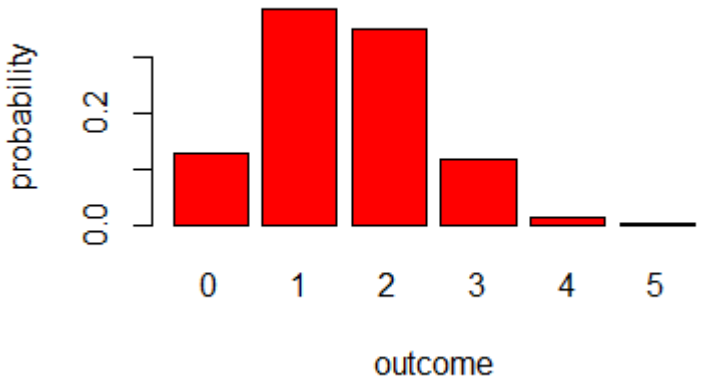
```
table(rgeom(100,0.1))
```

```
 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 17 18 19 20 21
10  9  6  6  4  3 11  7  2  7  3  3  4  5  4  1  3  1  1  2
22 24 25 28 30 38 45 59
 1  1  1  1  1  1  1  1
```

Section 7.4.4

Hide

```
ph <- dhyper(0:5,6,14,5)
barplot(ph,names=(0:5),col="red",xlab="outcome",ylab="probability")
```



Hide

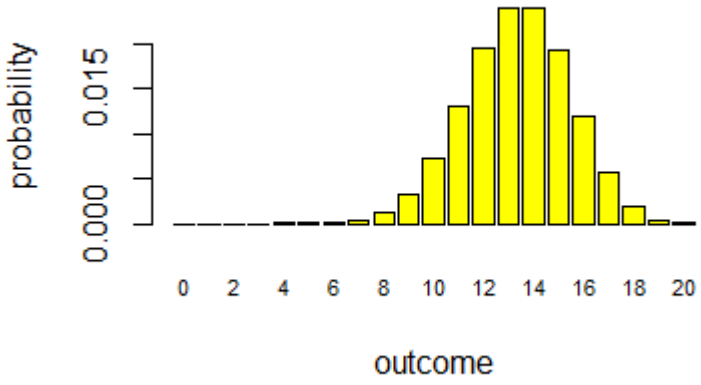
```
rhyper(20,6,14,5)
```

```
 [1] 2 3 1 1 1 1 2 2 2 2 1 2 2 1 2 2 1 2 3 2
```

Section 7.4.5

Hide

```
multi <- function(a,b,c) {
  factorial(a+b+c)/(factorial(a)*factorial(b)*factorial(c))*0.5^a*0.25^b*0.25^c}

barplot(sapply(0:20,function (i) multi(i,20-i,4)),names=0:20,cex.names=0.7,
        xlab="outcome",ylab="probability",col="yellow")
```



Section 7.4.6

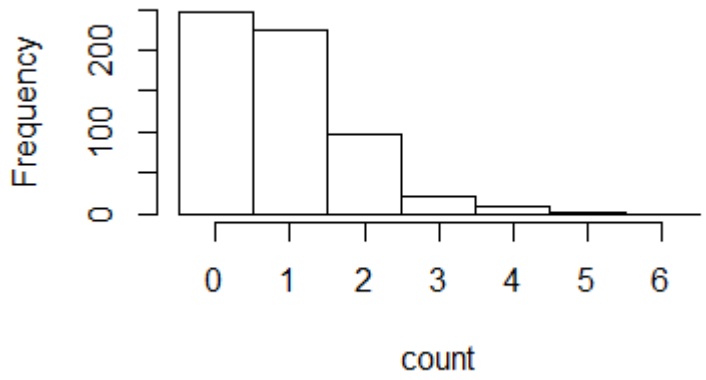
Hide

```
count <- rpois(600,0.9)
table(count)
```

```
count
  0   1   2   3   4   5
247 224  98  22   8   1
```

