Contrast coding in linear (mixed-effects) models

Nora Wickelmaier

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- For ANOVAs, so-called sum or effects coding is usually used

1. 2×2 example for linear mixed-effects model

Example by Brehm and Alday (2022)

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    2. *Foods* with levels *Salad* and *Soup*

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- The dependent measure is *RT* (speed of eating in minutes)
- We will simulate 20 subjects and 10 food ingredients (items)
- The data is generated by the following model

$$RT = \beta_0 + \beta_1 \mathit{Utensils} + \beta_2 \mathit{Foods} + \beta_3 \mathit{Utensils} \times \mathit{Foods} + \upsilon_0 + \eta_0 + \varepsilon$$

with $\upsilon_0 \sim N(0, \sigma_\upsilon^2)$, $\eta_0 \sim N(0, \sigma_\eta^2)$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, all i.i.d.

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with $\upsilon_0 \sim N(0, \sigma_\upsilon^2)$, $\eta_0 \sim N(0, \sigma_\eta^2)$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, all i.i.d.

Example by Brehm and Alday (2022)

```
library(lme4)
set.seed(1012)

# Average time to eat in min
SpoonSoup  <- 5
ForkSoup   <- 10
SpoonSalad <- 10
ForkSalad  <- 5
# Standard deviation for all groups
Groupsd <- 2
# Number of subjects (ps) and ingredients (ii)
ps <- 20
ii <- 10
```

Example by Brehm and Alday (2022)

```r
# Create data frame for given design
ds <- data.frame(
  Utensils    = factor(rep(c("Spoon", "Fork"), each = ps * ii, times = 2)),
  Foods       = factor(rep(c("Soup", "Salad"), each = ps * ii * 2)),
  Participant = factor(rep(paste0("p", 1:ps), times = ii * 4)),
  Item        = factor(rep(paste0("i", 1:ii), each = ps, times = 4)))

# Simulate data
ds$RT <- c(rnorm(ps * ii, mean = SpoonSoup,  sd = Groupsd),
           rnorm(ps * ii, mean = ForkSoup,   sd = Groupsd),
           rnorm(ps * ii, mean = SpoonSalad, sd = Groupsd),
           rnorm(ps * ii, mean = ForkSalad,  sd = Groupsd))
psre <- rnorm(ps, mean = 0, sd = Groupsd / 5)
iire <- rnorm(ii, mean = 0, sd = Groupsd / 5)
ds$RT <- ds$RT + psre[ds$Participant] + iire[ds$Item]
```

Treatment coding

- The default coding in R is treatment (or dummy) coding
- The first level of each factor is assigned as the reference category

```
contrasts(ds$Utensils)
#        Spoon
# Fork       0
# Spoon      1
contrasts(ds$Foods)
#        Soup
# Salad     0
# Soup      1
```

Effects coding

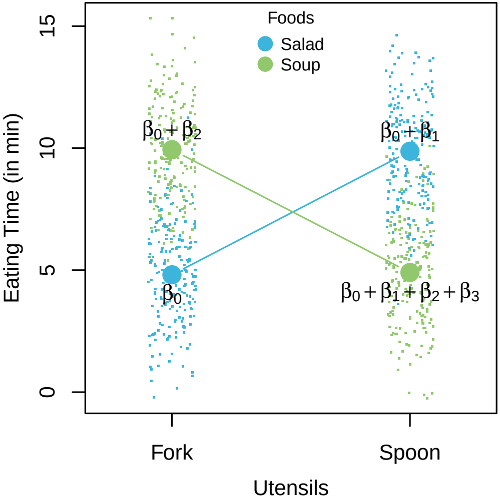
- In effects (or sum) coding, one level is set as negative and one positive, with zero as the mean of the two levels
- For more than two levels, there will also be a reference category, which is the *last* factor level in R

```
contrasts(ds$Utensils)
#        [,1]
# Fork      1
# Spoon    -1
contrasts(ds$Foods)
#        [,1]
# Salad     1
# Soup     -1
```

Exercise

- Fit the model on slide 4 to your simulated data
  1. with dummy coding
  2. with effects coding
- Interpret the fixed parameters and compare the variance components for the random effects for the two models
- What is the interpretation of the intercept for the two models? Why does it differ?

