Summarizing results from a simulation study

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[1] Parts of this slide set are a (modified) version of slides accompanying Chapter 3 of the book by Strobl, Henninger, Rothacher, and Debelak (2024)

Estimator properties

- One use of simulation studies it to examine the properties of statistical methods
- When we talk about "statistical methods," we usually mean the estimation of one or more model parameters
- To evaluate the quality of point estimators, several relevant criteria exist[2]
  - Unbiasedness
  - Consistency
  - Efficiency
  - Suffiency
- We will just look at two of them here: Unbiasedness and Consistency

[2] See, e. g., `https://en.wikipedia.org/wiki/Estimator` or any introductory statistics book

Quality of point estimators

Unbiasedness

- An estimator is called unbiased if the estimated values on average equal the true value
- If this only holds for large enough samples, the estimator is called asymptotically unbiased
- In our first simulation study, we examined the unbiasedness of the estimated slope coefficient in the regression model using plots and the bias
- Besides the bias, there are other statistics to examine unbiasedness

$$\widehat{Bias}_s = \frac{1}{niter} \sum_{i=1}^{niter} \left(\hat{\theta}_{is} - \theta_s \right)$$

Quality of point estimators

Unbiasedness

- Percent bias

$$\widehat{\%\,Bias}_s = \frac{\frac{1}{niter}\sum_{i=1}^{niter}(\hat{\theta}_{is} - \theta_s)}{\theta_s}$$

- Standardized bias

$$\widehat{stand\,Bias}_s = \frac{\frac{1}{niter}\sum_{i=1}^{niter}(\hat{\theta}_{is} - \theta_s)}{se_s\left(\widehat{Bias}\right)}$$

- Absolute bias

$$\widehat{abs\,Bias}_s = \frac{1}{niter}\sum_{i=1}^{niter}|\hat{\theta}_{is} - \theta_s|,$$

- Mean squared error (MSE) / Root mean squared error (RMSE)

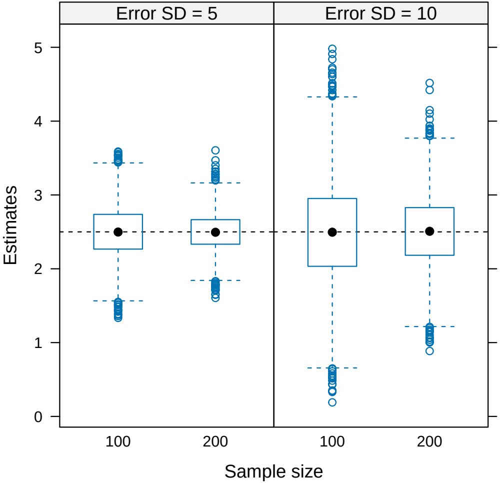
$$\widehat{MSE}_s = \frac{1}{niter}\sum_{i=1}^{niter}(\hat{\theta}_{is} - \theta_s)^2 \quad / \quad \widehat{RMSE}_s = \sqrt{\frac{1}{niter}\sum_{i=1}^{niter}(\hat{\theta}_{is} - \theta_s)^2}$$

$\rightarrow$ For unbiased estimators, all of the discussed statistics should be close to 0

Quality of point estimators

Consistency

- An estimator is consistent if the estimates scatter closer around the true value as the sample size increases
- If an estimator is consistent, it is also (asymptotically) unbiased
- The consistency of an estimator can be checked by graphically displaying the distribution of the individual estimates for different sample sizes



Quality of confidence intervals

- Confidence intervals (CIs) with a confidence level of e.g. 95% can be constructed around point estimators
- That means, if we were to repeatedly collect samples and would calculate the confidence interval from each sample, about 95% of the confidence intervals would cover the true value
- In a simulation study we can determine the actual proportion of confidence intervals that cover the true value
- This is called the estimated coverage probability

$$\widehat{CP}_s = \frac{\sum_{i=1}^{niter} I(\theta_s \in CI_{is})}{niter},$$

where I denotes the indicator function that takes the value 1 if the event specified in parentheses occurs, and otherwise takes the value 0

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  - What test for normality should we pick?
  - Anything else you would like to vary?
- How can we quantify the biasedness of our parameter estimate?
- How can we determine if the estimator is consistent?

References

Strobl, C., Henninger, M., Rothacher, Y., & Debelak, R. (2024). *Simulationsstudien in R: Design und praktische Durchführung*. Springer Berlin.