Hide

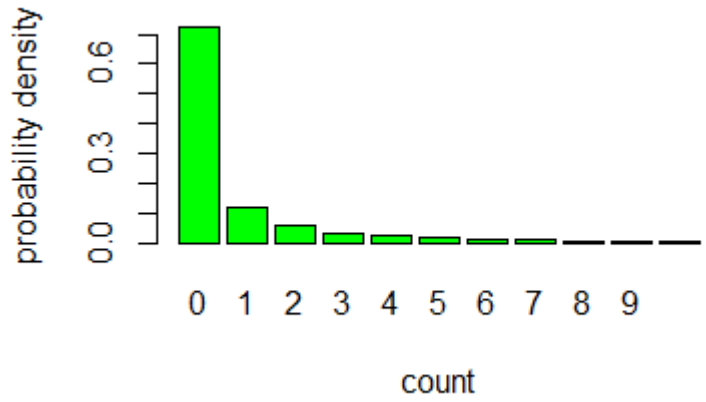
```
hist(count,breaks = - 0.5:6.5,main="")
```



Section 7.4.7

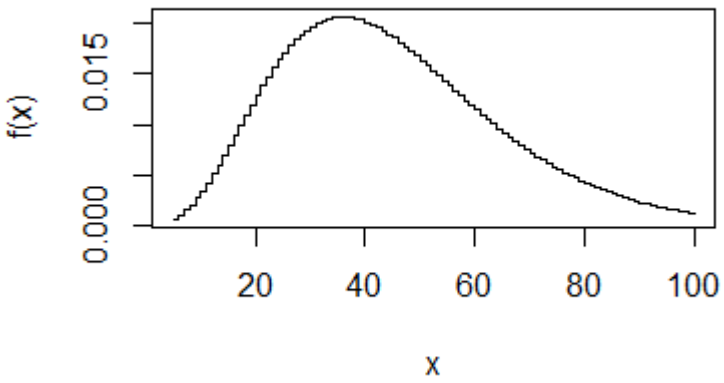
Hide

```
negbin <- function(x,u,k)
  (1+u/k)^(-k)*(u/(u+k))^x*gamma(k+x)/(factorial(x)*gamma(k))

xf <- sapply(0:10, function(i) negbin(i,0.8,0.2))
barplot(xf,names=0:10,xlab="count",ylab="probability density",col="green")
```



Hide

```
plot(5:100,dnbinom(5:100,5,0.1),type="s",xlab="x",ylab="f(x)")
```



Hide

```
count <- rnbinom(100,1,0.6)
table(count)
```

```
count
 0  1  2  3  4
63 17 15  3  2
```

Hide

```
mean(count)
```

```
[1] 0.64
```

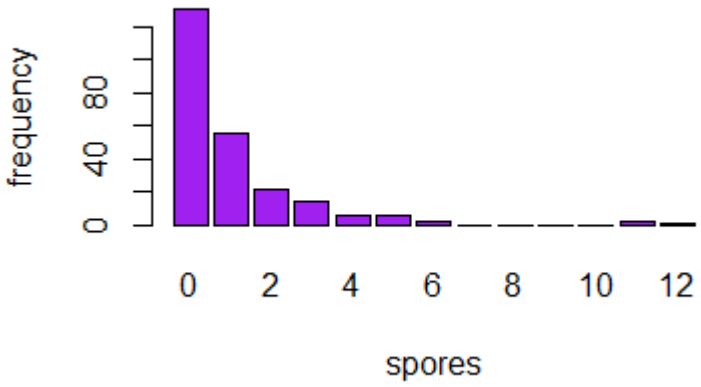
Hide

```
var(count)
```

```
[1] 0.96
```

Hide

```
x <- 0:12
freq <- c(131,55,21,14,6,6,2,0,0,0,0,2,1)
barplot(freq,names=x,ylab="frequency",xlab="spores",col="purple")
```



Hide

```
y <- rep(x,freq)
mean(y)
```

```
[1] 1.004202
```

Hide

```
var(y)
```

```
[1] 3.075932
```

Hide

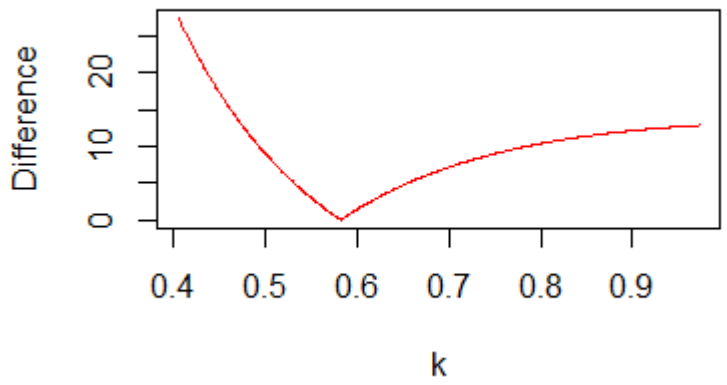
```
mean(y)^2/(var(y)-mean(y))
```

```
[1] 0.4867531
```

