CSCE 531 Discrete Mathematics Fall 2017 Homework 1

Section 1.4

1. (Inspired by Problem 10) Let $C(x)$be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

(a) A student in your class has a cat, a dog, and a ferret.

$\exists x\,(C(x) \wedge D(x) \wedge F(x))$

(e) For each of the three [types of animal (cats, dogs, and ferrets)], there is a student in your class who has one of these animals as a pet.

$\exists x\,(C(x) \vee D(x) \vee F(x))$

2. (Problem 36) Find counterexamples, if possible, to these universally quantified statements, where the domain for all statements consists of all real numbers. $\mathbb{R}$

(a) $\forall x(x^2 \neq x)$

False if $x=1$ $\quad x^2=x$

$(1)^2=1$

$1=1$ is a counterexample to $x^2 \neq x$

(b) $\forall x(x^2 \neq 2)$

False if $x=\sqrt{2}$ then $(\sqrt{2})^2=2$ this is a counterexample of $x^2 \neq 2$

(c) $\forall x(|x| > 0)$

False if $x=0$ then $|0|>0$ or $0>0$ (which is false)

Section 1.5

3. (Inspired by Problem 28) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. You may need to refer to Appendix 1.

(a) $\forall x \exists y(x^2 = y)$ = For every x there exists a y such that $x^2 = y$
True $[y = x^2]$
Case $y = 4$ $x = 2$ $(2)^2 = 4$ ✓

(b) $\forall x \exists y(x = y^2)$ = For every x there exists a y such that $x = y^2$
True for all $\mathbb{R}$: when $y = \sqrt{x}$
Case: If $x = -1$ there are no real numbers squared that equals -1

(c) $\exists x \forall y(xy = 0)$
There exists an x for every y such that $xy = 0$
True if $x = 0$ All $\mathbb{R}$ multiplied by zero is zero

(d) $\exists x \exists y(x + y \neq y + x)$
False: This would contradict the communitive property of adition

(e) $\forall x[x \neq 0 \rightarrow \exists y(xy = 1)]$ then there exists a y such that $xy = 1$
For every x, if $x \neq 0$
If $x \neq 0$ (which is given) and $y = \frac{1}{x}$ this is true. Multiplicative Inverse

(f) $\exists x \forall y(y \neq 0 \rightarrow xy = 1)$
There exists an x such that for every y, $xy = 1$ if $y \neq 0$.
This is false because you can't have $\forall y$ and $y \neq 0$

(g) $\forall x \exists y(x + y = 1)$
True $y = 1 - x$

(h) $\exists x \exists y(x + 2y = 2 \wedge 2x + 4y = 5)$
$2(x + 2y = 2)$
$2x + 4y = 5$
$2x + 4y = 4$
$2x + 4y = 5$
If $x = 2 - 2y$
Unsolvable Equations
$2(2 - 2y) + 4y = 5$
$4 - 4y + 4y = 5$ Uns
False → Equations cannot equal 2 and 5 at the same time for the same values of x and

(i) $\forall x \exists y(x + y = 2 \wedge 2x - y = 1)$
For all values of x there exists a y such that $x + y = 2$ and $2x - y = 1$
If $y = 2 - x$
$2x - (2 - x) = 1$
$2x - 2 + x = 1$
$3x = 3$ $x = 1$
If $x = 1$ $y = 2 - 1$ $y = 1$
$\boxed{\text{True if } x = 1 \text{ and } y = 1}$

(j) $\forall x \forall y \exists z \left(z = \frac{x+y}{2}\right)$
True for all $\mathbb{R}$
All numbers x and y can be averaged.