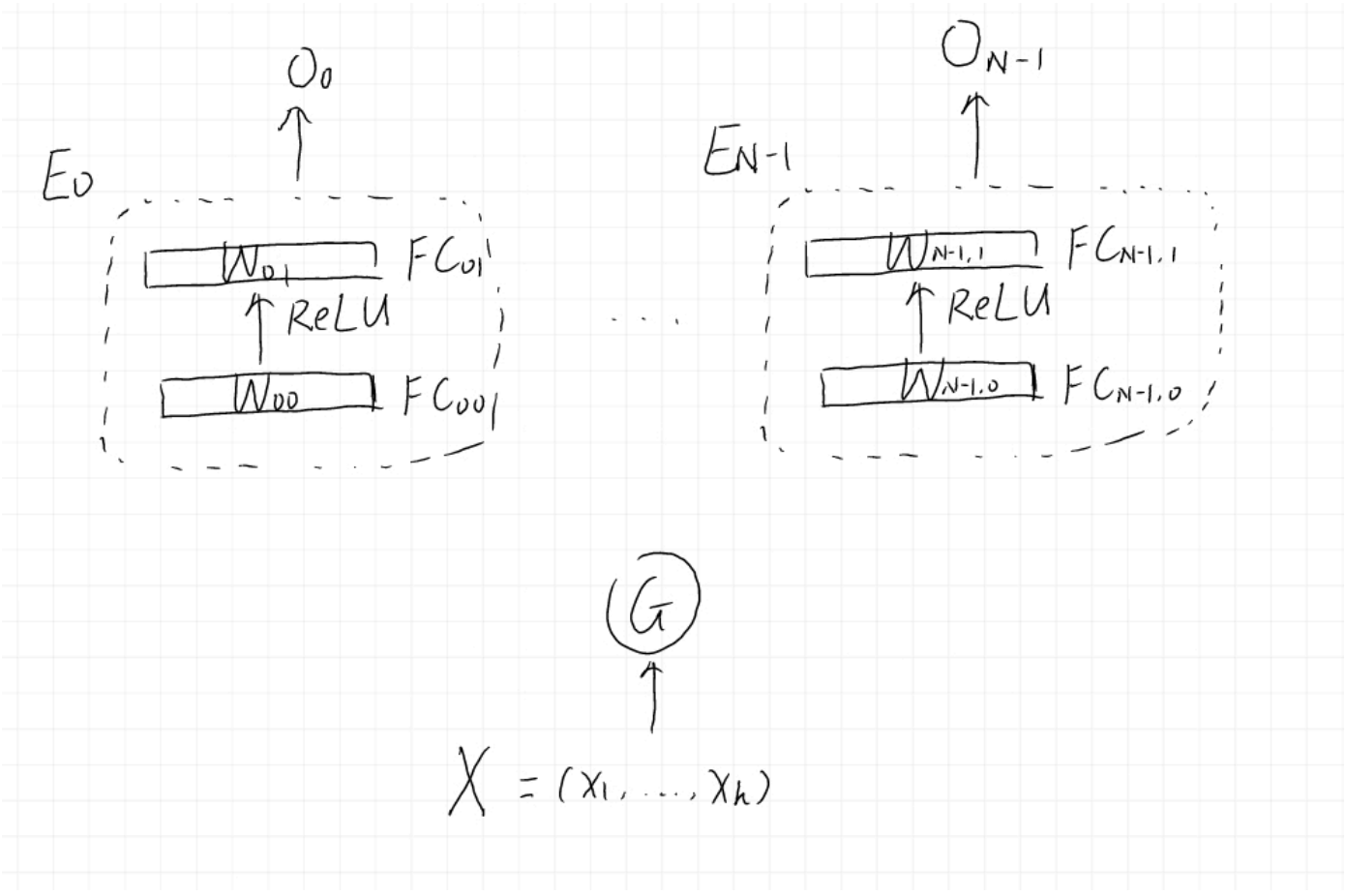
0. Notations:

- N: number of experts, $N > 1$
- d: number of slices per expert, $d > 1$
- h: length of embedding vectors
- m: dimension of the first fully connected layer
- n: dimension of the second fully connected layer
- b: batch size
- B_{comp}: computations per second
- B_{comm}: communications per second
- r: each token is routed to top-r experts ($1 \leq r < N$)
- T_0: time consumed to dispatch a batch of tokens when there is no slicing (it is also the time consumed to combine outputs of experts), note that $T_0 \leq \frac{\frac{b}{d} \cdot h(d-1)}{B_{comm}} = \frac{bh(d-1)}{dB_{comm}}$
- δ: time saved by doing expert-slicing per batch
- R: ratio of sliced MoE inference time over unsliced MoE inference time