Hide

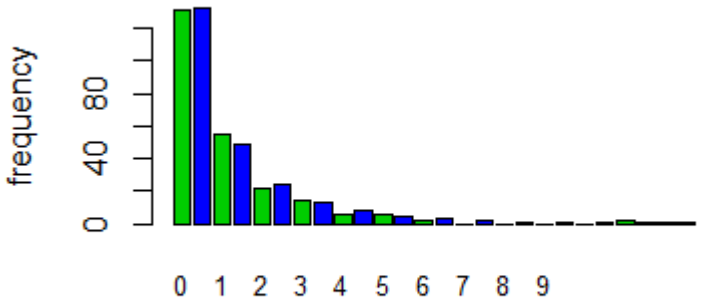
```
kfit <- function(x) {
  lhs <- numeric()
  rhs <- numeric()
  y <- 0:(length(x) - 1)
  j <- 0:(length(x)-2)
  m <- sum(x * y)/(sum(x))
  s2 <- (sum(x * y^2) - sum(x * y)^2/sum(x))/(sum(x)- 1)
  k1 <- m^2/(s2 - m)
  a <- numeric(length(x)-1)
  for(i in 1:(length(x) - 1)) a[i] <- sum(x [- c(1:i)])
  i <- 0
  for (k in seq(k1/1.2,2*k1,0.001)) {
    i <- i+1
    lhs[i] <- sum(x) * log(1 + m/k)
    rhs[i] <- sum(a/(k + j))
  }
  k <- seq(k1/1.2,2*k1,0.001)
  plot(k, abs(lhs-rhs),xlab="k",ylab="Difference",type="l",col="red")
  d <- min(abs(lhs-rhs))
  sdd <- which(abs(lhs-rhs)==d)
  k[sdd]
}

kfit(freq)
```

```
[1] 0.5826276
```



Hide

```
nb <- 238*(1+1.0042/0.582)^(-0.582)*factorial(.582+(0:12)-1)/
  (factorial(0:12)*factorial(0.582-1))*(1.0042/(1.0042+0.582))^(0:12)


both <- numeric(26)
both[1:26 %% 2 != 0] <- freq
both[1:26 %% 2 == 0] <- nb
labs <- as.character(rep(0:12,each=2))
labs[1:26%%2==0] <- ""

barplot(both,col=rep(c(3,4),13),ylab="frequency",names=labs,cex.names=0.8)
```



Hide

```
legend(locator(1),c("observed","expected"),fill=c(3,4))
```

```
Error in legend(locator(1), c("observed", "expected"), fill = c(3, 4)) :
  invalid coordinate lengths
```

Section 7.5

Hide

```
a <- matrix(c(1,0,4,2,-1,1),nrow=3)
a
```

```
     [,1] [,2]
[1,]    1    2
[2,]    0   -1
[3,]    4    1
```

Hide

```
b <- matrix(c(1,-1,2,1,1,0),nrow=2)
b
```

```
     [,1] [,2] [,3]
[1,]    1    2    1
[2,]   -1    1    0
```

Section 7.5.1

Hide

```
a[1,]
```

```
[1] 1 2
```

Hide

```
b[,1]
```

```
[1]  1 -1
```

Hide

```
a[1,]*b[,1]
```

```
[1]  1 -2
```

Hide

```
sum(a[1,]*b[,1])
```

```
[1] -1
```

Hide

```
a[1,]
```

```
[1] 1 2
```

Hide

```
b[,2]
```

```
[1] 2 1
```

Hide

```
a[1,]*b[,2]
```

```
[1] 2 2
```

Hide

```
sum(a[1,]*b[,2])
```

```
[1] 4
```

Hide

```
a[1,]*b[,3]
```

```
[1] 1 0
```

Hide

```
sum(a[1,]*b[,3])
```

```
[1] 1
```

Hide

```
a %*% b
```

```
     [,1] [,2] [,3]
[1,]   -1    4    1
[2,]    1   -1    0
[3,]    3    9    4
```