Treatment coding



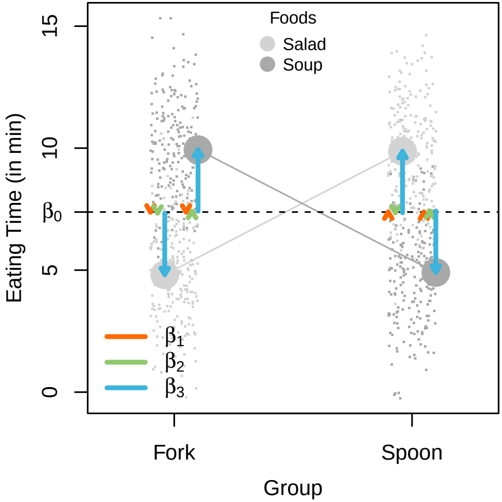
- Means from simulated data

	Salad	Soup
Fork	4.81	9.94
Spoon	9.87	4.90

- Reference categories: `Salad` and `Fork`
- Estimated coefficients

	β	SE	t
(Intercept)	4.81	0.21	23.45
UtensilsSpoon	5.06	0.20	25.19
FoodsSoup	5.13	0.20	25.52
UtensilsSpoon:FoodsSoup	-10.10	0.28	-35.52

Effects coding



- Means from simulated data

	Salad	Soup
Fork	4.81	9.94
Spoon	9.87	4.90

- Estimated coefficients

	β	SE	t
(Intercept)	7.38	0.16	44.97
Utensils1	-0.01	0.07	-0.09
Foods1	-0.04	0.07	-0.57
Utensils1:Foods1	-2.52	0.07	-35.52

2 Defining contrasts

General linear model

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta} + \boldsymbol{\varepsilon}_i$$

which corresponds to

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} 1 & x_{11} & x_{12} & \dots & x_{1p} \\ 1 & x_{21} & x_{22} & \dots & x_{2p} \\ 1 & x_{31} & x_{32} & \dots & x_{3p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{N1} & x_{N2} & \dots & x_{Np} \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

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- The four levels F_1 to F_4 reflect levels of word frequency with levels `low`, `medium-low`, `medium-high`, and `high` frequency words, and the dependent variable reflects some response time

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- In order to keep it reasonably complex, let us consider a design with one between-subjects factor with four levels
- The four levels F_1 to F_4 reflect levels of word frequency with levels `low`, `medium-low`, `medium-high`, and `high` frequency words, and the dependent variable reflects some response time
- From four factor levels, we can build three contrasts

Example by Schad et al. (2020)

- Let us consider 8 subjects for our 4-level between-subjects design

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta} + \boldsymbol{\varepsilon}_i$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_7 \\ \varepsilon_8 \end{pmatrix}$$

- For the four means of the factor levels, we get

$$\boldsymbol{\mu} = \mathbf{C} \boldsymbol{\beta}$$

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

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    2. Sum contrasts

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  2. Sum contrasts
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  5. Custom contrasts

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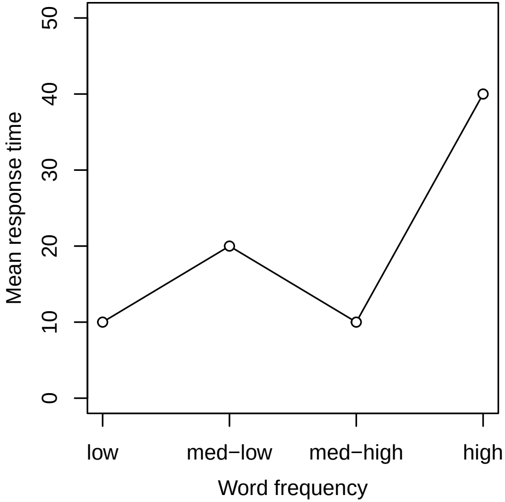


```r
design <- data.frame(
  F = factor(c("F1", "F2", "F3", "F4")),
  mu = c(10, 20, 10, 40))

beta <- lm(mu ~ F, design) |>
  coef() |>
  zapsmall()
# Contrast matrix
mm <- model.matrix( ~ F, design)

# Compare to mu
mm %*% beta
```

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- Hence, the hypothesis matrix is the inverse of the contrast matrix
- It is sometimes easier to define the hypothesis matrix and then obtain the contrast matrix based on it

Treatment contrasts

Contrast Matrix

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

$$\mu_1 = \beta_0$$
$$\mu_2 = \beta_0 + \beta_1$$
$$\mu_3 = \beta_0 + \beta_2$$
$$\mu_4 = \beta_0 + \beta_3$$