1. Structure of Moe (according to GShard):

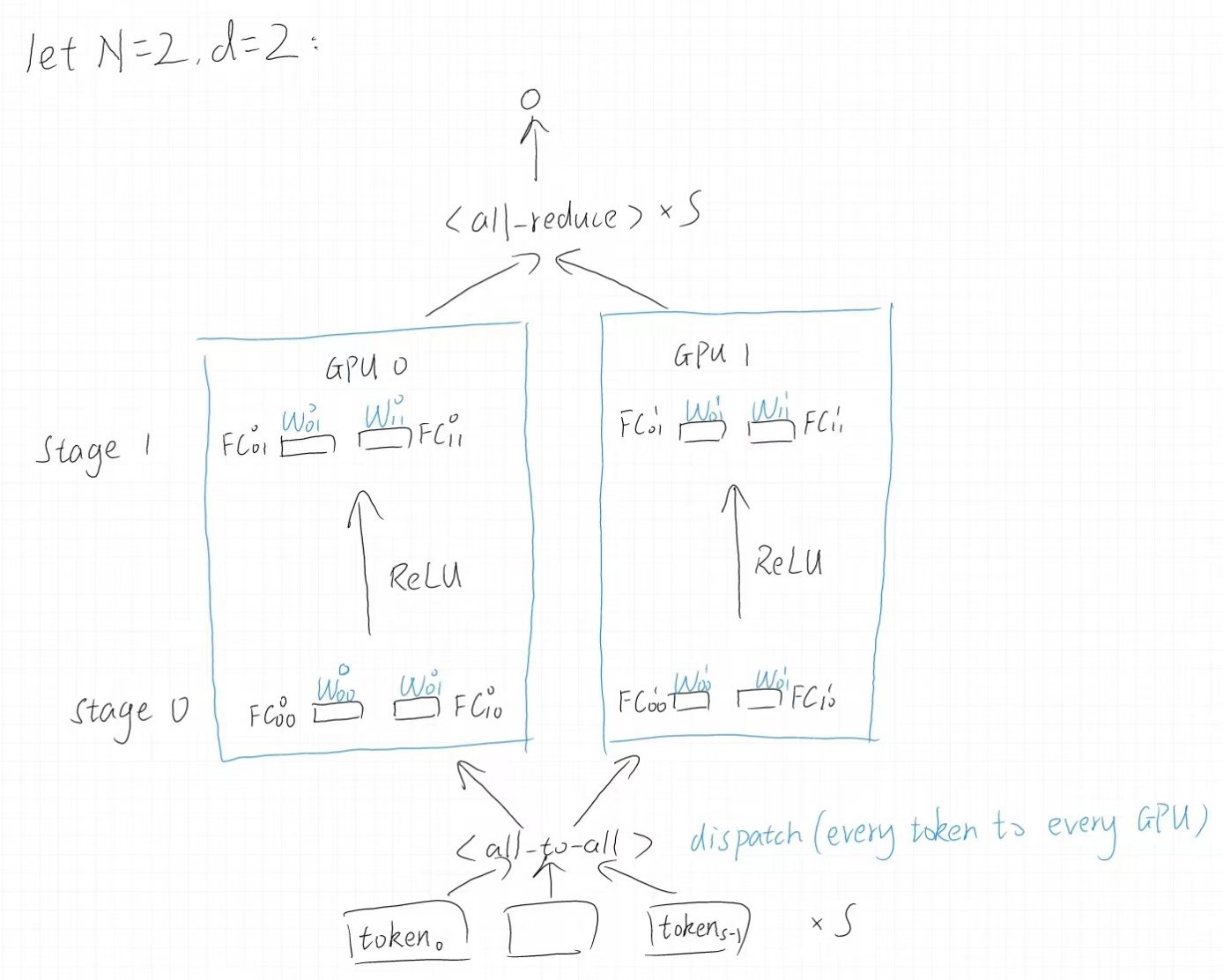


FC_{ij} stands for the j-th fully connected layer of the i-th expert, and W_{ij} denotes the matrix corresponding to FC_{ij}.