Hide

```
b %*% a
```

```
     [,1] [,2]
[1,]    5    1
[2,]   -1   -3
```

Section 7.5.2

Hide

```
(ym <- diag(1,3,3))
```

```
     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
```

Hide

```
diag(ym) <- 1:3
ym
```

```
     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    2    0
[3,]    0    0    3
```

Hide

```
diag(ym)
```

```
[1] 1 2 3
```

Hide

```
M <- cbind(X=1:5, Y=rnorm(5))
var(M)
```

```
          X         Y
X  2.5000000 -0.9805489
Y -0.9805489  0.7228564
```

Hide

```
diag(var(M))
```

```
       X         Y
2.5000000 0.7228564
```

Section 7.5.3

Hide

```
A <- matrix(c(1,2,4,2,1,1,3,1,2),nrow=3)
A
```

```
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    2    1    1
[3,]    4    1    2
```

Hide

```
det(A)
```

```
[1] -5
```

Hide

```
B <- A
B[3,] <- 3*B[3,]
B
```

```
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    2    1    1
[3,]   12    3    6
```

Hide

```
det(B)
```

```
[1] -15
```

Hide

```
C <- A
C[,2] <- 0
C
```

```
     [,1] [,2] [,3]
[1,]    1    0    3
[2,]    2    0    1
[3,]    4    0    2
```

Hide

```
det(C)
```

```
[1] 0
```

Section 7.5.4

Hide

```
library(MASS)
ginv(A)
```

```
             [,1] [,2] [,3]
[1,] -2.000000e-01  0.2  0.2
[2,] -5.828671e-16  2.0 -1.0
[3,]  4.000000e-01 -1.4  0.6
```

Hide

```
ginv(ginv(A))
```

```
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    2    1    1
[3,]    4    1    2
```

Hide

```
1/det(ginv(A))
```

```
[1] -5
```

Section 7.5.5

Hide

```
L <- c(0,0.7,0,0,6,0,0.5,0,3,0,0,0.3,1,0,0,0)
L <- matrix(L,nrow=4)
L
```

```
     [,1] [,2] [,3] [,4]
[1,]  0.0  6.0  3.0    1
[2,]  0.7  0.0  0.0    0
[3,]  0.0  0.5  0.0    0
[4,]  0.0  0.0  0.3    0
```

Hide

```
n <- c(45,20,17,3)
n <- matrix(n,ncol=1)
n
```

```
     [,1]
[1,]   45
[2,]   20
[3,]   17
[4,]    3
```

Hide

```
L %*% n
```

```
      [,1]
[1,] 174.0
[2,]  31.5
[3,]  10.0
[4,]   5.1
```

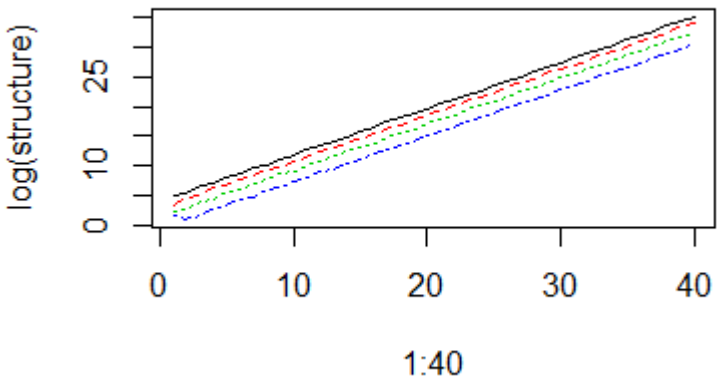
Hide

```
45*0+20*6+17*3+3*1
```

```
[1] 174
```

Hide

```
fun <- function(x) L %*% x
n <- c(45,20,17,3)
n <- matrix(n,ncol=1)
structure <- numeric(160)
dim(structure) <- c(40,4)
for (i in 1:40) {
  n <- fun(n)
  structure[i,] <- n
}
matplot(1:40,log(structure),type="l")
```