Hypothesis Matrix

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$$

$$\beta_0 = \mu_1$$
$$\beta_1 = \mu_2 - \mu_1$$
$$\beta_2 = \mu_3 - \mu_1$$
$$\beta_3 = \mu_4 - \mu_1$$

Hypotheses being tested: H_{01}: $\mu_1 = \mu_2$, H_{02}: $\mu_1 = \mu_3$, H_{03}: $\mu_1 = \mu_4$

Sum contrasts

Contrast Matrix

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & -1 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

$$\begin{aligned} \mu_1 &= \beta_0 + \beta_1 \\ \mu_2 &= \beta_0 + \beta_2 \\ \mu_3 &= \beta_0 + \beta_3 \\ \mu_4 &= \beta_0 - \beta_1 - \beta_2 - \beta_3 \end{aligned}$$

Hypothesis Matrix

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 3/4 & -1/4 & -1/4 & -1/4 \\ -1/4 & 3/4 & -1/4 & -1/4 \\ -1/4 & -1/4 & 3/4 & -1/4 \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$$

$$\begin{aligned} \beta_0 &= \frac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4} \\ \beta_1 &= \frac{3}{4}\mu_1 - \frac{1}{4}(\mu_2 + \mu_3 + \mu_4) \\ \beta_2 &= \frac{3}{4}\mu_2 - \frac{1}{4}(\mu_1 + \mu_3 + \mu_4) \\ \beta_3 &= \frac{3}{4}\mu_3 - \frac{1}{4}(\mu_1 + \mu_2 + \mu_4) \end{aligned}$$

Hypotheses being tested: H_{01}: $\frac{\mu_2+\mu_3+\mu_4}{3} = \mu_1$, H_{02}: $\frac{\mu_1+\mu_3+\mu_4}{3} = \mu_2$, H_{03}: $\frac{\mu_1+\mu_2+\mu_4}{3} = \mu_3$

Helmert contrasts

Contrast Matrix

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 0 & 2 & -1 \\ 1 & 0 & 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

$$\begin{aligned} \mu_1 &= \beta_0 - \beta_1 - \beta_2 - \beta_3 \\ \mu_2 &= \beta_0 + \beta_1 - \beta_2 - \beta_3 \\ \mu_3 &= \beta_0 + 2 \cdot \beta_2 - \beta_3 \\ \mu_4 &= \beta_0 + 3 \cdot \beta_3 \end{aligned}$$

Hypothesis Matrix

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ -1/6 & -1/6 & 1/3 & 0 \\ -1/12 & -1/12 & -1/12 & 1/4 \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$$

$$\begin{aligned} \beta_0 &= \frac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4} \\ \beta_1 &= -\frac{1}{2}\mu_1 + \frac{1}{2}\mu_2 \\ \beta_2 &= -\frac{1}{6}(\mu_1 + \mu_2) + \frac{1}{3}\mu_3 \\ \beta_3 &= -\frac{1}{12}(\mu_1 + \mu_2 + \mu_3) + \frac{1}{4}\mu_4 \end{aligned}$$

Hypotheses being tested: H_{01}: $\mu_1 = \mu_2$, H_{02}: $\frac{\mu_1+\mu_2}{2} = \mu_3$, H_{03}: $\frac{\mu_1+\mu_2+\mu_3}{3} = \mu_4$

Sequential difference contrasts

Contrast Matrix

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \begin{pmatrix} 1 & -3/4 & -1/2 & -1/4 \\ 1 & 1/4 & -1/2 & -1/4 \\ 1 & 1/4 & 1/2 & -1/4 \\ 1 & 1/4 & 1/2 & 3/4 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

$$\mu_1 = \beta_0 - \frac{3}{4}\beta_1 - \frac{1}{2}\beta_2 - \frac{1}{4}\beta_3$$
$$\mu_2 = \beta_0 + \frac{1}{4}\beta_1 - \frac{1}{2}\beta_2 - \frac{1}{4}\beta_3$$
$$\mu_3 = \beta_0 + \frac{1}{4}\beta_1 + \frac{1}{2}\beta_2 - \frac{1}{4}\beta_3$$
$$\mu_4 = \beta_0 + \frac{1}{4}\beta_1 + \frac{1}{2}\beta_2 + \frac{3}{4}\beta_3$$

Hypothesis Matrix

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$$

$$\beta_0 = \frac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4}$$
$$\beta_1 = \mu_2 - \mu_1$$
$$\beta_2 = \mu_3 - \mu_2$$
$$\beta_3 = \mu_4 - \mu_3$$

