Power Simulation for Logistic Regression

Last modified: 2025-03-02

Overview

- Large effects from subtle manipulations?
  - Decision biases from two-hand tapping
- Power simulation for logistic regression
- Exercises

Refresher: Framing

- Tversky and Kahneman (1981)

  "Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed" (p. 453)

  If Program A is adopted **200** people will be **saved** [109]

  If Program B is adopted there is 1/3 probability that **600** people will be **saved**, and 2/3 probability that **no people** will be **saved** [43]

  If Program C is adopted **400** people will **die** [34]

  If Program D is adopted there is 1/3 probability that **nobody** will **die**, and 2/3 probability that **600** people will **die** [121]

- Odds ratio (OR) $= 9.0$

Decision biases from two-hand tapping

- McElroy and Seta (2004), $n = 48$

  "a behavioral task of finger tapping was used to induce asymmetrical activation of the respective hemispheres . . . Framing effects were found when the right hemisphere was selectively activated whereas they were not observed when the left hemisphere was selectively activated" (p. 572)

	right-hand tapping		left-hand tapping		ratio of odds ratios (ROR)
	safe	risky	safe	risky	
gain	8	4	12	1	
loss	7	4	3	9	
OR		1.1		36	31.5

- Our replication (see Gelman, 2020), $n = 332$

gain	52	31	56	27	
loss	26	57	30	53	
OR		3.7		3.7	1.0

Example: How to fix the two-hand tapping study?

Power simulation

Let us go through the general steps

1. Specify the model including the effect of interest
2. Generate observations from the model
3. Test H_0
4. Repeat

Example: How to fix the two-hand tapping study?

1. Specify model

- Logit model with interaction

$$\log \frac{p}{1-p} = \beta_0 + \beta_1 \cdot \text{left hand} + \beta_2 \cdot \text{gain} + \beta_3 \cdot (\text{left hand} \times \text{gain})$$

- Suggest a minimum relevant effect
  - We can look at the original framing effect study and its many replications
  - Former study by McElroy and Seta (2003) found ROR $= 3.4$ for similar manipulation
  - Other studies investigating influencing factors (with RORs $\approx$ 2–3, e. g., foreign language effect, Costa et al., 2014; Wickelmaier, 2015)
- Underlying distribution: $X \sim \mathit{Binom}(n, p)$

Example: How to fix the two-hand tapping study?

1. Specify model

Moderate framing effect

safe:risky

gain	2:1
loss	1:2

OR = 4

left hand

safe:risky

gain	$\sqrt{8}$:1
loss	1:$\sqrt{8}$

OR = 8

right hand

safe:risky

gain	$\sqrt{2}$:1
loss	1:$\sqrt{2}$

OR = 2

ROR = 4

Translating into parameters

- $\exp(\beta_0) = \frac{1}{\sqrt{2}}$
  odds in reference categories: right and loss
- $\exp(\beta_1) = \frac{1}{2}$
  OR of switching to left hand
- $\exp(\beta_2) = 2$
  OR of switching to gain frame
- $\exp(\beta_3) = 4$
  ROR

$$\log \frac{p}{1-p} = \beta_0 + \beta_1 \cdot \text{left hand} + \beta_2 \cdot \text{gain} + \beta_3 \cdot (\text{left hand} \times \text{gain})$$

Example: How to fix the two-hand tapping study?

2. Generate observations

- Calculate logits for the model

```
dat <- read.table(header = TRUE, text = "
  hand frame
     r  gain
     r  loss
     l  gain
     l  loss")                                     # ref. cat.
dat$hand <- factor(dat$hand, levels = c("r", "l"))       #   right
dat$frame <- factor(dat$frame, levels = c("loss", "gain"))  #   loss

expbeta <- c(1/sqrt(2), 1/2, 2, 4)  # ROR = 4, linear on logit scale
logit <- model.matrix(~ hand * frame, dat) %*% log(expbeta)
```

Example: How to fix the two-hand tapping study?

2. Generate observations

- Simulate data from binomial distribution

```
n <- 100
y <- rbinom(4, size = n/4, prob = plogis(logit))
```

```
## Sim 1              Sim 2              ...
## hand frame  y      hand frame  y
##    r  gain  16       r  gain  15
##    r  loss  7        r  loss  13
##    l  gain  21       l  gain  19
##    l  loss  9        l  loss  7
```

Example: How to fix the two-hand tapping study?

3. Test H_0

- Fit null model to your generated observations, H_0: $\beta_3 = 0$

```
m1 <- glm(cbind(y, n/4 - y) ~ hand + frame, binomial, dat)
```

- Fit interaction model to your generated observations, H_1: $\beta_3 \neq 0$

```
m2 <- glm(cbind(y, n/4 - y) ~ hand * frame, binomial, dat)
## ROR estimate = 5.9
```

- Perform a likelihood ratio test of the interaction

```
anova(m1, m2, test = "LRT")
## Analysis of Deviance Table
##
## Model 1: cbind(y, n/4 - y) ~ hand + frame
## Model 2: cbind(y, n/4 - y) ~ hand * frame
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1         1     4.3436
## 2         0     0.0000  1   4.3436  0.03715
```

Example: How to fix the two-hand tapping study?

4. Repeat

- Do previous steps repeatedly
  - Calculate the proportion of significant tests ($=$ power)
  - Adjust n to reach the preset power criterion

References

Costa, A., Foucart, A., Arnon, I., Aparici, M., & Apesteguia, J. (2014). "Piensa" twice: On the foreign language effect in decision making. *Cognition*, *130*, 236–254. doi: 10.1016/j.cognition.2013.11.010

Gelman, A. (2020, October 22). *An odds ratio of 30, which they (sensibly) don't believe.* Retrieved from `https://statmodeling.stat.columbia.edu/2020/10/22/an-odds-ratio-of-30-which-they-sensibly-dont-believe/`

McElroy, T., & Seta, J. J. (2003). Framing effects: An analytic–holistic perspective. *Journal of Experimental Social Psychology*, *39*(6), 610–617. doi: 10.1016/S0022-1031(03)00036-2

McElroy, T., & Seta, J. J. (2004). On the other hand am I rational? Hemispheric activation and the framing effect. *Brain and Cognition*, *55*(3), 572–580. doi: 10.1016/j.bandc.2004.04.002

Tversky, A., & Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science*, *211*, 453–458. doi: 10.1126/science.7455683

Wickelmaier, F. (2015). On not testing the foreign-language effect: A comment on Costa, Foucart, Arnon, Aparici, and Apesteguia (2014). *ArXiv*. doi: 10.48550/arXiv.1506.07727