Hide

```
sum(structure[40,])/sum(structure[39,])
```

```
[1] 2.164035
```

Hide

```
structure[40,]/sum(structure[40,])
```

```
[1] 0.709769309 0.230139847 0.052750539 0.007340305
```

Hide

```
eigen(L)
```

```
eigen() decomposition
$values
[1]  2.1694041+0.0000000i -1.9186627+0.0000000i
[3] -0.1253707+0.0975105i -0.1253707-0.0975105i

$vectors
               [,1]           [,2]                    [,3]
[1,] 0.949264118+0i -0.93561508+0i -0.01336028-0.03054433i
[2,] 0.306298338+0i  0.34134741+0i -0.03616819+0.14241169i
[3,] 0.070595039+0i -0.08895451+0i  0.36511901-0.28398118i
[4,] 0.009762363+0i  0.01390883+0i -0.87369452+0.00000000i
                       [,4]
[1,] -0.01336028+0.03054433i
[2,] -0.03616819-0.14241169i
[3,]  0.36511901+0.28398118i
[4,] -0.87369452+0.00000000i
```

Hide

```
eigen(L)$vectors[,1]/sum(eigen(L)$vectors[,1])
```

```
[1] 0.710569659+0i 0.229278977+0i 0.052843768+0i
[4] 0.007307597+0i
```

Section 7.5.6

Hide

```
numbers <- read.table("tannin.txt",header=T)
attach(numbers)
names(numbers)
```

```
[1] "growth" "tannin"
```

Hide

```
growth
```

```
[1] 12 10  8 11  6  7  2  3  3
```

Hide

```
sum(growth)
```

```
[1] 62
```

Hide

```
growth^2
```

```
[1] 144 100  64 121  36  49   4   9   9
```

Hide

```
sum(growth^2)
```

```
[1] 536
```

Hide

```
tannin
```

```
[1] 0 1 2 3 4 5 6 7 8
```

Hide

```
sum(tannin)
```

```
[1] 36
```

Hide

```
tannin^2
```

```
[1]  0  1  4  9 16 25 36 49 64
```

Hide

```
sum(tannin^2)
```

```
[1] 204
```

Hide

```
growth*tannin
```

```
[1]  0 10 16 33 24 35 12 21 24
```

Hide

```
sum(growth*tannin)
```

```
[1] 175
```

Hide

```
growth %*% tannin
```

```
     [,1]
[1,]  175
```

Hide

```
growth %*% growth
```

```
     [,1]
[1,]  536
```

Hide

```
growth %*% t(growth)
```

```
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
 [1,]  144  120   96  132   72   84   24   36   36
 [2,]  120  100   80  110   60   70   20   30   30
 [3,]   96   80   64   88   48   56   16   24   24
 [4,]  132  110   88  121   66   77   22   33   33
 [5,]   72   60   48   66   36   42   12   18   18
 [6,]   84   70   56   77   42   49   14   21   21
 [7,]   24   20   16   22   12   14    4    6    6
 [8,]   36   30   24   33   18   21    6    9    9
 [9,]   36   30   24   33   18   21    6    9    9
```

Hide

```
tannin %*% tannin
```

```
     [,1]
[1,]  204
```

Hide

```
growth %*% rep(1,9)
```

```
     [,1]
[1,]   62
```

Hide

```
tannin %*% rep(1,9)
```

```
     [,1]
[1,]   36
```

Hide

```
rep(1,9 )%*% rep(1,9)
```

```
     [,1]
[1,]    9
```

Hide

```
a <- cbind(growth,tannin)
a
```

```
      growth tannin
 [1,]     12      0
 [2,]     10      1
 [3,]      8      2
 [4,]     11      3
 [5,]      6      4
 [6,]      7      5
 [7,]      2      6
 [8,]      3      7
 [9,]      3      8
```

Hide

```
t(a) %*% a
```

```
       growth tannin
growth    536    175
tannin    175    204
```

Hide

```
b <- cbind(1,growth,tannin)
b
```

```
        growth tannin
 [1,] 1     12      0
 [2,] 1     10      1
 [3,] 1      8      2
 [4,] 1     11      3
 [5,] 1      6      4
 [6,] 1      7      5
 [7,] 1      2      6
 [8,] 1      3      7
 [9,] 1      3      8
```