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- Hence, we assume a potential hypothetical outcome, such as $\mu_1 = 10$, $\mu_2 = 10$, $\mu_3 = 20$, and $\mu_4 = 30$

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- Let us assume for our example, we want to test if the first two means (F_1 and F_2) are identical, but that means for levels F_3 and F_4 increase linearly
- Hence, we assume a potential hypothetical outcome, such as $\mu_1 = 10$, $\mu_2 = 10$, $\mu_3 = 20$, and $\mu_4 = 30$
- These means can easily be turned into a contrast by centering them and then maybe do some scaling

μ_1	μ_2	μ_3	μ_4
10	10	20	30
−7.5	−7.5	2.5	12.5
−3	−3	1	5

Custom contrasts

Contrast Matrix

$$\begin{pmatrix}\mu_1\\\mu_2\\\mu_3\\\mu_4\end{pmatrix} = \begin{pmatrix}1 & -3\\1 & -3\\1 & 1\\1 & 5\end{pmatrix} \cdot \begin{pmatrix}\beta_0\\\beta_1\end{pmatrix}$$

$$\begin{aligned}\mu_1 &= \beta_0 - 3\beta_1\\\mu_2 &= \beta_0 - 3\beta_1\\\mu_3 &= \beta_0 + \beta_1\\\mu_4 &= \beta_0 + 5\beta_1\end{aligned}$$

Hypothesis Matrix

$$\begin{pmatrix}\beta_0\\\beta_1\end{pmatrix} = \begin{pmatrix}1/4 & 1/4 & 1/4 & 1/4\\-0.068 & -0.068 & 0.023 & 0.114\end{pmatrix} \cdot \begin{pmatrix}\mu_1\\\mu_2\\\mu_3\\\mu_4\end{pmatrix}$$

$$\begin{aligned}\beta_0 &= \frac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4}\\\beta_1 &= -\frac{3}{44}\mu_1 - \frac{3}{44}\mu_2 + \frac{1}{44}\mu_3 + \frac{5}{44}\mu_4\end{aligned}$$

Hypothesis being tested: H_{01}: $-3\mu_1 - 3\mu_2 + \mu_3 + 5\mu_4 = 0$

Custom contrasts

Contrast Matrix

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \begin{pmatrix} 1 & -3 & -0.511 & -0.533 \\ 1 & -3 & 0.131 & 0.727 \\ 1 & 1 & 0.760 & -0.387 \\ 1 & 5 & -0.380 & 0.194 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

- It is often a good idea to add orthogonal contrasts, so any variance in your data that is independent of your contrast can be accounted for
- R will automatically add orthogonal contrasts if you provide less than $n-1$ contrasts for a factor with n levels

Hypothesis Matrix

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ -0.068 & -0.068 & 0.023 & 0.114 \\ -0.511 & 0.131 & 0.760 & -0.380 \\ -0.533 & 0.727 & -0.387 & 0.194 \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$$

Hypothesis being tested: H_{01}: $-3\mu_1 - 3\mu_2 + \mu_3 + 5\mu_4 = 0$

Simulate data

```
set.seed(1700)

n <- 20
dat <- data.frame(id = 1:n,
                  F = factor(rep(c("F1", "F2", "F3", "F4"),
                                 times = n / 4)),
                 DV = rnorm(n, mean = c(10, 20, 10, 40), sd = 5))

datm <- aggregate(DV ~ F, dat, mean)
aggregate(DV ~ F, dat, sd)

lattice::xyplot(DV ~ F, dat, type = c("a", "p"))
```

Exercise

- Now fit linear models with all contrasts we looked at
  1. Treatment contrasts
  2. Sum contrasts
  3. Helmert contrasts
  4. Sequential difference contrasts
  5. Custom contrasts
- Interpret the fixed parameters
- Predict data with each model and compare the results
- Look again at the intercepts for all models and compare

References

Brehm, L., & Alday, P. M. (2022).Contrast coding choices in a decade of mixed models. *Journal of Memory and Language*, *125*, 104334. https://doi.org/10.1016/j.jml.2022.104334

Schad, D. J., Vasishth, S., Hohenstein, S., & Kliegl, R. (2020).How to capitalize on a priori contrasts in linear (mixed) models: A tutorial. *Journal of Memory and Language*, *110*, 104038. https://doi.org/10.1016/j.jml.2019.104038