Growth curve models

Nora Wickelmaier

Last modified: October 16, 2024

Example: Growth curve model

This example from linguistics is taken from Winter and Wieling (2016)

- Participants play a game of ‘vocal charades’
- At each round, a participant has to vocalize a meaning to the partner (e.g., ‘ugly’) without using language (e.g., through grunting or hissing)
- The partner has to guess the meaning of the vocalization
- This game is played repeatedly with the finding that over time, a dyad converges on a set of nonlinguistic vocalizations that assure a high degree of intelligibility between the two participants in the dyad

Example: Growth curve model

- Initially, participants may be struggling with the task and explore very different kinds of vocalizations
- Over time, they may converge on a more stable set of iconic vocalizations, that is vocalizations that resemble the intended referent (e.g., a high-pitched sound for ‘attractive’ and a low-pitched sound for ‘ugly’)
- Finally, after even more time, the dyad may conventionalize to idiosyncratic patterns that deviate from iconicity and become increasingly arbitrary

Example: Growth curve model

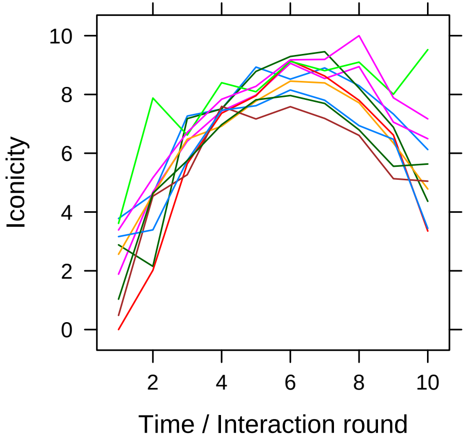
100 observations of three variables (simulated data set)

Variable	Description
`dyad`	different pairs of subjects playing the vocal charades game
`t`	sequential rounds for which the vocal charades game was played
`iconicity`	iconicity measure

For better interpretation, `t` will be centered

```
dat$t_c <- dat$t - mean(dat$t)
```

Visualization of data




```
xyplot(
  iconicity ~ t, dat,
  groups = dyad,
  type = "l",
  xlab = "Time/Interaction round",
  ylab = "Iconicity")
```

Mixed-effects model with quadratic trend

We will now consider a model with uncorrelated random effects

$$y_{ij} = \beta_0 + \beta_1 t_{ij} + \beta_2 t_{ij}^2 + \upsilon_{0i} + \upsilon_{1i} t_{ij} + \upsilon_{2i} t_{ij}^2 + \varepsilon_{ij}$$

with

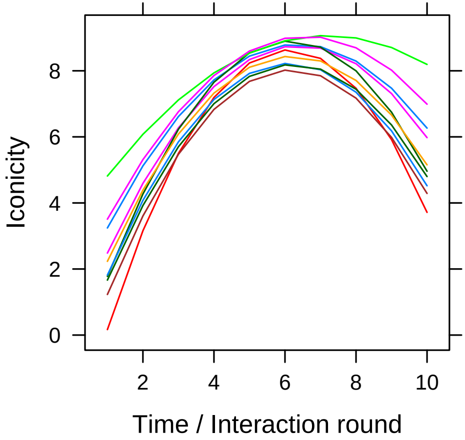
$$\begin{pmatrix} \upsilon_{0i} \\ \upsilon_{1i} \\ \upsilon_{2i} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\upsilon_0}^2 & 0 & 0 \\ 0 & \sigma_{\upsilon_1}^2 & 0 \\ 0 & 0 & \sigma_{\upsilon_2}^2 \end{pmatrix} \right) \text{ i.i.d.}$$

$$\varepsilon_i \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_{n_i}) \text{ i.i.d.}$$

This model is fitted by

```
gcm1 <- lmer(iconicity ~ t_c + I(t_c^2) +
  (1 | dyad) + (0 + t_c | dyad) + (0 + I(t_c^2) | dyad),
  data=dat, REML=F)
```

Visualization of model predictions




```
xyplot(
  predict(gcm1) ~ t, dat,
  groups = dyad,
  type = "l",
  xlab = "Time/Interaction round",
  ylab = "Iconicity")
```

ML estimates of parameters

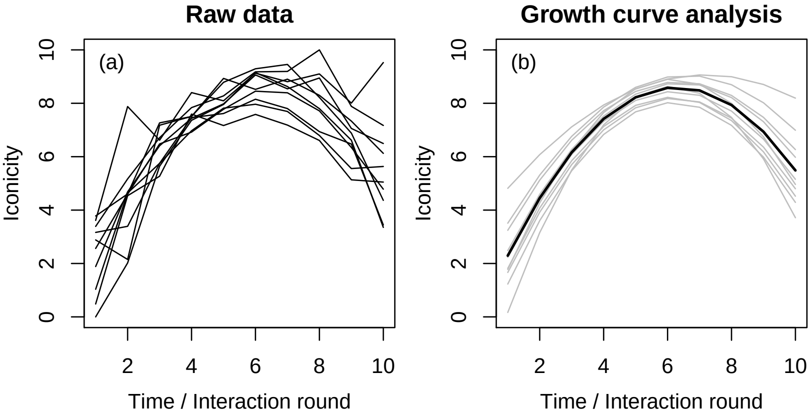
```
...
Random effects:
 Groups   Name        Variance Std.Dev.
 dyad     (Intercept) 0.152845 0.39095
 dyad.1   t_c         0.002595 0.05094
 dyad.2   I(t_c^2)    0.003504 0.05920
 Residual             0.429691 0.65551
Number of obs: 100, groups:  dyad, 10

Fixed effects:
            Estimate Std. Error t value
(Intercept)  8.45761    0.15849   53.36
t_c          0.35475    0.02793   12.70
I(t_c^2)    -0.22558    0.02078  -10.86
...
```

Interpretation of results

- There are now two slopes, one for the effect of linear time (t_c, $\beta_1 = 0.35$) and one for the effect of quadratic time (t_c^2, $\beta_2 = -0.23$), both of which are allowed to differ by dyad ($\sigma_{\upsilon_{1i}} = 0.05$ and $\sigma_{\upsilon_{2i}} = 0.06$)
- The negative value for the quadratic term indicates the inverse U-shape
- The point of reversal is $t = \bar{t} + \frac{-\hat{\beta}_1}{2 * \hat{\beta}_2} = 5.5 + \frac{-.35}{2 * (-.22)} = 6.29$
- The model assumes that the random intercept and slopes are all uncorrelated

Plots from the article



References

Winter, B., & Wieling, M. (2016).How to analyze linguistic change using mixed models, growth curve analysis and generalized additive modeling. *Journal of Language Evolution*, *1*(1), 7–18. https://doi.org/10.1093/jole/lzv003