Hide

```
dimnames(b)[[2]] [1] <- "sample"

t(b) %*% b
```

```
       sample growth tannin
sample      9     62     36
growth     62    536    175
tannin     36    175    204
```

Section 7.5.7

Hide

```
Y <- growth
one <- rep(1,9)

t(one) %*% one
```

```
     [,1]
[1,]    9
```

Hide

```
X <- cbind(1,tannin)
X
```

```
       tannin
 [1,] 1      0
 [2,] 1      1
 [3,] 1      2
 [4,] 1      3
 [5,] 1      4
 [6,] 1      5
 [7,] 1      6
 [8,] 1      7
 [9,] 1      8
```

Hide

```
t(Y) %*% Y
```

```
     [,1]
[1,]  536
```

Hide

```
t(one) %*% Y
```

```
     [,1]
[1,]   62
```

Hide

```
t(Y) %*% one %*% t(one) %*% Y
```

```
     [,1]
[1,] 3844
```

Hide

```
t(X) %*% X
```

```
          tannin
        9     36
tannin 36    204
```

Hide

```
t(X) %*% Y
```

```
       [,1]
         62
tannin  175
```

Hide

```
library(MASS)
ginv(t(X) %*% X)
```

```
           [,1]       [,2]
[1,]  0.37777778 -0.06666667
[2,] -0.06666667  0.01666667
```

Hide

```
ginv(t(X) %*% X) %*% t(X) %*% Y
```

```
         [,1]
[1,] 11.755556
[2,] -1.216667
```

Hide

```
CF <- t(Y) %*% one %*% t(one) %*% Y/9
CF
```

```
        [,1]
[1,] 427.1111
```

Hide

```
t(Y) %*% Y - CF
```

```
        [,1]
[1,] 108.8889
```

Hide

```
b <- ginv(t(X) %*% X) %*% t(X) %*% Y
t(b) %*% t(X) %*% Y - CF
```

```
        [,1]
[1,] 88.81667
```

Hide

```
t(Y) %*% Y - t(b) %*% t(X) %*% Y
```

```
        [,1]
[1,] 20.07222
```

Section 7.6

Hide

```
A <- matrix(c(3,1,4,2),nrow=2)
A
```

```
     [,1] [,2]
[1,]    3    4
[2,]    1    2
```

Hide

```
kv <- matrix(c(12,8),nrow=2)
kv
```

```
     [,1]
[1,]   12
[2,]    8
```

Hide

```
solve(A,kv)
```

```
     [,1]
[1,]   -4
[2,]    6
```

Section 7.7

Section 7.7.1

Hide

```
D(expression(2*x^3),"x")
```

```
2 * (3 * x^2)
```

Hide

```
D(expression(log(x)),"x")
```

```
1/x
```

Hide

```
D(expression(a*exp(-b * x)),"x")
```

```
-(a * (exp(-b * x) * b))
```

Hide

```
D(expression(a/(1+b*exp(-c * x))),"x")
```

```
a * (b * (exp(-c * x) * c))/(1 + b * exp(-c * x))^2
```

Hide

```
trig.exp <- expression(sin(cos(x + y^2)))
D(trig.exp, "x")
```

```
-(cos(cos(x + y^2)) * sin(x + y^2))
```