$o_i = W_{i1}\text{Activation}(\text{W}_{\text{i0}}\text{x}) \cdot \text{G}_\text{i}(\text{x}), \text{G}_\text{i}(\text{x}) \in [0, 1]$

2. Slicing (N=d=2)

2.1 Structure



Assume for token x is routed to expert E_0.

$$W_{00} = \begin{bmatrix} W_{00}^0 \\ W_{00}^1 \end{bmatrix}, W_{01} = \begin{bmatrix} W_{01}^0 & W_{01}^1 \end{bmatrix}$$

$$t = \text{Activation}(\text{W}_{00}\text{x}) = \begin{bmatrix} \text{Activation}(\text{W}_{00}^0\text{x}) \\ \text{Activation}(\text{W}_{00}^1\text{x}) \end{bmatrix}$$

$$o = W_{01}t = W_{01}^0\text{Activation}(\text{W}_{00}^0\text{x}) + \text{W}_{01}^1\text{Activation}(\text{W}_{00}^1\text{x})$$

2.2 Analysis

By doing expert-slicing, computation and communication cost change as follows:

	Before	After
Number of multiplications per GPU per token	$hm + mn$	$(hm + mn)/2$
Number of additions per GPU per token	$(h-1)m + (m-1)n$	$(h-1)m/2 + (m-1)n/2$
Number of activations per GPU per token	$m + n$	$m/2 + n/2$
Communication cost per batch	$2T_0$	$\frac{bh}{2B_{comm}} + \frac{bh}{2B_{comm}} = \frac{bh}{B_{comm}}$

3 General Situation

	Before	After
Number of multiplications per GPU per token	$(hm + mn) \cdot \frac{N}{d}$	$(hm + mn) \cdot \frac{N}{d^2}$
Number of additions per GPU per token	$[(h-1)m + (m-1)n] \cdot \frac{N}{d}$	$[(h-1)m + (m-1)n] \cdot \frac{N}{d^2}$
Number of activations per GPU per token	$(m + n) \cdot \frac{N}{d}$	$(m + n) \cdot \frac{N}{d^2}$
Communication cost per batch	$2T_0$	$\frac{bh(d-1)}{dB_{comm}} + \frac{bh(d-1)}{dB_{comm}} = \frac{2bh(d-1)}{dB_{comm}}$

$\delta = 2b(1 - \frac{1}{d})\frac{hm+mn}{B_{comp}} \cdot \frac{N}{d} + 2T_0 - \frac{2bh(d-1)}{dB_{comm}}$

Since $1 - \frac{1}{d} > 0$ while $T_0 \leq \frac{bh(d-1)}{dB_{comm}}$, there is a trade-off between less computation cost and more communication cost. If δ proves **positive**, then expert-slicing is effective.

Moment Estimation of T_0, δ, R

Denote the number of communications required for token $x_{ij} (0 \leq j \leq \frac{b}{d} - 1)$ on GPU $i (0 \leq i \leq d-1)$ as $G_{ij} \in [\frac{(r-1)N}{d}, \frac{rN}{d}]$, the number of communications between GPU i and other GPUs as M_i.

It is well-known that $\binom{d}{r} = \binom{d-1}{r-1} + \binom{d-1}{r}$.

$E(G_{ij}) = \frac{N}{d}[(r-1) \cdot \frac{\binom{d-1}{r-1}}{\binom{d}{r}} + r \cdot \frac{\binom{d-1}{r}}{\binom{d}{r}}] = \frac{Nr(d-1)}{d^2}$

$E(M_i) = E(\sum_{j=0}^{\frac{b}{d}-1} G_{ij}) = \sum_{j=0}^{\frac{b}{d}-1} E(G_{ij}) = \frac{bNr(d-1)}{d^3}$

$E(T_0) = E(\frac{hM_i}{B_{comm}}) = \frac{bNhr(d-1)}{d^3 B_{comm}}, 1 \leq r < N$

For sparsely-gated MoE network, assuming $m = 4h$, $n = h$, $r = 1$, we have

$\hat{\delta} = 2bh(1 - \frac{1}{d})[\frac{8h}{B_{comp}} \cdot \frac{N}{d} + \frac{1}{B_{comm}}(\frac{N}{d^2} - 1)]$

$\hat{\delta} > 0 \Leftrightarrow h > \frac{1}{8}\frac{B_{comp}}{B_{comm}}(1 - \frac{N}{d^2}) \cdot \frac{d}{N}$

$\hat{R} = \frac{\frac{16h^2 \cdot \frac{N}{d^2}}{B_{comp}} + \frac{2h(d-1)}{dB_{comm}}}{\frac{16h^2 \cdot \frac{N}{d}}{B_{comp}} + \frac{2Nh(d-1)}{d^3 B_{comm}}} \to \frac{1}{d} (h \to \infty)$