

Generalized linear mixed-effects models

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GLMMs

- Like linear models, mixed-effects models can be extended so that they allow for response variables that have arbitrary distributions
- A GLM(M) is a specific combination of a response distribution, a link function, and a linear predictor
- We can choose (almost) all the link functions that work with `glm()` for `glmer()`

<i>## Family name</i>	<i>Link functions</i>
binomial	logit, probit, log, cloglog
gaussian	identity, log, inverse
Gamma	identity, inverse, log
inverse.gaussian	1/mu ² , identity, inverse, log
poisson	log, identity, sqrt

Example: 1989 Bangladesh Fertility Survey¹

Subset containing variables about

- District where women live
- Age (mean centered)
- Living children (coded 1 = no children, 2 = one child, 3 = two children, 4 = three or more children)
- Use of artificial contraception (yes or no)
- Area where women live is rural or urban

¹Reanalysis of <https://repsychling.github.io/SMLP2024/glmm.html>

Exercise

- Read the data into R
- Create a plot showing the probability to use artificial contraception depending on age
- Draw separate lines for number of living children
- Make two panels: one for rural and one for urban
- Use `ggplot2` or `lattice`

Hint: Use `geom_smooth()` for `ggplot2` and `type = "smooth"` for `lattice::xyplot()`

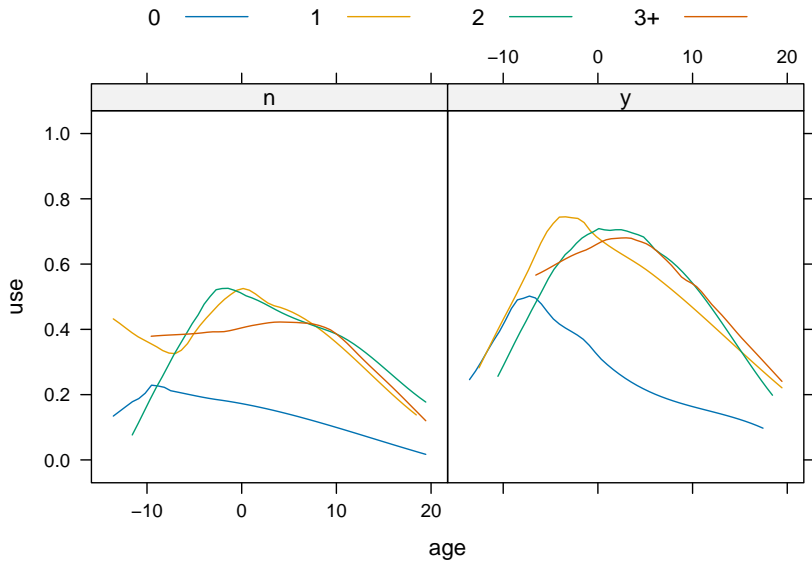
Data set contra

```
dat <- read.table("data/contr.dat", header = TRUE)

dat$districtID <- factor(dat$districtID)
dat$childCode <- factor(dat$childCode,
                        levels = 1:4,
                        labels = c("0", "1", "2", "3+"))
dat$isUrban <- factor(dat$isUrban,
                     levels = 0:1,
                     labels = c("n", "y"))

# simplify names
names(dat) <- c("dist", "use", "livch", "age", "urban")
```

Visualization



GLMM

- Fit the following model to the data

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 age + \beta_2 age^2 + \beta_3 urban + \beta_4 livch + v_0$$

with $v_0 \sim N(0, \sigma_{v_0}^2)$

- We are assuming that there are differences between the districts, so we add a random intercept for district
- What are your conclusions about differences between women with and without children?

Dichotomize livch

- We create a new factor `children` that indicates if a woman has children or not (independently of how many)
- Next, we add an interaction term for `children` and `age` to our model

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 \text{age} + \beta_2 \text{children} + \beta_3 (\text{age} \times \text{children}) + \beta_4 \text{age}^2 + \beta_5 \text{urban} + v_0$$

with $v_0 \sim N(0, \sigma_{v_0}^2)$

- How can we compare which model fits the data better?

Nested random effect

- It turns out that districts are very big and can incorporate rural as well as urban areas
- The districts can be very different depending on this
- Add a random intercept to the model taking this into account

```
xtabs( ~ urban + dist, dat)
xtabs( ~ urban + factor(urban:dist), dat)
xtabs( ~ dist + factor(dist:urban), dat) |> print(zero = ".")
```

- Compare the models – what is your conclusion?
- What exactly is the difference between these models?

Model predictions

Compare models

```
data.frame(models = c("gm1", "gm2", "gm3"),  
            df = AIC(gm1, gm2, gm3)[, 1],  
            AIC = AIC(gm1, gm2, gm3)[, 2],  
            BIC = BIC(gm1, gm2, gm3)[, 2],  
            deviance = c(deviance(gm1), deviance(gm2), deviance(gm3)))
```

Model predictions

```
newdat <- data.frame(children = factor(rep(c("true", "false"),  
                                           each = 2)),  
                     urban = factor(rep(c("y", "n"), times = 2)),  
                     age = 0)  
newdat$pre <- predict(gm3, type = "response", newdata = newdat,  
                     re.form = NA)
```

Exercise

- Create a new data frame with variables `children` ("true", "false"), `urban` ("yes", "no"), and `age` (ranging from -14 to 20)
- Add a prediction for each combination of the three variables
- Draw a plot of your predictions
- Interpret the results

Summary

- From the data plot we can see a quadratic trend in the probability by age.
- The patterns for women with children are similar and we do not need to distinguish between 1, 2, and 3+ children.
- We do distinguish between those women who do not have children and those with children. This shows up in a significant $age \times children$ interaction term.