Hide

```
-(cos(cos(x + y^2)) * sin(x + y^2))
```

```
longer object length is not a multiple of shorter object lengthlonger object length is not a mul
tiple of shorter object length
```

```
  [1]  0.00000000 -0.72160615 -0.83169192 -0.07743200
  [5]  0.60080541  0.92060272  0.16021139 -0.47895859
  [9] -0.97890422 -0.25256805  0.36348984  0.99998041
 [13]  0.35659995  0.00000000 -0.72160615 -0.83169192
 [17] -0.07743200  0.60080541  0.92060272  0.16021139
 [21] -0.47895859 -0.97890422 -0.25256805  0.36348984
 [25]  0.99998041  0.35659995  0.00000000 -0.72160615
 [29] -0.83169192 -0.07743200  0.60080541  0.92060272
 [33]  0.16021139 -0.47895859 -0.97890422 -0.25256805
 [37]  0.36348984  0.99998041  0.35659995  0.00000000
 [41] -0.72160615 -0.83169192 -0.07743200  0.60080541
 [45]  0.92060272  0.16021139 -0.47895859 -0.97890422
 [49] -0.25256805  0.36348984  0.99998041  0.35659995
 [53]  0.00000000 -0.72160615 -0.83169192 -0.07743200
 [57]  0.60080541  0.92060272  0.16021139 -0.47895859
 [61] -0.97890422 -0.25256805  0.36348984  0.99998041
 [65]  0.35659995  0.00000000 -0.72160615 -0.83169192
 [69] -0.07743200  0.60080541  0.92060272  0.16021139
 [73] -0.47895859 -0.97890422 -0.25256805  0.36348984
 [77]  0.99998041  0.35659995  0.00000000 -0.72160615
 [81] -0.83169192 -0.07743200  0.60080541  0.92060272
 [85]  0.16021139 -0.47895859 -0.97890422 -0.25256805
 [89]  0.36348984  0.99998041  0.35659995  0.00000000
 [93] -0.72160615 -0.83169192 -0.07743200  0.60080541
 [97]  0.92060272  0.16021139 -0.47895859 -0.97890422
[101] -0.25256805  0.36348984  0.99998041  0.35659995
[105]  0.00000000 -0.72160615 -0.83169192 -0.07743200
[109]  0.60080541  0.92060272  0.16021139 -0.47895859
[113] -0.97890422 -0.25256805  0.36348984  0.99998041
[117]  0.35659995  0.00000000 -0.72160615 -0.83169192
[121] -0.07743200  0.60080541  0.92060272  0.16021139
[125] -0.47895859 -0.97890422 -0.25256805  0.36348984
[129]  0.99998041  0.35659995  0.00000000 -0.83169192
[133] -0.07743200  0.60080541  0.92060272  0.16021139
[137] -0.47895859 -0.97890422 -0.25256805  0.36348984
[141]  0.99998041  0.35659995 -0.25872185 -0.72160615
[145] -0.83169192 -0.07743200  0.60080541  0.92060272
[149]  0.16021139 -0.47895859 -0.97890422 -0.25256805
[153]  0.36348984  0.99998041  0.35659995 -0.25872185
[157] -0.72160615 -0.83169192 -0.07743200  0.60080541
[161]  0.92060272  0.16021139 -0.47895859 -0.97890422
[165] -0.25256805  0.36348984  0.99998041  0.35659995
[169] -0.25872185 -0.72160615 -0.83169192 -0.07743200
[173]  0.60080541  0.92060272  0.16021139 -0.47895859
[177] -0.97890422 -0.25256805  0.36348984  0.99998041
[181]  0.35659995 -0.25872185 -0.72160615 -0.83169192
[185] -0.07743200  0.60080541 -0.97890422 -0.25256805
[189]  0.36348984  0.99998041  0.35659995 -0.25872185
[193] -0.98136105 -0.47149184  0.16566986  0.60080541
[197]  0.92060272  0.16021139 -0.47895859 -0.97890422
[201] -0.25256805  0.36348984  0.99998041  0.35659995
[205] -0.25872185 -0.98136105 -0.47149184 -0.71426062
[209] -0.25256805  0.36348984  0.99998041  0.35659995
```

```
[213] -0.25872185 -0.98136105 -0.47149184  0.16566986
[217]  0.92523052  0.59312824 -0.08239835 -0.83797751
[221] -0.71426062  0.16566986  0.92523052  0.59312824
[225] -0.08239835 -0.83797751 -0.71426062  0.24646084
[229] -0.37042264 -0.99982373 -0.34975360  0.26492241
[233]  0.98366988  0.61614758  0.98366988 -0.32281412
[237]  0.29019406 -0.42731375
```

Section 7.7.2

Hide

```
integrate(dnorm,0,Inf)
```

```
0.5 with absolute error < 4.7e-05
```

Hide

```
integrate(dnorm,-Inf,Inf)
```

```
1 with absolute error < 9.4e-05
```

Hide

```
integrate(function(x) rep(2, length(x)), 0, 1)
```

```
2 with absolute error < 2.2e-14
```

