CSCE 531 Discrete Mathematics Fall 2017 Homework 1

<u>Section 1.4</u>

1. (Inspired by Problem 10) Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

    (a) A student in your class has a cat, a dog, and a ferret.

    (e) For each of the three [types of animal (cats, dogs, and ferrets)], there is a student in your class who has one of these animals as a pet.

2. (Problem 36) Find counterexamples, if possible, to these universally quantified statements, where the domain for all statements consists of all real numbers.

    (a) $\forall x(x^2 \neq x)$

    (b) $\forall x(x^2 \neq 2)$

    (c) $\forall x(|x| > 0)$

3. (Inspired by Problem 28) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. You may need to refer to Appendix 1.

(a) $\forall x \exists y (x^2 = y)$

(b) $\forall x \exists y (x = y^2)$

(c) $\exists x \forall y (xy = 0)$

(d) $\exists x \exists y (x + y \neq y + x)$

(e) $\forall x [x \neq 0 \rightarrow \exists y (xy = 1)]$

(f) $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$

(g) $\forall x \exists y (x + y = 1)$

(h) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$

(i) $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$

(j) $\forall x \forall y \exists z \left(z = \frac{x+y}{2} \right)$