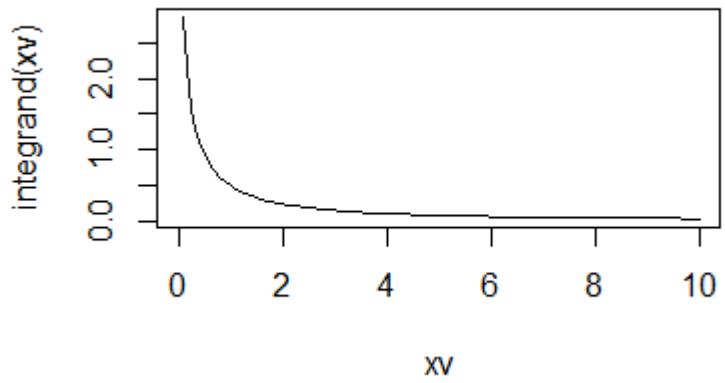
Hide

```
integrand <- function(x) {1/((x+1)*sqrt(x))}
integrate(integrand, lower = 0, upper = Inf)
```

```
3.141593 with absolute error < 2.7e-05
```

Hide

```
xv <- seq(0,10,0.1)
plot(xv,integrand(xv),type="l")
```



Section 7.7.3

Hide

```
install.packages("deSolve")
```

```
WARNING: Rtools is required to build R packages but is not currently installed. Please download
and install the appropriate version of Rtools before proceeding:

https://cran.rstudio.com/bin/windows/Rtools/
Installing package into 愀鞄C:/Users/Nathan/Documents/R/win-library/3.6愀牷
(as 愀鞄lib愀牷 is unspecified)
trying URL 'https://cran.rstudio.com/bin/windows/contrib/3.6/deSolve_1.24.zip'
Content type 'application/zip' length 2912323 bytes (2.8 MB)
downloaded 2.8 MB
```

```
package ‘deSolve’ successfully unpacked and MD5 sums checked

The downloaded binary packages are in
    C:\Users\Nathan\AppData\Local\Temp\RtmpwTf5lf\downloaded_packages
```

Hide

```
library(deSolve)

phmodel <- function(t,state,parameters){
  with(as.list(c(state,parameters)),{
    dv <- r*v*(K-v)/K-b*v*n
    dn <- c*v*n-d*n
    result <- c(dv,dn)
    list(result)
  })}

times <- seq(0,500,length=501)
parameters <- c(r=0.4,K=1000,b=0.02,c=0.01,d=0.3)
initial <- c(v=50,n=10)
output <- ode(y=initial,time=times,func=phmodel,parms=parameters)
head(output)
```

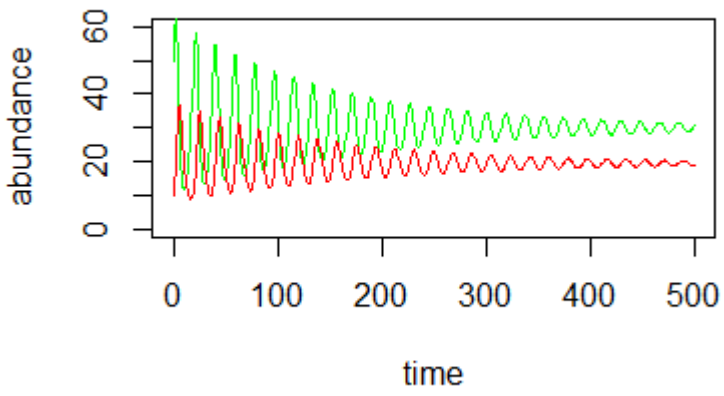
```
     time        v        n
[1,]    0 50.00000 10.00000
[2,]    1 58.29220 12.75106
[3,]    2 62.99695 17.40172
[4,]    3 60.70065 24.09264
[5,]    4 50.79407 31.32860
[6,]    5 37.68312 36.12636
```

Hide

```
plot(output[,1],output[,2],
     ylim=c(0,60),type="n",ylab="abundance",xlab="time")
lines(output[,1],output[,2],col="green")
```

Hide

```
lines(output[,1],output[,3],col="red")
```



Hide

```
plot(output[,3],output[,2],
     ylim=c(0,70),xlim=c(0,70),type="n",ylab="plant",xlab="herbivore")
lines(output[,2],output[,3],col="red")
```